Computer algebra independent integration tests

4-Trig-functions/4.1-Sine/4.1.1.3-g-tan-^p-a+b-sin-^m

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June 29, 2021 Compiled on June 29, 2021 at 8:31pm

Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [208]. This is test number [72].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 99.04 (206)	% 0.96 (2)
Mathematica	% 97.60 (203)	% 2.40 (5)
Maple	% 85.58 (178)	% 14.42 (30)
Maxima	% 68.27 (142)	% 31.73 (66)
Fricas	% 85.58 (178)	% 14.42 (30)
Sympy	% 1.92 (4)	% 98.08 (204)
Giac	% 61.06 (127)	% 38.94 (81)
Mupad	% 74.04 (154)	% 25.96 (54)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

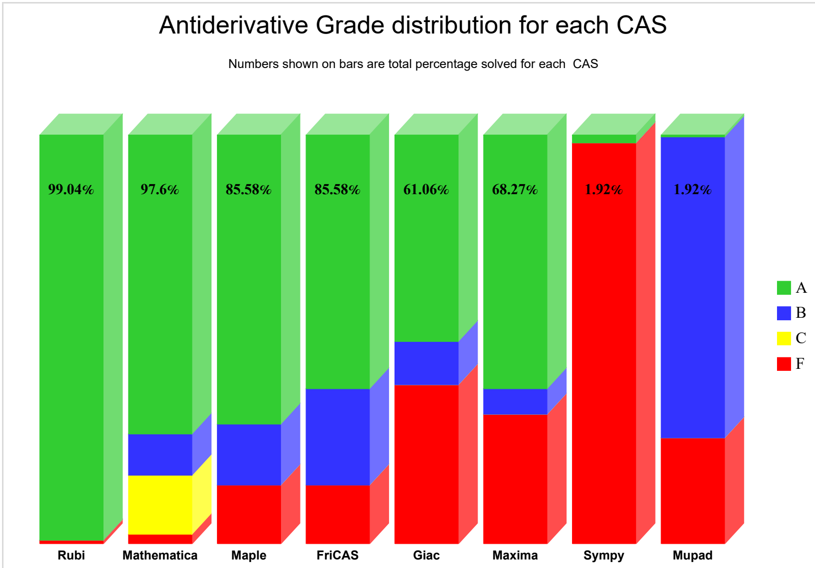
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

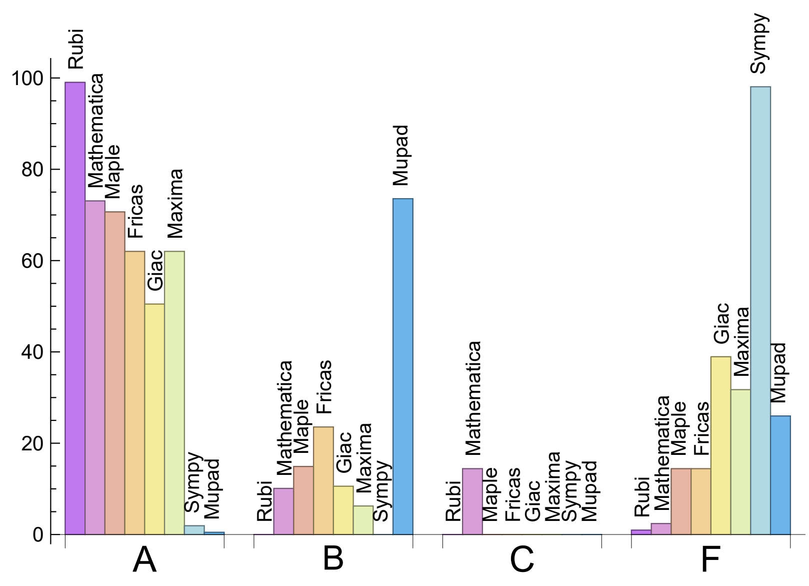
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.04	0.00	0.00	0.96
Mathematica	73.08	10.10	14.42	2.40
Maple	70.67	14.90	0.00	14.42
Maxima	62.02	6.25	0.00	31.73
Fricas	62.02	23.56	0.00	14.42
Sympy	1.92	0.00	0.00	98.08
Giac	50.48	10.58	0.00	38.94
Mupad	0.48	73.56	0.00	25.96

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	2	100.00 %	0.00 %	0.00 %
Mathematica	5	100.00 %	0.00 %	0.00 %
Maple	30	100.00 %	0.00 %	0.00 %
Maxima	66	63.64 %	13.64 %	22.73 %
Fricas	30	86.67 %	13.33 %	0.00 %
Sympy	204	90.69 %	9.31 %	0.00 %
Giac	81	39.51 %	44.44 %	16.05 %
Mupad	54	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

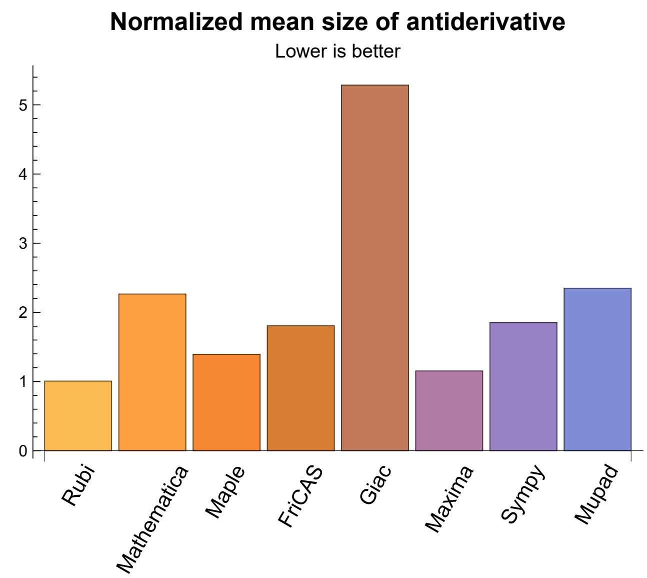
1.3 Performance

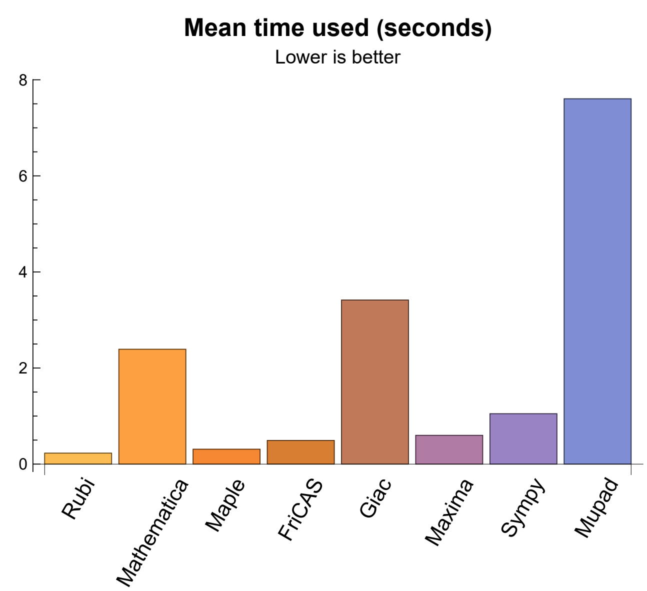
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.23	133.35	1.01	113.00	1.00
Mathematica	2.39	304.28	2.26	113.00	1.00
Maple	0.31	183.64	1.39	136.00	1.30
Maxima	0.60	125.20	1.15	95.00	1.00
Fricas	0.49	263.02	1.80	157.50	1.46
Sympy	1.05	112.50	1.85	99.50	1.83
Giac	3.41	417.33	5.29	138.00	1.36
Mupad	7.60	306.68	2.35	231.50	2.31

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{208}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {110, 114, 115, 117, 118, 123, 124, 126, 127, 128, 129, 136, 138, 158, 168, 179, 190, 201, 203, 204, 205, 206, 207}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://`

ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

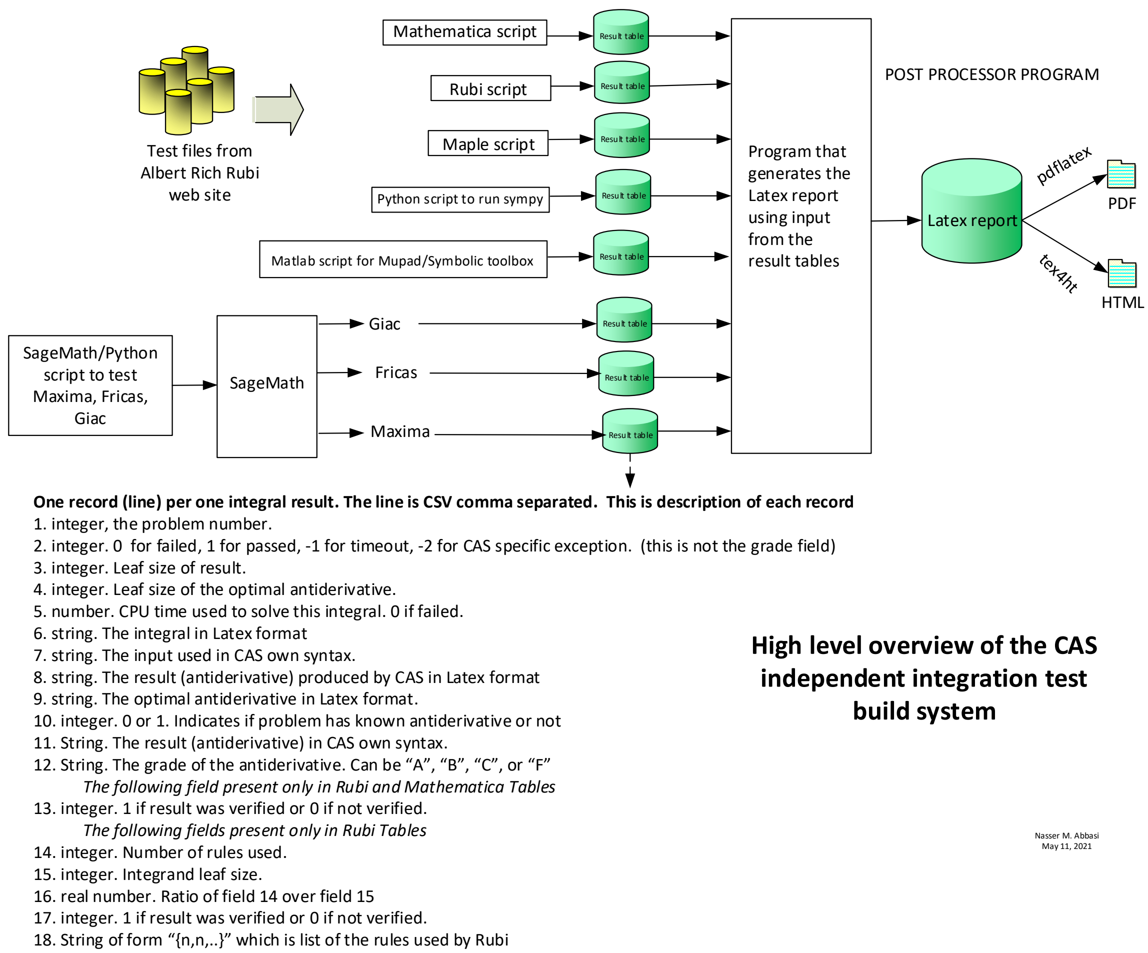
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 208 }

B grade: { }

C grade: { }

F grade: { 206, 207 }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 94, 96, 97, 98, 99, 100, 101, 102, 120, 130, 131, 132, 133, 134, 137, 140, 141, 142, 143, 144, 145, 146, 150, 151, 152, 153, 154, 155, 156, 157, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 207, 208 }

B grade: { 21, 55, 56, 57, 58, 59, 60, 88, 89, 90, 93, 105, 106, 110, 126, 127, 128, 129, 158, 179, 206 }

C grade: { 11, 12, 13, 91, 92, 95, 103, 104, 107, 108, 109, 111, 112, 113, 114, 115, 116, 117, 118, 119, 123, 124, 136, 138, 147, 148, 149, 203, 204, 205 }

F grade: { 121, 122, 125, 135, 139 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 16, 19, 20, 21, 22, 23, 27, 28, 31, 32, 33, 36, 37, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 55, 56, 57, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 163, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 181, 182, 183, 184, 185, 186, 187, 188, 192, 193, 194, 195, 196, 197, 208 }

B grade: { 14, 15, 17, 18, 24, 25, 26, 29, 30, 34, 35, 38, 39, 53, 54, 58, 59, 60, 160, 164, 178, 179, 180, 189, 190, 191, 198, 199, 200, 201, 202 }

C grade: { }

F grade: { 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 203, 204, 205, 206, 207 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 55, 56, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 96, 100, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 182, 183, 184, 185, 186, 193, 194, 195, 196, 197, 208 }

B grade: { 53, 54, 57, 58, 59, 60, 87, 88, 89, 90, 99, 181, 192 }

C grade: { }

F grade: { 91, 92, 93, 94, 95, 97, 98, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 176, 177, 178, 179, 180, 187, 188, 189, 190, 191, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 14, 15, 16, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 91, 92, 95, 96, 99, 100, 103, 107, 111, 112, 140, 141, 142, 143, 144, 145, 146, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 183, 184, 187, 188, 198, 199, 208 }

B grade: { 10, 11, 12, 13, 17, 24, 30, 57, 58, 59, 60, 74, 84, 88, 89, 90, 93, 94, 97, 98, 101, 102, 104, 105, 106, 108, 109, 110, 113, 114, 147, 148, 149, 181, 182, 185, 186, 189, 190, 191, 192, 193, 194, 195, 196, 197, 200, 201, 202 }

C grade: { }

F grade: { 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 203, 204, 205, 206, 207 }

2.1.6 Sympy

A grade: { 22, 32, 40, 56 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208 }

2.1.7 Giac

A grade: { 4, 5, 6, 7, 12, 13, 17, 18, 22, 28, 32, 33, 37, 40, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 142, 143, 144, 148, 149, 152, 153, 154, 157, 158, 159, 162, 163, 164, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 195, 196, 197, 198, 199, 200, 201, 202, 208 }

B grade: { 3, 10, 11, 16, 21, 23, 24, 42, 53, 57, 58, 59, 60, 141, 146, 147, 151, 156, 167, 168, 181, 193 }

C grade: { }

F grade: { 1, 2, 8, 9, 14, 15, 19, 20, 25, 26, 27, 29, 30, 31, 34, 35, 36, 38, 39, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 145, 150, 155, 160, 161, 165, 166, 203, 204, 205, 206, 207 }

2.1.8 Mupad

A grade: { 208 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202 }

C grade: { }

F grade: { 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 203, 204, 205, 206, 207 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	123	147	95	159	0	0	235
normalized size	1	1.00	1.07	1.28	0.83	1.38	0.00	0.00	2.04
time (sec)	N/A	0.072	0.447	0.154	0.326	0.445	0.000	0.000	6.628

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	77	96	51	87	0	0	154
normalized size	1	1.00	1.08	1.35	0.72	1.23	0.00	0.00	2.17
time (sec)	N/A	0.048	0.117	0.148	0.307	0.462	0.000	0.000	6.469

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	38	29	25	27	0	1456	43
normalized size	1	1.00	1.27	0.97	0.83	0.90	0.00	48.53	1.43
time (sec)	N/A	0.021	0.022	0.100	0.301	0.424	0.000	1.175	6.623

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	25	22	24	0	23	38
normalized size	1	1.00	1.08	1.04	0.92	1.00	0.00	0.96	1.58
time (sec)	N/A	0.020	0.033	0.089	0.355	0.424	0.000	0.219	6.578

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	60	83	45	69	0	60	146
normalized size	1	1.00	1.11	1.54	0.83	1.28	0.00	1.11	2.70
time (sec)	N/A	0.037	0.118	0.204	0.302	0.435	0.000	0.413	6.540

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	87	136	69	110	0	82	207
normalized size	1	1.00	1.07	1.68	0.85	1.36	0.00	1.01	2.56
time (sec)	N/A	0.048	0.197	0.170	0.647	0.445	0.000	0.548	6.697

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	111	195	91	158	0	104	267
normalized size	1	1.00	0.97	1.70	0.79	1.37	0.00	0.90	2.32
time (sec)	N/A	0.059	0.389	0.191	0.305	0.467	0.000	1.464	7.368

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	110	135	87	116	0	0	351
normalized size	1	1.00	1.09	1.34	0.86	1.15	0.00	0.00	3.48
time (sec)	N/A	0.092	0.060	0.243	0.405	0.425	0.000	0.000	11.223

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	81	98	65	88	0	0	231
normalized size	1	1.00	1.12	1.36	0.90	1.22	0.00	0.00	3.21
time (sec)	N/A	0.074	0.046	0.207	0.418	0.452	0.000	0.000	9.418

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	47	59	39	80	0	1008	111
normalized size	1	1.00	1.21	1.51	1.00	2.05	0.00	25.85	2.85
time (sec)	N/A	0.104	0.039	0.171	0.396	0.404	0.000	4.949	6.768

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	75	57	54	84	0	108	108
normalized size	1	1.00	1.83	1.39	1.32	2.05	0.00	2.63	2.63
time (sec)	N/A	0.052	0.043	0.101	0.421	0.452	0.000	0.565	6.906

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	125	106	92	160	0	141	228
normalized size	1	1.00	1.52	1.29	1.12	1.95	0.00	1.72	2.78
time (sec)	N/A	0.080	0.052	0.120	0.402	0.432	0.000	0.356	6.602

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	164	159	125	222	0	199	291
normalized size	1	1.00	1.34	1.30	1.02	1.82	0.00	1.63	2.39
time (sec)	N/A	0.096	0.067	0.127	0.410	0.454	0.000	0.459	6.673

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	75	261	96	168	0	0	283
normalized size	1	1.00	0.63	2.19	0.81	1.41	0.00	0.00	2.38
time (sec)	N/A	0.082	0.243	0.197	0.306	0.469	0.000	0.000	6.567

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	54	162	58	90	0	0	204
normalized size	1	1.00	0.75	2.25	0.81	1.25	0.00	0.00	2.83
time (sec)	N/A	0.062	0.100	0.174	0.304	0.456	0.000	0.000	7.111

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	40	69	43	45	0	6695	178
normalized size	1	1.00	0.77	1.33	0.83	0.87	0.00	128.75	3.42
time (sec)	N/A	0.038	0.038	0.141	0.305	0.430	0.000	19.303	6.669

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	28	94	53	76	0	47	56
normalized size	1	1.00	0.93	3.13	1.77	2.53	0.00	1.57	1.87
time (sec)	N/A	0.039	0.041	0.223	0.309	0.411	0.000	0.389	6.669

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	86	313	107	206	0	121	392
normalized size	1	1.00	0.65	2.37	0.81	1.56	0.00	0.92	2.97
time (sec)	N/A	0.075	0.220	0.212	0.315	0.457	0.000	0.958	11.182

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	174	251	152	152	0	0	392
normalized size	1	1.00	1.17	1.68	1.02	1.02	0.00	0.00	2.63
time (sec)	N/A	0.165	0.839	0.307	0.422	0.455	0.000	0.000	10.917

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	159	186	120	196	0	0	287
normalized size	1	1.00	1.32	1.55	1.00	1.63	0.00	0.00	2.39
time (sec)	N/A	0.203	1.308	0.292	0.403	0.427	0.000	0.000	10.072

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	145	117	84	125	0	5370	213
normalized size	1	1.00	2.04	1.65	1.18	1.76	0.00	75.63	3.00
time (sec)	N/A	0.089	0.434	0.245	0.414	0.421	0.000	24.522	8.693

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	34	52	47	41	78	38	123
normalized size	1	1.00	0.76	1.16	1.04	0.91	1.73	0.84	2.73
time (sec)	N/A	0.014	0.190	0.072	0.302	0.420	0.417	0.324	6.583

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	94	89	79	105	0	143	201
normalized size	1	1.00	1.27	1.20	1.07	1.42	0.00	1.93	2.72
time (sec)	N/A	0.102	0.577	0.129	0.649	0.436	0.000	0.255	6.520

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	191	190	139	192	0	209	293
normalized size	1	1.00	1.95	1.94	1.42	1.96	0.00	2.13	2.99
time (sec)	N/A	0.161	5.275	0.222	0.457	0.466	0.000	0.552	6.517

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	99	445	133	240	0	0	398
normalized size	1	1.00	0.62	2.78	0.83	1.50	0.00	0.00	2.49
time (sec)	N/A	0.109	0.552	0.232	0.316	0.455	0.000	0.000	6.469

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	66	205	72	104	0	0	262
normalized size	1	1.00	0.73	2.25	0.79	1.14	0.00	0.00	2.88
time (sec)	N/A	0.071	0.158	0.183	0.303	0.435	0.000	0.000	7.460

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	52	85	57	61	0	0	281
normalized size	1	1.00	0.74	1.21	0.81	0.87	0.00	0.00	4.01
time (sec)	N/A	0.045	0.045	0.144	0.310	0.427	0.000	0.000	7.265

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	67	109	80	118	0	94	253
normalized size	1	1.00	0.68	1.11	0.82	1.20	0.00	0.96	2.58
time (sec)	N/A	0.065	0.186	0.240	0.301	0.433	0.000	1.431	6.760

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	243	359	209	289	0	0	438
normalized size	1	1.00	1.35	1.99	1.16	1.61	0.00	0.00	2.43
time (sec)	N/A	0.357	5.117	0.377	0.402	0.424	0.000	0.000	11.047

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	177	266	165	220	0	0	371
normalized size	1	1.00	1.49	2.24	1.39	1.85	0.00	0.00	3.12
time (sec)	N/A	0.194	2.104	0.359	0.422	0.463	0.000	0.000	10.500

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	115	167	117	154	0	0	288
normalized size	1	1.00	1.29	1.88	1.31	1.73	0.00	0.00	3.24
time (sec)	N/A	0.125	0.483	0.314	0.437	0.428	0.000	0.000	10.331

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	44	74	72	54	121	55	156
normalized size	1	1.00	0.70	1.17	1.14	0.86	1.92	0.87	2.48
time (sec)	N/A	0.054	0.315	0.136	0.293	0.401	1.054	0.368	8.919

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	106	105	93	121	0	162	264
normalized size	1	1.00	1.15	1.14	1.01	1.32	0.00	1.76	2.87
time (sec)	N/A	0.137	1.088	0.140	0.399	0.451	0.000	1.395	6.774

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	83	387	109	154	0	0	379
normalized size	1	1.00	0.64	3.00	0.84	1.19	0.00	0.00	2.94
time (sec)	N/A	0.095	0.446	0.235	0.295	0.451	0.000	0.000	7.880

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	76	245	85	116	0	0	320
normalized size	1	1.00	0.71	2.29	0.79	1.08	0.00	0.00	2.99
time (sec)	N/A	0.079	0.151	0.201	0.294	0.459	0.000	0.000	7.564

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	62	101	70	74	0	0	131
normalized size	1	1.00	0.70	1.15	0.80	0.84	0.00	0.00	1.49
time (sec)	N/A	0.053	0.069	0.155	0.295	0.437	0.000	0.000	6.633

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	78	125	82	131	0	96	298
normalized size	1	1.00	0.76	1.23	0.80	1.28	0.00	0.94	2.92
time (sec)	N/A	0.067	0.130	0.244	0.309	0.445	0.000	0.548	6.398

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	252	360	238	247	0	0	437
normalized size	1	1.00	1.76	2.52	1.66	1.73	0.00	0.00	3.06
time (sec)	N/A	0.199	1.653	0.408	0.422	0.432	0.000	0.000	11.050

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	125	231	181	179	0	0	363
normalized size	1	1.00	1.11	2.04	1.60	1.58	0.00	0.00	3.21
time (sec)	N/A	0.161	1.101	0.312	0.454	0.443	0.000	0.000	10.255

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	57	111	108	70	224	72	237
normalized size	1	1.00	0.66	1.28	1.24	0.80	2.57	0.83	2.72
time (sec)	N/A	0.082	0.399	0.195	0.296	0.423	1.893	1.531	8.591

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	136	127	117	135	0	194	295
normalized size	1	1.00	1.17	1.09	1.01	1.16	0.00	1.67	2.54
time (sec)	N/A	0.159	1.598	0.137	0.400	0.446	0.000	0.714	6.779

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	209	190	218	219	0	274	384
normalized size	1	1.00	1.49	1.36	1.56	1.56	0.00	1.96	2.74
time (sec)	N/A	0.225	5.279	0.233	0.400	0.453	0.000	1.292	6.709

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	283	293	313	291	0	339	454
normalized size	1	1.00	1.43	1.48	1.58	1.47	0.00	1.71	2.29
time (sec)	N/A	0.428	1.578	0.231	0.405	0.469	0.000	1.392	6.853

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	101	162	175	167	0	136	388
normalized size	1	1.00	0.78	1.25	1.35	1.28	0.00	1.05	2.98
time (sec)	N/A	0.161	0.983	0.182	0.312	0.450	0.000	9.206	10.740

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	84	126	130	147	0	116	281
normalized size	1	1.00	0.79	1.19	1.23	1.39	0.00	1.09	2.65
time (sec)	N/A	0.136	0.326	0.176	0.309	0.472	0.000	3.137	10.425

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	54	90	89	125	0	96	172
normalized size	1	1.00	0.66	1.10	1.09	1.52	0.00	1.17	2.10
time (sec)	N/A	0.116	0.164	0.169	0.312	0.423	0.000	0.775	9.111

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	58	28	54	47	58	0	58	61
normalized size	1	1.57	0.76	1.46	1.27	1.57	0.00	1.57	1.65
time (sec)	N/A	0.067	0.036	0.178	0.353	0.423	0.000	0.295	6.658

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	33	31	28	0	33	32
normalized size	1	1.00	1.00	1.03	0.97	0.88	0.00	1.03	1.00
time (sec)	N/A	0.039	0.018	0.116	0.302	0.416	0.000	1.101	6.531

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	24	30	26	30	0	26	23
normalized size	1	1.00	0.75	0.94	0.81	0.94	0.00	0.81	0.72
time (sec)	N/A	0.068	0.030	0.130	0.302	0.424	0.000	1.124	6.576

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	30	49	46	63	0	46	45
normalized size	1	1.00	0.59	0.96	0.90	1.24	0.00	0.90	0.88
time (sec)	N/A	0.089	0.046	0.233	0.307	0.402	0.000	0.271	6.560

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	61	67	66	96	0	66	63
normalized size	1	1.00	0.90	0.99	0.97	1.41	0.00	0.97	0.93
time (sec)	N/A	0.093	0.141	0.246	0.302	0.416	0.000	0.972	6.795

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	77	87	86	127	0	86	83
normalized size	1	1.00	0.92	1.04	1.02	1.51	0.00	1.02	0.99
time (sec)	N/A	0.100	0.206	0.277	0.305	0.427	0.000	0.374	6.767

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	146	175	338	95	0	172	99
normalized size	1	1.00	1.74	2.08	4.02	1.13	0.00	2.05	1.18
time (sec)	N/A	0.097	0.315	0.184	0.383	0.413	0.000	4.391	8.486

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	106	130	214	75	0	120	73
normalized size	1	1.00	1.54	1.88	3.10	1.09	0.00	1.74	1.06
time (sec)	N/A	0.092	0.325	0.170	0.319	0.417	0.000	6.776	6.732

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	106	70	90	47	0	68	47
normalized size	1	1.00	2.12	1.40	1.80	0.94	0.00	1.36	0.94
time (sec)	N/A	0.088	0.162	0.145	0.303	0.422	0.000	0.660	6.402

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	48	22	27	42	27	21	21
normalized size	1	1.00	2.09	0.96	1.17	1.83	1.17	0.91	0.91
time (sec)	N/A	0.012	0.041	0.074	0.331	0.422	0.830	0.341	6.428

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	69	56	70	62	0	65	25
normalized size	1	1.00	2.38	1.93	2.41	2.14	0.00	2.24	0.86
time (sec)	N/A	0.051	0.231	0.185	0.297	0.412	0.000	4.004	6.636

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	124	132	155	111	0	127	115
normalized size	1	1.00	2.14	2.28	2.67	1.91	0.00	2.19	1.98
time (sec)	N/A	0.088	0.509	0.200	0.298	0.431	0.000	0.228	6.635

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	189	208	234	155	0	187	183
normalized size	1	1.00	2.30	2.54	2.85	1.89	0.00	2.28	2.23
time (sec)	N/A	0.106	0.759	0.239	0.498	0.429	0.000	1.963	6.662

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	284	284	315	198	0	244	387
normalized size	1	1.00	2.68	2.68	2.97	1.87	0.00	2.30	3.65
time (sec)	N/A	0.127	0.924	0.324	0.327	0.436	0.000	0.317	8.221

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	112	180	202	218	0	146	444
normalized size	1	1.00	0.59	0.95	1.07	1.15	0.00	0.77	2.35
time (sec)	N/A	0.149	1.640	0.248	0.315	0.491	0.000	71.382	10.464

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	91	144	167	198	0	126	361
normalized size	1	1.00	0.62	0.99	1.14	1.36	0.00	0.86	2.47
time (sec)	N/A	0.108	0.447	0.235	0.316	0.464	0.000	19.849	10.504

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	70	108	110	178	0	102	240
normalized size	1	1.00	0.67	1.04	1.06	1.71	0.00	0.98	2.31
time (sec)	N/A	0.086	0.317	0.236	0.299	0.427	0.000	2.056	10.052

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	36	72	70	104	0	90	116
normalized size	1	1.00	0.60	1.20	1.17	1.73	0.00	1.50	1.93
time (sec)	N/A	0.048	0.081	0.239	0.296	0.411	0.000	0.997	7.640

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	36	50	46	59	0	45	87
normalized size	1	1.00	0.69	0.96	0.88	1.13	0.00	0.87	1.67
time (sec)	N/A	0.050	0.057	0.152	0.297	0.413	0.000	0.395	6.590

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	49	66	55	76	0	115	103
normalized size	1	1.00	0.75	1.02	0.85	1.17	0.00	1.77	1.58
time (sec)	N/A	0.059	0.068	0.323	0.300	0.416	0.000	0.489	6.528

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	38	39	36	57	0	36	36
normalized size	1	1.00	0.69	0.71	0.65	1.04	0.00	0.65	0.65
time (sec)	N/A	0.051	0.067	0.247	0.298	0.393	0.000	0.604	6.335

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	49	46	94	0	46	46
normalized size	1	1.00	1.00	0.67	0.63	1.29	0.00	0.63	0.63
time (sec)	N/A	0.057	0.074	0.287	0.308	0.436	0.000	0.605	6.375

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	78	79	76	127	0	76	76
normalized size	1	1.00	0.61	0.62	0.60	1.00	0.00	0.60	0.60
time (sec)	N/A	0.074	0.151	0.325	0.327	0.426	0.000	0.770	6.542

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	88	89	86	162	0	86	85
normalized size	1	1.00	0.61	0.61	0.59	1.12	0.00	0.59	0.59
time (sec)	N/A	0.081	0.214	0.378	0.309	0.437	0.000	1.754	6.633

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	118	119	116	195	0	116	116
normalized size	1	1.00	0.59	0.60	0.58	0.98	0.00	0.58	0.58
time (sec)	N/A	0.102	0.342	0.422	0.304	0.468	0.000	1.327	6.849

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	102	162	188	248	0	136	418
normalized size	1	1.00	0.60	0.95	1.10	1.45	0.00	0.80	2.44
time (sec)	N/A	0.123	0.676	0.259	0.321	0.457	0.000	24.625	10.051

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	82	126	146	226	0	114	302
normalized size	1	1.00	0.65	1.00	1.16	1.79	0.00	0.90	2.40
time (sec)	N/A	0.091	0.361	0.265	0.299	0.445	0.000	1.912	9.925

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	52	90	98	154	0	81	186
normalized size	1	1.00	0.63	1.10	1.20	1.88	0.00	0.99	2.27
time (sec)	N/A	0.057	0.151	0.244	0.297	0.437	0.000	0.790	8.819

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	52	68	72	104	0	59	148
normalized size	1	1.00	0.70	0.92	0.97	1.41	0.00	0.80	2.00
time (sec)	N/A	0.058	0.182	0.167	0.300	0.451	0.000	0.641	6.648

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	61	84	80	147	0	154	169
normalized size	1	1.00	0.71	0.98	0.93	1.71	0.00	1.79	1.97
time (sec)	N/A	0.070	0.191	0.474	0.338	0.446	0.000	0.484	6.701

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	69	97	75	131	0	174	171
normalized size	1	1.00	0.72	1.01	0.78	1.36	0.00	1.81	1.78
time (sec)	N/A	0.071	0.314	0.343	0.373	0.434	0.000	0.963	6.717

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	48	49	46	84	0	46	46
normalized size	1	1.00	0.66	0.67	0.63	1.15	0.00	0.63	0.63
time (sec)	N/A	0.057	0.097	0.331	0.303	0.414	0.000	0.813	6.693

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	68	69	66	117	0	66	66
normalized size	1	1.00	0.62	0.63	0.61	1.07	0.00	0.61	0.61
time (sec)	N/A	0.069	0.076	0.355	0.331	0.419	0.000	2.039	6.629

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	88	89	86	152	0	86	86
normalized size	1	1.00	0.61	0.61	0.59	1.05	0.00	0.59	0.59
time (sec)	N/A	0.081	0.111	0.385	0.306	0.436	0.000	2.483	6.811

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	88	89	86	185	0	86	85
normalized size	1	1.00	0.61	0.61	0.59	1.28	0.00	0.59	0.59
time (sec)	N/A	0.078	0.120	0.427	0.305	0.464	0.000	6.359	6.822

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	112	180	213	290	0	146	476
normalized size	1	1.00	0.57	0.92	1.09	1.49	0.00	0.75	2.44
time (sec)	N/A	0.135	1.446	0.240	0.534	0.468	0.000	7.591	10.661

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	50	81	95	102	0	76	172
normalized size	1	1.00	0.38	0.61	0.72	0.77	0.00	0.58	1.30
time (sec)	N/A	0.089	0.104	0.239	0.369	0.451	0.000	2.337	7.508

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	62	108	121	198	0	91	240
normalized size	1	1.00	0.59	1.03	1.15	1.89	0.00	0.87	2.29
time (sec)	N/A	0.065	0.256	0.249	0.331	0.443	0.000	0.718	10.070

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	73	101	103	196	0	185	228
normalized size	1	1.00	0.69	0.95	0.97	1.85	0.00	1.75	2.15
time (sec)	N/A	0.083	0.802	0.337	0.327	0.445	0.000	1.178	6.610

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	89	130	95	186	0	232	235
normalized size	1	1.00	0.66	0.96	0.70	1.38	0.00	1.72	1.74
time (sec)	N/A	0.086	0.161	0.334	0.316	0.447	0.000	1.122	6.807

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	124	158	356	129	0	146	231
normalized size	1	1.00	0.98	1.24	2.80	1.02	0.00	1.15	1.82
time (sec)	N/A	0.310	0.428	0.213	0.378	0.445	0.000	1.808	7.577

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	315	161	288	369	0	135	203
normalized size	1	1.00	2.92	1.49	2.67	3.42	0.00	1.25	1.88
time (sec)	N/A	0.318	0.423	0.306	0.328	0.439	0.000	0.702	12.052

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	589	195	285	445	0	179	171
normalized size	1	1.00	4.91	1.62	2.38	3.71	0.00	1.49	1.42
time (sec)	N/A	0.249	6.088	0.315	0.331	0.450	0.000	0.418	7.826

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	733	229	279	439	0	204	209
normalized size	1	1.00	5.51	1.72	2.10	3.30	0.00	1.53	1.57
time (sec)	N/A	0.249	6.132	0.337	0.323	0.472	0.000	1.288	7.672

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	195	394	172	0	200	0	0	-1
normalized size	1	1.20	2.43	1.06	0.00	1.23	0.00	0.00	-0.01
time (sec)	N/A	0.923	5.568	0.697	0.000	0.471	0.000	0.000	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	114	89	0	169	0	0	-1
normalized size	1	1.00	1.13	0.88	0.00	1.67	0.00	0.00	-0.01
time (sec)	N/A	0.185	0.336	0.728	0.000	0.430	0.000	0.000	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	206	125	0	279	0	0	-1
normalized size	1	1.00	2.31	1.40	0.00	3.13	0.00	0.00	-0.01
time (sec)	N/A	0.192	0.994	0.793	0.000	0.440	0.000	0.000	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	309	170	0	380	0	0	-1
normalized size	1	1.00	1.90	1.04	0.00	2.33	0.00	0.00	-0.01
time (sec)	N/A	0.377	1.603	0.929	0.000	0.443	0.000	0.000	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	195	141	139	0	239	0	0	-1
normalized size	1	1.17	0.84	0.83	0.00	1.43	0.00	0.00	-0.01
time (sec)	N/A	0.975	5.547	0.750	0.000	0.441	0.000	0.000	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	46	55	145	48	0	0	-1
normalized size	1	1.00	0.52	0.62	1.65	0.55	0.00	0.00	-0.01
time (sec)	N/A	0.196	4.152	0.580	0.584	0.414	0.000	0.000	0.000

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	233	144	0	315	0	0	-1
normalized size	1	1.00	1.93	1.19	0.00	2.60	0.00	0.00	-0.01
time (sec)	N/A	0.321	0.756	0.898	0.000	0.430	0.000	0.000	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	334	196	0	424	0	0	-1
normalized size	1	1.00	1.70	0.99	0.00	2.15	0.00	0.00	-0.01
time (sec)	N/A	0.496	1.628	1.002	0.000	0.451	0.000	0.000	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	208	112	87	277	98	0	0	-1
normalized size	1	1.38	0.74	0.58	1.83	0.65	0.00	0.00	-0.01
time (sec)	N/A	0.979	5.471	0.693	1.291	0.430	0.000	0.000	0.000

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	60	67	191	70	0	0	-1
normalized size	1	1.00	0.51	0.57	1.62	0.59	0.00	0.00	-0.01
time (sec)	N/A	0.214	5.473	0.491	0.482	0.426	0.000	0.000	0.000

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	261	162	0	363	0	0	-1
normalized size	1	1.00	1.73	1.07	0.00	2.40	0.00	0.00	-0.01
time (sec)	N/A	0.429	1.261	0.726	0.000	0.446	0.000	0.000	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	360	222	0	485	0	0	-1
normalized size	1	1.00	1.59	0.98	0.00	2.14	0.00	0.00	-0.00
time (sec)	N/A	0.627	1.744	0.909	0.000	0.443	0.000	0.000	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	241	118	231	0	229	0	0	-1
normalized size	1	1.61	0.79	1.54	0.00	1.53	0.00	0.00	-0.01
time (sec)	N/A	0.933	0.689	1.010	0.000	0.437	0.000	0.000	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	118	130	0	200	0	0	-1
normalized size	1	1.00	1.10	1.21	0.00	1.87	0.00	0.00	-0.01
time (sec)	N/A	0.195	0.256	0.576	0.000	0.432	0.000	0.000	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	138	103	0	263	0	0	-1
normalized size	1	1.00	2.23	1.66	0.00	4.24	0.00	0.00	-0.02
time (sec)	N/A	0.110	0.321	0.691	0.000	0.430	0.000	0.000	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	292	144	0	369	0	0	-1
normalized size	1	1.00	2.16	1.07	0.00	2.73	0.00	0.00	-0.01
time (sec)	N/A	0.622	0.608	0.822	0.000	0.443	0.000	0.000	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	195	334	289	0	270	0	0	-1
normalized size	1	1.10	1.89	1.63	0.00	1.53	0.00	0.00	-0.01
time (sec)	N/A	1.199	0.373	0.802	0.000	0.481	0.000	0.000	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	128	202	0	237	0	0	-1
normalized size	1	1.00	0.96	1.51	0.00	1.77	0.00	0.00	-0.01
time (sec)	N/A	0.223	0.456	0.698	0.000	0.466	0.000	0.000	0.000

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	206	134	0	421	0	0	-1
normalized size	1	1.00	1.82	1.19	0.00	3.73	0.00	0.00	-0.01
time (sec)	N/A	0.229	2.115	0.801	0.000	0.459	0.000	0.000	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-1)	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	294	144	0	383	0	0	-1
normalized size	1	1.00	2.04	1.00	0.00	2.66	0.00	0.00	-0.01
time (sec)	N/A	0.553	0.764	0.892	0.000	0.447	0.000	0.000	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	260	394	353	0	307	0	0	-1
normalized size	1	1.26	1.90	1.71	0.00	1.48	0.00	0.00	-0.00
time (sec)	N/A	1.434	0.541	0.989	0.000	0.484	0.000	0.000	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	284	266	0	279	0	0	-1
normalized size	1	1.00	1.70	1.59	0.00	1.67	0.00	0.00	-0.01
time (sec)	N/A	0.303	0.402	0.958	0.000	0.447	0.000	0.000	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	451	219	0	539	0	0	-1
normalized size	1	1.00	3.20	1.55	0.00	3.82	0.00	0.00	-0.01
time (sec)	N/A	0.346	0.740	0.688	0.000	0.468	0.000	0.000	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	332	182	0	564	0	0	-1
normalized size	1	1.00	1.74	0.95	0.00	2.95	0.00	0.00	-0.01
time (sec)	N/A	0.959	2.411	1.007	0.000	0.461	0.000	0.000	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	982	982	318	0	0	0	0	0	-1
normalized size	1	1.00	0.32	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.271	3.237	0.273	0.000	0.456	0.000	0.000	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	290	0	0	0	0	0	-1
normalized size	1	1.00	2.36	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.193	2.709	0.222	0.000	0.444	0.000	0.000	0.000

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	2692	0	0	0	0	0	-1
normalized size	1	1.00	33.65	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.101	23.710	0.195	0.000	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	2796	0	0	0	0	0	-1
normalized size	1	1.00	34.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.094	22.335	0.227	0.000	0.000	0.000	0.000	0.000

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	551	551	128	0	0	0	0	0	-1
normalized size	1	1.00	0.23	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.494	0.788	0.233	0.000	1.312	0.000	0.000	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	100	0	0	0	0	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.216	0.499	0.213	0.000	1.690	0.000	0.000	0.000

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	12.571	0.199	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.094	8.942	0.253	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	5199	0	0	0	0	0	-1
normalized size	1	1.00	19.33	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.345	58.504	2.675	0.000	1.995	0.000	0.000	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	2054	0	0	0	0	0	-1
normalized size	1	1.00	10.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.227	18.142	2.243	0.000	1.065	0.000	0.000	0.000

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.139	2.058	1.522	0.000	0.648	0.000	0.000	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	232	0	0	0	0	0	-1
normalized size	1	1.00	2.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.126	3.925	0.466	0.000	0.486	0.000	0.000	0.000

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	667	0	0	0	0	0	-1
normalized size	1	1.00	4.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.275	13.956	1.022	0.000	0.480	0.000	0.000	0.000

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	1276	0	0	0	0	0	-1
normalized size	1	1.00	5.15	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.422	27.816	1.002	0.000	0.453	0.000	0.000	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	367	0	0	0	0	0	-1
normalized size	1	1.00	3.31	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.124	2.178	1.324	0.000	0.473	0.000	0.000	0.000

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	105	0	0	0	0	0	-1
normalized size	1	1.00	0.64	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.149	0.257	0.265	0.000	0.438	0.000	0.000	0.000

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	63	0	0	0	0	0	-1
normalized size	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.069	1.368	0.000	0.481	0.000	0.000	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.044	0.055	1.425	0.000	0.452	0.000	0.000	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	68	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.192	0.418	0.000	0.436	0.000	0.000	0.000

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	83	0	0	0	0	0	-1
normalized size	1	1.00	0.67	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.263	0.529	0.000	0.458	0.000	0.000	0.000

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.355	1.123	0.259	0.000	0.441	0.000	0.000	0.000

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	4043	0	0	0	0	0	-1
normalized size	1	1.00	25.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.248	6.325	0.219	0.000	0.442	0.000	0.000	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	90	0	0	0	0	0	-1
normalized size	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.154	0.450	0.000	0.480	0.000	0.000	0.000

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	5048	0	0	0	0	0	-1
normalized size	1	1.00	56.72	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.102	26.447	0.351	0.000	0.450	0.000	0.000	0.000

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.781	0.442	0.000	0.503	0.000	0.000	0.000

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	77	96	73	90	0	0	176
normalized size	1	1.00	0.88	1.09	0.83	1.02	0.00	0.00	2.00
time (sec)	N/A	0.077	0.126	0.095	1.625	0.504	0.000	0.000	6.735

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	38	46	43	45	0	1456	74
normalized size	1	1.00	0.69	0.84	0.78	0.82	0.00	26.47	1.35
time (sec)	N/A	0.038	0.017	0.084	1.153	0.430	0.000	4.249	6.636

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	43	25	22	24	0	23	47
normalized size	1	1.00	1.79	1.04	0.92	1.00	0.00	0.96	1.96
time (sec)	N/A	0.021	0.036	0.080	0.606	0.448	0.000	1.274	6.564

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	60	83	45	69	0	60	146
normalized size	1	1.00	1.11	1.54	0.83	1.28	0.00	1.11	2.70
time (sec)	N/A	0.041	0.212	0.189	0.957	0.463	0.000	1.790	6.631

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	87	136	69	110	0	82	207
normalized size	1	1.00	1.07	1.68	0.85	1.36	0.00	1.01	2.56
time (sec)	N/A	0.052	0.244	0.177	1.402	0.465	0.000	1.844	6.670

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	81	98	65	73	0	0	110
normalized size	1	1.00	1.12	1.36	0.90	1.01	0.00	0.00	1.53
time (sec)	N/A	0.078	0.037	0.155	1.703	0.472	0.000	0.000	10.183

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	47	59	39	47	0	1008	55
normalized size	1	1.00	1.24	1.55	1.03	1.24	0.00	26.53	1.45
time (sec)	N/A	0.059	0.035	0.136	1.367	0.446	0.000	35.578	6.595

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	75	57	54	84	0	108	158
normalized size	1	1.00	1.83	1.39	1.32	2.05	0.00	2.63	3.85
time (sec)	N/A	0.053	0.037	0.098	1.381	0.465	0.000	0.285	6.597

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	125	106	92	160	0	141	225
normalized size	1	1.00	1.52	1.29	1.12	1.95	0.00	1.72	2.74
time (sec)	N/A	0.080	0.046	0.115	1.819	0.467	0.000	0.575	6.294

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	164	159	125	222	0	199	288
normalized size	1	1.00	1.34	1.30	1.02	1.82	0.00	1.63	2.36
time (sec)	N/A	0.097	0.054	0.129	1.885	0.498	0.000	0.683	6.299

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	108	172	105	140	0	0	232
normalized size	1	1.00	0.97	1.55	0.95	1.26	0.00	0.00	2.09
time (sec)	N/A	0.173	0.441	0.146	0.464	0.453	0.000	0.000	6.722

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	64	82	70	74	0	7855	150
normalized size	1	1.00	0.82	1.05	0.90	0.95	0.00	100.71	1.92
time (sec)	N/A	0.071	0.132	0.148	0.372	0.473	0.000	12.160	6.366

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	45	40	42	0	41	117
normalized size	1	1.00	1.00	0.98	0.87	0.91	0.00	0.89	2.54
time (sec)	N/A	0.039	0.025	0.099	0.485	0.461	0.000	0.417	6.436

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	70	120	69	115	0	99	221
normalized size	1	1.00	0.83	1.43	0.82	1.37	0.00	1.18	2.63
time (sec)	N/A	0.073	0.259	0.237	0.770	0.486	0.000	0.745	6.385

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	107	220	105	177	0	138	310
normalized size	1	1.00	0.85	1.75	0.83	1.40	0.00	1.10	2.46
time (sec)	N/A	0.104	0.732	0.278	0.985	0.497	0.000	0.873	6.444

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	176	185	119	118	0	0	235
normalized size	1	1.00	1.18	1.24	0.80	0.79	0.00	0.00	1.58
time (sec)	N/A	0.162	0.724	0.197	1.246	0.445	0.000	0.000	10.036

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	77	116	83	81	0	7670	147
normalized size	1	1.00	0.82	1.23	0.88	0.86	0.00	81.60	1.56
time (sec)	N/A	0.122	0.485	0.227	1.037	0.440	0.000	45.199	9.287

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	116	102	79	118	0	148	277
normalized size	1	1.00	1.49	1.31	1.01	1.51	0.00	1.90	3.55
time (sec)	N/A	0.086	0.411	0.131	0.727	0.459	0.000	0.199	7.273

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	293	199	138	218	0	241	584
normalized size	1	1.00	2.20	1.50	1.04	1.64	0.00	1.81	4.39
time (sec)	N/A	0.149	6.214	0.246	1.027	0.482	0.000	0.353	8.997

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	351	302	183	306	0	337	888
normalized size	1	1.00	1.74	1.50	0.91	1.51	0.00	1.67	4.40
time (sec)	N/A	0.169	1.120	0.233	2.125	0.520	0.000	0.505	11.280

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	141	279	162	194	0	0	366
normalized size	1	1.00	0.94	1.86	1.08	1.29	0.00	0.00	2.44
time (sec)	N/A	0.242	0.249	0.161	0.401	0.492	0.000	0.000	6.981

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	90	139	113	116	0	0	226
normalized size	1	1.00	0.86	1.32	1.08	1.10	0.00	0.00	2.15
time (sec)	N/A	0.114	0.199	0.145	0.303	0.470	0.000	0.000	6.746

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	64	57	66	0	58	118
normalized size	1	1.00	1.00	0.96	0.85	0.99	0.00	0.87	1.76
time (sec)	N/A	0.045	0.027	0.104	1.198	0.467	0.000	0.922	6.678

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	97	165	98	153	0	131	312
normalized size	1	1.00	0.84	1.42	0.84	1.32	0.00	1.13	2.69
time (sec)	N/A	0.094	0.289	0.257	1.141	0.466	0.000	0.940	6.946

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	144	316	142	225	0	185	424
normalized size	1	1.00	0.87	1.92	0.86	1.36	0.00	1.12	2.57
time (sec)	N/A	0.141	1.037	0.230	0.639	0.498	0.000	2.447	6.969

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	226	268	167	157	0	0	297
normalized size	1	1.00	1.03	1.22	0.76	0.71	0.00	0.00	1.35
time (sec)	N/A	0.224	0.708	0.252	1.263	0.437	0.000	0.000	9.197

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	113	169	119	116	0	0	249
normalized size	1	1.00	0.77	1.16	0.82	0.79	0.00	0.00	1.71
time (sec)	N/A	0.169	0.769	0.286	2.155	0.423	0.000	0.000	9.152

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	143	125	95	143	0	199	289
normalized size	1	1.00	1.40	1.23	0.93	1.40	0.00	1.95	2.83
time (sec)	N/A	0.112	1.323	0.143	0.905	0.468	0.000	1.621	6.857

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	355	264	187	293	0	421	405
normalized size	1	1.00	1.83	1.36	0.96	1.51	0.00	2.17	2.09
time (sec)	N/A	0.184	6.234	0.228	1.840	0.486	0.000	0.689	6.809

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	346	415	252	412	0	471	507
normalized size	1	1.00	1.19	1.43	0.87	1.42	0.00	1.62	1.74
time (sec)	N/A	0.231	2.574	0.258	1.890	0.573	0.000	1.051	7.058

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	184	304	288	261	0	343	498
normalized size	1	1.00	0.90	1.49	1.41	1.28	0.00	1.68	2.44
time (sec)	N/A	0.363	1.302	0.180	1.896	0.631	0.000	10.464	7.426

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	117	164	142	157	0	177	217
normalized size	1	1.00	0.93	1.30	1.13	1.25	0.00	1.40	1.72
time (sec)	N/A	0.191	0.492	0.180	0.690	0.543	0.000	2.104	7.035

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	87	76	65	63	0	71	91
normalized size	1	1.00	1.18	1.03	0.88	0.85	0.00	0.96	1.23
time (sec)	N/A	0.066	0.085	0.180	0.448	0.459	0.000	0.405	6.732

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	35	33	31	0	35	48
normalized size	1	1.00	1.00	1.03	0.97	0.91	0.00	1.03	1.41
time (sec)	N/A	0.040	0.019	0.112	0.652	0.484	0.000	0.718	6.362

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	65	106	77	118	0	114	144
normalized size	1	1.00	0.77	1.26	0.92	1.40	0.00	1.36	1.71
time (sec)	N/A	0.089	0.158	0.221	2.084	0.477	0.000	0.381	6.632

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	115	216	139	271	0	201	281
normalized size	1	1.00	0.78	1.46	0.94	1.83	0.00	1.36	1.90
time (sec)	N/A	0.138	1.060	0.202	0.608	0.473	0.000	0.455	6.409

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	195	269	0	476	0	241	372
normalized size	1	1.00	1.10	1.52	0.00	2.69	0.00	1.36	2.10
time (sec)	N/A	0.239	1.401	0.193	0.000	0.539	0.000	3.699	9.547

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	152	117	0	305	0	107	148
normalized size	1	1.00	1.58	1.22	0.00	3.18	0.00	1.11	1.54
time (sec)	N/A	0.099	0.197	0.158	0.000	0.464	0.000	1.790	6.369

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	108	155	0	314	0	129	204
normalized size	1	1.00	1.35	1.94	0.00	3.92	0.00	1.61	2.55
time (sec)	N/A	0.243	0.245	0.172	0.000	0.526	0.000	0.416	7.000

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	350	348	0	633	0	273	654
normalized size	1	1.00	2.27	2.26	0.00	4.11	0.00	1.77	4.25
time (sec)	N/A	0.426	6.121	0.198	0.000	0.636	0.000	0.390	7.144

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	504	629	0	1079	0	490	1099
normalized size	1	1.00	1.64	2.05	0.00	3.51	0.00	1.60	3.58
time (sec)	N/A	1.106	1.414	0.206	0.000	1.041	0.000	0.597	7.130

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	240	318	505	555	0	494	755
normalized size	1	1.00	0.99	1.31	2.09	2.29	0.00	2.04	3.12
time (sec)	N/A	0.633	6.255	0.270	1.061	0.970	0.000	9.560	7.782

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	145	182	274	388	0	248	351
normalized size	1	1.00	0.90	1.13	1.70	2.41	0.00	1.54	2.18
time (sec)	N/A	0.312	0.794	0.264	1.452	0.680	0.000	1.981	7.245

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	162	132	124	195	0	156	158
normalized size	1	1.00	1.49	1.21	1.14	1.79	0.00	1.43	1.45
time (sec)	N/A	0.096	0.299	0.243	0.924	0.542	0.000	0.460	6.758

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	42	54	47	69	0	51	105
normalized size	1	1.00	0.79	1.02	0.89	1.30	0.00	0.96	1.98
time (sec)	N/A	0.053	0.081	0.146	0.604	0.508	0.000	0.416	6.376

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	96	150	116	259	0	165	235
normalized size	1	1.00	0.84	1.32	1.02	2.27	0.00	1.45	2.06
time (sec)	N/A	0.115	0.631	0.291	0.304	0.533	0.000	0.431	6.379

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	187	282	189	542	0	278	439
normalized size	1	1.00	0.99	1.50	1.01	2.88	0.00	1.48	2.34
time (sec)	N/A	0.181	6.139	0.273	0.806	0.548	0.000	0.416	6.900

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	341	382	0	815	0	406	722
normalized size	1	1.00	1.02	1.15	0.00	2.45	0.00	1.22	2.17
time (sec)	N/A	0.627	2.027	0.259	0.000	0.575	0.000	2.980	9.895

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	169	282	0	569	0	251	313
normalized size	1	1.00	0.84	1.41	0.00	2.84	0.00	1.26	1.56
time (sec)	N/A	0.301	1.057	0.306	0.000	0.552	0.000	0.874	8.675

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	139	245	0	768	0	218	1616
normalized size	1	1.00	1.21	2.13	0.00	6.68	0.00	1.90	14.05
time (sec)	N/A	0.453	0.764	0.279	0.000	0.660	0.000	0.396	8.317

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	403	527	0	1149	0	356	973
normalized size	1	1.00	1.69	2.21	0.00	4.83	0.00	1.50	4.09
time (sec)	N/A	0.695	6.198	0.287	0.000	0.715	0.000	0.542	6.830

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	424	424	361	897	0	2011	0	596	1424
normalized size	1	1.00	0.85	2.12	0.00	4.74	0.00	1.41	3.36
time (sec)	N/A	1.493	1.566	0.323	0.000	1.063	0.000	0.899	6.905

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	304	465	730	981	0	585	1229
normalized size	1	1.00	0.95	1.45	2.27	3.06	0.00	1.82	3.83
time (sec)	N/A	0.877	6.341	0.327	1.438	1.349	0.000	6.814	10.735

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	196	323	441	788	0	464	690
normalized size	1	1.00	0.84	1.39	1.90	3.40	0.00	2.00	2.97
time (sec)	N/A	0.482	2.277	0.305	0.740	0.861	0.000	2.390	7.464

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	213	198	228	462	0	257	304
normalized size	1	1.00	1.43	1.33	1.53	3.10	0.00	1.72	2.04
time (sec)	N/A	0.132	2.161	0.293	0.659	0.661	0.000	0.530	6.854

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	60	74	81	154	0	69	369
normalized size	1	1.00	0.80	0.99	1.08	2.05	0.00	0.92	4.92
time (sec)	N/A	0.060	0.262	0.175	0.687	0.518	0.000	1.083	6.537

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	121	194	156	404	0	154	334
normalized size	1	1.00	0.83	1.34	1.08	2.79	0.00	1.06	2.30
time (sec)	N/A	0.133	0.940	0.329	0.735	0.581	0.000	0.536	6.790

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	195	348	236	754	0	327	563
normalized size	1	1.00	0.88	1.57	1.07	3.41	0.00	1.48	2.55
time (sec)	N/A	0.211	5.306	0.350	0.700	0.631	0.000	0.835	7.262

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	474	474	351	922	0	1249	0	632	1099
normalized size	1	1.00	0.74	1.95	0.00	2.64	0.00	1.33	2.32
time (sec)	N/A	0.872	1.045	0.316	0.000	0.641	0.000	2.416	11.220

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	350	212	766	0	934	0	384	627
normalized size	1	1.00	0.61	2.19	0.00	2.67	0.00	1.10	1.79
time (sec)	N/A	0.539	3.221	0.289	0.000	0.592	0.000	1.821	10.358

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	195	729	0	1394	0	339	1762
normalized size	1	1.00	0.97	3.61	0.00	6.90	0.00	1.68	8.72
time (sec)	N/A	0.788	5.783	0.303	0.000	1.038	0.000	0.500	7.853

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	459	780	0	2027	0	451	1261
normalized size	1	1.00	1.59	2.70	0.00	7.01	0.00	1.56	4.36
time (sec)	N/A	1.072	6.211	0.332	0.000	1.045	0.000	0.920	7.311

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	492	492	448	1252	0	2571	0	731	1614
normalized size	1	1.00	0.91	2.54	0.00	5.23	0.00	1.49	3.28
time (sec)	N/A	2.127	1.758	0.411	0.000	1.175	0.000	1.518	7.301

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	4791	0	0	0	0	0	-1
normalized size	1	1.00	17.68	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.380	16.096	2.629	0.000	0.496	0.000	0.000	0.000

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	2464	0	0	0	0	0	-1
normalized size	1	1.00	13.25	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.242	14.236	2.489	0.000	0.490	0.000	0.000	0.000

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	849	0	0	0	0	0	-1
normalized size	1	1.00	6.58	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.148	9.534	1.533	0.000	0.463	0.000	0.000	0.000

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	B	F	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	284	0	858	0	0	0	0	0	-1
normalized size	1	0.00	3.02	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.048	13.590	0.796	0.000	0.459	0.000	0.000	0.000

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	F	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	737	0	866	0	0	0	0	0	-1
normalized size	1	0.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.045	14.397	1.567	0.000	0.596	0.000	0.000	0.000

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.040	3.129	1.523	0.000	0.714	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [159] had the largest ratio of [.4762]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.00	19	0.105
2	A	3	2	1.00	19	0.105
3	A	3	2	1.00	17	0.118
4	A	3	2	1.00	17	0.118
5	A	3	2	1.00	19	0.105
6	A	3	2	1.00	19	0.105
7	A	3	2	1.00	19	0.105
8	A	9	5	1.00	19	0.263
9	A	8	5	1.00	19	0.263
10	A	5	5	1.00	19	0.263
11	A	7	6	1.00	19	0.316
12	A	9	7	1.00	19	0.368
13	A	11	7	1.00	19	0.368
14	A	3	2	1.00	21	0.095
15	A	3	2	1.00	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
16	A	3	2	1.00	19	0.105
17	A	2	2	1.00	21	0.095
18	A	3	2	1.00	21	0.095
19	A	14	9	1.00	21	0.429
20	A	4	4	1.00	21	0.190
21	A	6	5	1.00	21	0.238
22	A	1	1	1.00	12	0.083
23	A	8	6	1.00	21	0.286
24	A	12	7	1.00	21	0.333
25	A	3	2	1.00	21	0.095
26	A	3	2	1.00	21	0.095
27	A	3	2	1.00	19	0.105
28	A	3	2	1.00	21	0.095
29	A	9	7	1.00	21	0.333
30	A	10	7	1.00	21	0.333
31	A	8	6	1.00	21	0.286
32	A	7	5	1.00	12	0.417
33	A	10	7	1.00	21	0.333
34	A	3	2	1.00	21	0.095
35	A	3	2	1.00	21	0.095
36	A	3	2	1.00	19	0.105
37	A	3	2	1.00	21	0.095
38	A	13	7	1.00	21	0.333
39	A	11	6	1.00	21	0.286
40	A	10	5	1.00	12	0.417
41	A	12	6	1.00	21	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
42	A	17	8	1.00	21	0.381
43	A	21	8	1.00	21	0.381
44	A	8	5	1.00	21	0.238
45	A	7	5	1.00	21	0.238
46	A	6	5	1.00	21	0.238
47	A	5	5	1.57	19	0.263
48	A	4	4	1.00	19	0.210
49	A	5	4	1.00	21	0.190
50	A	5	4	1.00	21	0.190
51	A	6	5	1.00	21	0.238
52	A	6	5	1.00	21	0.238
53	A	6	5	1.00	21	0.238
54	A	6	5	1.00	21	0.238
55	A	5	4	1.00	21	0.190
56	A	1	1	1.00	12	0.083
57	A	4	4	1.00	21	0.190
58	A	5	5	1.00	21	0.238
59	A	6	5	1.00	21	0.238
60	A	7	5	1.00	21	0.238
61	A	4	3	1.00	21	0.143
62	A	4	3	1.00	21	0.143
63	A	4	3	1.00	21	0.143
64	A	4	3	1.00	19	0.158
65	A	3	2	1.00	19	0.105
66	A	3	2	1.00	21	0.095
67	A	3	2	1.00	21	0.095

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
68	A	3	2	1.00	21	0.095
69	A	3	2	1.00	21	0.095
70	A	3	2	1.00	21	0.095
71	A	3	2	1.00	21	0.095
72	A	4	3	1.00	21	0.143
73	A	4	3	1.00	21	0.143
74	A	4	3	1.00	19	0.158
75	A	3	2	1.00	19	0.105
76	A	3	2	1.00	21	0.095
77	A	3	2	1.00	21	0.095
78	A	3	2	1.00	21	0.095
79	A	3	2	1.00	21	0.095
80	A	3	2	1.00	21	0.095
81	A	3	2	1.00	21	0.095
82	A	4	3	1.00	21	0.143
83	A	3	2	1.00	21	0.095
84	A	4	3	1.00	19	0.158
85	A	3	2	1.00	21	0.095
86	A	3	2	1.00	21	0.095
87	A	17	5	1.00	21	0.238
88	A	14	8	1.00	21	0.381
89	A	14	8	1.00	21	0.381
90	A	16	6	1.00	21	0.286
91	A	15	10	1.20	23	0.435
92	A	4	4	1.00	23	0.174
93	A	4	4	1.00	23	0.174

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
94	A	7	7	1.00	23	0.304
95	A	14	9	1.17	23	0.391
96	A	3	3	1.00	23	0.130
97	A	5	5	1.00	23	0.217
98	A	8	8	1.00	23	0.348
99	A	10	7	1.38	23	0.304
100	A	4	4	1.00	23	0.174
101	A	6	5	1.00	23	0.217
102	A	10	8	1.00	23	0.348
103	A	17	9	1.61	23	0.391
104	A	4	4	1.00	23	0.174
105	A	4	4	1.00	23	0.174
106	A	11	7	1.00	23	0.304
107	A	20	9	1.10	23	0.391
108	A	5	5	1.00	23	0.217
109	A	6	5	1.00	23	0.217
110	A	10	6	1.00	23	0.261
111	A	23	9	1.26	23	0.391
112	A	6	6	1.00	23	0.261
113	A	7	6	1.00	23	0.261
114	A	16	8	1.00	23	0.348
115	A	10	9	1.00	23	0.391
116	A	4	4	1.00	23	0.174
117	A	3	3	1.00	23	0.130
118	A	3	3	1.00	23	0.130
119	A	8	7	1.00	23	0.304

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
120	A	4	4	1.00	23	0.174
121	A	3	3	1.00	23	0.130
122	A	3	3	1.00	23	0.130
123	A	10	6	1.00	23	0.261
124	A	8	6	1.00	23	0.261
125	A	6	5	1.00	21	0.238
126	A	4	4	1.00	23	0.174
127	A	10	6	1.00	23	0.261
128	A	13	6	1.00	23	0.261
129	A	4	3	1.00	23	0.130
130	A	4	4	1.00	21	0.190
131	A	3	3	1.00	19	0.158
132	A	2	2	1.00	19	0.105
133	A	3	3	1.00	21	0.143
134	A	4	4	1.00	21	0.190
135	A	6	6	1.00	21	0.286
136	A	5	5	1.00	21	0.238
137	A	2	2	1.00	12	0.167
138	A	3	3	1.00	21	0.143
139	A	3	3	1.00	21	0.143
140	A	6	5	1.00	19	0.263
141	A	5	4	1.00	17	0.235
142	A	3	2	1.00	17	0.118
143	A	3	2	1.00	19	0.105
144	A	3	2	1.00	19	0.105
145	A	8	5	1.00	19	0.263

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
146	A	7	5	1.00	19	0.263
147	A	7	6	1.00	19	0.316
148	A	9	7	1.00	19	0.368
149	A	11	7	1.00	19	0.368
150	A	7	5	1.00	21	0.238
151	A	6	4	1.00	19	0.210
152	A	3	2	1.00	19	0.105
153	A	3	2	1.00	21	0.095
154	A	3	2	1.00	21	0.095
155	A	13	9	1.00	21	0.429
156	A	11	9	1.00	21	0.429
157	A	9	7	1.00	21	0.333
158	A	13	9	1.00	21	0.429
159	A	16	10	1.00	21	0.476
160	A	7	5	1.00	21	0.238
161	A	6	4	1.00	19	0.210
162	A	3	2	1.00	19	0.105
163	A	3	2	1.00	21	0.095
164	A	3	2	1.00	21	0.095
165	A	16	9	1.00	21	0.429
166	A	14	10	1.00	21	0.476
167	A	11	9	1.00	21	0.429
168	A	17	10	1.00	21	0.476
169	A	21	10	1.00	21	0.476
170	A	5	3	1.00	21	0.143
171	A	4	3	1.00	21	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
172	A	3	2	1.00	19	0.105
173	A	4	4	1.00	19	0.210
174	A	3	2	1.00	21	0.095
175	A	3	2	1.00	21	0.095
176	A	13	9	1.00	21	0.429
177	A	8	7	1.00	21	0.333
178	A	7	7	1.00	21	0.333
179	A	7	7	1.00	21	0.333
180	A	9	7	1.00	21	0.333
181	A	5	3	1.00	21	0.143
182	A	4	3	1.00	21	0.143
183	A	3	2	1.00	19	0.105
184	A	3	2	1.00	19	0.105
185	A	3	2	1.00	21	0.095
186	A	3	2	1.00	21	0.095
187	A	16	8	1.00	21	0.381
188	A	12	7	1.00	21	0.333
189	A	8	7	1.00	21	0.333
190	A	8	7	1.00	21	0.333
191	A	10	7	1.00	21	0.333
192	A	5	3	1.00	21	0.143
193	A	4	3	1.00	21	0.143
194	A	3	2	1.00	19	0.105
195	A	3	2	1.00	19	0.105
196	A	3	2	1.00	21	0.095
197	A	3	2	1.00	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
198	A	22	9	1.00	21	0.429
199	A	18	8	1.00	21	0.381
200	A	9	8	1.00	21	0.381
201	A	9	7	1.00	21	0.333
202	A	11	7	1.00	21	0.333
203	A	10	6	1.00	23	0.261
204	A	8	6	1.00	23	0.261
205	A	6	5	1.00	21	0.238
206	F	0	0	N/A	0	N/A
207	F	0	0	N/A	0	N/A
208	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

3.1 $\int (a + a\sin(c + dx))\tan^5(c + dx)\,dx$

Optimal. Leaf size=115

$$\frac{a^3}{8d(a - a\sin(c + dx))^2} - \frac{a^2}{d(a - a\sin(c + dx))} + \frac{a^2}{8d(a\sin(c + dx) + a)} - \frac{a\sin(c + dx)}{d} - \frac{23a\log(1 - \sin(c + dx))}{16d} + \frac{7a}{}$$

```
[Out] -23/16*a*ln(1-sin(d*x+c))/d+7/16*a*ln(1+sin(d*x+c))/d-a*sin(d*x+c)/d+1/8*a^
3/d/(a-a*sin(d*x+c))^2-a^2/d/(a-a*sin(d*x+c))+1/8*a^2/d/(a+a*sin(d*x+c))
```

Rubi [A] time = 0.07, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2707, 88}

$$\frac{a^3}{8d(a - a\sin(c + dx))^2} - \frac{a^2}{d(a - a\sin(c + dx))} + \frac{a^2}{8d(a\sin(c + dx) + a)} - \frac{a\sin(c + dx)}{d} - \frac{23a\log(1 - \sin(c + dx))}{16d} + \frac{7a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] (-23*a*Log[1 - Sin[c + d*x]])/(16*d) + (7*a*Log[1 + Sin[c + d*x]])/(16*d) -
 (a*Sin[c + d*x])/d + a^3/(8*d*(a - a*Sin[c + d*x])^2) - a^2/(d*(a - a*Sin[
c + d*x])) + a^2/(8*d*(a + a*Sin[c + d*x]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int (a + a\sin(c + dx))\tan^5(c + dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(a-x)^3(a+x)^2}\,dx, x, a\sin(c + dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-1 + \frac{a^3}{4(a-x)^3} - \frac{a^2}{(a-x)^2} + \frac{23a}{16(a-x)} - \frac{a^2}{8(a+x)^2} + \frac{7a}{16(a+x)}\right) dx, x, a\sin(c\right.}{d}$$
$$= -\frac{23a\log(1 - \sin(c + dx))}{16d} + \frac{7a\log(1 + \sin(c + dx))}{16d} - \frac{a\sin(c + dx)}{d} + \frac{}{8a}$$

Mathematica [A] time = 0.45, size = 123, normalized size = 1.07

$$-\frac{a\sin(c + dx)\tan^4(c + dx)}{d} - \frac{a\left(-\tan^4(c + dx) + 2\tan^2(c + dx) + 4\log(\cos(c + dx))\right)}{4d} - \frac{5a\left(6\tan(c + dx)\sec^3(c +\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] -((a*Sin[c + d*x]*Tan[c + d*x]^4)/d) - (a*(4*Log[Cos[c + d*x]] + 2*Tan[c +
d*x]^2 - Tan[c + d*x]^4))/(4*d) - (5*a*(6*Sec[c + d*x]^3*Tan[c + d*x] - 8*S
ec[c + d*x]*Tan[c + d*x]^3 - 3*(ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c
+ d*x])))/(8*d)
```

fricas [A] time = 0.44, size = 159, normalized size = 1.38

$$\frac{16\,a\cos(dx + c)^4 + 2\,a\cos(dx + c)^2 + 7\left(a\cos(dx + c)^2\sin(dx + c) - a\cos(dx + c)^2\right)\log(\sin(dx + c) + 1) - 23}{16\left(d\cos(dx + c)^2\sin(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/16*(16*a*cos(d*x + c)^4 + 2*a*cos(d*x + c)^2 + 7*(a*cos(d*x + c)^2*sin(d*
x + c) - a*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 23*(a*cos(d*x + c)^2*sin
(d*x + c) - a*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) + 2*(8*a*cos(d*x + c)^
2 + a)*sin(d*x + c) - 6*a)/(d*cos(d*x + c)^2*sin(d*x + c) - d*cos(d*x + c)^
2)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.15, size = 147, normalized size = 1.28

$$\frac{a\left(\sin^7(dx+c)\right)}{4d\cos(dx+c)^4}-\frac{3a\left(\sin^7(dx+c)\right)}{8d\cos(dx+c)^2}-\frac{3a\left(\sin^5(dx+c)\right)}{8d}-\frac{5a\left(\sin^3(dx+c)\right)}{8d}-\frac{15a\sin(dx+c)}{8d}+\frac{15a\ln(\sec(dx+}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))*tan(d*x+c)^5,x)
```

```
[Out] 1/4/d*a*sin(d*x+c)^7/cos(d*x+c)^4-3/8/d*a*sin(d*x+c)^7/cos(d*x+c)^2-3/8*a*s
in(d*x+c)^5/d-5/8*a*sin(d*x+c)^3/d-15/8*a*sin(d*x+c)/d+15/8/d*a*ln(sec(d*x+
c)+tan(d*x+c))+1/4/d*a*tan(d*x+c)^4-1/2/d*a*tan(d*x+c)^2-1/d*a*ln(cos(d*x+c
))
```

maxima [A] time = 0.33, size = 95, normalized size = 0.83

$$\frac{7\,a\log(\sin(dx+c)+1)-23\,a\log(\sin(dx+c)-1)-16\,a\sin(dx+c)+\frac{2\left(9\,a\sin(dx+c)^2-a\sin(dx+c)-6\,a\right)}{\sin(dx+c)^3-\sin(dx+c)^2-\sin(dx+c)+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] 1/16*(7*a*log(sin(d*x + c) + 1) - 23*a*log(sin(d*x + c) - 1) - 16*a*sin(d*x
 + c) + 2*(9*a*sin(d*x + c)^2 - a*sin(d*x + c) - 6*a)/(sin(d*x + c)^3 - sin
(d*x + c)^2 - sin(d*x + c) + 1))/d
```

mupad [B] time = 6.63, size = 235, normalized size = 2.04

$$-\frac{-\frac{15\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{2}+\frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}-5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}+\frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}-\frac{15\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5*(a + a*sin(c + d*x)),x)
```

```
[Out] ((11*a*tan(c/2 + (d*x)/2)^2)/2 - (15*a*tan(c/2 + (d*x)/2))/4 + (11*a*tan(c/
2 + (d*x)/2)^3)/4 - 5*a*tan(c/2 + (d*x)/2)^4 + (11*a*tan(c/2 + (d*x)/2)^5)/
4 + (11*a*tan(c/2 + (d*x)/2)^6)/2 - (15*a*tan(c/2 + (d*x)/2)^7)/4)/(d*(2*ta
n(c/2 + (d*x)/2)^3 - 2*tan(c/2 + (d*x)/2) - 2*tan(c/2 + (d*x)/2)^4 + 2*tan(
c/2 + (d*x)/2)^5 - 2*tan(c/2 + (d*x)/2)^7 + tan(c/2 + (d*x)/2)^8 + 1)) - (2
3*a*log(tan(c/2 + (d*x)/2) - 1))/(8*d) + (7*a*log(tan(c/2 + (d*x)/2) + 1))/
(8*d) + (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sin\left(c+dx\right)\tan^5\left(c+dx\right)dx+\int\tan^5\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)**5,x)
```

```
[Out] a*(Integral(sin(c + d*x)*tan(c + d*x)**5, x) + Integral(tan(c + d*x)**5, x)
)
```

3.2 $\int (a+a\sin(c+dx))\tan^3(c+dx)\,dx$

Optimal. Leaf size=71

$$\frac{a^2}{2d(a-a\sin(c+dx))}+\frac{a\sin(c+dx)}{d}+\frac{5a\log(1-\sin(c+dx))}{4d}-\frac{a\log(\sin(c+dx)+1)}{4d}$$

```
[Out] 5/4*a*ln(1-sin(d*x+c))/d-1/4*a*ln(1+sin(d*x+c))/d+a*sin(d*x+c)/d+1/2*a^2/d/
(a-a*sin(d*x+c))
```

Rubi [A] time = 0.05, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2707, 88}

$$\frac{a^2}{2d(a-a\sin(c+dx))}+\frac{a\sin(c+dx)}{d}+\frac{5a\log(1-\sin(c+dx))}{4d}-\frac{a\log(\sin(c+dx)+1)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])*Tan[c + d*x]^3,x]
```

```
[Out] (5*a*Log[1 - Sin[c + d*x]])/(4*d) - (a*Log[1 + Sin[c + d*x]])/(4*d) + (a*Si
n[c + d*x])/d + a^2/(2*d*(a - a*Sin[c + d*x]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int (a+a\sin(c+dx))\tan^3(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{(a-x)^2(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{a^2}{2(a-x)^2}-\frac{5a}{4(a-x)}-\frac{a}{4(a+x)}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{5a\log(1-\sin(c+dx))}{4d} - \frac{a\log(1+\sin(c+dx))}{4d} + \frac{a\sin(c+dx)}{d} + \frac{}{2d(a-}$$

Mathematica [**A**] time = 0.12, size = 77, normalized size = 1.08

$$-\frac{a\sin(c+dx)\tan^2(c+dx)}{d}+\frac{a\left(\tan^2(c+dx)+2\log(\cos(c+dx))\right)}{2d}-\frac{3a\left(\tanh^{-1}(\sin(c+dx))-\tan(c+dx)\sec(c\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])*Tan[c + d*x]^3,x]
```

```
[Out] -((a*Sin[c + d*x]*Tan[c + d*x]^2)/d) - (3*a*(ArcTanh[Sin[c + d*x]] - Sec[c
+ d*x]*Tan[c + d*x]))/(2*d) + (a*(2*Log[Cos[c + d*x]] + Tan[c + d*x]^2))/(2
*d)
```

fricas [**A**] time = 0.46, size = 87, normalized size = 1.23

$$-\frac{4\,a\cos(dx+c)^2+(a\sin(dx+c)-a)\log(\sin(dx+c)+1)-5\,(a\sin(dx+c)-a)\log(-\sin(dx+c)+1)+4\,a}{4\,(d\sin(dx+c)-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(4*a*cos(d*x + c)^2 + (a*sin(d*x + c) - a)*log(sin(d*x + c) + 1) - 5*(
a*sin(d*x + c) - a)*log(-sin(d*x + c) + 1) + 4*a*sin(d*x + c) - 2*a)/(d*sin
(d*x + c) - d)
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.15, size = 96, normalized size = 1.35

$$\frac{a\left(\sin^5(dx+c)\right)}{2d\cos(dx+c)^2}+\frac{a\left(\sin^3(dx+c)\right)}{2d}+\frac{3a\sin(dx+c)}{2d}-\frac{3a\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{a\left(\tan^2(dx+c)\right)}{2d}+\frac{a\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))*tan(d*x+c)^3,x)
```

```
[Out] 1/2/d*a*sin(d*x+c)^5/cos(d*x+c)^2+1/2*a*sin(d*x+c)^3/d+3/2*a*sin(d*x+c)/d-3
/2/d*a*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*a*tan(d*x+c)^2+1/d*a*ln(cos(d*x+c))
```

maxima [A] time = 0.31, size = 51, normalized size = 0.72

$$-\frac{a\log(\sin(dx+c)+1)-5a\log(\sin(dx+c)-1)-4a\sin(dx+c)+\frac{2a}{\sin(dx+c)-1}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(a*log(sin(d*x + c) + 1) - 5*a*log(sin(d*x + c) - 1) - 4*a*sin(d*x + c
) + 2*a/(sin(d*x + c) - 1))/d
```

mupad [B] time = 6.47, size = 154, normalized size = 2.17

$$\frac{5a\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)}{2d}-\frac{a\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{2d}+\frac{3a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-4a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+3a\tan\left(\right.}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + a*sin(c + d*x)),x)
```

```
[Out] (5*a*log(tan(c/2 + (d*x)/2) - 1))/(2*d) - (a*log(tan(c/2 + (d*x)/2) + 1))/(
2*d) + (3*a*tan(c/2 + (d*x)/2) - 4*a*tan(c/2 + (d*x)/2)^2 + 3*a*tan(c/2 + (
d*x)/2)^3)/(d*(2*tan(c/2 + (d*x)/2)^2 - 2*tan(c/2 + (d*x)/2) - 2*tan(c/2 +
(d*x)/2)^3 + tan(c/2 + (d*x)/2)^4 + 1)) - (a*log(tan(c/2 + (d*x)/2)^2 + 1))
/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sin(c+dx)\tan^3(c+dx)\,dx+\int\tan^3(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)**3,x)

[Out] a*(Integral(sin(c + d*x)*tan(c + d*x)**3, x) + Integral(tan(c + d*x)**3, x)
)
```

3.3 $\int (a + a\sin(c + dx))\tan(c + dx)\,dx$

Optimal. Leaf size=30

$$-\frac{a\sin(c+dx)}{d} - \frac{a\log(1-\sin(c+dx))}{d}$$

[Out] -a*ln(1-sin(d*x+c))/d-a*sin(d*x+c)/d

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2707, 43}

$$-\frac{a\sin(c+dx)}{d} - \frac{a\log(1-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[c + d*x])*Tan[c + d*x],x]

[Out] -((a*Log[1 - Sin[c + d*x]])/d) - (a*Sin[c + d*x])/d

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int (a+a\sin(c+dx))\tan(c+dx)\,dx &= \frac{\text{Subst}\left(\int \frac{x}{a-x}\,dx, x, a\sin(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \left(-1+\frac{a}{a-x}\right)dx, x, a\sin(c+dx)\right)}{d} \\
&= -\frac{a\log(1-\sin(c+dx))}{d} - \frac{a\sin(c+dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 1.27

$$-\frac{a\sin(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])*Tan[c + d*x],x]
[Out] (a*ArcTanh[Sin[c + d*x]])/d - (a*Log[Cos[c + d*x]])/d - (a*Sin[c + d*x])/d
```

fricas [A] time = 0.42, size = 27, normalized size = 0.90

$$-\frac{a\log\left(-\sin\left(dx+c\right)+1\right)+a\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c),x, algorithm="fricas")
[Out] -(a*log(-sin(d*x + c) + 1) + a*sin(d*x + c))/d
```

giac [B] time = 1.18, size = 1456, normalized size = 48.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c),x, algorithm="giac")
[Out] -1/2*(a*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) +
2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c
)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + t
an(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/
2*d*x)^2*tan(1/2*c)^2 - a*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*
x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/
2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*
tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2
*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + a*log(4*(tan(d*x)^4*tan(c)^2 - 2*
tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) +
1)/(tan(c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + a*log(2*(tan(1/2*d*x)^4*ta
n(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 +
tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1
/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2
*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2 - a*log(2*(tan(1/2*d*x)
^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)
^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*
```

```
tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x
) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2 + a*log(4*(tan(d*x
)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*t
an(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(1/2*d*x)^2 - 4*a*tan(1/2*d*x)^2*tan
(1/2*c) + a*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c
) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1
/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2
 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*ta
n(1/2*c)^2 - a*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/
2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*ta
n(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x
)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))
*tan(1/2*c)^2 + a*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*
x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(1/2
*c)^2 - 4*a*tan(1/2*d*x)*tan(1/2*c)^2 + a*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^
2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d
*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*t
an(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*
c) + 1)/(tan(1/2*c)^2 + 1)) - a*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(
1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*
tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^
2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(t
an(1/2*c)^2 + 1)) + a*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + ta
n(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) + 4
*a*tan(1/2*d*x) + 4*a*tan(1/2*c))/(d*tan(1/2*d*x)^2*tan(1/2*c)^2 + d*tan(1/
2*d*x)^2 + d*tan(1/2*c)^2 + d)
```

maple [A] time = 0.10, size = 29, normalized size = 0.97

$$-\frac{a\sin\left(dx+c\right)}{d}-\frac{a\ln\left(\sin\left(dx+c\right)-1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))*tan(d*x+c),x)
```

```
[Out] -a*sin(d*x+c)/d-a/d*ln(sin(d*x+c)-1)
```

maxima [A] time = 0.30, size = 25, normalized size = 0.83

$$-\frac{a\log\left(\sin\left(dx+c\right)-1\right)+a\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] -(a*log(sin(d*x + c) - 1) + a*sin(d*x + c))/d
```

mupad [B] time = 6.62, size = 43, normalized size = 1.43

$$-\frac{a\left(\sin\left(c+d\,x\right)+2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)-\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + a*sin(c + d*x)),x)
[Out] -(a*(sin(c + d*x) + 2*log(tan(c/2 + (d*x)/2) - 1) - log(tan(c/2 + (d*x)/2)^
2 + 1)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sin\left(c+dx\right)\tan\left(c+dx\right)dx+\int\tan\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c),x)
[Out] a*(Integral(sin(c + d*x)*tan(c + d*x), x) + Integral(tan(c + d*x), x))
```

3.4 $\int \cot(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=24

$$\frac{a\sin(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

```
[Out] a*ln(sin(d*x+c))/d+a*sin(d*x+c)/d
```

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2707, 43}

$$\frac{a\sin(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Log[Sin[c + d*x]])/d + (a*Sin[c + d*x])/d
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)(a+a\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int \frac{a+x}{x}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(1+\frac{a}{x}\right)\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{a\log(\sin(c+dx))}{d}+\frac{a\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.03, size = 26, normalized size = 1.08

$$\frac{a(\sin(c+dx)+\log(\tan(c+dx))+\log(\cos(c+dx)))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
[Out] (a*(Log[Cos[c + d*x]] + Log[Tan[c + d*x]] + Sin[c + d*x]))/d
```

fricas [A] time = 0.42, size = 24, normalized size = 1.00

$$\frac{a\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)+a\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] (a*log(1/2*sin(d*x + c)) + a*sin(d*x + c))/d
```

giac [A] time = 0.22, size = 23, normalized size = 0.96

$$\frac{a\log\left(|\sin\left(dx+c\right)|\right)+a\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] (a*log(abs(sin(d*x + c))) + a*sin(d*x + c))/d
```

maple [A] time = 0.09, size = 25, normalized size = 1.04

$$\frac{a\ln\left(\sin\left(dx+c\right)\right)}{d}+\frac{a\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+a*sin(d*x+c)),x)
[Out] a*ln(sin(d*x+c))/d+a*sin(d*x+c)/d
```

maxima [A] time = 0.36, size = 22, normalized size = 0.92

$$\frac{a\log\left(\sin\left(dx+c\right)\right)+a\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (a*log(sin(d*x + c)) + a*sin(d*x + c))/d
```

mupad [B] time = 6.58, size = 38, normalized size = 1.58

$$\frac{a\left(\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)+\sin\left(c+d\,x\right)-\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + a*sin(c + d*x)),x)
```

```
[Out] (a*(log(tan(c/2 + (d*x)/2)) + sin(c + d*x) - log(tan(c/2 + (d*x)/2)^2 + 1))
)/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \sin\left(c+dx\right)\cot\left(c+dx\right)dx+\int \cot\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(sin(c + d*x)*cot(c + d*x), x) + Integral(cot(c + d*x), x))
```

3.5 $\int \cot^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=54

$$-\frac{a\sin(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{a\csc(c+dx)}{d}-\frac{a\log(\sin(c+dx))}{d}$$

```
[Out] -a*csc(d*x+c)/d-1/2*a*csc(d*x+c)^2/d-a*ln(sin(d*x+c))/d-a*sin(d*x+c)/d
```

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2707, 75}

$$-\frac{a\sin(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{a\csc(c+dx)}{d}-\frac{a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) - (a*Csc[c + d*x]^2)/(2*d) - (a*Log[Sin[c + d*x]])/d
- (a*Sin[c + d*x])/d
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \cot^3(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)(a+x)^2}{x^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-1+\frac{a^3}{x^3}+\frac{a^2}{x^2}-\frac{a}{x}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{a\csc(c+dx)}{d} - \frac{a\csc^2(c+dx)}{2d} - \frac{a\log(\sin(c+dx))}{d} - \frac{a\sin(c+dx)}{d}$$

Mathematica [A] time = 0.12, size = 60, normalized size = 1.11

$$-\frac{a\sin(c+dx)}{d} - \frac{a\csc(c+dx)}{d} - \frac{a\left(\cot^2(c+dx)+2\log(\tan(c+dx))+2\log(\cos(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) - (a*(Cot[c + d*x]^2 + 2*Log[Cos[c + d*x]] + 2*Log[Ta
n[c + d*x]]))/(2*d) - (a*Sin[c + d*x])/d
```

fricas [A] time = 0.44, size = 69, normalized size = 1.28

$$-\frac{2\left(a\cos(dx+c)^2-a\right)\log\left(\frac{1}{2}\sin(dx+c)\right)+2\left(a\cos(dx+c)^2-2a\right)\sin(dx+c)-a}{2\left(d\cos(dx+c)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a*cos(d*x + c)^2 - a)*log(1/2*sin(d*x + c)) + 2*(a*cos(d*x + c)^2
- 2*a)*sin(d*x + c) - a)/(d*cos(d*x + c)^2 - d)
```

giac [A] time = 0.41, size = 60, normalized size = 1.11

$$-\frac{2a\log(|\sin(dx+c)|)+2a\sin(dx+c)-\frac{3a\sin(dx+c)^2-2a\sin(dx+c)-a}{\sin(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*a*log(abs(sin(d*x + c))) + 2*a*sin(d*x + c) - (3*a*sin(d*x + c)^2 -
 2*a*sin(d*x + c) - a)/sin(d*x + c)^2)/d
```

maple [A] time = 0.20, size = 83, normalized size = 1.54

$$-\frac{a\left(\cos^{4}\left(dx+c\right)\right)}{d\sin\left(dx+c\right)}-\frac{\left(\cos^{2}\left(dx+c\right)\right)\sin\left(dx+c\right)a}{d}-\frac{2a\sin\left(dx+c\right)}{d}-\frac{a\left(\cot^{2}\left(dx+c\right)\right)}{2d}-\frac{a\ln\left(\sin\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/d*a/sin(d*x+c)*cos(d*x+c)^4-1/d*cos(d*x+c)^2*sin(d*x+c)*a-2*a*sin(d*x+c)
/d-1/2/d*a*cot(d*x+c)^2-a*ln(sin(d*x+c))/d
```

maxima [A] time = 0.30, size = 45, normalized size = 0.83

$$-\frac{2\,a\log\left(\sin\left(dx+c\right)\right)+2\,a\sin\left(dx+c\right)+\frac{2\,a\sin\left(dx+c\right)+a}{\sin\left(dx+c\right)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*log(sin(d*x + c)) + 2*a*sin(d*x + c) + (2*a*sin(d*x + c) + a)/sin
(d*x + c)^2)/d
```

mupad [B] time = 6.54, size = 146, normalized size = 2.70

$$\frac{a\,\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{10\,a\tan{\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+\frac{a\tan{\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{2}+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a}{2}}{d\left(4\tan{\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+4\tan{\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\right)}-\frac{a\tan\left(\frac{c}{2}+\frac{d}{}\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + a*sin(c + d*x)),x)
```

```
[Out] (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (a*tan(c/2 + (d*x)/2))/(2*d) - (a/2 +
 2*a*tan(c/2 + (d*x)/2) + (a*tan(c/2 + (d*x)/2)^2)/2 + 10*a*tan(c/2 + (d*x)
/2)^3)/(d*(4*tan(c/2 + (d*x)/2)^2 + 4*tan(c/2 + (d*x)/2)^4)) - (a*tan(c/2 +
 (d*x)/2)^2)/(8*d) - (a*log(tan(c/2 + (d*x)/2)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sin\left(c+dx\right)\cot^{3}\left(c+dx\right)\,dx+\int\cot^{3}\left(c+dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(sin(c + d*x)*cot(c + d*x)**3, x) + Integral(cot(c + d*x)**3, x)
)
```

3.6 $\int \cot^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$\frac{a\sin(c+dx)}{d}-\frac{a\csc^4(c+dx)}{4d}-\frac{a\csc^3(c+dx)}{3d}+\frac{a\csc^2(c+dx)}{d}+\frac{2a\csc(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

```
[Out] 2*a*csc(d*x+c)/d+a*csc(d*x+c)^2/d-1/3*a*csc(d*x+c)^3/d-1/4*a*csc(d*x+c)^4/d
+a*ln(sin(d*x+c))/d+a*sin(d*x+c)/d
```

Rubi [A] time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2707, 88}

$$\frac{a\sin(c+dx)}{d}-\frac{a\csc^4(c+dx)}{4d}-\frac{a\csc^3(c+dx)}{3d}+\frac{a\csc^2(c+dx)}{d}+\frac{2a\csc(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*a*Csc[c + d*x])/d + (a*Csc[c + d*x]^2)/d - (a*Csc[c + d*x]^3)/(3*d) - (a
*Csc[c + d*x]^4)/(4*d) + (a*Log[Sin[c + d*x]])/d + (a*Sin[c + d*x])/d
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \cot^5(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^3}{x^5}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{a^5}{x^5}+\frac{a^4}{x^4}-\frac{2a^3}{x^3}-\frac{2a^2}{x^2}+\frac{a}{x}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{2a\csc(c+dx)}{d}+\frac{a\csc^2(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc^4(c+dx)}{4d}+\frac{a\,\mathrm{lc}}{}$$

Mathematica [A] time = 0.20, size = 87, normalized size = 1.07

$$\frac{a\sin(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}+\frac{2a\csc(c+dx)}{d}+\frac{a\left(-\cot^4(c+dx)+2\cot^2(c+dx)+4\log(\tan(c+dx))+4\log(c\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/(3*d) + (a*(2*Cot[c + d*x]^2 - Co
t[c + d*x]^4 + 4*Log[Cos[c + d*x]] + 4*Log[Tan[c + d*x]]))/(4*d) + (a*Sin[c
 + d*x])/d
```

fricas [A] time = 0.45, size = 110, normalized size = 1.36

$$-\frac{12\,a\cos(dx+c)^2-12\left(a\cos(dx+c)^4-2\,a\cos(dx+c)^2+a\right)\log\left(\frac{1}{2}\sin(dx+c)\right)-4\left(3\,a\cos(dx+c)^4-12\,a\right.}{12\left(d\cos(dx+c)^4-2\,d\cos(dx+c)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(12*a*cos(d*x + c)^2 - 12*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)
*log(1/2*sin(d*x + c)) - 4*(3*a*cos(d*x + c)^4 - 12*a*cos(d*x + c)^2 + 8*a)
*sin(d*x + c) - 9*a)/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

giac [A] time = 0.55, size = 82, normalized size = 1.01

$$\frac{12\,a\log\left(|\sin(dx+c)|\right)+12\,a\sin(dx+c)-\frac{25\,a\sin(dx+c)^4-24\,a\sin(dx+c)^3-12\,a\sin(dx+c)^2+4\,a\sin(dx+c)+3\,a}{\sin(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(12*a*log(abs(sin(d*x + c))) + 12*a*sin(d*x + c) - (25*a*sin(d*x + c)^
4 - 24*a*sin(d*x + c)^3 - 12*a*sin(d*x + c)^2 + 4*a*sin(d*x + c) + 3*a)/sin
(d*x + c)^4)/d
```

maple [A] time = 0.17, size = 136, normalized size = 1.68

$$-\frac{a\left(\cos^{6}(dx+c)\right)}{3d\sin(dx+c)^{3}}+\frac{a\left(\cos^{6}(dx+c)\right)}{d\sin(dx+c)}+\frac{8a\sin(dx+c)}{3d}+\frac{\left(\cos^{4}(dx+c)\right)\sin(dx+c)a}{d}+\frac{4\left(\cos^{2}(dx+c)\right)\sin(dx+}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/3/d*a/sin(d*x+c)^3*cos(d*x+c)^6+1/d*a/sin(d*x+c)*cos(d*x+c)^6+8/3*a*sin(
d*x+c)/d+1/d*cos(d*x+c)^4*sin(d*x+c)*a+4/3/d*cos(d*x+c)^2*sin(d*x+c)*a-1/4/
d*a*cot(d*x+c)^4+1/2/d*a*cot(d*x+c)^2+a*ln(sin(d*x+c))/d
```

maxima [A] time = 0.65, size = 69, normalized size = 0.85

$$\frac{12\,a\log\left(\sin\left(dx+c\right)\right)+12\,a\sin\left(dx+c\right)+\frac{24\,a\sin(dx+c)^{3}+12\,a\sin(dx+c)^{2}-4\,a\sin(dx+c)-3\,a}{\sin(dx+c)^{4}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(12*a*log(sin(d*x + c)) + 12*a*sin(d*x + c) + (24*a*sin(d*x + c)^3 + 1
2*a*sin(d*x + c)^2 - 4*a*sin(d*x + c) - 3*a)/sin(d*x + c)^4)/d
```

mupad [B] time = 6.70, size = 207, normalized size = 2.56

$$\frac{7\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}+\frac{46\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}+3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+\frac{40\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{3}+\frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{4}-\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}-\frac{a}{4}}{d\left(16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}\right)}-\frac{a\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5*(a + a*sin(c + d*x)),x)
```

```
[Out] (7*a*tan(c/2 + (d*x)/2))/(8*d) + ((11*a*tan(c/2 + (d*x)/2)^2)/4 - (2*a*tan(
c/2 + (d*x)/2))/3 - a/4 + (40*a*tan(c/2 + (d*x)/2)^3)/3 + 3*a*tan(c/2 + (d*
x)/2)^4 + 46*a*tan(c/2 + (d*x)/2)^5)/(d*(16*tan(c/2 + (d*x)/2)^4 + 16*tan(c
/2 + (d*x)/2)^6)) - (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d + (3*a*tan(c/2 + (d
```

```
*x)/2)^2)/(16*d) - (a*tan(c/2 + (d*x)/2)^3)/(24*d) - (a*tan(c/2 + (d*x)/2)^
4)/(64*d) + (a*log(tan(c/2 + (d*x)/2)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \sin\left(c+dx\right)\cot^{5}\left(c+dx\right)dx+\int \cot^{5}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(sin(c + d*x)*cot(c + d*x)**5, x) + Integral(cot(c + d*x)**5, x)
)
```

3.7 $\int \cot^7(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=115

$$-\frac{a\sin(c+dx)}{d}-\frac{a\csc^6(c+dx)}{6d}-\frac{a\csc^5(c+dx)}{5d}+\frac{3a\csc^4(c+dx)}{4d}+\frac{a\csc^3(c+dx)}{d}-\frac{3a\csc^2(c+dx)}{2d}-\frac{3a\csc(c+dx)}{d}$$

```
[Out] -3*a*csc(d*x+c)/d-3/2*a*csc(d*x+c)^2/d+a*csc(d*x+c)^3/d+3/4*a*csc(d*x+c)^4/
d-1/5*a*csc(d*x+c)^5/d-1/6*a*csc(d*x+c)^6/d-a*ln(sin(d*x+c))/d-a*sin(d*x+c)
/d
```

Rubi [A] time = 0.06, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2707, 88}

$$-\frac{a\sin(c+dx)}{d}-\frac{a\csc^6(c+dx)}{6d}-\frac{a\csc^5(c+dx)}{5d}+\frac{3a\csc^4(c+dx)}{4d}+\frac{a\csc^3(c+dx)}{d}-\frac{3a\csc^2(c+dx)}{2d}-\frac{3a\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*a*Csc[c + d*x])/d - (3*a*Csc[c + d*x]^2)/(2*d) + (a*Csc[c + d*x]^3)/d +
 (3*a*Csc[c + d*x]^4)/(4*d) - (a*Csc[c + d*x]^5)/(5*d) - (a*Csc[c + d*x]^6)
/(6*d) - (a*Log[Sin[c + d*x]])/d - (a*Sin[c + d*x])/d
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \cot^7(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)^4}{x^7}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-1+\frac{a^7}{x^7}+\frac{a^6}{x^6}-\frac{3a^5}{x^5}-\frac{3a^4}{x^4}+\frac{3a^3}{x^3}+\frac{3a^2}{x^2}-\frac{a}{x}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{3a\csc(c+dx)}{d}-\frac{3a\csc^2(c+dx)}{2d}+\frac{a\csc^3(c+dx)}{d}+\frac{3a\csc^4(c+dx)}{4d}-$$

Mathematica [A] time = 0.39, size = 111, normalized size = 0.97

$$-\frac{a\sin(c+dx)}{d}-\frac{a\csc^5(c+dx)}{5d}+\frac{a\csc^3(c+dx)}{d}-\frac{3a\csc(c+dx)}{d}-\frac{a\left(2\cot^6(c+dx)-3\cot^4(c+dx)+6\cot^2(c+\right.}{1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*a*Csc[c + d*x])/d + (a*Csc[c + d*x]^3)/d - (a*Csc[c + d*x]^5)/(5*d) - (
a*(6*Cot[c + d*x]^2 - 3*Cot[c + d*x]^4 + 2*Cot[c + d*x]^6 + 12*Log[Cos[c +
d*x]] + 12*Log[Tan[c + d*x]]))/(12*d) - (a*Sin[c + d*x])/d
```

fricas [A] time = 0.47, size = 158, normalized size = 1.37

$$\frac{90\,a\cos(dx+c)^4-135\,a\cos(dx+c)^2-60\left(a\cos(dx+c)^6-3\,a\cos(dx+c)^4+3\,a\cos(dx+c)^2-a\right)\log\left(\frac{1}{2}\,\text{s}\right.}{60\left(d\cos(dx+c)^6-3\,d\cos(dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/60*(90*a*cos(d*x + c)^4 - 135*a*cos(d*x + c)^2 - 60*(a*cos(d*x + c)^6 - 3
*a*cos(d*x + c)^4 + 3*a*cos(d*x + c)^2 - a)*log(1/2*sin(d*x + c)) - 12*(5*a
*cos(d*x + c)^6 - 30*a*cos(d*x + c)^4 + 40*a*cos(d*x + c)^2 - 16*a)*sin(d*x
 + c) + 55*a)/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 -
 d)
```

giac [A] time = 1.46, size = 104, normalized size = 0.90

$$-\frac{60\,a\log(|\sin(dx+c)|)+60\,a\sin(dx+c)-\frac{147\,a\sin(dx+c)^6-180\,a\sin(dx+c)^5-90\,a\sin(dx+c)^4+60\,a\sin(dx+c)^3+45\,a\sin(dx+c)^2-}{\sin(dx+c)^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/60*(60*a*log(abs(sin(d*x + c))) + 60*a*sin(d*x + c) - (147*a*sin(d*x + c
)^6 - 180*a*sin(d*x + c)^5 - 90*a*sin(d*x + c)^4 + 60*a*sin(d*x + c)^3 + 45
*a*sin(d*x + c)^2 - 12*a*sin(d*x + c) - 10*a)/sin(d*x + c)^6)/d
```

maple [A] time = 0.19, size = 195, normalized size = 1.70

$$-\frac{a\left(\cos^{8}(dx+c)\right)}{5d\sin(dx+c)^{5}}+\frac{a\left(\cos^{8}(dx+c)\right)}{5d\sin(dx+c)^{3}}-\frac{a\left(\cos^{8}(dx+c)\right)}{d\sin(dx+c)}-\frac{16a\sin(dx+c)}{5d}-\frac{\left(\cos^{6}(dx+c)\right)\sin(dx+c)\,a}{d}-\frac{6\left(\cos^{4}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^7*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/5/d*a/sin(d*x+c)^5*cos(d*x+c)^8+1/5/d*a/sin(d*x+c)^3*cos(d*x+c)^8-1/d*a/
sin(d*x+c)*cos(d*x+c)^8-16/5*a*sin(d*x+c)/d-1/d*cos(d*x+c)^6*sin(d*x+c)*a-6
/5/d*cos(d*x+c)^4*sin(d*x+c)*a-8/5/d*cos(d*x+c)^2*sin(d*x+c)*a-1/6/d*a*cot(
d*x+c)^6+1/4/d*a*cot(d*x+c)^4-1/2/d*a*cot(d*x+c)^2-a*ln(sin(d*x+c))/d
```

maxima [A] time = 0.30, size = 91, normalized size = 0.79

$$-\frac{60\,a\log\left(\sin\left(dx+c\right)\right)+60\,a\sin\left(dx+c\right)+\frac{180\,a\sin(dx+c)^{5}+90\,a\sin(dx+c)^{4}-60\,a\sin(dx+c)^{3}-45\,a\sin(dx+c)^{2}+12\,a\sin(dx+c)+10\,a}{\sin(dx+c)^{6}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*(60*a*log(sin(d*x + c)) + 60*a*sin(d*x + c) + (180*a*sin(d*x + c)^5 +
 90*a*sin(d*x + c)^4 - 60*a*sin(d*x + c)^3 - 45*a*sin(d*x + c)^2 + 12*a*sin
(d*x + c) + 10*a)/sin(d*x + c)^6)/d
```

mupad [B] time = 7.37, size = 267, normalized size = 2.32

$$\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{32\,d}-\frac{29\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{128\,d}-\frac{19\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}}{32\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{160\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{384\,d}-\frac{a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^7*(a + a*sin(c + d*x)),x)
```

```
[Out] (3*a*tan(c/2 + (d*x)/2)^3)/(32*d) - (29*a*tan(c/2 + (d*x)/2)^2)/(128*d) - (
19*a*tan(c/2 + (d*x)/2))/(16*d) + (a*tan(c/2 + (d*x)/2)^4)/(32*d) - (a*tan(
c/2 + (d*x)/2)^5)/(160*d) - (a*tan(c/2 + (d*x)/2)^6)/(384*d) - (a*(1920*log
(tan(c/2 + (d*x)/2)) - 1920*log(tan(c/2 + (d*x)/2)^2 + 1)))/(1920*d) - (cot
(c/2 + (d*x)/2)^6*(a/384 + (a*tan(c/2 + (d*x)/2))/160 - (11*a*tan(c/2 + (d*
x)/2)^2)/384 - (7*a*tan(c/2 + (d*x)/2)^3)/80 + (25*a*tan(c/2 + (d*x)/2)^4)/
128 + (35*a*tan(c/2 + (d*x)/2)^5)/32 + (29*a*tan(c/2 + (d*x)/2)^6)/128 + (5
1*a*tan(c/2 + (d*x)/2)^7)/16))/(d*(tan(c/2 + (d*x)/2)^2 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \sin\left(c+dx\right)\cot^{7}\left(c+dx\right)dx+\int \cot^{7}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**7*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(sin(c + d*x)*cot(c + d*x)**7, x) + Integral(cot(c + d*x)**7, x)
)
```

3.8 $\int (a + a\sin(c + dx))\tan^6(c + dx)\,dx$

Optimal. Leaf size=101

$$\frac{a\cos(c+dx)}{d}+\frac{a\tan^5(c+dx)}{5d}-\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{a\sec^5(c+dx)}{5d}-\frac{a\sec^3(c+dx)}{d}+\frac{3a\sec(c+dx)}{d}-ax$$

```
[Out] -a*x+a*cos(d*x+c)/d+3*a*sec(d*x+c)/d-a*sec(d*x+c)^3/d+1/5*a*sec(d*x+c)^5/d+
a*tan(d*x+c)/d-1/3*a*tan(d*x+c)^3/d+1/5*a*tan(d*x+c)^5/d
```

Rubi [A] time = 0.09, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2710, 3473, 8, 2590, 270}

$$\frac{a\cos(c+dx)}{d}+\frac{a\tan^5(c+dx)}{5d}-\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{a\sec^5(c+dx)}{5d}-\frac{a\sec^3(c+dx)}{d}+\frac{3a\sec(c+dx)}{d}-ax$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])*Tan[c + d*x]^6,x]
```

```
[Out] -(a*x) + (a*Cos[c + d*x])/d + (3*a*Sec[c + d*x])/d - (a*Sec[c + d*x]^3)/d +
 (a*Sec[c + d*x]^5)/(5*d) + (a*Tan[c + d*x])/d - (a*Tan[c + d*x]^3)/(3*d) +
 (a*Tan[c + d*x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2710

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
```

```
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int (a+a\sin(c+dx))\tan^6(c+dx)\,dx = \int \left(a\tan^6(c+dx)+a\sin(c+dx)\tan^6(c+dx)\right)\,dx$$
$$= a\int \tan^6(c+dx)\,dx + a\int \sin(c+dx)\tan^6(c+dx)\,dx$$
$$= \frac{a\tan^5(c+dx)}{5d} - a\int \tan^4(c+dx)\,dx - \frac{a\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{x^6}\,dx, x, \cos(c+\right.}{d}$$
$$= -\frac{a\tan^3(c+dx)}{3d} + \frac{a\tan^5(c+dx)}{5d} + a\int \tan^2(c+dx)\,dx - \frac{a\,\text{Subst}\left(\int \left(\right.\right.}{}$$
$$= \frac{a\cos(c+dx)}{d} + \frac{3a\sec(c+dx)}{d} - \frac{a\sec^3(c+dx)}{d} + \frac{a\sec^5(c+dx)}{5d} + \frac{a\,\text{ta}}{}$$
$$= -ax + \frac{a\cos(c+dx)}{d} + \frac{3a\sec(c+dx)}{d} - \frac{a\sec^3(c+dx)}{d} + \frac{a\sec^5(c+dx)}{5d}$$

Mathematica [A] time = 0.06, size = 110, normalized size = 1.09

$$\frac{a\cos(c+dx)}{d} - \frac{a\tan^{-1}(\tan(c+dx))}{d} + \frac{a\tan^5(c+dx)}{5d} - \frac{a\tan^3(c+dx)}{3d} + \frac{a\tan(c+dx)}{d} + \frac{a\sec^5(c+dx)}{5d} - \frac{a\sec^3(c+}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])*Tan[c + d*x]^6,x]
```

```
[Out] -((a*ArcTan[Tan[c + d*x]])/d) + (a*Cos[c + d*x])/d + (3*a*Sec[c + d*x])/d -
 (a*Sec[c + d*x]^3)/d + (a*Sec[c + d*x]^5)/(5*d) + (a*Tan[c + d*x])/d - (a*
Tan[c + d*x]^3)/(3*d) + (a*Tan[c + d*x]^5)/(5*d)
```

fricas [A] time = 0.42, size = 116, normalized size = 1.15

$$\frac{15\,adx\cos(dx+c)^3 - 38\,a\cos(dx+c)^4 - 11\,a\cos(dx+c)^2 - \left(15\,adx\cos(dx+c)^3 - 15\,a\cos(dx+c)^4 - 22\,a\,\text{c}\right.}{15\left(d\cos(dx+c)^3\sin(dx+c) - d\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] 1/15*(15*a*d*x*cos(d*x + c)^3 - 38*a*cos(d*x + c)^4 - 11*a*cos(d*x + c)^2 -
 (15*a*d*x*cos(d*x + c)^3 - 15*a*cos(d*x + c)^4 - 22*a*cos(d*x + c)^2 + 4*a
)*sin(d*x + c) + a)/(d*cos(d*x + c)^3*sin(d*x + c) - d*cos(d*x + c)^3)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^6,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.24, size = 135, normalized size = 1.34

$$\frac{a\left(\frac{\sin^8(dx+c)}{5\cos(dx+c)^5}-\frac{\sin^8(dx+c)}{5\cos(dx+c)^3}+\frac{\sin^8(dx+c)}{\cos(dx+c)}+\left(\frac{16}{5}+\sin^6(dx+c)+\frac{6\left(\sin^4(dx+c)\right)}{5}+\frac{8\left(\sin^2(dx+c)\right)}{5}\right)\cos(dx+c)\right)+a\left(\frac{\left(\tan^5(dx\right.}{5}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))*tan(d*x+c)^6,x)
```

```
[Out] 1/d*(a*(1/5*sin(d*x+c)^8/cos(d*x+c)^5-1/5*sin(d*x+c)^8/cos(d*x+c)^3+sin(d*x
+c)^8/cos(d*x+c)+(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(
d*x+c))+a*(1/5*tan(d*x+c)^5-1/3*tan(d*x+c)^3+tan(d*x+c)-d*x-c))
```

maxima [A] time = 0.41, size = 87, normalized size = 0.86

$$\frac{\left(3\tan(dx+c)^5-5\tan(dx+c)^3-15\,dx-15\,c+15\tan(dx+c)\right)a+3\,a\left(\frac{15\cos(dx+c)^4-5\cos(dx+c)^2+1}{\cos(dx+c)^5}+5\cos(dx+\right.}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/15*((3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(d*x + c
))*a + 3*a*((15*cos(d*x + c)^4 - 5*cos(d*x + c)^2 + 1)/cos(d*x + c)^5 + 5*c
os(d*x + c)))/d
```

mupad [B] time = 11.22, size = 351, normalized size = 3.48

$$\left(\frac{a\,\left(30\,c+30\,d\,x-30\right)}{15}-2\,a\,\left(c+d\,x\right)\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(\frac{a\,\left(15\,c+15\,d\,x+60\right)}{15}-a\,\left(c+d\,x\right)\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\left(4\,a\,\left(c+d\,x\right)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^6*(a + a*sin(c + d*x)),x)
```

```
[Out] ((a*(15*c + 15*d*x - 96))/15 - tan(c/2 + (d*x)/2)*((a*(30*c + 30*d*x - 162)
)/15 - 2*a*(c + d*x)) - a*(c + d*x) + tan(c/2 + (d*x)/2)^8*((a*(15*c + 15*d
*x + 60))/15 - a*(c + d*x)) + tan(c/2 + (d*x)/2)^9*((a*(30*c + 30*d*x - 30)
)/15 - 2*a*(c + d*x)) - tan(c/2 + (d*x)/2)^4*((a*(30*c + 30*d*x - 52))/15 -
 2*a*(c + d*x)) - tan(c/2 + (d*x)/2)^7*((a*(60*c + 60*d*x - 40))/15 - 4*a*(
c + d*x)) - tan(c/2 + (d*x)/2)^2*((a*(15*c + 15*d*x - 156))/15 - a*(c + d*x
)) + tan(c/2 + (d*x)/2)^6*((a*(30*c + 30*d*x - 140))/15 - 2*a*(c + d*x)) +
tan(c/2 + (d*x)/2)^3*((a*(60*c + 60*d*x - 344))/15 - 4*a*(c + d*x)) + (44*a
*tan(c/2 + (d*x)/2)^5)/15)/(d*(tan(c/2 + (d*x)/2) - 1)^5*(tan(c/2 + (d*x)/2
) + 1)^3*(tan(c/2 + (d*x)/2)^2 + 1)) - a*x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sin\left(c+dx\right)\tan^6\left(c+dx\right)dx+\int\tan^6\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)**6,x)
```

```
[Out] a*(Integral(sin(c + d*x)*tan(c + d*x)**6, x) + Integral(tan(c + d*x)**6, x)
)
```

3.9 $\int (a + a\sin(c + dx))\tan^4(c + dx)\,dx$

Optimal. Leaf size=72

$$-\frac{a\cos(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}-\frac{a\tan(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}-\frac{2a\sec(c+dx)}{d}+ax$$

```
[Out] a*x-a*cos(d*x+c)/d-2*a*sec(d*x+c)/d+1/3*a*sec(d*x+c)^3/d-a*tan(d*x+c)/d+1/3
*a*tan(d*x+c)^3/d
```

Rubi [A] time = 0.07, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2710, 3473, 8, 2590, 270}

$$-\frac{a\cos(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}-\frac{a\tan(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}-\frac{2a\sec(c+dx)}{d}+ax$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] a*x - (a*Cos[c + d*x])/d - (2*a*Sec[c + d*x])/d + (a*Sec[c + d*x]^3)/(3*d)
- (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2710

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
```

```
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int (a+a\sin(c+dx))\tan^4(c+dx)\,dx &= \int \left(a\tan^4(c+dx)+a\sin(c+dx)\tan^4(c+dx)\right) dx\\
&= a\int \tan^4(c+dx)\,dx + a\int \sin(c+dx)\tan^4(c+dx)\,dx\\
&= \frac{a\tan^3(c+dx)}{3d} - a\int \tan^2(c+dx)\,dx - \frac{a\,\mathrm{Subst}\left(\int \frac{(1-x^2)^2}{x^4}\,dx, x, \cos(c+\right.}{d}\\
&= -\frac{a\tan(c+dx)}{d} + \frac{a\tan^3(c+dx)}{3d} + a\int 1\,dx - \frac{a\,\mathrm{Subst}\left(\int \left(1+\frac{1}{x^4}-\frac{2}{x^2}\right)\right.}{d}\\
&= ax - \frac{a\cos(c+dx)}{d} - \frac{2a\sec(c+dx)}{d} + \frac{a\sec^3(c+dx)}{3d} - \frac{a\tan(c+dx)}{d} +
\end{aligned}$$

Mathematica [A] time = 0.05, size = 81, normalized size = 1.12

$$-\frac{a\cos(c+dx)}{d}+\frac{a\tan^{-1}(\tan(c+dx))}{d}+\frac{a\tan^3(c+dx)}{3d}-\frac{a\tan(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}-\frac{2a\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] (a*ArcTan[Tan[c + d*x]])/d - (a*Cos[c + d*x])/d - (2*a*Sec[c + d*x])/d + (a
*Sec[c + d*x]^3)/(3*d) - (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

fricas [A] time = 0.45, size = 88, normalized size = 1.22

$$-\frac{3\,adx\cos(dx+c)-7\,a\cos(dx+c)^2-\left(3\,adx\cos(dx+c)-3\,a\cos(dx+c)^2-2\,a\right)\sin(dx+c)-a}{3\,(d\cos(dx+c)\sin(dx+c)-d\cos(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*a*d*x*cos(d*x + c) - 7*a*cos(d*x + c)^2 - (3*a*d*x*cos(d*x + c) - 3
*a*cos(d*x + c)^2 - 2*a)*sin(d*x + c) - a)/(d*cos(d*x + c)*sin(d*x + c) - d
*cos(d*x + c))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.21, size = 98, normalized size = 1.36

$$\frac{a\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^6(dx+c)}{\cos(dx+c)}-\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)\right)+a\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+dx+c\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))*tan(d*x+c)^4,x)
```

```
[Out] 1/d*(a*(1/3*sin(d*x+c)^6/cos(d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+
c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+a*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c))
```

maxima [A] time = 0.42, size = 65, normalized size = 0.90

$$\frac{\left(\tan(dx+c)^3+3\,dx+3\,c-3\,\tan(dx+c)\right)a-a\left(\frac{6\,\cos(dx+c)^2-1}{\cos(dx+c)^3}+3\,\cos(dx+c)\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/3*((tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a - a*((6*cos(d*x + c)
^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x + c)))/d
```

mupad [B] time = 9.42, size = 231, normalized size = 3.21

$$a\,x+\frac{\left(\frac{2\,a\,(3\,c+3\,d\,x)}{3}-\frac{a\,(6\,c+6\,d\,x-6)}{3}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(\frac{a\,(3\,c+3\,d\,x-12)}{3}-\frac{a\,(3\,c+3\,d\,x)}{3}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+\left(\frac{a\,(3}{}\right.}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)^3\left(\tan\left(\frac{c}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + a*sin(c + d*x)),x)
```

```
[Out] a*x + ((a*(3*c + 3*d*x))/3 - (a*(3*c + 3*d*x - 16))/3 + tan(c/2 + (d*x)/2)^
2*((a*(3*c + 3*d*x))/3 - (a*(3*c + 3*d*x - 4))/3) - tan(c/2 + (d*x)/2)^4*((
a*(3*c + 3*d*x))/3 - (a*(3*c + 3*d*x - 12))/3) + tan(c/2 + (d*x)/2)^5*((2*a
*(3*c + 3*d*x))/3 - (a*(6*c + 6*d*x - 6))/3) - tan(c/2 + (d*x)/2)*((2*a*(3*
c + 3*d*x))/3 - (a*(6*c + 6*d*x - 26))/3) + (4*a*tan(c/2 + (d*x)/2)^3)/3)/(
d*(tan(c/2 + (d*x)/2) - 1)^3*(tan(c/2 + (d*x)/2) + 1)*(tan(c/2 + (d*x)/2)^2
 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \sin\left(c+dx\right)\tan^4\left(c+dx\right)dx+\int \tan^4\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)**4,x)
```

```
[Out] a*(Integral(sin(c + d*x)*tan(c + d*x)**4, x) + Integral(tan(c + d*x)**4, x)
)
```

3.10 $\int (a + a\sin(c + dx))\tan^2(c + dx)\,dx$

Optimal. Leaf size=39

$$\frac{a\cos(c + dx)}{d} + \frac{a\cos(c + dx)}{d(1 - \sin(c + dx))} - ax$$

```
[Out] -a*x+a*cos(d*x+c)/d+a*cos(d*x+c)/d/(1-sin(d*x+c))
```

Rubi [A] time = 0.10, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2708, 2746, 12, 2735, 2648}

$$\frac{a\cos(c + dx)}{d} + \frac{a\cos(c + dx)}{d(1 - \sin(c + dx))} - ax$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -(a*x) + (a*Cos[c + d*x])/d + (a*Cos[c + d*x])/(d*(1 - Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2708

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_), x_Symbol] :> Dist[a^p, Int[Sin[e + f*x]^p/(a - b*Sin[e + f*x])^m, x], x
] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[
p, 2*m]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2746

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x])/(d*f), x] + Dist[1/d, Int
[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}
\int (a+a\sin(c+dx))\tan^2(c+dx)\,dx &= a^2\int \frac{\sin^2(c+dx)}{a-a\sin(c+dx)}\,dx\\
&= \frac{a\cos(c+dx)}{d}+a\int \frac{a\sin(c+dx)}{a-a\sin(c+dx)}\,dx\\
&= \frac{a\cos(c+dx)}{d}+a^2\int \frac{\sin(c+dx)}{a-a\sin(c+dx)}\,dx\\
&= -ax+\frac{a\cos(c+dx)}{d}+a^2\int \frac{1}{a-a\sin(c+dx)}\,dx\\
&= -ax+\frac{a\cos(c+dx)}{d}+\frac{a^2\cos(c+dx)}{d(a-a\sin(c+dx))}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 47, normalized size = 1.21

$$\frac{a\cos(c+dx)}{d}-\frac{a\tan^{-1}(\tan(c+dx))}{d}+\frac{a\tan(c+dx)}{d}+\frac{a\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -((a*ArcTan[Tan[c + d*x]])/d) + (a*Cos[c + d*x])/d + (a*Sec[c + d*x])/d + (
a*Tan[c + d*x])/d
```

fricas [B] time = 0.40, size = 80, normalized size = 2.05

$$-\frac{adx-a\cos\left(dx+c\right)^2+(adx-2\,a)\cos\left(dx+c\right)-(adx-a\cos\left(dx+c\right)+a)\sin\left(dx+c\right)-a}{d\cos\left(dx+c\right)-d\sin\left(dx+c\right)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -(a*d*x - a*cos(d*x + c)^2 + (a*d*x - 2*a)*cos(d*x + c) - (a*d*x - a*cos(d*
x + c) + a)*sin(d*x + c) - a)/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

giac [B] time = 4.95, size = 1008, normalized size = 25.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -(a*d*x*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) - a*d*x*tan(1/2*d*x)^4*
tan(1/2*c)^4 - 4*a*d*x*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) - 2*a*ta
n(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) + a*tan(d*x)*tan(1/2*d*x)^4*tan(1
/2*c)^4 + a*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) + 4*a*d*x*tan(1/2*d*x)^3*tan
(1/2*c)^3 + 2*a*tan(1/2*d*x)^4*tan(1/2*c)^4 - a*d*x*tan(d*x)*tan(1/2*d*x)^4
*tan(c) - 4*a*d*x*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 4*a*d*x*tan(d
*x)*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) + 8*a*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*
c)^3*tan(c) - a*d*x*tan(d*x)*tan(1/2*c)^4*tan(c) - 4*a*tan(d*x)*tan(1/2*d*x
)^3*tan(1/2*c)^3 - 4*a*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) + a*d*x*tan(1/2*d
*x)^4 + 4*a*d*x*tan(1/2*d*x)^3*tan(1/2*c) + 4*a*d*x*tan(1/2*d*x)*tan(1/2*c)
^3 - 8*a*tan(1/2*d*x)^3*tan(1/2*c)^3 + a*d*x*tan(1/2*c)^4 - 2*a*tan(d*x)*ta
n(1/2*d*x)^4*tan(c) - 4*a*d*x*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*tan(c) - 8*a
*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 24*a*tan(d*x)*tan(1/2*d*x)^2*t
an(1/2*c)^2*tan(c) - 8*a*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - 2*a*ta
n(d*x)*tan(1/2*c)^4*tan(c) - a*tan(d*x)*tan(1/2*d*x)^4 - 4*a*tan(d*x)*tan(1
/2*d*x)^3*tan(1/2*c) - 4*a*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3 - a*tan(d*x)*
tan(1/2*c)^4 - a*tan(1/2*d*x)^4*tan(c) - 4*a*tan(1/2*d*x)^3*tan(1/2*c)*tan(
c) - 4*a*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - a*tan(1/2*c)^4*tan(c) + 2*a*tan
(1/2*d*x)^4 + 4*a*d*x*tan(1/2*d*x)*tan(1/2*c) + 8*a*tan(1/2*d*x)^3*tan(1/2*
c) + 24*a*tan(1/2*d*x)^2*tan(1/2*c)^2 + 8*a*tan(1/2*d*x)*tan(1/2*c)^3 + 2*a
*tan(1/2*c)^4 + a*d*x*tan(d*x)*tan(c) + 8*a*tan(d*x)*tan(1/2*d*x)*tan(1/2*c
)*tan(c) - 4*a*tan(d*x)*tan(1/2*d*x)*tan(1/2*c) - 4*a*tan(1/2*d*x)*tan(1/2*
c)*tan(c) - a*d*x - 8*a*tan(1/2*d*x)*tan(1/2*c) - 2*a*tan(d*x)*tan(c) + a*t
an(d*x) + a*tan(c) + 2*a)/(d*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) -
d*tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*d*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*ta
n(c) + 4*d*tan(1/2*d*x)^3*tan(1/2*c)^3 - d*tan(d*x)*tan(1/2*d*x)^4*tan(c) -
 4*d*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 4*d*tan(d*x)*tan(1/2*d*x)*
tan(1/2*c)^3*tan(c) - d*tan(d*x)*tan(1/2*c)^4*tan(c) + d*tan(1/2*d*x)^4 + 4
*d*tan(1/2*d*x)^3*tan(1/2*c) + 4*d*tan(1/2*d*x)*tan(1/2*c)^3 + d*tan(1/2*c)
^4 - 4*d*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*tan(c) + 4*d*tan(1/2*d*x)*tan(1/2
*c) + d*tan(d*x)*tan(c) - d)
```

maple [A] time = 0.17, size = 59, normalized size = 1.51

$$\frac{a\left(\frac{\sin^4(dx+c)}{\cos(dx+c)} + \left(2+\sin^2(dx+c)\right)\cos(dx+c)\right) + a\left(\tan(dx+c) - dx - c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))*tan(d*x+c)^2,x)
```

```
[Out] 1/d*(a*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+a*(tan(d*x+c)-
d*x-c))
```

maxima [A] time = 0.40, size = 39, normalized size = 1.00

$$-\frac{(dx+c-\tan(dx+c))a-a\left(\frac{1}{\cos(dx+c)}+\cos(dx+c)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -((d*x + c - tan(d*x + c))*a - a*(1/cos(d*x + c) + cos(d*x + c)))/d
```

mupad [B] time = 6.77, size = 111, normalized size = 2.85

$$\frac{(a\ (c+d\,x-2)-a\ (c+d\,x))\ \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+(a\ (c+d\,x)-a\ (c+d\,x-2))\ \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a\ (c+d\,x)+a\ (c+}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + a*sin(c + d*x)),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(a*(c + d*x) - a*(c + d*x - 2)) - tan(c/2 + (d*x)/2)^2*
(a*(c + d*x) - a*(c + d*x - 2)) - a*(c + d*x) + a*(c + d*x - 4))/(d*(tan(c/
2 + (d*x)/2) - 1)*(tan(c/2 + (d*x)/2)^2 + 1)) - a*x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sin(c+dx)\tan^2(c+dx)\,dx+\int\tan^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)**2,x)
```

```
[Out] a*(Integral(sin(c + d*x)*tan(c + d*x)**2, x) + Integral(tan(c + d*x)**2, x)
)
```

3.11 $\int \cot^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=41

$$\frac{a\cos(c+dx)}{d}-\frac{a\cot(c+dx)}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}-ax$$

```
[Out] -a*x-a*arctanh(cos(d*x+c))/d+a*cos(d*x+c)/d-a*cot(d*x+c)/d
```

Rubi [A] time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2710, 2592, 321, 206, 3473, 8}

$$\frac{a\cos(c+dx)}{d}-\frac{a\cot(c+dx)}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}-ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*x) - (a*ArcTanh[Cos[c + d*x]])/d + (a*Cos[c + d*x])/d - (a*Cot[c + d*x]
)/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
```

```
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2710

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int \cot^2(c+dx)(a+a\sin(c+dx))\,dx &= \int \left(a\cos(c+dx)\cot(c+dx)+a\cot^2(c+dx)\right)\,dx \\
&= a\int \cos(c+dx)\cot(c+dx)\,dx + a\int \cot^2(c+dx)\,dx \\
&= -\frac{a\cot(c+dx)}{d} - a\int 1\,dx - \frac{a\,\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \cos(c+dx)\right)}{d} \\
&= -ax + \frac{a\cos(c+dx)}{d} - \frac{a\cot(c+dx)}{d} - \frac{a\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(c+dx)\right)}{d} \\
&= -ax - \frac{a\tanh^{-1}(\cos(c+dx))}{d} + \frac{a\cos(c+dx)}{d} - \frac{a\cot(c+dx)}{d}
\end{aligned}$$

Mathematica [C] time = 0.04, size = 75, normalized size = 1.83

$$-\frac{a\cot(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(c+dx)\right)}{d}+\frac{a\cos(c+dx)}{d}+\frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}-\frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Cos[c + d*x])/d - (a*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c
 + d*x]^2])/d - (a*Log[Cos[(c + d*x)/2]])/d + (a*Log[Sin[(c + d*x)/2]])/d
```

fricas [B] time = 0.45, size = 84, normalized size = 2.05

$$-\frac{a\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-a\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+2\,a\cos\left(dx+c\right)+2\,(adx-a}{2\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(a*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - a*log(-1/2*cos(d*x + c)
+ 1/2)*sin(d*x + c) + 2*a*cos(d*x + c) + 2*(a*d*x - a*cos(d*x + c))*sin(d*x
 + c))/(d*sin(d*x + c))
```

giac [B] time = 0.56, size = 108, normalized size = 2.63

$$-\frac{6\,(dx+c)a-6\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-10\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(6*(d*x + c)*a - 6*a*log(abs(tan(1/2*d*x + 1/2*c))) - 3*a*tan(1/2*d*x
+ 1/2*c) + (2*a*tan(1/2*d*x + 1/2*c)^3 + 3*a*tan(1/2*d*x + 1/2*c)^2 - 10*a*
tan(1/2*d*x + 1/2*c) + 3*a)/(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))
)/d
```

maple [A] time = 0.10, size = 57, normalized size = 1.39

$$-ax+\frac{a\cos\left(dx+c\right)}{d}-\frac{a\cot\left(dx+c\right)}{d}+\frac{a\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)}{d}-\frac{ca}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+a*sin(d*x+c)),x)
```

```
[Out] -a*x+a*cos(d*x+c)/d-a*cot(d*x+c)/d+1/d*a*ln(csc(d*x+c)-cot(d*x+c))-1/d*c*a
```

maxima [A] time = 0.42, size = 54, normalized size = 1.32

$$-\frac{2\left(dx+c+\frac{1}{\tan\left(dx+c\right)}\right)a-a\left(2\cos\left(dx+c\right)-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*(d*x + c + 1/tan(d*x + c))*a - a*(2*cos(d*x + c) - log(cos(d*x + c)
 + 1) + log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 6.91, size = 108, normalized size = 2.63

$$\frac{2\,a\,\mathrm{atan}\left(\frac{\sqrt{2}\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,\cos\left(\frac{c}{2}-\frac{\pi}{4}+\frac{d\,x}{2}\right)}\right)+a\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{a\,\cos\left(c+d\,x\right)-\frac{a\,\sin(2\,c+2\,d\,x)}{2}}{d\,\sin\left(c+d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + a*sin(c + d*x)),x)
```

```
[Out] (2*a*atan((2^(1/2)*(cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2)))/(2*cos(c/2 -
pi/4 + (d*x)/2))) + a*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (a*co
s(c + d*x) - (a*sin(2*c + 2*d*x))/2)/(d*sin(c + d*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sin\left(c+dx\right)\cot^{2}\left(c+dx\right)dx+\int\cot^{2}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(sin(c + d*x)*cot(c + d*x)**2, x) + Integral(cot(c + d*x)**2, x)
)
```

3.12 $\int \cot^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=82

$$-\frac{3a\cos(c+dx)}{2d}-\frac{a\cot^3(c+dx)}{3d}+\frac{a\cot(c+dx)}{d}-\frac{a\cos(c+dx)\cot^2(c+dx)}{2d}+\frac{3a\tanh^{-1}(\cos(c+dx))}{2d}+ax$$

```
[Out] a*x+3/2*a*arctanh(cos(d*x+c))/d-3/2*a*cos(d*x+c)/d+a*cot(d*x+c)/d-1/2*a*cos
(d*x+c)*cot(d*x+c)^2/d-1/3*a*cot(d*x+c)^3/d
```

Rubi [A] time = 0.08, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {2710, 2592, 288, 321, 206, 3473, 8}

$$-\frac{3a\cos(c+dx)}{2d}-\frac{a\cot^3(c+dx)}{3d}+\frac{a\cot(c+dx)}{d}-\frac{a\cos(c+dx)\cot^2(c+dx)}{2d}+\frac{3a\tanh^{-1}(\cos(c+dx))}{2d}+ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] a*x + (3*a*ArcTanh[Cos[c + d*x]])/(2*d) - (3*a*Cos[c + d*x])/(2*d) + (a*Cot
[c + d*x])/d - (a*Cos[c + d*x]*Cot[c + d*x]^2)/(2*d) - (a*Cot[c + d*x]^3)/(
3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2710

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \cot^4(c+dx)(a+a\sin(c+dx))\,dx = \int \left(a\cos(c+dx)\cot^3(c+dx)+a\cot^4(c+dx)\right)dx$$
$$= a\int \cos(c+dx)\cot^3(c+dx)\,dx + a\int \cot^4(c+dx)\,dx$$
$$= -\frac{a\cot^3(c+dx)}{3d} - a\int \cot^2(c+dx)\,dx - \frac{a\,\mathrm{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx, x, \cos(c+$$
$$= \frac{a\cot(c+dx)}{d} - \frac{a\cos(c+dx)\cot^2(c+dx)}{2d} - \frac{a\cot^3(c+dx)}{3d} + a\int 1\,dx +$$
$$= ax - \frac{3a\cos(c+dx)}{2d} + \frac{a\cot(c+dx)}{d} - \frac{a\cos(c+dx)\cot^2(c+dx)}{2d} - \frac{a\cot^3$$
$$= ax + \frac{3a\tanh^{-1}(\cos(c+dx))}{2d} - \frac{3a\cos(c+dx)}{2d} + \frac{a\cot(c+dx)}{d} - \frac{a\cos(c$$

Mathematica [C] time = 0.05, size = 125, normalized size = 1.52

$$-\frac{a\cot^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2(c+dx)\right)}{3d} - \frac{a\cos(c+dx)}{d} - \frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{3a\log\left(\mathrm{si}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Cos[c + d*x])/d) - (a*Csc[(c + d*x)/2]^2)/(8*d) - (a*Cot[c + d*x]^3*Hy
pergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])/(3*d) + (3*a*Log[Cos[(c +
d*x)/2]])/(2*d) - (3*a*Log[Sin[(c + d*x)/2]])/(2*d) + (a*Sec[(c + d*x)/2]^2
)/(8*d)
```

fricas [B] time = 0.43, size = 160, normalized size = 1.95

$$\frac{16\,a\cos(dx+c)^3 + 9\left(a\cos(dx+c)^2-a\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c) - 9\left(a\cos(dx+c)^2-a\right)\log\left(-\frac{1}{2}\right.}{12\left(d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(16*a*cos(d*x + c)^3 + 9*(a*cos(d*x + c)^2 - a)*log(1/2*cos(d*x + c) +
 1/2)*sin(d*x + c) - 9*(a*cos(d*x + c)^2 - a)*log(-1/2*cos(d*x + c) + 1/2)*
```

```
sin(d*x + c) - 12*a*cos(d*x + c) + 6*(2*a*d*x*cos(d*x + c)^2 - 2*a*cos(d*x
+ c)^3 - 2*a*d*x + 3*a*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*
sin(d*x + c))
```

giac [A] time = 0.36, size = 141, normalized size = 1.72

$$\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+24\,(dx+c)a-36\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(a*tan(1/2*d*x + 1/2*c)^3 + 3*a*tan(1/2*d*x + 1/2*c)^2 + 24*(d*x + c)*
a - 36*a*log(abs(tan(1/2*d*x + 1/2*c))) - 15*a*tan(1/2*d*x + 1/2*c) - 48*a/
(tan(1/2*d*x + 1/2*c)^2 + 1) + (66*a*tan(1/2*d*x + 1/2*c)^3 + 15*a*tan(1/2*
d*x + 1/2*c)^2 - 3*a*tan(1/2*d*x + 1/2*c) - a)/tan(1/2*d*x + 1/2*c)^3)/d
```

maple [A] time = 0.12, size = 106, normalized size = 1.29

$$-\frac{a\left(\cos^{5}(dx+c)\right)}{2d\sin(dx+c)^{2}}-\frac{a\left(\cos^{3}(dx+c)\right)}{2d}-\frac{3a\cos(dx+c)}{2d}-\frac{3a\ln(\csc(dx+c)-\cot(dx+c))}{2d}-\frac{a\left(\cot^{3}(dx+c)\right)}{3d}+\frac{a\,c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/2/d*a/sin(d*x+c)^2*cos(d*x+c)^5-1/2*a*cos(d*x+c)^3/d-3/2*a*cos(d*x+c)/d-
3/2/d*a*ln(csc(d*x+c)-cot(d*x+c))-1/3*a*cot(d*x+c)^3/d+a*cot(d*x+c)/d+a*x+1
/d*c*a
```

maxima [A] time = 0.40, size = 92, normalized size = 1.12

$$\frac{4\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^{2}-1}{\tan(dx+c)^{3}}\right)a+3\,a\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^{2}-1}-4\,\cos(dx+c)+3\,\log(\cos(dx+c)+1)-3\,\log(\cos(dx+c)\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(4*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a + 3*a*(2*co
s(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x + c) + 1)
- 3*log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 6.60, size = 228, normalized size = 2.78

$$\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-\frac{-5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+17\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\frac{14\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a}{3}}{d\left(8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}-\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + a*sin(c + d*x)),x)
```

```
[Out] (a*tan(c/2 + (d*x)/2)^2)/(8*d) - (a/3 + a*tan(c/2 + (d*x)/2) - (14*a*tan(c/
2 + (d*x)/2)^2)/3 + 17*a*tan(c/2 + (d*x)/2)^3 - 5*a*tan(c/2 + (d*x)/2)^4)/(
d*(8*tan(c/2 + (d*x)/2)^3 + 8*tan(c/2 + (d*x)/2)^5)) - (5*a*tan(c/2 + (d*x)
/2))/(8*d) + (a*tan(c/2 + (d*x)/2)^3)/(24*d) - (3*a*log(tan(c/2 + (d*x)/2))
)/(2*d) - (2*a*atan((4*a^2)/(6*a^2 + 4*a^2*tan(c/2 + (d*x)/2)) - (6*a^2*tan
(c/2 + (d*x)/2))/(6*a^2 + 4*a^2*tan(c/2 + (d*x)/2))))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sin\left(c+dx\right)\cot^4\left(c+dx\right)dx+\int\cot^4\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(sin(c + d*x)*cot(c + d*x)**4, x) + Integral(cot(c + d*x)**4, x)
)
```

3.13 $\int \cot^6(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=122

$$\frac{15a\cos(c+dx)}{8d}-\frac{a\cot^5(c+dx)}{5d}+\frac{a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-\frac{a\cos(c+dx)\cot^4(c+dx)}{4d}+\frac{5a\cos(c+dx)\cot^2}{8d}$$

```
[Out] -a*x-15/8*a*arctanh(cos(d*x+c))/d+15/8*a*cos(d*x+c)/d-a*cot(d*x+c)/d+5/8*a*
cos(d*x+c)*cot(d*x+c)^2/d+1/3*a*cot(d*x+c)^3/d-1/4*a*cos(d*x+c)*cot(d*x+c)^
4/d-1/5*a*cot(d*x+c)^5/d
```

Rubi [A] time = 0.10, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {2710, 2592, 288, 321, 206, 3473, 8}

$$\frac{15a\cos(c+dx)}{8d}-\frac{a\cot^5(c+dx)}{5d}+\frac{a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-\frac{a\cos(c+dx)\cot^4(c+dx)}{4d}+\frac{5a\cos(c+dx)\cot^2}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*x) - (15*a*ArcTanh[Cos[c + d*x]])/(8*d) + (15*a*Cos[c + d*x])/(8*d) - (
a*Cot[c + d*x])/d + (5*a*Cos[c + d*x]*Cot[c + d*x]^2)/(8*d) + (a*Cot[c + d*
x]^3)/(3*d) - (a*Cos[c + d*x]*Cot[c + d*x]^4)/(4*d) - (a*Cot[c + d*x]^5)/(5
*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2710

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \cot^6(c+dx)(a+a\sin(c+dx))\,dx = \int \left(a\cos(c+dx)\cot^5(c+dx)+a\cot^6(c+dx)\right)\,dx$$
$$= a\int \cos(c+dx)\cot^5(c+dx)\,dx + a\int \cot^6(c+dx)\,dx$$
$$= -\frac{a\cot^5(c+dx)}{5d} - a\int \cot^4(c+dx)\,dx - \frac{a\,\mathrm{Subst}\left(\int \frac{x^6}{\left(1-x^2\right)^3}\,dx, x, \cos(c+\right.}{d}$$
$$= \frac{a\cot^3(c+dx)}{3d} - \frac{a\cos(c+dx)\cot^4(c+dx)}{4d} - \frac{a\cot^5(c+dx)}{5d} + a\int \cot^2$$
$$= -\frac{a\cot(c+dx)}{d} + \frac{5a\cos(c+dx)\cot^2(c+dx)}{8d} + \frac{a\cot^3(c+dx)}{3d} - \frac{a\cos(c}{}$$
$$= -ax + \frac{15a\cos(c+dx)}{8d} - \frac{a\cot(c+dx)}{d} + \frac{5a\cos(c+dx)\cot^2(c+dx)}{8d} +$$
$$= -ax - \frac{15a\tanh^{-1}(\cos(c+dx))}{8d} + \frac{15a\cos(c+dx)}{8d} - \frac{a\cot(c+dx)}{d} + \frac{5a}{}$$

Mathematica [C] time = 0.07, size = 164, normalized size = 1.34

$$-\frac{a\cot^5(c+dx)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-\tan^2(c+dx)\right)}{5d}+\frac{a\cos(c+dx)}{d}-\frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{9a\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{a\sec^4}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Cos[c + d*x])/d + (9*a*Csc[(c + d*x)/2]^2)/(32*d) - (a*Csc[(c + d*x)/2]^
4)/(64*d) - (a*Cot[c + d*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[c + d*x
]^2])/(5*d) - (15*a*Log[Cos[(c + d*x)/2]])/(8*d) + (15*a*Log[Sin[(c + d*x)/
2]])/(8*d) - (9*a*Sec[(c + d*x)/2]^2)/(32*d) + (a*Sec[(c + d*x)/2]^4)/(64*d
)
```

fricas [B] time = 0.45, size = 222, normalized size = 1.82

$$-\frac{368\,a\cos(dx+c)^5-560\,a\cos(dx+c)^3+225\left(a\cos(dx+c)^4-2\,a\cos(dx+c)^2+a\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/240*(368*a*cos(d*x + c)^5 - 560*a*cos(d*x + c)^3 + 225*(a*cos(d*x + c)^4
 - 2*a*cos(d*x + c)^2 + a)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 225*(
a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)*log(-1/2*cos(d*x + c) + 1/2)*sin
(d*x + c) + 240*a*cos(d*x + c) + 30*(8*a*d*x*cos(d*x + c)^4 - 8*a*cos(d*x +
 c)^5 - 16*a*d*x*cos(d*x + c)^2 + 25*a*cos(d*x + c)^3 + 8*a*d*x - 15*a*cos(
d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*
x + c))
```

giac [A] time = 0.46, size = 199, normalized size = 1.63

$$\frac{6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-70\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-240\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-960\,(dx+c)a+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/960*(6*a*tan(1/2*d*x + 1/2*c)^5 + 15*a*tan(1/2*d*x + 1/2*c)^4 - 70*a*tan(
1/2*d*x + 1/2*c)^3 - 240*a*tan(1/2*d*x + 1/2*c)^2 - 960*(d*x + c)*a + 1800*
a*log(abs(tan(1/2*d*x + 1/2*c))) + 660*a*tan(1/2*d*x + 1/2*c) + 1920*a/(tan
(1/2*d*x + 1/2*c)^2 + 1) - (4110*a*tan(1/2*d*x + 1/2*c)^5 + 660*a*tan(1/2*d
*x + 1/2*c)^4 - 240*a*tan(1/2*d*x + 1/2*c)^3 - 70*a*tan(1/2*d*x + 1/2*c)^2
+ 15*a*tan(1/2*d*x + 1/2*c) + 6*a)/tan(1/2*d*x + 1/2*c)^5)/d
```

maple [A] time = 0.13, size = 159, normalized size = 1.30

$$-\frac{a\left(\cos^{7}(dx+c)\right)}{4d\sin(dx+c)^{4}}+\frac{3a\left(\cos^{7}(dx+c)\right)}{8d\sin(dx+c)^{2}}+\frac{3a\left(\cos^{5}(dx+c)\right)}{8d}+\frac{5a\left(\cos^{3}(dx+c)\right)}{8d}+\frac{15a\cos(dx+c)}{8d}+\frac{15a\ln(\csc(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/4/d*a/sin(d*x+c)^4*cos(d*x+c)^7+3/8/d*a/sin(d*x+c)^2*cos(d*x+c)^7+3/8*a*
cos(d*x+c)^5/d+5/8*a*cos(d*x+c)^3/d+15/8*a*cos(d*x+c)/d+15/8/d*a*ln(csc(d*x
+c)-cot(d*x+c))-1/5*a*cot(d*x+c)^5/d+1/3*a*cot(d*x+c)^3/d-a*cot(d*x+c)/d-a*
x-1/d*c*a
```

maxima [A] time = 0.41, size = 125, normalized size = 1.02

$$-\frac{16\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^{4}-5\,\tan(dx+c)^{2}+3}{\tan(dx+c)^{5}}\right)a+15\,a\left(\frac{2\left(9\,\cos(dx+c)^{3}-7\,\cos(dx+c)\right)}{\cos(dx+c)^{4}-2\,\cos(dx+c)^{2}+1}-16\,\cos(dx+c)+15\,\log(\cos(d}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/240*(16*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(
d*x + c)^5)*a + 15*a*(2*(9*cos(d*x + c)^3 - 7*cos(d*x + c))/(cos(d*x + c)^4
 - 2*cos(d*x + c)^2 + 1) - 16*cos(d*x + c) + 15*log(cos(d*x + c) + 1) - 15*
log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 6.67, size = 291, normalized size = 2.39

$$\frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d}-\frac{22\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-72\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\frac{59\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}-\frac{15\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}-\frac{32\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{15}+\frac{a}{\cdots}}{d\left(32\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+32\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6*(a + a*sin(c + d*x)),x)
```

```
[Out] (11*a*tan(c/2 + (d*x)/2))/(16*d) - (a/5 + (a*tan(c/2 + (d*x)/2))/2 - (32*a*
tan(c/2 + (d*x)/2)^2)/15 - (15*a*tan(c/2 + (d*x)/2)^3)/2 + (59*a*tan(c/2 +
(d*x)/2)^4)/3 - 72*a*tan(c/2 + (d*x)/2)^5 + 22*a*tan(c/2 + (d*x)/2)^6)/(d*(
32*tan(c/2 + (d*x)/2)^5 + 32*tan(c/2 + (d*x)/2)^7)) - (a*tan(c/2 + (d*x)/2)
^2)/(4*d) - (7*a*tan(c/2 + (d*x)/2)^3)/(96*d) + (a*tan(c/2 + (d*x)/2)^4)/(6
4*d) + (a*tan(c/2 + (d*x)/2)^5)/(160*d) + (15*a*log(tan(c/2 + (d*x)/2)))/(8
*d) + (2*a*atan((4*a^2)/((15*a^2)/2 + 4*a^2*tan(c/2 + (d*x)/2)) - (15*a^2*t
an(c/2 + (d*x)/2))/(2*((15*a^2)/2 + 4*a^2*tan(c/2 + (d*x)/2)))))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sin\left(c+dx\right)\cot^{6}\left(c+dx\right)dx+\int\cot^{6}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(sin(c + d*x)*cot(c + d*x)**6, x) + Integral(cot(c + d*x)**6, x)
)
```

3.14 $\int (a + a\sin(c + dx))^2 \tan^5(c + dx)\, dx$

Optimal. Leaf size=119

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}-\frac{9a^3}{4d(a-a\sin(c+dx))}-\frac{a^2\sin^2(c+dx)}{2d}-\frac{2a^2\sin(c+dx)}{d}-\frac{31a^2\log(1-\sin(c+dx))}{8d}-\frac{a^2\log}{}$$

```
[Out] -31/8*a^2*ln(1-sin(d*x+c))/d-1/8*a^2*ln(1+sin(d*x+c))/d-2*a^2*sin(d*x+c)/d-
1/2*a^2*sin(d*x+c)^2/d+1/4*a^4/d/(a-a*sin(d*x+c))^2-9/4*a^3/d/(a-a*sin(d*x+
c))
```

Rubi [A] time = 0.08, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 88}

$$-\frac{a^2\sin^2(c+dx)}{2d}+\frac{a^4}{4d(a-a\sin(c+dx))^2}-\frac{9a^3}{4d(a-a\sin(c+dx))}-\frac{2a^2\sin(c+dx)}{d}-\frac{31a^2\log(1-\sin(c+dx))}{8d}-\frac{a^2 l}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^5,x]
```

```
[Out] (-31*a^2*Log[1 - Sin[c + d*x]])/(8*d) - (a^2*Log[1 + Sin[c + d*x]])/(8*d) -
 (2*a^2*Sin[c + d*x])/d - (a^2*Sin[c + d*x]^2)/(2*d) + a^4/(4*d*(a - a*Sin[
c + d*x])^2) - (9*a^3)/(4*d*(a - a*Sin[c + d*x]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int (a+a\sin(c+dx))^2\tan^5(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(a-x)^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-2a+\frac{a^4}{2(a-x)^3}-\frac{9a^3}{4(a-x)^2}+\frac{31a^2}{8(a-x)}-x-\frac{a^2}{8(a+x)}\right)dx, x, a\sin(c+\right.}{d}$$
$$= -\frac{31a^2\log(1-\sin(c+dx))}{8d}-\frac{a^2\log(1+\sin(c+dx))}{8d}-\frac{2a^2\sin(c+dx}{d}$$

Mathematica [A] time = 0.24, size = 75, normalized size = 0.63

$$-\frac{a^2\left(4\sin^2(c+dx)+16\sin(c+dx)-\frac{18}{\sin(c+dx)-1}-\frac{2}{(\sin(c+dx)-1)^2}+31\log(1-\sin(c+dx))+\log(\sin(c+dx)+1)\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^5,x]
```

```
[Out] -1/8*(a^2*(31*Log[1 - Sin[c + d*x]] + Log[1 + Sin[c + d*x]] - 2/(-1 + Sin[c
 + d*x])^2 - 18/(-1 + Sin[c + d*x]) + 16*Sin[c + d*x] + 4*Sin[c + d*x]^2))/
d
```

fricas [A] time = 0.47, size = 168, normalized size = 1.41

$$\frac{4\,a^2\cos(dx+c)^4+22\,a^2\cos(dx+c)^2-12\,a^2-\left(a^2\cos(dx+c)^2+2\,a^2\sin(dx+c)-2\,a^2\right)\log(\sin(dx+c)+}{8\left(d\cos(dx+c)^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/8*(4*a^2*cos(d*x + c)^4 + 22*a^2*cos(d*x + c)^2 - 12*a^2 - (a^2*cos(d*x +
 c)^2 + 2*a^2*sin(d*x + c) - 2*a^2)*log(sin(d*x + c) + 1) - 31*(a^2*cos(d*x
 + c)^2 + 2*a^2*sin(d*x + c) - 2*a^2)*log(-sin(d*x + c) + 1) - 2*(4*a^2*cos
(d*x + c)^2 - 5*a^2)*sin(d*x + c))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2
*d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.20, size = 261, normalized size = 2.19

$$\frac{a^2\left(\sin^8(dx+c)\right)}{4d\cos(dx+c)^4}-\frac{a^2\left(\sin^8(dx+c)\right)}{2d\cos(dx+c)^2}-\frac{a^2\left(\sin^6(dx+c)\right)}{2d}-\frac{3a^2\left(\sin^4(dx+c)\right)}{4d}-\frac{3a^2\left(\sin^2(dx+c)\right)}{2d}-\frac{4a^2\ln(\cos(dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^2*tan(d*x+c)^5,x)
```

```
[Out] 1/4/d*a^2*sin(d*x+c)^8/cos(d*x+c)^4-1/2/d*a^2*sin(d*x+c)^8/cos(d*x+c)^2-1/2
/d*a^2*sin(d*x+c)^6-3/4/d*a^2*sin(d*x+c)^4-3/2*a^2*sin(d*x+c)^2/d-4/d*a^2*l
n(cos(d*x+c))+1/2/d*a^2*sin(d*x+c)^7/cos(d*x+c)^4-3/4/d*a^2*sin(d*x+c)^7/co
s(d*x+c)^2-3/4/d*a^2*sin(d*x+c)^5-5/4/d*a^2*sin(d*x+c)^3-15/4*a^2*sin(d*x+c
)/d+15/4/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/4/d*a^2*tan(d*x+c)^4-1/2/d*a^2*t
an(d*x+c)^2
```

maxima [A] time = 0.31, size = 96, normalized size = 0.81

$$-\frac{4\,a^2\sin(dx+c)^2+a^2\log(\sin(dx+c)+1)+31\,a^2\log(\sin(dx+c)-1)+16\,a^2\sin(dx+c)-\frac{2\left(9\,a^2\sin(dx+c)-8\right.}{\sin(dx+c)^2-2\,\sin(dx}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/8*(4*a^2*sin(d*x + c)^2 + a^2*log(sin(d*x + c) + 1) + 31*a^2*log(sin(d*x
 + c) - 1) + 16*a^2*sin(d*x + c) - 2*(9*a^2*sin(d*x + c) - 8*a^2)/(sin(d*x
+ c)^2 - 2*sin(d*x + c) + 1))/d
```

mupad [B] time = 6.57, size = 283, normalized size = 2.38

$$\frac{4\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{4\,d}-\frac{\frac{15\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2}-22\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\frac{61\,a^2\tan}{2}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (4*a^2*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (a^2*log(tan(c/2 + (d*x)/2) + 1))
/(4*d) - ((61*a^2*tan(c/2 + (d*x)/2)^3)/2 - 22*a^2*tan(c/2 + (d*x)/2)^2 - 3
```

```
6*a^2*tan(c/2 + (d*x)/2)^4 + (61*a^2*tan(c/2 + (d*x)/2)^5)/2 - 22*a^2*tan(c
/2 + (d*x)/2)^6 + (15*a^2*tan(c/2 + (d*x)/2)^7)/2 + (15*a^2*tan(c/2 + (d*x)
/2))/2)/(d*(8*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2) - 12*tan(c/2 + (d
*x)/2)^3 + 14*tan(c/2 + (d*x)/2)^4 - 12*tan(c/2 + (d*x)/2)^5 + 8*tan(c/2 +
(d*x)/2)^6 - 4*tan(c/2 + (d*x)/2)^7 + tan(c/2 + (d*x)/2)^8 + 1)) - (31*a^2*
log(tan(c/2 + (d*x)/2) - 1))/(4*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\sin(c+dx)\tan^5(c+dx)\,dx+\int \sin^2(c+dx)\tan^5(c+dx)\,dx+\int \tan^5(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**2*tan(d*x+c)**5,x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)*tan(c + d*x)**5, x) + Integral(sin(c + d*x)**
2*tan(c + d*x)**5, x) + Integral(tan(c + d*x)**5, x))
```

3.15 $\int (a+a\sin(c+dx))^2\tan^3(c+dx)\,dx$

Optimal. Leaf size=72

$$\frac{a^3}{d(a-a\sin(c+dx))}+\frac{a^2\sin^2(c+dx)}{2d}+\frac{2a^2\sin(c+dx)}{d}+\frac{3a^2\log(1-\sin(c+dx))}{d}$$

```
[Out] 3*a^2*ln(1-sin(d*x+c))/d+2*a^2*sin(d*x+c)/d+1/2*a^2*sin(d*x+c)^2/d+a^3/d/(a
-a*sin(d*x+c))
```

Rubi [A] time = 0.06, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 43}

$$\frac{a^2\sin^2(c+dx)}{2d}+\frac{a^3}{d(a-a\sin(c+dx))}+\frac{2a^2\sin(c+dx)}{d}+\frac{3a^2\log(1-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] (3*a^2*Log[1 - Sin[c + d*x]])/d + (2*a^2*Sin[c + d*x])/d + (a^2*Sin[c + d*x
]^2)/(2*d) + a^3/(d*(a - a*Sin[c + d*x]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int (a+a\sin(c+dx))^2\tan^3(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{(a-x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(2a+\frac{a^3}{(a-x)^2}-\frac{3a^2}{a-x}+x\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{3a^2\log(1-\sin(c+dx))}{d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin^2(c+dx)}{2d}+\frac{}{d(a-a}$$

Mathematica [A] time = 0.10, size = 54, normalized size = 0.75

$$\frac{a^2\left(\sin^2(c+dx)+4\sin(c+dx)+\frac{2}{1-\sin(c+dx)}+6\log(1-\sin(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] (a^2*(6*Log[1 - Sin[c + d*x]] + 2/(1 - Sin[c + d*x]) + 4*Sin[c + d*x] + Sin
[c + d*x]^2))/(2*d)
```

fricas [A] time = 0.46, size = 90, normalized size = 1.25

$$-\frac{6\,a^2\cos(dx+c)^2-3\,a^2-12\left(a^2\sin(dx+c)-a^2\right)\log(-\sin(dx+c)+1)+\left(2\,a^2\cos(dx+c)^2+7\,a^2\right)\sin(dx}{4\,(d\sin(dx+c)-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(6*a^2*cos(d*x + c)^2 - 3*a^2 - 12*(a^2*sin(d*x + c) - a^2)*log(-sin(d
*x + c) + 1) + (2*a^2*cos(d*x + c)^2 + 7*a^2)*sin(d*x + c))/(d*sin(d*x + c)
 - d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.17, size = 162, normalized size = 2.25

$$\frac{a^2\left(\sin^6(dx+c)\right)}{2d\cos(dx+c)^2}+\frac{a^2\left(\sin^4(dx+c)\right)}{2d}+\frac{a^2\left(\sin^2(dx+c)\right)}{d}+\frac{3a^2\ln(\cos(dx+c))}{d}+\frac{a^2\left(\sin^5(dx+c)\right)}{d\cos(dx+c)^2}+\frac{a^2\left(\sin^3(dx\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^2*tan(d*x+c)^3,x)
```

```
[Out] 1/2/d*a^2*sin(d*x+c)^6/cos(d*x+c)^2+1/2/d*a^2*sin(d*x+c)^4+a^2*sin(d*x+c)^2
/d+3/d*a^2*ln(cos(d*x+c))+1/d*a^2*sin(d*x+c)^5/cos(d*x+c)^2+1/d*a^2*sin(d*x
+c)^3+3*a^2*sin(d*x+c)/d-3/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*a^2*tan(d*
x+c)^2
```

maxima [A] time = 0.30, size = 58, normalized size = 0.81

$$\frac{a^2\sin(dx+c)^2+6\,a^2\log(\sin(dx+c)-1)+4\,a^2\sin(dx+c)-\frac{2\,a^2}{\sin(dx+c)-1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(a^2*sin(d*x + c)^2 + 6*a^2*log(sin(d*x + c) - 1) + 4*a^2*sin(d*x + c)
- 2*a^2/(sin(d*x + c) - 1))/d
```

mupad [B] time = 7.11, size = 204, normalized size = 2.83

$$\frac{6\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-6\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+8\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-6\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+6\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (8*a^2*tan(c/2 + (d*x)/2)^3 - 6*a^2*tan(c/2 + (d*x)/2)^2 - 6*a^2*tan(c/2 +
(d*x)/2)^4 + 6*a^2*tan(c/2 + (d*x)/2)^5 + 6*a^2*tan(c/2 + (d*x)/2))/(d*(3*t
an(c/2 + (d*x)/2)^2 - 2*tan(c/2 + (d*x)/2) - 4*tan(c/2 + (d*x)/2)^3 + 3*tan
(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^5 + tan(c/2 + (d*x)/2)^6 + 1)) + (
6*a^2*log(tan(c/2 + (d*x)/2) - 1))/d - (3*a^2*log(tan(c/2 + (d*x)/2)^2 + 1)
)/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\sin(c+dx)\tan^3(c+dx)\,dx+\int \sin^2(c+dx)\tan^3(c+dx)\,dx+\int \tan^3(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**2*tan(d*x+c)**3,x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)*tan(c + d*x)**3, x) + Integral(sin(c + d*x)**
2*tan(c + d*x)**3, x) + Integral(tan(c + d*x)**3, x))
```

3.16 $\int (a + a\sin(c + dx))^2 \tan(c + dx)\,dx$

Optimal. Leaf size=52

$$-\frac{a^2\sin^2(c+dx)}{2d}-\frac{2a^2\sin(c+dx)}{d}-\frac{2a^2\log(1-\sin(c+dx))}{d}$$

```
[Out] -2*a^2*ln(1-sin(d*x+c))/d-2*a^2*sin(d*x+c)/d-1/2*a^2*sin(d*x+c)^2/d
```

Rubi [A] time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2707, 77}

$$-\frac{a^2\sin^2(c+dx)}{2d}-\frac{2a^2\sin(c+dx)}{d}-\frac{2a^2\log(1-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] (-2*a^2*Log[1 - Sin[c + d*x]])/d - (2*a^2*Sin[c + d*x])/d - (a^2*Sin[c + d*
x]^2)/(2*d)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int (a+a\sin(c+dx))^2\tan(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x(a+x)}{a-x}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-2a+\frac{2a^2}{a-x}-x\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{2a^2\log(1-\sin(c+dx))}{d} - \frac{2a^2\sin(c+dx)}{d} - \frac{a^2\sin^2(c+dx)}{2d}$$

Mathematica [A] time = 0.04, size = 40, normalized size = 0.77

$$-\frac{a^2\left(\sin^2(c+dx)+4\sin(c+dx)+4\log(1-\sin(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] -1/2*(a^2*(4*Log[1 - Sin[c + d*x]] + 4*Sin[c + d*x] + Sin[c + d*x]^2))/d
```

fricas [A] time = 0.43, size = 45, normalized size = 0.87

$$\frac{a^2\cos(dx+c)^2 - 4a^2\log(-\sin(dx+c)+1) - 4a^2\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*cos(d*x + c)^2 - 4*a^2*log(-sin(d*x + c) + 1) - 4*a^2*sin(d*x + c)
)/d
```

giac [B] time = 19.30, size = 6695, normalized size = 128.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c),x, algorithm="giac")
```

```
[Out] -1/4*(4*a^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c
) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1
/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2
 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*ta
```

```
n(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 - 4*a^2*log(2*(tan(1/2*d*x)^4
*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2
 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*ta
n(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x)
+ 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c
)^2*tan(c)^2 + 4*a^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan
(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(
d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 - a^2*tan(d*x)^2*tan(1/2*d*x)^2
*tan(1/2*c)^2*tan(c)^2 + 4*a^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1
/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*t
an(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2
 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(ta
n(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*a^2*log(2*(tan(
1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*ta
n(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x
)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan
(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^2
*tan(1/2*c)^2 + 4*a^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + ta
n(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan
(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*a^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c
)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2
*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)
*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/
2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(c)^2 - 4*a^2*lo
g(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*
d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*ta
n(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^
2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1
/2*d*x)^2*tan(c)^2 + 4*a^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c)
 + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)
)*tan(d*x)^2*tan(1/2*d*x)^2*tan(c)^2 - 16*a^2*tan(d*x)^2*tan(1/2*d*x)^2*tan
(1/2*c)*tan(c)^2 + 4*a^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x
)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2
*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*t
an(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*
c)^2 + 1))*tan(d*x)^2*tan(1/2*c)^2*tan(c)^2 - 4*a^2*log(2*(tan(1/2*d*x)^4*t
an(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 +
 tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(
1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) +
2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*c)^2*tan(c)^2 + 4*
a^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2
+ tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(1/2*c)
^2*tan(c)^2 - 16*a^2*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^2*tan(c)^2 + 4*a^2*
log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/
2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*
```

```
tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c
)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2
*tan(1/2*c)^2*tan(c)^2 - 4*a^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1
/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*t
an(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2
 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(ta
n(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + 4*a^2*log(4*(tan(d*
x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*
tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 +
 a^2*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*a^2*tan(d*x)*tan(1/2*d*x)^2
*tan(1/2*c)^2*tan(c) - a^2*tan(d*x)^2*tan(1/2*d*x)^2*tan(c)^2 - a^2*tan(d*x
)^2*tan(1/2*c)^2*tan(c)^2 + a^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + 4*a^
2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(
1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 -
2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2
*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*t
an(1/2*d*x)^2 - 4*a^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4
*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*
x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(
1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^
2 + 1))*tan(d*x)^2*tan(1/2*d*x)^2 + 4*a^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*ta
n(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)
/(tan(c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^2 - 16*a^2*tan(d*x)^2*tan(1/2*d*x)
^2*tan(1/2*c) + 4*a^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4
*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*
x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(
1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^
2 + 1))*tan(d*x)^2*tan(1/2*c)^2 - 4*a^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2
- 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x
)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan
(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c)
 + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*c)^2 + 4*a^2*log(4*(tan(d*x)^4
*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(
d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(1/2*c)^2 - 16*a^2*tan(d*x)^
2*tan(1/2*d*x)*tan(1/2*c)^2 + 4*a^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*
tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4
+ 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2
*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1
)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*a^2*log(2*(tan(1/2*d*
x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*
c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 -
2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d
*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4
*a^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2
 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(1/2*d*x)^2*tan(1
```

/2*c)^2 + 4*a^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1
/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*t
an(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*
x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1)
)*tan(d*x)^2*tan(c)^2 - 4*a^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/
2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*ta
n(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2
+ 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan
(1/2*c)^2 + 1))*tan(d*x)^2*tan(c)^2 + 4*a^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*
tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) +
1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 + 16*a^2*tan(d*x)^2*tan(1/2*d*x)*tan
(c)^2 + 4*a^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2
*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan
(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)
^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*
tan(1/2*d*x)^2*tan(c)^2 - 4*a^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(
1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*
tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^
2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(t
an(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(c)^2 + 4*a^2*log(4*(tan(d*x)^4*tan(c)^
2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan
(c) + 1)/(tan(c)^2 + 1))*tan(1/2*d*x)^2*tan(c)^2 + 16*a^2*tan(d*x)^2*tan(1/
2*c)*tan(c)^2 - 16*a^2*tan(1/2*d*x)^2*tan(1/2*c)*tan(c)^2 + 4*a^2*log(2*(ta
n(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*
tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d
*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*t
an(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*c)^2*tan(c)^2 -
4*a^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2
*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)
^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + ta
n(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2
*c)^2*tan(c)^2 + 4*a^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + t
an(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*ta
n(1/2*c)^2*tan(c)^2 - 16*a^2*tan(1/2*d*x)*tan(1/2*c)^2*tan(c)^2 + a^2*tan(d
*x)^2*tan(1/2*d*x)^2 + a^2*tan(d*x)^2*tan(1/2*c)^2 - a^2*tan(1/2*d*x)^2*tan
(1/2*c)^2 + 4*a^2*tan(d*x)*tan(1/2*d*x)^2*tan(c) + 4*a^2*tan(d*x)*tan(1/2*c
)^2*tan(c) - a^2*tan(d*x)^2*tan(c)^2 + a^2*tan(1/2*d*x)^2*tan(c)^2 + a^2*ta
n(1/2*c)^2*tan(c)^2 + 4*a^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*
d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(
1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 +
2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1
/2*c)^2 + 1))*tan(d*x)^2 - 4*a^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan
(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2
*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)
^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(

tan(1/2*c)^2 + 1))*tan(d*x)^2 + 4*a^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2 + 16*a^2*tan(d*x)^2*tan(1/2*d*x) + 4*a^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2 - 4*a^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2 + 4*a^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(1/2*d*x)^2 + 16*a^2*tan(d*x)^2*tan(1/2*c) - 16*a^2*tan(1/2*d*x)^2*tan(1/2*c) + 4*a^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*c)^2 - 4*a^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*c)^2 + 4*a^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(1/2*c)^2 - 16*a^2*tan(1/2*d*x)*tan(1/2*c)^2 + 4*a^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(c)^2 - 4*a^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(c)^2 + 4*a^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(c)^2 + 16*a^2*tan(1/2*d*x)*tan(c)^2 + 16*a^2*tan(1/2*c)*tan(c)^2 + a^2*tan(d*x)^2 - a^2*tan(1/2*d*x)^2 - a^2*tan(1/2*c)^2 + 4*a^2*tan(d*x)*tan(c) + a^2*tan(c)^2 + 4*a^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1)) - 4*a^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1)) + 4*a^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3

```
*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)
^2 + 1)) + 16*a^2*tan(1/2*d*x) + 16*a^2*tan(1/2*c) - a^2)/(d*tan(d*x)^2*tan
(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + d*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^
2 + d*tan(d*x)^2*tan(1/2*d*x)^2*tan(c)^2 + d*tan(d*x)^2*tan(1/2*c)^2*tan(c)
^2 + d*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + d*tan(d*x)^2*tan(1/2*d*x)^2 +
 d*tan(d*x)^2*tan(1/2*c)^2 + d*tan(1/2*d*x)^2*tan(1/2*c)^2 + d*tan(d*x)^2*t
an(c)^2 + d*tan(1/2*d*x)^2*tan(c)^2 + d*tan(1/2*c)^2*tan(c)^2 + d*tan(d*x)^
2 + d*tan(1/2*d*x)^2 + d*tan(1/2*c)^2 + d*tan(c)^2 + d)
```

maple [A] time = 0.14, size = 69, normalized size = 1.33

$$-\frac{a^2\left(\sin^2(dx+c)\right)}{2d}-\frac{2a^2\ln(\cos(dx+c))}{d}-\frac{2a^2\sin(dx+c)}{d}+\frac{2a^2\ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^2*tan(d*x+c),x)
```

```
[Out] -1/2*a^2*sin(d*x+c)^2/d-2/d*a^2*ln(cos(d*x+c))-2*a^2*sin(d*x+c)/d+2/d*a^2*l
n(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.31, size = 43, normalized size = 0.83

$$-\frac{a^2\sin(dx+c)^2+4\,a^2\log(\sin(dx+c)-1)+4\,a^2\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*(a^2*sin(d*x + c)^2 + 4*a^2*log(sin(d*x + c) - 1) + 4*a^2*sin(d*x + c)
)/d
```

mupad [B] time = 6.67, size = 178, normalized size = 3.42

$$-\frac{4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(4\,a^2\,\left(2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)-\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)\right)-2\,a^2\,\left(4\,\ln\left(\tan\left(\right.\right.\right.\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + a*sin(c + d*x))^2,x)
```

```
[Out] - (4*a^2*tan(c/2 + (d*x)/2)^3 - tan(c/2 + (d*x)/2)^2*(4*a^2*(2*log(tan(c/2
+ (d*x)/2) - 1) - log(tan(c/2 + (d*x)/2)^2 + 1)) - 2*a^2*(4*log(tan(c/2 + (
```

```
d*x)/2) - 1) - 2*log(tan(c/2 + (d*x)/2)^2 + 1) + 1)) + 4*a^2*tan(c/2 + (d*x
)/2))/(d*(tan(c/2 + (d*x)/2)^2 + 1)^2) - (2*a^2*(2*log(tan(c/2 + (d*x)/2) -
 1) - log(tan(c/2 + (d*x)/2)^2 + 1)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\sin\left(c+dx\right)\tan\left(c+dx\right)dx+\int \sin^2\left(c+dx\right)\tan\left(c+dx\right)dx+\int \tan\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**2*tan(d*x+c),x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)*tan(c + d*x), x) + Integral(sin(c + d*x)**2*t
an(c + d*x), x) + Integral(tan(c + d*x), x))
```

3.17 $\int \cot^3(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=30

$$-\frac{\csc^2(c+dx)(a\sin(c+dx)+a)^4}{2a^2d}$$

```
[Out] -1/2*csc(d*x+c)^2*(a+a*sin(d*x+c))^4/a^2/d
```

Rubi [A] time = 0.04, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 74}

$$-\frac{\csc^2(c+dx)(a\sin(c+dx)+a)^4}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(Csc[c + d*x]^2*(a + a*Sin[c + d*x])^4)/(2*a^2*d)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cot^3(c+dx)(a+a\sin(c+dx))^2\,dx &= \frac{\text{Subst}\left(\int \frac{(a-x)(a+x)^3}{x^3}\,dx, x, a\sin(c+dx)\right)}{d}\\ &= -\frac{\csc^2(c+dx)(a+a\sin(c+dx))^4}{2a^2d}\end{aligned}$$

Mathematica [A] time = 0.04, size = 28, normalized size = 0.93

$$-\frac{a^2(\sin(c+dx)+1)^4\csc^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/2*(a^2*Csc[c + d*x]^2*(1 + Sin[c + d*x])^4)/d
```

fricas [B] time = 0.41, size = 76, normalized size = 2.53

$$\frac{2\,a^2\cos\left(dx+c\right)^4-3\,a^2\cos\left(dx+c\right)^2+3\,a^2-8\left(a^2\cos\left(dx+c\right)^2-2\,a^2\right)\sin\left(dx+c\right)}{4\left(d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^2*cos(d*x + c)^4 - 3*a^2*cos(d*x + c)^2 + 3*a^2 - 8*(a^2*cos(d*x +
 c)^2 - 2*a^2)*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

giac [A] time = 0.39, size = 47, normalized size = 1.57

$$-\frac{a^2\left(\frac{1}{\sin(dx+c)}+\sin\left(dx+c\right)\right)^2+4\,a^2\left(\frac{1}{\sin(dx+c)}+\sin\left(dx+c\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(a^2*(1/sin(d*x + c) + sin(d*x + c))^2 + 4*a^2*(1/sin(d*x + c) + sin(d
*x + c)))/d
```

maple [B] time = 0.22, size = 94, normalized size = 3.13

$$\frac{a^2\left(\cos^2\left(dx+c\right)\right)}{2d}-\frac{2a^2\left(\cos^4\left(dx+c\right)\right)}{d\sin\left(dx+c\right)}-\frac{2a^2\left(\cos^2\left(dx+c\right)\right)\sin\left(dx+c\right)}{d}-\frac{4a^2\sin\left(dx+c\right)}{d}-\frac{a^2\left(\cot^2\left(dx+c\right)\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/2/d*a^2*cos(d*x+c)^2-2/d*a^2/sin(d*x+c)*cos(d*x+c)^4-2/d*a^2*cos(d*x+c)^2
*sin(d*x+c)-4*a^2*sin(d*x+c)/d-1/2/d*a^2*cot(d*x+c)^2
```

maxima [A] time = 0.31, size = 53, normalized size = 1.77

$$-\frac{a^2\sin(dx+c)^2+4a^2\sin(dx+c)+\frac{4a^2\sin(dx+c)+a^2}{\sin(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a^2*sin(d*x + c)^2 + 4*a^2*sin(d*x + c) + (4*a^2*sin(d*x + c) + a^2)/
sin(d*x + c)^2)/d
```

mupad [B] time = 6.67, size = 56, normalized size = 1.87

$$-\frac{a^2\left(2\sin(c+d\,x)^4+8\sin(c+d\,x)^3-\sin(c+d\,x)^2+8\,\sin(c+d\,x)+2\right)}{4\,d\sin(c+d\,x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + a*sin(c + d*x))^2,x)
```

```
[Out] -(a^2*(8*sin(c + d*x) - sin(c + d*x)^2 + 8*sin(c + d*x)^3 + 2*sin(c + d*x)^
4 + 2))/(4*d*sin(c + d*x)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\sin(c+dx)\cot^3(c+dx)\,dx+\int\sin^2(c+dx)\cot^3(c+dx)\,dx+\int\cot^3(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)*cot(c + d*x)**3, x) + Integral(sin(c + d*x)**
2*cot(c + d*x)**3, x) + Integral(cot(c + d*x)**3, x))
```

3.18 $\int \cot^7(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=132

$$-\frac{a^2\sin^2(c+dx)}{2d}-\frac{2a^2\sin(c+dx)}{d}-\frac{a^2\csc^6(c+dx)}{6d}-\frac{2a^2\csc^5(c+dx)}{5d}+\frac{a^2\csc^4(c+dx)}{2d}+\frac{2a^2\csc^3(c+dx)}{d}-\frac{6a^2\text{cs}}{}$$

```
[Out] -6*a^2*csc(d*x+c)/d+2*a^2*csc(d*x+c)^3/d+1/2*a^2*csc(d*x+c)^4/d-2/5*a^2*csc
(d*x+c)^5/d-1/6*a^2*csc(d*x+c)^6/d+2*a^2*ln(sin(d*x+c))/d-2*a^2*sin(d*x+c)/
d-1/2*a^2*sin(d*x+c)^2/d
```

Rubi [A] time = 0.08, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 88}

$$-\frac{a^2\sin^2(c+dx)}{2d}-\frac{2a^2\sin(c+dx)}{d}-\frac{a^2\csc^6(c+dx)}{6d}-\frac{2a^2\csc^5(c+dx)}{5d}+\frac{a^2\csc^4(c+dx)}{2d}+\frac{2a^2\csc^3(c+dx)}{d}-\frac{6a^2\text{cs}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-6*a^2*Csc[c + d*x])/d + (2*a^2*Csc[c + d*x]^3)/d + (a^2*Csc[c + d*x]^4)/(
2*d) - (2*a^2*Csc[c + d*x]^5)/(5*d) - (a^2*Csc[c + d*x]^6)/(6*d) + (2*a^2*L
og[Sin[c + d*x]])/d - (2*a^2*Sin[c + d*x])/d - (a^2*Sin[c + d*x]^2)/(2*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \cot^7(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)^5}{x^7}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-2a+\frac{a^8}{x^7}+\frac{2a^7}{x^6}-\frac{2a^6}{x^5}-\frac{6a^5}{x^4}+\frac{6a^3}{x^2}+\frac{2a^2}{x}-x\right)dx, x, a\sin(c+dx\right.}{d}$$
$$= -\frac{6a^2\csc(c+dx)}{d}+\frac{2a^2\csc^3(c+dx)}{d}+\frac{a^2\csc^4(c+dx)}{2d}-\frac{2a^2\csc^5(c+dx}{5d}$$

Mathematica [A] time = 0.22, size = 86, normalized size = 0.65

$$-\frac{a^2\left(15\sin^2(c+dx)+60\sin(c+dx)+5\csc^6(c+dx)+12\csc^5(c+dx)-15\csc^4(c+dx)-60\csc^3(c+dx)+180\right.}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/30*(a^2*(180*Csc[c + d*x] - 60*Csc[c + d*x]^3 - 15*Csc[c + d*x]^4 + 12*C
sc[c + d*x]^5 + 5*Csc[c + d*x]^6 - 60*Log[Sin[c + d*x]] + 60*Sin[c + d*x] +
 15*Sin[c + d*x]^2))/d
```

fricas [A] time = 0.46, size = 206, normalized size = 1.56

$$\frac{30\,a^2\cos(dx+c)^8-105\,a^2\cos(dx+c)^6+135\,a^2\cos(dx+c)^4-45\,a^2\cos(dx+c)^2-5\,a^2+120\left(a^2\cos(dx+c)\right.}{60\left(d\cos(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/60*(30*a^2*cos(d*x + c)^8 - 105*a^2*cos(d*x + c)^6 + 135*a^2*cos(d*x + c)
^4 - 45*a^2*cos(d*x + c)^2 - 5*a^2 + 120*(a^2*cos(d*x + c)^6 - 3*a^2*cos(d*
x + c)^4 + 3*a^2*cos(d*x + c)^2 - a^2)*log(1/2*sin(d*x + c)) - 24*(5*a^2*co
s(d*x + c)^6 - 30*a^2*cos(d*x + c)^4 + 40*a^2*cos(d*x + c)^2 - 16*a^2)*sin(
d*x + c))/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
```

giac [A] time = 0.96, size = 121, normalized size = 0.92

$$-\frac{15\,a^2\sin(dx+c)^2-60\,a^2\log(|\sin(dx+c)|)+60\,a^2\sin(dx+c)+\frac{147\,a^2\sin(dx+c)^6+180\,a^2\sin(dx+c)^5-60\,a^2\sin(dx+c)^3-15}{\sin(dx+c)^6}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/30*(15*a^2*sin(d*x + c)^2 - 60*a^2*log(abs(sin(d*x + c))) + 60*a^2*sin(d
*x + c) + (147*a^2*sin(d*x + c)^6 + 180*a^2*sin(d*x + c)^5 - 60*a^2*sin(d*x
 + c)^3 - 15*a^2*sin(d*x + c)^2 + 12*a^2*sin(d*x + c) + 5*a^2)/sin(d*x + c)
^6)/d
```

maple [B] time = 0.21, size = 313, normalized size = 2.37

$$-\frac{a^2\left(\cos^8(dx+c)\right)}{4d\sin(dx+c)^4}+\frac{a^2\left(\cos^8(dx+c)\right)}{2d\sin(dx+c)^2}+\frac{a^2\left(\cos^6(dx+c)\right)}{2d}+\frac{3a^2\left(\cos^4(dx+c)\right)}{4d}+\frac{3a^2\left(\cos^2(dx+c)\right)}{2d}+\frac{2a^2\ln(\mathrm{si}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^7*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/4/d*a^2/sin(d*x+c)^4*cos(d*x+c)^8+1/2/d*a^2/sin(d*x+c)^2*cos(d*x+c)^8+1/
2/d*a^2*cos(d*x+c)^6+3/4/d*a^2*cos(d*x+c)^4+3/2/d*a^2*cos(d*x+c)^2+2*a^2*ln
(sin(d*x+c))/d-2/5/d*a^2/sin(d*x+c)^5*cos(d*x+c)^8+2/5/d*a^2/sin(d*x+c)^3*c
os(d*x+c)^8-2/d*a^2/sin(d*x+c)*cos(d*x+c)^8-32/5*a^2*sin(d*x+c)/d-2/d*a^2*s
in(d*x+c)*cos(d*x+c)^6-12/5/d*a^2*sin(d*x+c)*cos(d*x+c)^4-16/5/d*a^2*cos(d*
x+c)^2*sin(d*x+c)-1/6/d*a^2*cot(d*x+c)^6+1/4/d*a^2*cot(d*x+c)^4-1/2/d*a^2*c
ot(d*x+c)^2
```

maxima [A] time = 0.31, size = 107, normalized size = 0.81

$$-\frac{15\,a^2\sin(dx+c)^2-60\,a^2\log(\sin(dx+c))+60\,a^2\sin(dx+c)+\frac{180\,a^2\sin(dx+c)^5-60\,a^2\sin(dx+c)^3-15\,a^2\sin(dx+c)^2+12}{\sin(dx+c)^6}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/30*(15*a^2*sin(d*x + c)^2 - 60*a^2*log(sin(d*x + c)) + 60*a^2*sin(d*x +
c) + (180*a^2*sin(d*x + c)^5 - 60*a^2*sin(d*x + c)^3 - 15*a^2*sin(d*x + c)^
2 + 12*a^2*sin(d*x + c) + 5*a^2)/sin(d*x + c)^6)/d
```

mupad [B] time = 11.18, size = 392, normalized size = 2.97

$$-\frac{a^2\left(24\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-20\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-312\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-220\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3864\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-360\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^7*(a + a*sin(c + d*x))^2,x)
```

```
[Out] -(a^2*(24*tan(c/2 + (d*x)/2) - 20*tan(c/2 + (d*x)/2)^2 - 312*tan(c/2 + (d*x
)/2)^3 - 220*tan(c/2 + (d*x)/2)^4 + 3864*tan(c/2 + (d*x)/2)^5 - 360*tan(c/2
 + (d*x)/2)^6 + 21000*tan(c/2 + (d*x)/2)^7 + 3510*tan(c/2 + (d*x)/2)^8 + 21
000*tan(c/2 + (d*x)/2)^9 - 360*tan(c/2 + (d*x)/2)^10 + 3864*tan(c/2 + (d*x)
/2)^11 - 220*tan(c/2 + (d*x)/2)^12 - 312*tan(c/2 + (d*x)/2)^13 - 20*tan(c/2
 + (d*x)/2)^14 + 24*tan(c/2 + (d*x)/2)^15 + 5*tan(c/2 + (d*x)/2)^16 + 3840*
tan(c/2 + (d*x)/2)^6*log(tan(c/2 + (d*x)/2)^2 + 1) + 7680*tan(c/2 + (d*x)/2
)^8*log(tan(c/2 + (d*x)/2)^2 + 1) + 3840*tan(c/2 + (d*x)/2)^10*log(tan(c/2
+ (d*x)/2)^2 + 1) - 3840*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^6 - 768
0*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^8 - 3840*log(tan(c/2 + (d*x)/2
))*tan(c/2 + (d*x)/2)^10 + 5))/(1920*d*tan(c/2 + (d*x)/2)^6*(tan(c/2 + (d*x
)/2)^2 + 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2 \left(\int 2 \sin(c + dx) \cot^7(c + dx)\, dx + \int \sin^2(c + dx) \cot^7(c + dx)\, dx + \int \cot^7(c + dx)\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**7*(a+a*sin(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)*cot(c + d*x)**7, x) + Integral(sin(c + d*x)**
2*cot(c + d*x)**7, x) + Integral(cot(c + d*x)**7, x))
```

3.19 $\int (a + a\sin(c + dx))^2 \tan^6(c + dx)\, dx$

Optimal. Leaf size=149

$$\frac{2a^2\cos(c+dx)}{d}+\frac{9a^2\tan^5(c+dx)}{10d}-\frac{3a^2\tan^3(c+dx)}{2d}+\frac{9a^2\tan(c+dx)}{2d}+\frac{2a^2\sec^5(c+dx)}{5d}-\frac{2a^2\sec^3(c+dx)}{d}+\frac{6a^2}{}$$

```
[Out] -9/2*a^2*x+2*a^2*cos(d*x+c)/d+6*a^2*sec(d*x+c)/d-2*a^2*sec(d*x+c)^3/d+2/5*a
^2*sec(d*x+c)^5/d+9/2*a^2*tan(d*x+c)/d-3/2*a^2*tan(d*x+c)^3/d+9/10*a^2*tan(
d*x+c)^5/d-1/2*a^2*sin(d*x+c)^2*tan(d*x+c)^5/d
```

Rubi [A] time = 0.17, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2710, 3473, 8, 2590, 270, 2591, 288, 302, 203}

$$\frac{2a^2\cos(c+dx)}{d}+\frac{9a^2\tan^5(c+dx)}{10d}-\frac{3a^2\tan^3(c+dx)}{2d}+\frac{9a^2\tan(c+dx)}{2d}+\frac{2a^2\sec^5(c+dx)}{5d}-\frac{2a^2\sec^3(c+dx)}{d}+\frac{6a^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^6,x]
```

```
[Out] (-9*a^2*x)/2 + (2*a^2*Cos[c + d*x])/d + (6*a^2*Sec[c + d*x])/d - (2*a^2*Sec
[c + d*x]^3)/d + (2*a^2*Sec[c + d*x]^5)/(5*d) + (9*a^2*Tan[c + d*x])/(2*d)
- (3*a^2*Tan[c + d*x]^3)/(2*d) + (9*a^2*Tan[c + d*x]^5)/(10*d) - (a^2*Sin[c
 + d*x]^2*Tan[c + d*x]^5)/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
```

```
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2710

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int (a+a\sin(c+dx))^2\tan^6(c+dx)\,dx = \int \left(a^2\tan^6(c+dx)+2a^2\sin(c+dx)\tan^6(c+dx)+a^2\sin^2(c+dx)\tan\right.$$
$$= a^2\int \tan^6(c+dx)\,dx + a^2\int \sin^2(c+dx)\tan^6(c+dx)\,dx + \left(2a^2\right)\int \mathrm{si}$$
$$= \frac{a^2\tan^5(c+dx)}{5d} - a^2\int \tan^4(c+dx)\,dx + \frac{a^2\,\mathrm{Subst}\left(\int \frac{x^8}{(1+x^2)^2}\,dx, x, \tan\right.}{d}$$
$$= -\frac{a^2\tan^3(c+dx)}{3d} + \frac{a^2\tan^5(c+dx)}{5d} - \frac{a^2\sin^2(c+dx)\tan^5(c+dx)}{2d} + a$$
$$= \frac{2a^2\cos(c+dx)}{d} + \frac{6a^2\sec(c+dx)}{d} - \frac{2a^2\sec^3(c+dx)}{d} + \frac{2a^2\sec^5(c+d}{5d}$$
$$= -a^2x + \frac{2a^2\cos(c+dx)}{d} + \frac{6a^2\sec(c+dx)}{d} - \frac{2a^2\sec^3(c+dx)}{d} + \frac{2a^2\,\mathrm{se}}{}$$
$$= -\frac{9a^2x}{2} + \frac{2a^2\cos(c+dx)}{d} + \frac{6a^2\sec(c+dx)}{d} - \frac{2a^2\sec^3(c+dx)}{d} + \frac{2a^2\,\mathrm{s}}{}$$

Mathematica [A] time = 0.84, size = 174, normalized size = 1.17

$$-\frac{a^2\sec^5(c+dx)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4(250\sin(c+dx)-720c\sin(2(c+dx))-720dx\sin(2(c+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^6,x]
```

```
[Out] -1/160*(a^2*Sec[c + d*x]^5*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(-500 +
10*(103 + 90*c + 90*d*x)*Cos[c + d*x] - 544*Cos[2*(c + d*x)] - 206*Cos[3*(c
 + d*x)] - 180*c*Cos[3*(c + d*x)] - 180*d*x*Cos[3*(c + d*x)] + 20*Cos[4*(c
+ d*x)] + 250*Sin[c + d*x] - 824*Sin[2*(c + d*x)] - 720*c*Sin[2*(c + d*x)]
- 720*d*x*Sin[2*(c + d*x)] + 351*Sin[3*(c + d*x)] + 5*Sin[5*(c + d*x)]))/d
```

fricas [A] time = 0.46, size = 152, normalized size = 1.02

$$-\frac{45\,a^2dx\cos(dx+c)^3-10\,a^2\cos(dx+c)^4-90\,a^2dx\cos(dx+c)+78\,a^2\cos(dx+c)^2-4\,a^2-\left(5\,a^2\cos(dx+\right.}{10\left(d\cos(dx+c)^3+2\,d\cos(dx+c)\sin(dx+c)-2\,d\,\mathrm{c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] -1/10*(45*a^2*d*x*cos(d*x + c)^3 - 10*a^2*cos(d*x + c)^4 - 90*a^2*d*x*cos(d
*x + c) + 78*a^2*cos(d*x + c)^2 - 4*a^2 - (5*a^2*cos(d*x + c)^4 - 90*a^2*d*
x*cos(d*x + c) + 84*a^2*cos(d*x + c)^2 - 6*a^2)*sin(d*x + c))/(d*cos(d*x +
c)^3 + 2*d*cos(d*x + c)*sin(d*x + c) - 2*d*cos(d*x + c))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^6,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.31, size = 251, normalized size = 1.68

$$\frac{a^2\left(\frac{\sin^9(dx+c)}{5\cos(dx+c)^5}-\frac{4\left(\sin^9(dx+c)\right)}{15\cos(dx+c)^3}+\frac{8\left(\sin^9(dx+c)\right)}{5\cos(dx+c)}+\frac{8\left(\sin^7(dx+c)+\frac{7\left(\sin^5(dx+c)\right)}{6}+\frac{35\left(\sin^3(dx+c)\right)}{24}+\frac{35\sin(dx+c)}{16}\right)\cos(dx+c)}{5}-\frac{7dx}{2}-\frac{7c}{2}\right)+2a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^2*tan(d*x+c)^6,x)
```

```
[Out] 1/d*(a^2*(1/5*sin(d*x+c)^9/cos(d*x+c)^5-4/15*sin(d*x+c)^9/cos(d*x+c)^3+8/5*
sin(d*x+c)^9/cos(d*x+c)+8/5*(sin(d*x+c)^7+7/6*sin(d*x+c)^5+35/24*sin(d*x+c)
^3+35/16*sin(d*x+c))*cos(d*x+c)-7/2*d*x-7/2*c)+2*a^2*(1/5*sin(d*x+c)^8/cos(
d*x+c)^5-1/5*sin(d*x+c)^8/cos(d*x+c)^3+sin(d*x+c)^8/cos(d*x+c)+(16/5+sin(d*
x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))+a^2*(1/5*tan(d*x+c)^5
-1/3*tan(d*x+c)^3+tan(d*x+c)-d*x-c))
```

maxima [A] time = 0.42, size = 152, normalized size = 1.02

$$\frac{\left(6\tan(dx+c)^5-20\tan(dx+c)^3-105\,dx-105\,c+\frac{15\tan(dx+c)}{\tan(dx+c)^2+1}+90\tan(dx+c)\right)a^2+2\left(3\tan(dx+c)^5-5\ \mathrm{t}\right.}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/30*((6*tan(d*x + c)^5 - 20*tan(d*x + c)^3 - 105*d*x - 105*c + 15*tan(d*x
+ c)/(tan(d*x + c)^2 + 1) + 90*tan(d*x + c))*a^2 + 2*(3*tan(d*x + c)^5 - 5*
```

```
tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(d*x + c))*a^2 + 12*a^2*((15*cos(d*x
 + c)^4 - 5*cos(d*x + c)^2 + 1)/cos(d*x + c)^5 + 5*cos(d*x + c)))/d
```

mupad [B] time = 10.92, size = 392, normalized size = 2.63

$$-\frac{9a^2x}{2}-\frac{\frac{9a^2(c+dx)}{2}-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(18a^2(c+dx)-\frac{a^2(180c+180dx-422)}{10}\right)+\frac{174a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{5}-\frac{a^2(45c+45dx-128)}{10}+\text{t}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^6*(a + a*sin(c + d*x))^2,x)
```

```
[Out] - (9*a^2*x)/2 - ((9*a^2*(c + d*x))/2 - tan(c/2 + (d*x)/2)*(18*a^2*(c + d*x)
 - (a^2*(180*c + 180*d*x - 422))/10) + (174*a^2*tan(c/2 + (d*x)/2)^5)/5 - (
a^2*(45*c + 45*d*x - 128))/10 + tan(c/2 + (d*x)/2)^9*(18*a^2*(c + d*x) - (a
^2*(180*c + 180*d*x - 90))/10) + tan(c/2 + (d*x)/2)^4*(27*a^2*(c + d*x) - (
a^2*(270*c + 270*d*x - 168))/10) - tan(c/2 + (d*x)/2)^8*((63*a^2*(c + d*x))
/2 - (a^2*(315*c + 315*d*x - 360))/10) - tan(c/2 + (d*x)/2)^6*(27*a^2*(c +
d*x) - (a^2*(270*c + 270*d*x - 600))/10) - tan(c/2 + (d*x)/2)^3*(36*a^2*(c
+ d*x) - (a^2*(360*c + 360*d*x - 424))/10) + tan(c/2 + (d*x)/2)^2*((63*a^2*
(c + d*x))/2 - (a^2*(315*c + 315*d*x - 536))/10) + tan(c/2 + (d*x)/2)^7*(36
*a^2*(c + d*x) - (a^2*(360*c + 360*d*x - 600))/10))/(d*(tan(c/2 + (d*x)/2)
- 1)^5*(tan(c/2 + (d*x)/2) + 1)*(tan(c/2 + (d*x)/2)^2 + 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\sin(c+dx)\tan^6(c+dx)\,dx+\int \sin^2(c+dx)\tan^6(c+dx)\,dx+\int \tan^6(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**2*tan(d*x+c)**6,x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)*tan(c + d*x)**6, x) + Integral(sin(c + d*x)**
2*tan(c + d*x)**6, x) + Integral(tan(c + d*x)**6, x))
```

3.20 $\int (a + a\sin(c + dx))^2 \tan^4(c + dx)\,dx$

Optimal. Leaf size=120

$$\frac{a^4 \sin^3(c+dx)\cos(c+dx)}{3d(a-a\sin(c+dx))^2}-\frac{16a^2\cos(c+dx)}{3d}-\frac{8a^2\sin^2(c+dx)\cos(c+dx)}{3d(1-\sin(c+dx))}-\frac{7a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{7a^2x}{2}$$

```
[Out] 7/2*a^2*x-16/3*a^2*cos(d*x+c)/d-7/2*a^2*cos(d*x+c)*sin(d*x+c)/d-8/3*a^2*cos
(d*x+c)*sin(d*x+c)^2/d/(1-sin(d*x+c))+1/3*a^4*cos(d*x+c)*sin(d*x+c)^3/d/(a-
a*sin(d*x+c))^2
```

Rubi [A] time = 0.20, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2708, 2765, 2977, 2734}

$$-\frac{16a^2\cos(c+dx)}{3d}+\frac{a^4 \sin^3(c+dx)\cos(c+dx)}{3d(a-a\sin(c+dx))^2}-\frac{8a^2\sin^2(c+dx)\cos(c+dx)}{3d(1-\sin(c+dx))}-\frac{7a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{7a^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^4,x]
```

```
[Out] (7*a^2*x)/2 - (16*a^2*Cos[c + d*x])/(3*d) - (7*a^2*Cos[c + d*x]*Sin[c + d*x
])/(2*d) - (8*a^2*Cos[c + d*x]*Sin[c + d*x]^2)/(3*d*(1 - Sin[c + d*x])) + (
a^4*Cos[c + d*x]*Sin[c + d*x]^3)/(3*d*(a - a*Sin[c + d*x])^2)
```

Rule 2708

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_), x_Symbol] :> Dist[a^p, Int[Sin[e + f*x]^p/(a - b*Sin[e + f*x])^m, x], x
] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[
p, 2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
```

```
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int (a+a\sin(c+dx))^2\tan^4(c+dx)\,dx = a^4\int \frac{\sin^4(c+dx)}{(a-a\sin(c+dx))^2}\,dx$$
$$= \frac{a^4\cos(c+dx)\sin^3(c+dx)}{3d(a-a\sin(c+dx))^2}+\frac{1}{3}a^2\int \frac{\sin^2(c+dx)(-3a-5a\sin(c+dx)}{a-a\sin(c+dx)}$$
$$= -\frac{8a^2\cos(c+dx)\sin^2(c+dx)}{3d(1-\sin(c+dx))}+\frac{a^4\cos(c+dx)\sin^3(c+dx)}{3d(a-a\sin(c+dx))^2}-\frac{1}{3}\int \sin$$
$$= \frac{7a^2x}{2}-\frac{16a^2\cos(c+dx)}{3d}-\frac{7a^2\cos(c+dx)\sin(c+dx)}{2d}-\frac{8a^2\cos(c+}{3d(1-\sin}$$

Mathematica [A] time = 1.31, size = 159, normalized size = 1.32

$$-\frac{a^2\left(-21(12c+12dx+7)\cos\left(\frac{1}{2}(c+dx)\right)+(84c+84dx+239)\cos\left(\frac{3}{2}(c+dx)\right)+3\left(-5\cos\left(\frac{5}{2}(c+dx)\right)+\cos\left(\right.\right.\right.}{48d\left(\cos\left(\frac{1}{2}(c+\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^4,x]
```

```
[Out] -1/48*(a^2*(-21*(7 + 12*c + 12*d*x)*Cos[(c + d*x)/2] + (239 + 84*c + 84*d*x
)*Cos[(3*(c + d*x))/2] + 3*(-5*Cos[(5*(c + d*x))/2] + Cos[(7*(c + d*x))/2]
```

```
+ 2*(50 + 56*c + 56*d*x + (-27 + 28*c + 28*d*x)*Cos[c + d*x] - 6*Cos[2*(c +
 d*x)] - Cos[3*(c + d*x)])*Sin[(c + d*x)/2])))/(d*(Cos[(c + d*x)/2] - Sin[(
c + d*x)/2])^3)
```

fricas [A] time = 0.43, size = 196, normalized size = 1.63

$$\frac{3\,a^2\cos\left(dx+c\right)^4-6\,a^2\cos\left(dx+c\right)^3-42\,a^2dx+\left(21\,a^2dx+31\,a^2\right)\cos\left(dx+c\right)^2-2\,a^2-\left(21\,a^2dx-38\,a^2\right)\cos\left(\cdots\right.}{6\left(d\cos\left(dx+c\right)^2-d\cos\left(dx+c\right)+\left(d\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*a^2*cos(d*x + c)^4 - 6*a^2*cos(d*x + c)^3 - 42*a^2*d*x + (21*a^2*d*x
 + 31*a^2)*cos(d*x + c)^2 - 2*a^2 - (21*a^2*d*x - 38*a^2)*cos(d*x + c) - (3
*a^2*cos(d*x + c)^3 - 42*a^2*d*x + 9*a^2*cos(d*x + c)^2 + 2*a^2 - (21*a^2*d
*x - 40*a^2)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2 - d*cos(d*x + c)
 + (d*cos(d*x + c) + 2*d)*sin(d*x + c) - 2*d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.29, size = 186, normalized size = 1.55

$$\frac{a^2\left(\frac{\sin^7(dx+c)}{3\cos(dx+c)^3}-\frac{4\left(\sin^7(dx+c)\right)}{3\cos(dx+c)}-\frac{4\left(\sin^5(dx+c)+\frac{5\left(\sin^3(dx+c)\right)}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{3}+\frac{5dx}{2}+\frac{5c}{2}\right)+2a^2\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^6(dx+c)}{\cos(dx+c)}-\cdots\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^2*tan(d*x+c)^4,x)
```

```
[Out] 1/d*(a^2*(1/3*sin(d*x+c)^7/cos(d*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/3*(si
n(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c)+2*a^
2*(1/3*sin(d*x+c)^6/cos(d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+
4/3*sin(d*x+c)^2)*cos(d*x+c))+a^2*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c))
```

maxima [A] time = 0.40, size = 120, normalized size = 1.00

$$\frac{\left(2\tan\left(dx+c\right)^{3}+15\,dx+15\,c-\frac{3\tan(dx+c)}{\tan(dx+c)^{2}+1}-12\tan\left(dx+c\right)\right)a^{2}+2\left(\tan\left(dx+c\right)^{3}+3\,dx+3\,c-3\tan\left(dx+c\right.\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/6*((2*tan(d*x + c)^3 + 15*d*x + 15*c - 3*tan(d*x + c)/(tan(d*x + c)^2 + 1
) - 12*tan(d*x + c))*a^2 + 2*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c)
)*a^2 - 4*a^2*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x + c)))/d
```

mupad [B] time = 10.07, size = 287, normalized size = 2.39

$$\frac{7\,a^2\,x}{2}+\frac{\frac{7\,a^2\,(c+d\,x)}{2}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{21\,a^2\,(c+d\,x)}{2}-\frac{a^2\,(63\,c+63\,d\,x-150)}{6}\right)-\frac{a^2\,(21\,c+21\,d\,x-64)}{6}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(\frac{21\,a^2\,(c+d\,x)}{2}-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (7*a^2*x)/2 + ((7*a^2*(c + d*x))/2 - tan(c/2 + (d*x)/2)*((21*a^2*(c + d*x))
/2 - (a^2*(63*c + 63*d*x - 150))/6) - (a^2*(21*c + 21*d*x - 64))/6 + tan(c/
2 + (d*x)/2)^6*((21*a^2*(c + d*x))/2 - (a^2*(63*c + 63*d*x - 42))/6) - tan(
c/2 + (d*x)/2)^5*((35*a^2*(c + d*x))/2 - (a^2*(105*c + 105*d*x - 126))/6) +
 tan(c/2 + (d*x)/2)^2*((35*a^2*(c + d*x))/2 - (a^2*(105*c + 105*d*x - 194))
/6) + tan(c/2 + (d*x)/2)^4*((49*a^2*(c + d*x))/2 - (a^2*(147*c + 147*d*x -
196))/6) - tan(c/2 + (d*x)/2)^3*((49*a^2*(c + d*x))/2 - (a^2*(147*c + 147*d
*x - 252))/6))/(d*(tan(c/2 + (d*x)/2) - 1)^3*(tan(c/2 + (d*x)/2)^2 + 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int 2\sin\left(c+dx\right)\tan^{4}\left(c+dx\right)dx+\int\sin^{2}\left(c+dx\right)\tan^{4}\left(c+dx\right)dx+\int\tan^{4}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**2*tan(d*x+c)**4,x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)*tan(c + d*x)**4, x) + Integral(sin(c + d*x)**
2*tan(c + d*x)**4, x) + Integral(tan(c + d*x)**4, x))
```

3.21 $\int (a + a\sin(c + dx))^2 \tan^2(c + dx)\,dx$

Optimal. Leaf size=71

$$\frac{2a^2\cos(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{2a^2\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{5a^2x}{2}$$

```
[Out] -5/2*a^2*x+2*a^2*cos(d*x+c)/d+2*a^2*cos(d*x+c)/d/(1-sin(d*x+c))+1/2*a^2*cos
(d*x+c)*sin(d*x+c)/d
```

Rubi [A] time = 0.09, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2709, 2648, 2638, 2635, 8}

$$\frac{2a^2\cos(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{2a^2\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{5a^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] (-5*a^2*x)/2 + (2*a^2*Cos[c + d*x])/d + (2*a^2*Cos[c + d*x])/(d*(1 - Sin[c
+ d*x])) + (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(c+dx))^2\tan^2(c+dx)\,dx &= a^2\int\left(-2-\frac{2}{-1+\sin(c+dx)}-2\sin(c+dx)-\sin^2(c+dx)\right)dx\\ &= -2a^2x-a^2\int\sin^2(c+dx)\,dx-\left(2a^2\right)\int\frac{1}{-1+\sin(c+dx)}\,dx-\left(2a^2\right)\int\\ &= -2a^2x+\frac{2a^2\cos(c+dx)}{d}+\frac{2a^2\cos(c+dx)}{d(1-\sin(c+dx))}+\frac{a^2\cos(c+dx)\sin(c+}{2d}\\ &= -\frac{5a^2x}{2}+\frac{2a^2\cos(c+dx)}{d}+\frac{2a^2\cos(c+dx)}{d(1-\sin(c+dx))}+\frac{a^2\cos(c+dx)\sin(c+}{2d}\end{aligned}$$

Mathematica [B] time = 0.43, size = 145, normalized size = 2.04

$$-\frac{a^2(\sin(c+dx)+1)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)(10(c+dx)-\sin(2(c+dx))-8\cos(c+dx))+\sin\left(\frac{1}{2}(c+dx)\right)(-2(5c+}{4d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] -1/4*(a^2*(1 + Sin[c + d*x])^2*(Cos[(c + d*x)/2]*(10*(c + d*x) - 8*Cos[c +
d*x] - Sin[2*(c + d*x)]) + Sin[(c + d*x)/2]*(-2*(8 + 5*c + 5*d*x) + 8*Cos[c
 + d*x] + Sin[2*(c + d*x)])))/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos
[(c + d*x)/2] + Sin[(c + d*x)/2])^4)
```

fricas [A] time = 0.42, size = 125, normalized size = 1.76

$$\frac{a^2\cos(dx+c)^3-5a^2dx+4a^2\cos(dx+c)^2+4a^2-\left(5a^2dx-7a^2\right)\cos(dx+c)+\left(5a^2dx+a^2\cos(dx+c)^2-\right.}{2\left(d\cos(dx+c)-d\sin(dx+c)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*cos(d*x + c)^3 - 5*a^2*d*x + 4*a^2*cos(d*x + c)^2 + 4*a^2 - (5*a^2
*d*x - 7*a^2)*cos(d*x + c) + (5*a^2*d*x + a^2*cos(d*x + c)^2 - 3*a^2*cos(d*
x + c) + 4*a^2)*sin(d*x + c))/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

giac [B] time = 24.52, size = 5370, normalized size = 75.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(5*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 + 5*a^2*d*x
*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) - 5*a^2*d*x*tan(d*x)^2*tan(1
/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 - 20*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^3*tan(
1/2*c)^3*tan(c)^3 + 5*a^2*d*x*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3
 - 8*a^2*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 + 5*a^2*tan(d*x)^3
*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 + 5*a^2*tan(d*x)^2*tan(1/2*d*x)^4*tan
(1/2*c)^4*tan(c)^3 - 5*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4 - 20*
a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) + 5*a^2*d*x*tan(d*x)*
tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) - 8*a^2*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/
2*c)^4*tan(c) + 20*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2
- 5*a^2*d*x*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 + 8*a^2*tan(d*x)^2*tan(1/2
*d*x)^4*tan(1/2*c)^4*tan(c)^2 - 5*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^4*tan(c)^
3 - 20*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^3 - 20*a^2*d*x*t
an(d*x)^3*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^3 - 20*a^2*d*x*tan(d*x)*tan(1/2*
d*x)^3*tan(1/2*c)^3*tan(c)^3 + 32*a^2*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)^
3*tan(c)^3 - 5*a^2*d*x*tan(d*x)^3*tan(1/2*c)^4*tan(c)^3 - 8*a^2*tan(d*x)*ta
n(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 + 4*a^2*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/
2*c)^4 + 2*a^2*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) - 20*a^2*tan(d
*x)^3*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 + 2*a^2*tan(d*x)*tan(1/2*d*x)^4*
tan(1/2*c)^4*tan(c)^2 - 20*a^2*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c
)^3 + 4*a^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 + 20*a^2*d*x*tan(d*x)^2*ta
n(1/2*d*x)^3*tan(1/2*c)^3 - 5*a^2*d*x*tan(1/2*d*x)^4*tan(1/2*c)^4 + 8*a^2*t
an(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4 - 5*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^4
*tan(c) - 20*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 20*a^2*d
*x*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - 20*a^2*d*x*tan(d*x)*tan(1/
2*d*x)^3*tan(1/2*c)^3*tan(c) + 32*a^2*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)^
3*tan(c) - 5*a^2*d*x*tan(d*x)^3*tan(1/2*c)^4*tan(c) - 8*a^2*tan(d*x)*tan(1/
2*d*x)^4*tan(1/2*c)^4*tan(c) + 5*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*tan(c)^2
 + 20*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^2 + 20*a^2*d*x*ta
n(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^2 + 20*a^2*d*x*tan(1/2*d*x)^3*tan
(1/2*c)^3*tan(c)^2 - 32*a^2*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2
 + 5*a^2*d*x*tan(d*x)^2*tan(1/2*c)^4*tan(c)^2 + 8*a^2*tan(1/2*d*x)^4*tan(1/
```

2*c)^4*tan(c)^2 - 5*a^2*d*x*tan(d*x)*tan(1/2*d*x)^4*tan(c)^3 - 8*a^2*tan(d*x)^3*tan(1/2*d*x)^4*tan(c)^3 - 20*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)*tan(c)^3 - 20*a^2*d*x*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^3 - 32*a^2*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^3 - 96*a^2*tan(d*x)^3*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^3 - 20*a^2*d*x*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^3 - 32*a^2*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^3 + 32*a^2*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^3 - 5*a^2*d*x*tan(d*x)*tan(1/2*c)^4*tan(c)^3 - 8*a^2*tan(d*x)^3*tan(1/2*c)^4*tan(c)^3 - 16*a^2*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)^3 + 5*a^2*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4 - 8*a^2*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) + 5*a^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) - 5*a^2*tan(d*x)^3*tan(1/2*d*x)^4*tan(c)^2 - 20*a^2*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^2 - 20*a^2*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^2 - 8*a^2*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 - 5*a^2*tan(d*x)^3*tan(1/2*c)^4*tan(c)^2 - 5*a^2*tan(d*x)^2*tan(1/2*d*x)^4*tan(c)^3 - 20*a^2*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^3 - 20*a^2*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^3 - 16*a^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^3 - 5*a^2*tan(d*x)^2*tan(1/2*c)^4*tan(c)^3 + 5*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4 + 20*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c) + 20*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^3 + 20*a^2*d*x*tan(1/2*d*x)^3*tan(1/2*c)^3 - 32*a^2*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3 + 5*a^2*d*x*tan(d*x)^2*tan(1/2*c)^4 + 8*a^2*tan(1/2*d*x)^4*tan(1/2*c)^4 - 5*a^2*d*x*tan(d*x)*tan(1/2*d*x)^4*tan(c) - 8*a^2*tan(d*x)^3*tan(1/2*d*x)^4*tan(c) - 20*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)*tan(c) - 20*a^2*d*x*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 32*a^2*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 96*a^2*tan(d*x)^3*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c) - 20*a^2*d*x*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - 32*a^2*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) + 32*a^2*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) - 5*a^2*d*x*tan(d*x)*tan(1/2*c)^4*tan(c) - 8*a^2*tan(d*x)^3*tan(1/2*c)^4*tan(c) + 5*a^2*d*x*tan(1/2*d*x)^4*tan(c)^2 + 8*a^2*tan(d*x)^2*tan(1/2*d*x)^4*tan(c)^2 + 20*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)*tan(c)^2 + 20*a^2*d*x*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^2 + 32*a^2*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^2 + 96*a^2*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + 20*a^2*d*x*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^2 + 32*a^2*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^2 - 32*a^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 + 5*a^2*d*x*tan(1/2*c)^4*tan(c)^2 + 8*a^2*tan(d*x)^2*tan(1/2*c)^4*tan(c)^2 + 5*a^2*d*x*tan(d*x)^3*tan(c)^3 - 8*a^2*tan(d*x)*tan(1/2*d*x)^4*tan(c)^3 - 20*a^2*d*x*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*tan(c)^3 + 32*a^2*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)*tan(c)^3 - 32*a^2*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^3 - 96*a^2*tan(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^3 - 32*a^2*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^3 - 8*a^2*tan(d*x)*tan(1/2*c)^4*tan(c)^3 - 4*a^2*tan(d*x)^3*tan(1/2*d*x)^4 - 16*a^2*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c) - 16*a^2*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)^3 - 20*a^2*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3 - 4*a^2*tan(d*x)^3*tan(1/2*c)^4 - 2*a^2*tan(d*x)^2*tan(1/2*d*x)^4*tan(c) - 8*a^2*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 8*a^2*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - 20*a^2*tan(1/2*d*x)^3*

```
tan(1/2*c)^3*tan(c) - 2*a^2*tan(d*x)^2*tan(1/2*c)^4*tan(c) - 2*a^2*tan(d*x)
*tan(1/2*d*x)^4*tan(c)^2 - 20*a^2*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)*tan(c)
^2 - 8*a^2*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^2 - 8*a^2*tan(d*x)*tan
(1/2*d*x)*tan(1/2*c)^3*tan(c)^2 - 2*a^2*tan(d*x)*tan(1/2*c)^4*tan(c)^2 - 4*
a^2*tan(1/2*d*x)^4*tan(c)^3 - 20*a^2*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)*tan
(c)^3 - 16*a^2*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^3 - 16*a^2*tan(1/2*d*x)*tan
(1/2*c)^3*tan(c)^3 - 4*a^2*tan(1/2*c)^4*tan(c)^3 + 5*a^2*d*x*tan(1/2*d*x)^4
 + 8*a^2*tan(d*x)^2*tan(1/2*d*x)^4 + 20*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)*tan
(1/2*c) + 20*a^2*d*x*tan(1/2*d*x)^3*tan(1/2*c) + 32*a^2*tan(d*x)^2*tan(1/2*
d*x)^3*tan(1/2*c) + 96*a^2*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 20*a^2*
d*x*tan(1/2*d*x)*tan(1/2*c)^3 + 32*a^2*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^3
 - 32*a^2*tan(1/2*d*x)^3*tan(1/2*c)^3 + 5*a^2*d*x*tan(1/2*c)^4 + 8*a^2*tan(
d*x)^2*tan(1/2*c)^4 + 5*a^2*d*x*tan(d*x)^3*tan(c) - 8*a^2*tan(d*x)*tan(1/2*
d*x)^4*tan(c) - 20*a^2*d*x*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*tan(c) + 32*a^2
*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)*tan(c) - 32*a^2*tan(d*x)*tan(1/2*d*x)^3
*tan(1/2*c)*tan(c) - 96*a^2*tan(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c) - 3
2*a^2*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - 8*a^2*tan(d*x)*tan(1/2*c)
^4*tan(c) - 5*a^2*d*x*tan(d*x)^2*tan(c)^2 + 8*a^2*tan(1/2*d*x)^4*tan(c)^2 +
 20*a^2*d*x*tan(1/2*d*x)*tan(1/2*c)*tan(c)^2 - 32*a^2*tan(d*x)^2*tan(1/2*d*
x)*tan(1/2*c)*tan(c)^2 + 32*a^2*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^2 + 96*a^2
*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + 32*a^2*tan(1/2*d*x)*tan(1/2*c)^3*ta
n(c)^2 + 8*a^2*tan(1/2*c)^4*tan(c)^2 + 5*a^2*d*x*tan(d*x)*tan(c)^3 - 8*a^2*
tan(d*x)^3*tan(c)^3 + 32*a^2*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*tan(c)^3 - 5*
a^2*tan(d*x)*tan(1/2*d*x)^4 - 16*a^2*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c) - 2
0*a^2*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c) - 20*a^2*tan(d*x)*tan(1/2*d*x)*tan
(1/2*c)^3 - 5*a^2*tan(d*x)*tan(1/2*c)^4 - 5*a^2*tan(1/2*d*x)^4*tan(c) - 8*a
^2*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)*tan(c) - 20*a^2*tan(1/2*d*x)^3*tan(1/
2*c)*tan(c) - 20*a^2*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - 5*a^2*tan(1/2*c)^4*
tan(c) + 5*a^2*tan(d*x)^3*tan(c)^2 - 8*a^2*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)
*tan(c)^2 + 5*a^2*tan(d*x)^2*tan(c)^3 - 16*a^2*tan(1/2*d*x)*tan(1/2*c)*tan(
c)^3 - 5*a^2*d*x*tan(d*x)^2 + 8*a^2*tan(1/2*d*x)^4 + 20*a^2*d*x*tan(1/2*d*x
)*tan(1/2*c) - 32*a^2*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c) + 32*a^2*tan(1/2*d
*x)^3*tan(1/2*c) + 96*a^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 32*a^2*tan(1/2*d*x)
*tan(1/2*c)^3 + 8*a^2*tan(1/2*c)^4 + 5*a^2*d*x*tan(d*x)*tan(c) - 8*a^2*tan(
d*x)^3*tan(c) + 32*a^2*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*tan(c) - 5*a^2*d*x*
tan(c)^2 + 8*a^2*tan(d*x)^2*tan(c)^2 - 32*a^2*tan(1/2*d*x)*tan(1/2*c)*tan(c
)^2 - 8*a^2*tan(d*x)*tan(c)^3 + 4*a^2*tan(d*x)^3 - 20*a^2*tan(d*x)*tan(1/2*
d*x)*tan(1/2*c) + 2*a^2*tan(d*x)^2*tan(c) - 20*a^2*tan(1/2*d*x)*tan(1/2*c)*
tan(c) + 2*a^2*tan(d*x)*tan(c)^2 + 4*a^2*tan(c)^3 - 5*a^2*d*x + 8*a^2*tan(d
*x)^2 - 32*a^2*tan(1/2*d*x)*tan(1/2*c) - 8*a^2*tan(d*x)*tan(c) + 8*a^2*tan(
c)^2 + 5*a^2*tan(d*x) + 5*a^2*tan(c) + 8*a^2)/(d*tan(d*x)^3*tan(1/2*d*x)^4*
tan(1/2*c)^4*tan(c)^3 + d*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) - d
*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 - 4*d*tan(d*x)^3*tan(1/2*d
*x)^3*tan(1/2*c)^3*tan(c)^3 + d*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)
^3 - d*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*d*tan(d*x)^3*tan(1/2*d*x)
```

```
^3*tan(1/2*c)^3*tan(c) + d*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) + 4*
d*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 - d*tan(1/2*d*x)^4*tan(1/
2*c)^4*tan(c)^2 - d*tan(d*x)^3*tan(1/2*d*x)^4*tan(c)^3 - 4*d*tan(d*x)^3*tan
(1/2*d*x)^3*tan(1/2*c)*tan(c)^3 - 4*d*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)^3*
tan(c)^3 - 4*d*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^3 - d*tan(d*x)^3
*tan(1/2*c)^4*tan(c)^3 + 4*d*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3 - d*tan
(1/2*d*x)^4*tan(1/2*c)^4 - d*tan(d*x)^3*tan(1/2*d*x)^4*tan(c) - 4*d*tan(d*x
)^3*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 4*d*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*
c)^3*tan(c) - 4*d*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) - d*tan(d*x)^
3*tan(1/2*c)^4*tan(c) + d*tan(d*x)^2*tan(1/2*d*x)^4*tan(c)^2 + 4*d*tan(d*x)
^2*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^2 + 4*d*tan(d*x)^2*tan(1/2*d*x)*tan(1/2
*c)^3*tan(c)^2 + 4*d*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 + d*tan(d*x)^2*ta
n(1/2*c)^4*tan(c)^2 - d*tan(d*x)*tan(1/2*d*x)^4*tan(c)^3 - 4*d*tan(d*x)^3*t
an(1/2*d*x)*tan(1/2*c)*tan(c)^3 - 4*d*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*ta
n(c)^3 - 4*d*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^3 - d*tan(d*x)*tan(1
/2*c)^4*tan(c)^3 + d*tan(d*x)^2*tan(1/2*d*x)^4 + 4*d*tan(d*x)^2*tan(1/2*d*x
)^3*tan(1/2*c) + 4*d*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^3 + 4*d*tan(1/2*d*x
)^3*tan(1/2*c)^3 + d*tan(d*x)^2*tan(1/2*c)^4 - d*tan(d*x)*tan(1/2*d*x)^4*ta
n(c) - 4*d*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)*tan(c) - 4*d*tan(d*x)*tan(1/2
*d*x)^3*tan(1/2*c)*tan(c) - 4*d*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) -
 d*tan(d*x)*tan(1/2*c)^4*tan(c) + d*tan(1/2*d*x)^4*tan(c)^2 + 4*d*tan(d*x)^
2*tan(1/2*d*x)*tan(1/2*c)*tan(c)^2 + 4*d*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^2
 + 4*d*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^2 + d*tan(1/2*c)^4*tan(c)^2 + d*tan
(d*x)^3*tan(c)^3 - 4*d*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*tan(c)^3 + d*tan(1/
2*d*x)^4 + 4*d*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c) + 4*d*tan(1/2*d*x)^3*tan(
1/2*c) + 4*d*tan(1/2*d*x)*tan(1/2*c)^3 + d*tan(1/2*c)^4 + d*tan(d*x)^3*tan(
c) - 4*d*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*tan(c) - d*tan(d*x)^2*tan(c)^2 +
4*d*tan(1/2*d*x)*tan(1/2*c)*tan(c)^2 + d*tan(d*x)*tan(c)^3 - d*tan(d*x)^2 +
 4*d*tan(1/2*d*x)*tan(1/2*c) + d*tan(d*x)*tan(c) - d*tan(c)^2 - d)
```

maple [A] time = 0.24, size = 117, normalized size = 1.65

$$\frac{a^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+2a^2\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+\left(2+\sin^2(dx+c)\right)\cos(dx$$

$$d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^2*tan(d*x+c)^2,x)
```

```
[Out] 1/d*(a^2*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-
3/2*d*x-3/2*c)+2*a^2*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+
a^2*(tan(d*x+c)-d*x-c))
```

maxima [A] time = 0.41, size = 84, normalized size = 1.18

$$-\frac{\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\,\tan\left(dx+c\right)\right)a^2+2\left(dx+c-\tan\left(dx+c\right)\right)a^2-4\,a^2\left(\frac{1}{\cos(dx+c)}+\cos\left(dx+c\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))*a^
2 + 2*(d*x + c - tan(d*x + c))*a^2 - 4*a^2*(1/cos(d*x + c) + cos(d*x + c)))
/d
```

mupad [B] time = 8.69, size = 213, normalized size = 3.00

$$-\frac{5\,a^2\,x}{2}-\frac{\frac{5\,a^2\,(c+d\,x)}{2}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{5\,a^2\,(c+d\,x)}{2}-\frac{a^2\,(5\,c+5\,d\,x-6)}{2}\right)-\frac{a^2\,(5\,c+5\,d\,x-16)}{2}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(\frac{5\,a^2\,(c+d\,x)}{2}-\frac{a^2\,(5\,c+\cdots}{\cdots}\right.\cdots}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + a*sin(c + d*x))^2,x)
```

```
[Out] - (5*a^2*x)/2 - ((5*a^2*(c + d*x))/2 - tan(c/2 + (d*x)/2)*((5*a^2*(c + d*x)
)/2 - (a^2*(5*c + 5*d*x - 6))/2) - (a^2*(5*c + 5*d*x - 16))/2 + tan(c/2 + (
d*x)/2)^4*((5*a^2*(c + d*x))/2 - (a^2*(5*c + 5*d*x - 10))/2) - tan(c/2 + (d
*x)/2)^3*(5*a^2*(c + d*x) - (a^2*(10*c + 10*d*x - 10))/2) + tan(c/2 + (d*x)
/2)^2*(5*a^2*(c + d*x) - (a^2*(10*c + 10*d*x - 22))/2))/(d*(tan(c/2 + (d*x)
/2) - 1)*(tan(c/2 + (d*x)/2)^2 + 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\sin\left(c+dx\right)\tan^2\left(c+dx\right)dx+\int\sin^2\left(c+dx\right)\tan^2\left(c+dx\right)dx+\int\tan^2\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**2*tan(d*x+c)**2,x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)*tan(c + d*x)**2, x) + Integral(sin(c + d*x)**
2*tan(c + d*x)**2, x) + Integral(tan(c + d*x)**2, x))
```

3.22 $\int (a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=45

$$-\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{3a^2x}{2}$$

```
[Out] 3/2*a^2*x-2*a^2*cos(d*x+c)/d-1/2*a^2*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2644}

$$-\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{3a^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*a^2*x)/2 - (2*a^2*Cos[c + d*x])/d - (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d
)
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int (a+a\sin(c+dx))^2\,dx=\frac{3a^2x}{2}-\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cos(c+dx)\sin(c+dx)}{2d}$$

Mathematica [A] time = 0.19, size = 34, normalized size = 0.76

$$-\frac{a^2(-6(c+dx)+\sin(2(c+dx))+8\cos(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/4*(a^2*(-6*(c + d*x) + 8*Cos[c + d*x] + Sin[2*(c + d*x)]))/d
```

fricas [A] time = 0.42, size = 41, normalized size = 0.91

$$\frac{3 a^2 dx - a^2 \cos(dx + c) \sin(dx + c) - 4 a^2 \cos(dx + c)}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2,x, algorithm="fricas")
[Out] 1/2*(3*a^2*d*x - a^2*cos(d*x + c)*sin(d*x + c) - 4*a^2*cos(d*x + c))/d
```

giac [A] time = 0.32, size = 38, normalized size = 0.84

$$\frac{3}{2} a^2 x - \frac{2 a^2 \cos(dx + c)}{d} - \frac{a^2 \sin(2 dx + 2 c)}{4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2,x, algorithm="giac")
[Out] 3/2*a^2*x - 2*a^2*cos(d*x + c)/d - 1/4*a^2*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.07, size = 52, normalized size = 1.16

$$\frac{a^2 \left(-\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right) - 2a^2 \cos(dx + c) + a^2 (dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^2,x)
[Out] 1/d*(a^2*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)-2*a^2*cos(d*x+c)+a^2*(d
*x+c))
```

maxima [A] time = 0.30, size = 47, normalized size = 1.04

$$a^2 x + \frac{(2 dx + 2 c - \sin(2 dx + 2 c))a^2}{4 d} - \frac{2 a^2 \cos(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2,x, algorithm="maxima")
[Out] a^2*x + 1/4*(2*d*x + 2*c - sin(2*d*x + 2*c))*a^2/d - 2*a^2*cos(d*x + c)/d
```

mupad [B] time = 6.58, size = 123, normalized size = 2.73

$$\frac{3a^2x}{2} - \frac{a^2\left(\frac{3c}{2}+\frac{3dx}{2}\right) - a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 - a^2\left(\frac{3c}{2}+\frac{3dx}{2}-4\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(2a^2\left(\frac{3c}{2}+\frac{3dx}{2}\right) - a^2(3c+3d\right.}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2,x)
```

```
[Out] (3*a^2*x)/2 - (a^2*((3*c)/2 + (3*d*x)/2) - a^2*tan(c/2 + (d*x)/2)^3 - a^2*(
(3*c)/2 + (3*d*x)/2 - 4) + tan(c/2 + (d*x)/2)^2*(2*a^2*((3*c)/2 + (3*d*x)/2
) - a^2*(3*c + 3*d*x - 4)) + a^2*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)
^2 + 1)^2)
```

sympy [A] time = 0.42, size = 78, normalized size = 1.73

$$\begin{cases} \frac{a^2x\sin^2(c+dx)}{2} + \frac{a^2x\cos^2(c+dx)}{2} + a^2x - \frac{a^2\sin(c+dx)\cos(c+dx)}{2d} - \frac{2a^2\cos(c+dx)}{d} & \text{for } d \neq 0 \\ x(a\sin(c)+a)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(c + d*x)**2/2 + a**2*x*cos(c + d*x)**2/2 + a**2*x - a
**2*sin(c + d*x)*cos(c + d*x)/(2*d) - 2*a**2*cos(c + d*x)/d, Ne(d, 0)), (x*
(a*sin(c) + a)**2, True))
```

3.23 $\int \cot^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=74

$$\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}-\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^2x}{2}$$

```
[Out] -1/2*a^2*x-2*a^2*arctanh(cos(d*x+c))/d+2*a^2*cos(d*x+c)/d-a^2*cot(d*x+c)/d+
1/2*a^2*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A] time = 0.10, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2709, 3770, 3767, 8, 2638, 2635}

$$\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}-\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*x)/2 - (2*a^2*ArcTanh[Cos[c + d*x]])/d + (2*a^2*Cos[c + d*x])/d - (a^
2*Cot[c + d*x])/d + (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
```

```
p/2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\int \left(2a^4\csc(c+dx)+a^4\csc^2(c+dx)-2a^4\sin(c+dx)-a^4\sin^2(c+dx\right.}{a^2}$$
$$= a^2\int \csc^2(c+dx)\,dx - a^2\int \sin^2(c+dx)\,dx + \left(2a^2\right)\int \csc(c+dx)\,dx -$$
$$= -\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{2a^2\cos(c+dx)}{d} + \frac{a^2\cos(c+dx)\sin(c+}{2d}$$
$$= -\frac{a^2x}{2} - \frac{2a^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{2a^2\cos(c+dx)}{d} - \frac{a^2\cot(c+dx)}{d} +$$

Mathematica [A] time = 0.58, size = 94, normalized size = 1.27

$$-\frac{a^2\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(7\cos(c+dx)+\cos(3(c+dx))+4\sin(c+dx)\left(-4\cos(c+dx)-4\log\left(\sin\right.\right.\right.}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/16*(a^2*Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(7*Cos[c + d*x] + Cos[3*(c + d
*x)] + 4*(c + d*x - 4*Cos[c + d*x] + 4*Log[Cos[(c + d*x)/2]] - 4*Log[Sin[(c
 + d*x)/2]])*Sin[c + d*x]))/d
```

fricas [A] time = 0.44, size = 105, normalized size = 1.42

$$-\frac{a^2\cos(dx+c)^3+2\,a^2\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-2\,a^2\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+a^2\,c}{2\,d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*cos(d*x + c)^3 + 2*a^2*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) -
 2*a^2*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + a^2*cos(d*x + c) + (a^2*
d*x - 4*a^2*cos(d*x + c))*sin(d*x + c))/(d*sin(d*x + c))
```

giac [B] time = 0.26, size = 143, normalized size = 1.93

$$-\frac{(dx+c)a^2 - 4\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right) - a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + \frac{4\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)} + \frac{2\left(a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 4\,a^2\tan\left(\frac{1}{2}\,d\right.}{\left(\tan\left(\frac{1}{2}\,dx+\right.\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*((d*x + c)*a^2 - 4*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - a^2*tan(1/2*d*
x + 1/2*c) + (4*a^2*tan(1/2*d*x + 1/2*c) + a^2)/tan(1/2*d*x + 1/2*c) + 2*(a
^2*tan(1/2*d*x + 1/2*c)^3 - 4*a^2*tan(1/2*d*x + 1/2*c)^2 - a^2*tan(1/2*d*x
+ 1/2*c) - 4*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

maple [A] time = 0.13, size = 89, normalized size = 1.20

$$\frac{a^2\cos(dx+c)\sin(dx+c)}{2d} - \frac{a^2x}{2} - \frac{a^2c}{2d} + \frac{2a^2\cos(dx+c)}{d} + \frac{2a^2\ln(\csc(dx+c)-\cot(dx+c))}{d} - \frac{a^2\cot(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/2*a^2*cos(d*x+c)*sin(d*x+c)/d-1/2*a^2*x-1/2/d*a^2*c+2*a^2*cos(d*x+c)/d+2/
d*a^2*ln(csc(d*x+c)-cot(d*x+c))-a^2*cot(d*x+c)/d
```

maxima [A] time = 0.65, size = 79, normalized size = 1.07

$$\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))a^2 - 4\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^2 + 4\,a^2\left(2\,\cos(dx+c) - \log(\cos(dx+c)+1) + \log(\mathrm{co}\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^2 - 4*(d*x + c + 1/tan(d*x + c))*a^
2 + 4*a^2*(2*cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)))
/d
```

mupad [B] time = 6.52, size = 201, normalized size = 2.72

$$\frac{2\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{-3\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+8\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+8\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a^2}{d\,\left(2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}+\frac{a^2\,\mathrm{atan}\left(\frac{a^4}{4\,a^4+a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (2*a^2*log(tan(c/2 + (d*x)/2)))/d + (8*a^2*tan(c/2 + (d*x)/2)^3 - 3*a^2*tan
(c/2 + (d*x)/2)^4 - a^2 + 8*a^2*tan(c/2 + (d*x)/2))/(d*(2*tan(c/2 + (d*x)/2
) + 4*tan(c/2 + (d*x)/2)^3 + 2*tan(c/2 + (d*x)/2)^5)) + (a^2*atan(a^4/(4*a^
4 + a^4*tan(c/2 + (d*x)/2)) - (4*a^4*tan(c/2 + (d*x)/2))/(4*a^4 + a^4*tan(c
/2 + (d*x)/2))))/d + (a^2*tan(c/2 + (d*x)/2))/(2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\sin(c+dx)\cot^2(c+dx)\,dx+\int \sin^2(c+dx)\cot^2(c+dx)\,dx+\int \cot^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)*cot(c + d*x)**2, x) + Integral(sin(c + d*x)**
2*cot(c + d*x)**2, x) + Integral(cot(c + d*x)**2, x))
```

3.24 $\int \cot^4(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=98

$$-\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{3a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^2\cot(c+dx)\csc(c+a}{d}$$

```
[Out] -1/2*a^2*x+3*a^2*arctanh(cos(d*x+c))/d-2*a^2*cos(d*x+c)/d-1/3*a^2*cot(d*x+c
)^3/d-a^2*cot(d*x+c)*csc(d*x+c)/d-1/2*a^2*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A] time = 0.16, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2709, 3770, 3767, 8, 3768, 2638, 2635}

$$-\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{3a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^2\cot(c+dx)\csc(c+a}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*x)/2 + (3*a^2*ArcTanh[Cos[c + d*x]])/d - (2*a^2*Cos[c + d*x])/d - (a^
2*Cot[c + d*x]^3)/(3*d) - (a^2*Cot[c + d*x]*Csc[c + d*x])/d - (a^2*Cos[c +
d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
```

```
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\int \left(-a^6-4a^6\csc(c+dx)-a^6\csc^2(c+dx)+2a^6\csc^3(c+dx)+a^6\csc^4\right.}{a^4}$$
$$=-a^2x-a^2\int \csc^2(c+dx)\,dx+a^2\int \csc^4(c+dx)\,dx+a^2\int \sin^2(c+dx$$
$$=-a^2x+\frac{4a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot(c+dx)\csc}{d}$$
$$=-\frac{a^2x}{2}+\frac{3a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}-$$

Mathematica [A] time = 5.27, size = 191, normalized size = 1.95

$$\frac{a^2(\sin(c+dx)+1)^2\left(-12(c+dx)-6\sin(2(c+dx))-48\cos(c+dx)-4\tan\left(\frac{1}{2}(c+dx)\right)+4\cot\left(\frac{1}{2}(c+dx)\right)-6\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(1 + Sin[c + d*x])^2*(-12*(c + d*x) - 48*Cos[c + d*x] + 4*Cot[(c + d*x
)/2] - 6*Csc[(c + d*x)/2]^2 + 72*Log[Cos[(c + d*x)/2]] - 72*Log[Sin[(c + d*
x)/2]] + 6*Sec[(c + d*x)/2]^2 + 8*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 - (Csc[
(c + d*x)/2]^4*Sin[c + d*x])/2 - 6*Sin[2*(c + d*x)] - 4*Tan[(c + d*x)/2]))/
(24*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)
```

fricas [B] time = 0.47, size = 192, normalized size = 1.96

$$\frac{3\,a^2\cos\left(dx+c\right)^5 - 4\,a^2\cos\left(dx+c\right)^3 + 3\,a^2\cos\left(dx+c\right) + 9\left(a^2\cos\left(dx+c\right)^2 - a^2\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right) + \frac{1}{2}\right)\sin\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*a^2*cos(d*x + c)^5 - 4*a^2*cos(d*x + c)^3 + 3*a^2*cos(d*x + c) + 9*(
a^2*cos(d*x + c)^2 - a^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 9*(a^2
*cos(d*x + c)^2 - a^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*(a^2*d
*x*cos(d*x + c)^2 + 4*a^2*cos(d*x + c)^3 - a^2*d*x - 6*a^2*cos(d*x + c))*si
n(d*x + c))/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

giac [B] time = 0.55, size = 209, normalized size = 2.13

$$\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 12\,(dx+c)a^2 - 72\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right) - 3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{24\,}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*tan(1/2*d*x + 1/2*c)^2 - 12*(d*x +
 c)*a^2 - 72*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 3*a^2*tan(1/2*d*x + 1/2*c
) + 24*(a^2*tan(1/2*d*x + 1/2*c)^3 - 4*a^2*tan(1/2*d*x + 1/2*c)^2 - a^2*tan
(1/2*d*x + 1/2*c) - 4*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2 + (132*a^2*tan(1/
2*d*x + 1/2*c)^3 + 3*a^2*tan(1/2*d*x + 1/2*c)^2 - 6*a^2*tan(1/2*d*x + 1/2*c
) - a^2)/tan(1/2*d*x + 1/2*c)^3)/d
```

maple [B] time = 0.22, size = 190, normalized size = 1.94

$$-\frac{a^2\left(\cos^5\left(dx+c\right)\right)}{d\sin\left(dx+c\right)} - \frac{a^2\left(\cos^3\left(dx+c\right)\right)\sin\left(dx+c\right)}{d} - \frac{3a^2\cos\left(dx+c\right)\sin\left(dx+c\right)}{2d} - \frac{a^2x}{2} - \frac{a^2c}{2d} - \frac{a^2\left(\cos^5\left(dx+c\right)\right)}{d\sin\left(dx+c\right)^2} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/d*a^2/sin(d*x+c)*cos(d*x+c)^5-a^2*cos(d*x+c)^3*sin(d*x+c)/d-3/2*a^2*cos(
d*x+c)*sin(d*x+c)/d-1/2*a^2*x-1/2/d*a^2*c-1/d*a^2/sin(d*x+c)^2*cos(d*x+c)^5
-a^2*cos(d*x+c)^3/d-3*a^2*cos(d*x+c)/d-3/d*a^2*ln(csc(d*x+c)-cot(d*x+c))-1/
3*a^2*cot(d*x+c)^3/d+a^2*cot(d*x+c)/d
```

maxima [A] time = 0.46, size = 139, normalized size = 1.42

$$-\frac{3\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)}\right)a^2-2\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^2-3\,a^2\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}-4\,\cos\left(dx+c\right)+3\,\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/6*(3*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x + c
)))*a^2 - 2*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^2 - 3*a
^2*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x +
c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 6.52, size = 293, normalized size = 2.99

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{3\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a^2\,\mathrm{atan}\left(\frac{a^4}{6\,a^4-a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{6\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{6\,a^4-a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{-9\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^2)/(4*d) + (a^2*tan(c/2 + (d*x)/2)^3)/(24*d) - (3*a
^2*log(tan(c/2 + (d*x)/2)))/d - (a^2*atan(a^4/(6*a^4 - a^4*tan(c/2 + (d*x)/
2)) + (6*a^4*tan(c/2 + (d*x)/2))/(6*a^4 - a^4*tan(c/2 + (d*x)/2))))/d - (36
*a^2*tan(c/2 + (d*x)/2)^3 - (a^2*tan(c/2 + (d*x)/2)^2)/3 + (19*a^2*tan(c/2
+ (d*x)/2)^4)/3 + 34*a^2*tan(c/2 + (d*x)/2)^5 - 9*a^2*tan(c/2 + (d*x)/2)^6
+ a^2/3 + 2*a^2*tan(c/2 + (d*x)/2))/(d*(8*tan(c/2 + (d*x)/2)^3 + 16*tan(c/2
 + (d*x)/2)^5 + 8*tan(c/2 + (d*x)/2)^7)) - (a^2*tan(c/2 + (d*x)/2))/(8*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\sin\left(c+dx\right)\cot^4\left(c+dx\right)dx+\int \sin^2\left(c+dx\right)\cot^4\left(c+dx\right)dx+\int \cot^4\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+a*sin(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)*cot(c + d*x)**4, x) + Integral(sin(c + d*x)**
2*cot(c + d*x)**4, x) + Integral(cot(c + d*x)**4, x))
```

3.25 $\int (a+a\sin(c+dx))^3\tan^7(c+dx)\,dx$

Optimal. Leaf size=160

$$\frac{a^6}{6d(a-a\sin(c+dx))^3}-\frac{13a^5}{8d(a-a\sin(c+dx))^2}+\frac{71a^4}{8d(a-a\sin(c+dx))}+\frac{a^3\sin^3(c+dx)}{3d}+\frac{3a^3\sin^2(c+dx)}{2d}+\frac{7a^3\mathrm{si}}{}$$

```
[Out] 209/16*a^3*ln(1-sin(d*x+c))/d-1/16*a^3*ln(1+sin(d*x+c))/d+7*a^3*sin(d*x+c)/
d+3/2*a^3*sin(d*x+c)^2/d+1/3*a^3*sin(d*x+c)^3/d+1/6*a^6/d/(a-a*sin(d*x+c))^
3-13/8*a^5/d/(a-a*sin(d*x+c))^2+71/8*a^4/d/(a-a*sin(d*x+c))
```

Rubi [A] time = 0.11, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 88}

$$\frac{a^3\sin^3(c+dx)}{3d}+\frac{3a^3\sin^2(c+dx)}{2d}+\frac{a^6}{6d(a-a\sin(c+dx))^3}-\frac{13a^5}{8d(a-a\sin(c+dx))^2}+\frac{71a^4}{8d(a-a\sin(c+dx))}+\frac{7a^3\mathrm{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^7,x]
```

```
[Out] (209*a^3*Log[1 - Sin[c + d*x]])/(16*d) - (a^3*Log[1 + Sin[c + d*x]])/(16*d)
 + (7*a^3*Sin[c + d*x])/d + (3*a^3*Sin[c + d*x]^2)/(2*d) + (a^3*Sin[c + d*x
]^3)/(3*d) + a^6/(6*d*(a - a*Sin[c + d*x])^3) - (13*a^5)/(8*d*(a - a*Sin[c
+ d*x])^2) + (71*a^4)/(8*d*(a - a*Sin[c + d*x]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int (a+a\sin(c+dx))^3\tan^7(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^7}{(a-x)^4(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(7a^2+\frac{a^6}{2(a-x)^4}-\frac{13a^5}{4(a-x)^3}+\frac{71a^4}{8(a-x)^2}-\frac{209a^3}{16(a-x)}+3ax+x^2-\frac{a^3}{16(a+x)}\right.\right.}{d}$$
$$= \frac{209a^3\log(1-\sin(c+dx))}{16d}-\frac{a^3\log(1+\sin(c+dx))}{16d}+\frac{7a^3\sin(c+dx)}{d}$$

Mathematica [A] time = 0.55, size = 99, normalized size = 0.62

$$\frac{a^3\left(16\sin^3(c+dx)+72\sin^2(c+dx)+336\sin(c+dx)-\frac{426}{\sin(c+dx)-1}-\frac{78}{(\sin(c+dx)-1)^2}-\frac{8}{(\sin(c+dx)-1)^3}+627\log(1-\sin\right.}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^7,x]
```

```
[Out] (a^3*(627*Log[1 - Sin[c + d*x]] - 3*Log[1 + Sin[c + d*x]] - 8/(-1 + Sin[c +
 d*x])^3 - 78/(-1 + Sin[c + d*x])^2 - 426/(-1 + Sin[c + d*x]) + 336*Sin[c +
 d*x] + 72*Sin[c + d*x]^2 + 16*Sin[c + d*x]^3))/(48*d)
```

fricas [A] time = 0.45, size = 240, normalized size = 1.50

$$-\frac{16\,a^3\cos(dx+c)^6-216\,a^3\cos(dx+c)^4+1002\,a^3\cos(dx+c)^2-482\,a^3+3\left(3\,a^3\cos(dx+c)^2-4\,a^3-\left(a^3\cos\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] -1/48*(16*a^3*cos(d*x + c)^6 - 216*a^3*cos(d*x + c)^4 + 1002*a^3*cos(d*x +
c)^2 - 482*a^3 + 3*(3*a^3*cos(d*x + c)^2 - 4*a^3 - (a^3*cos(d*x + c)^2 - 4*
a^3)*sin(d*x + c))*log(sin(d*x + c) + 1) - 627*(3*a^3*cos(d*x + c)^2 - 4*a^
3 - (a^3*cos(d*x + c)^2 - 4*a^3)*sin(d*x + c))*log(-sin(d*x + c) + 1) - 2*(
12*a^3*cos(d*x + c)^4 + 398*a^3*cos(d*x + c)^2 - 245*a^3)*sin(d*x + c))/(3*
d*cos(d*x + c)^2 - (d*cos(d*x + c)^2 - 4*d)*sin(d*x + c) - 4*d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^7,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.23, size = 445, normalized size = 2.78

$$\frac{35a^3\left(\sin^9(dx+c)\right)}{48d}+\frac{3a^3\left(\sin^8(dx+c)\right)}{2d}+\frac{15a^3\left(\sin^7(dx+c)\right)}{8d}+\frac{a^3\left(\sin^{10}(dx+c)\right)}{2d\cos(dx+c)^6}-\frac{a^3\left(\sin^{10}(dx+c)\right)}{2d\cos(dx+c)^4}+\frac{3a^3\left(\mathrm{si}\right.}{2d\,\mathrm{co}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^3*tan(d*x+c)^7,x)
```

```
[Out] 35/48/d*a^3*sin(d*x+c)^9+3/2/d*a^3*sin(d*x+c)^8+15/8/d*a^3*sin(d*x+c)^7+1/2
/d*a^3*sin(d*x+c)^10/cos(d*x+c)^6-1/2/d*a^3*sin(d*x+c)^10/cos(d*x+c)^4+3/2/
d*a^3*sin(d*x+c)^10/cos(d*x+c)^2+1/2/d*a^3*sin(d*x+c)^9/cos(d*x+c)^6-3/8/d*
a^3*sin(d*x+c)^9/cos(d*x+c)^4+15/16/d*a^3*sin(d*x+c)^9/cos(d*x+c)^2+1/6/d*a
^3*sin(d*x+c)^11/cos(d*x+c)^6-5/24/d*a^3*sin(d*x+c)^11/cos(d*x+c)^4+35/48/d
*a^3*sin(d*x+c)^11/cos(d*x+c)^2+21/8/d*a^3*sin(d*x+c)^5+35/8*a^3*sin(d*x+c)
^3/d+105/8*a^3*sin(d*x+c)/d-105/8/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+1/6/d*a^3
*tan(d*x+c)^6-1/4/d*a^3*tan(d*x+c)^4+1/2/d*a^3*tan(d*x+c)^2+2/d*a^3*sin(d*x
+c)^6+3/d*a^3*sin(d*x+c)^4+6*a^3*sin(d*x+c)^2/d+13/d*a^3*ln(cos(d*x+c))
```

maxima [A] time = 0.32, size = 133, normalized size = 0.83

$$\frac{16\,a^3\sin(dx+c)^3+72\,a^3\sin(dx+c)^2-3\,a^3\log(\sin(dx+c)+1)+627\,a^3\log(\sin(dx+c)-1)+336\,a^3\sin(}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/48*(16*a^3*sin(d*x + c)^3 + 72*a^3*sin(d*x + c)^2 - 3*a^3*log(sin(d*x + c
) + 1) + 627*a^3*log(sin(d*x + c) - 1) + 336*a^3*sin(d*x + c) - 2*(213*a^3*
sin(d*x + c)^2 - 387*a^3*sin(d*x + c) + 178*a^3)/(sin(d*x + c)^3 - 3*sin(d*
x + c)^2 + 3*sin(d*x + c) - 1))/d
```

mupad [B] time = 6.47, size = 398, normalized size = 2.49

$$\frac{\frac{105\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{4}-\frac{263\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{2}+\frac{1301\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{4}-582\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\frac{1657\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{2}-}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+18\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-38\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}+63\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-84\tan\left(\frac{c}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^7*(a + a*sin(c + d*x))^3,x)
```

```
[Out] ((1301*a^3*tan(c/2 + (d*x)/2)^3)/4 - (263*a^3*tan(c/2 + (d*x)/2)^2)/2 - 582
*a^3*tan(c/2 + (d*x)/2)^4 + (1657*a^3*tan(c/2 + (d*x)/2)^5)/2 - (2767*a^3*t
an(c/2 + (d*x)/2)^6)/3 + (1657*a^3*tan(c/2 + (d*x)/2)^7)/2 - 582*a^3*tan(c/
2 + (d*x)/2)^8 + (1301*a^3*tan(c/2 + (d*x)/2)^9)/4 - (263*a^3*tan(c/2 + (d*
x)/2)^10)/2 + (105*a^3*tan(c/2 + (d*x)/2)^11)/4 + (105*a^3*tan(c/2 + (d*x)/
2))/4)/(d*(18*tan(c/2 + (d*x)/2)^2 - 6*tan(c/2 + (d*x)/2) - 38*tan(c/2 + (d
*x)/2)^3 + 63*tan(c/2 + (d*x)/2)^4 - 84*tan(c/2 + (d*x)/2)^5 + 92*tan(c/2 +
 (d*x)/2)^6 - 84*tan(c/2 + (d*x)/2)^7 + 63*tan(c/2 + (d*x)/2)^8 - 38*tan(c/
2 + (d*x)/2)^9 + 18*tan(c/2 + (d*x)/2)^10 - 6*tan(c/2 + (d*x)/2)^11 + tan(c
/2 + (d*x)/2)^12 + 1)) + (209*a^3*log(tan(c/2 + (d*x)/2) - 1))/(8*d) - (a^3
*log(tan(c/2 + (d*x)/2) + 1))/(8*d) - (13*a^3*log(tan(c/2 + (d*x)/2)^2 + 1)
)/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**3*tan(d*x+c)**7,x)
```

```
[Out] Timed out
```

3.26 $\int (a+a\sin(c+dx))^3 \tan^3(c+dx)\,dx$

Optimal. Leaf size=91

$$\frac{2a^4}{d(a-a\sin(c+dx))}+\frac{a^3\sin^3(c+dx)}{3d}+\frac{3a^3\sin^2(c+dx)}{2d}+\frac{5a^3\sin(c+dx)}{d}+\frac{7a^3\log(1-\sin(c+dx))}{d}$$

```
[Out] 7*a^3*ln(1-sin(d*x+c))/d+5*a^3*sin(d*x+c)/d+3/2*a^3*sin(d*x+c)^2/d+1/3*a^3*
sin(d*x+c)^3/d+2*a^4/d/(a-a*sin(d*x+c))
```

Rubi [A] time = 0.07, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 77}

$$\frac{a^3\sin^3(c+dx)}{3d}+\frac{3a^3\sin^2(c+dx)}{2d}+\frac{2a^4}{d(a-a\sin(c+dx))}+\frac{5a^3\sin(c+dx)}{d}+\frac{7a^3\log(1-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^3,x]
```

```
[Out] (7*a^3*Log[1 - Sin[c + d*x]])/d + (5*a^3*Sin[c + d*x])/d + (3*a^3*Sin[c + d
*x]^2)/(2*d) + (a^3*Sin[c + d*x]^3)/(3*d) + (2*a^4)/(d*(a - a*Sin[c + d*x])
)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int (a+a\sin(c+dx))^3\tan^3(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^3(a+x)}{(a-x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(5a^2+\frac{2a^4}{(a-x)^2}-\frac{7a^3}{a-x}+3ax+x^2\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{7a^3\log(1-\sin(c+dx))}{d}+\frac{5a^3\sin(c+dx)}{d}+\frac{3a^3\sin^2(c+dx)}{2d}+\frac{a^3\sin^3}{3}$$

Mathematica [A] time = 0.16, size = 66, normalized size = 0.73

$$\frac{a^3\left(2\sin^3(c+dx)+9\sin^2(c+dx)+30\sin(c+dx)+\frac{12}{1-\sin(c+dx)}+42\log(1-\sin(c+dx))\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^3,x]
```

```
[Out] (a^3*(42*Log[1 - Sin[c + d*x]] + 12/(1 - Sin[c + d*x]) + 30*Sin[c + d*x] +
9*Sin[c + d*x]^2 + 2*Sin[c + d*x]^3))/(6*d)
```

fricas [A] time = 0.43, size = 104, normalized size = 1.14

$$\frac{4\,a^3\cos(dx+c)^4-50\,a^3\cos(dx+c)^2+31\,a^3+84\left(a^3\sin(dx+c)-a^3\right)\log(-\sin(dx+c)+1)-\left(14\,a^3\cos(dx\right.}{12\,(d\sin(dx+c)-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(4*a^3*cos(d*x + c)^4 - 50*a^3*cos(d*x + c)^2 + 31*a^3 + 84*(a^3*sin(d
*x + c) - a^3)*log(-sin(d*x + c) + 1) - (14*a^3*cos(d*x + c)^2 + 55*a^3)*si
n(d*x + c))/(d*sin(d*x + c) - d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.18, size = 205, normalized size = 2.25

$$\frac{a^3\left(\sin^7(dx+c)\right)}{2d\cos(dx+c)^2}+\frac{a^3\left(\sin^5(dx+c)\right)}{2d}+\frac{7a^3\left(\sin^3(dx+c)\right)}{3d}+\frac{7a^3\sin(dx+c)}{d}-\frac{7a^3\ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^3*tan(d*x+c)^3,x)
```

```
[Out] 1/2/d*a^3*sin(d*x+c)^7/cos(d*x+c)^2+1/2/d*a^3*sin(d*x+c)^5+7/3*a^3*sin(d*x+
c)^3/d+7*a^3*sin(d*x+c)/d-7/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+3/2/d*a^3*sin(d
*x+c)^6/cos(d*x+c)^2+3/2/d*a^3*sin(d*x+c)^4+3*a^3*sin(d*x+c)^2/d+7/d*a^3*ln
(cos(d*x+c))+3/2/d*a^3*sin(d*x+c)^5/cos(d*x+c)^2+1/2/d*a^3*tan(d*x+c)^2
```

maxima [A] time = 0.30, size = 72, normalized size = 0.79

$$\frac{2\,a^3\sin(dx+c)^3+9\,a^3\sin(dx+c)^2+42\,a^3\log(\sin(dx+c)-1)+30\,a^3\sin(dx+c)-\frac{12\,a^3}{\sin(dx+c)-1}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*a^3*sin(d*x + c)^3 + 9*a^3*sin(d*x + c)^2 + 42*a^3*log(sin(d*x + c)
- 1) + 30*a^3*sin(d*x + c) - 12*a^3/(sin(d*x + c) - 1))/d
```

mupad [B] time = 7.46, size = 262, normalized size = 2.88

$$\frac{14\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{d}+\frac{14\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-14\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\frac{98\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{3}-\frac{100\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+6\tan\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (14*a^3*log(tan(c/2 + (d*x)/2) - 1))/d + ((98*a^3*tan(c/2 + (d*x)/2)^3)/3 -
 14*a^3*tan(c/2 + (d*x)/2)^2 - (100*a^3*tan(c/2 + (d*x)/2)^4)/3 + (98*a^3*t
an(c/2 + (d*x)/2)^5)/3 - 14*a^3*tan(c/2 + (d*x)/2)^6 + 14*a^3*tan(c/2 + (d*
x)/2)^7 + 14*a^3*tan(c/2 + (d*x)/2))/(d*(4*tan(c/2 + (d*x)/2)^2 - 2*tan(c/2
 + (d*x)/2) - 6*tan(c/2 + (d*x)/2)^3 + 6*tan(c/2 + (d*x)/2)^4 - 6*tan(c/2 +
 (d*x)/2)^5 + 4*tan(c/2 + (d*x)/2)^6 - 2*tan(c/2 + (d*x)/2)^7 + tan(c/2 + (
d*x)/2)^8 + 1)) - (7*a^3*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\sin\left(c+dx\right)\tan^3\left(c+dx\right)dx+\int 3\sin^2\left(c+dx\right)\tan^3\left(c+dx\right)dx+\int \sin^3\left(c+dx\right)\tan^3\left(c+dx\right)dx+\int \mathrm{t}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**3*tan(d*x+c)**3,x)
```

```
[Out] a**3*(Integral(3*sin(c + d*x)*tan(c + d*x)**3, x) + Integral(3*sin(c + d*x)
**2*tan(c + d*x)**3, x) + Integral(sin(c + d*x)**3*tan(c + d*x)**3, x) + In
tegral(tan(c + d*x)**3, x))
```

3.27 $\int (a+a\sin(c+dx))^3 \tan(c+dx)\,dx$

Optimal. Leaf size=70

$$-\frac{a^3\sin^3(c+dx)}{3d}-\frac{3a^3\sin^2(c+dx)}{2d}-\frac{4a^3\sin(c+dx)}{d}-\frac{4a^3\log(1-\sin(c+dx))}{d}$$

```
[Out] -4*a^3*ln(1-sin(d*x+c))/d-4*a^3*sin(d*x+c)/d-3/2*a^3*sin(d*x+c)^2/d-1/3*a^3
*sin(d*x+c)^3/d
```

Rubi [A] time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2707, 77}

$$-\frac{a^3\sin^3(c+dx)}{3d}-\frac{3a^3\sin^2(c+dx)}{2d}-\frac{4a^3\sin(c+dx)}{d}-\frac{4a^3\log(1-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3*Tan[c + d*x],x]
```

```
[Out] (-4*a^3*Log[1 - Sin[c + d*x]])/d - (4*a^3*Sin[c + d*x])/d - (3*a^3*Sin[c +
d*x]^2)/(2*d) - (a^3*Sin[c + d*x]^3)/(3*d)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int (a+a\sin(c+dx))^3\tan(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x(a+x)^2}{a-x}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-4a^2+\frac{4a^3}{a-x}-3ax-x^2\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{4a^3\log(1-\sin(c+dx))}{d} - \frac{4a^3\sin(c+dx)}{d} - \frac{3a^3\sin^2(c+dx)}{2d} - \frac{a^3\sin^3}{3}$$

Mathematica [A] time = 0.04, size = 52, normalized size = 0.74

$$-\frac{a^3\left(2\sin^3(c+dx)+9\sin^2(c+dx)+24\sin(c+dx)+24\log(1-\sin(c+dx))\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3*Tan[c + d*x],x]
```

```
[Out] -1/6*(a^3*(24*Log[1 - Sin[c + d*x]] + 24*Sin[c + d*x] + 9*Sin[c + d*x]^2 +
2*Sin[c + d*x]^3))/d
```

fricas [A] time = 0.43, size = 61, normalized size = 0.87

$$\frac{9\,a^3\cos(dx+c)^2-24\,a^3\log(-\sin(dx+c)+1)+2\left(a^3\cos(dx+c)^2-13\,a^3\right)\sin(dx+c)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/6*(9*a^3*cos(d*x + c)^2 - 24*a^3*log(-sin(d*x + c) + 1) + 2*(a^3*cos(d*x
+ c)^2 - 13*a^3)*sin(d*x + c))/d
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.14, size = 85, normalized size = 1.21

$$-\frac{a^3\left(\sin^3(dx+c)\right)}{3d}-\frac{4a^3\sin(dx+c)}{d}+\frac{4a^3\ln(\sec(dx+c)+\tan(dx+c))}{d}-\frac{3a^3\left(\sin^2(dx+c)\right)}{2d}-\frac{4a^3\ln(\cos(dx+ \cdots}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^3*tan(d*x+c),x)
```

```
[Out] -1/3*a^3*sin(d*x+c)^3/d-4*a^3*sin(d*x+c)/d+4/d*a^3*ln(sec(d*x+c)+tan(d*x+c)
)-3/2*a^3*sin(d*x+c)^2/d-4/d*a^3*ln(cos(d*x+c))
```

maxima [A] time = 0.31, size = 57, normalized size = 0.81

$$-\frac{2\,a^3\sin(dx+c)^3+9\,a^3\sin(dx+c)^2+24\,a^3\log(\sin(dx+c)-1)+24\,a^3\sin(dx+c)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^3*sin(d*x + c)^3 + 9*a^3*sin(d*x + c)^2 + 24*a^3*log(sin(d*x + c)
 - 1) + 24*a^3*sin(d*x + c))/d
```

mupad [B] time = 7.26, size = 281, normalized size = 4.01

$$-\frac{\frac{56\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+8\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(2\,a^3\left(12\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)-6\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1 \cdots\right.\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + a*sin(c + d*x))^3,x)
```

```
[Out] - ((56*a^3*tan(c/2 + (d*x)/2)^3)/3 + 8*a^3*tan(c/2 + (d*x)/2)^5 - tan(c/2 +
 (d*x)/2)^2*(2*a^3*(12*log(tan(c/2 + (d*x)/2) - 1) - 6*log(tan(c/2 + (d*x)/
2)^2 + 1)) - (2*a^3*(36*log(tan(c/2 + (d*x)/2) - 1) - 18*log(tan(c/2 + (d*x
)/2)^2 + 1) + 9))/3) - tan(c/2 + (d*x)/2)^4*(2*a^3*(12*log(tan(c/2 + (d*x)/
2) - 1) - 6*log(tan(c/2 + (d*x)/2)^2 + 1)) - (2*a^3*(36*log(tan(c/2 + (d*x)
/2) - 1) - 18*log(tan(c/2 + (d*x)/2)^2 + 1) + 9))/3) + 8*a^3*tan(c/2 + (d*x
)/2))/(d*(tan(c/2 + (d*x)/2)^2 + 1)^3) - (2*a^3*(12*log(tan(c/2 + (d*x)/2)
- 1) - 6*log(tan(c/2 + (d*x)/2)^2 + 1)))/(3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\sin(c+dx)\tan(c+dx)\,dx+\int 3\sin^2(c+dx)\tan(c+dx)\,dx+\int \sin^3(c+dx)\tan(c+dx)\,dx+\int \tan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**3*tan(d*x+c),x)
```

```
[Out] a**3*(Integral(3*sin(c + d*x)*tan(c + d*x), x) + Integral(3*sin(c + d*x)**2
*tan(c + d*x), x) + Integral(sin(c + d*x)**3*tan(c + d*x), x) + Integral(ta
n(c + d*x), x))
```

3.28 $\int \cot^3(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=98

$$-\frac{a^3\sin^3(c+dx)}{3d}-\frac{3a^3\sin^2(c+dx)}{2d}-\frac{2a^3\sin(c+dx)}{d}-\frac{a^3\csc^2(c+dx)}{2d}-\frac{3a^3\csc(c+dx)}{d}+\frac{2a^3\log(\sin(c+dx))}{d}$$

```
[Out] -3*a^3*csc(d*x+c)/d-1/2*a^3*csc(d*x+c)^2/d+2*a^3*ln(sin(d*x+c))/d-2*a^3*sin
(d*x+c)/d-3/2*a^3*sin(d*x+c)^2/d-1/3*a^3*sin(d*x+c)^3/d
```

Rubi [A] time = 0.06, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 75}

$$-\frac{a^3\sin^3(c+dx)}{3d}-\frac{3a^3\sin^2(c+dx)}{2d}-\frac{2a^3\sin(c+dx)}{d}-\frac{a^3\csc^2(c+dx)}{2d}-\frac{3a^3\csc(c+dx)}{d}+\frac{2a^3\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a^3*Csc[c + d*x])/d - (a^3*Csc[c + d*x]^2)/(2*d) + (2*a^3*Log[Sin[c + d
*x]])/d - (2*a^3*Sin[c + d*x])/d - (3*a^3*Sin[c + d*x]^2)/(2*d) - (a^3*Sin[
c + d*x]^3)/(3*d)
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \cot^3(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)(a+x)^4}{x^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-2a^2+\frac{a^5}{x^3}+\frac{3a^4}{x^2}+\frac{2a^3}{x}-3ax-x^2\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{3a^3\csc(c+dx)}{d}-\frac{a^3\csc^2(c+dx)}{2d}+\frac{2a^3\log(\sin(c+dx))}{d}-\frac{2a^3\sin(c+$$

Mathematica [A] time = 0.19, size = 67, normalized size = 0.68

$$-\frac{a^3\left(2\sin^3(c+dx)+9\sin^2(c+dx)+12\sin(c+dx)+3\csc^2(c+dx)+18\csc(c+dx)-12\log(\sin(c+dx))+30\right.}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/6*(a^3*(30 + 18*Csc[c + d*x] + 3*Csc[c + d*x]^2 - 12*Log[Sin[c + d*x]] +
 12*Sin[c + d*x] + 9*Sin[c + d*x]^2 + 2*Sin[c + d*x]^3))/d
```

fricas [A] time = 0.43, size = 118, normalized size = 1.20

$$\frac{18\,a^3\cos(dx+c)^4-27\,a^3\cos(dx+c)^2+15\,a^3+24\left(a^3\cos(dx+c)^2-a^3\right)\log\left(\frac{1}{2}\sin(dx+c)\right)+4\left(a^3\cos(dx+\right.}{12\left(d\cos(dx+c)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(18*a^3*cos(d*x + c)^4 - 27*a^3*cos(d*x + c)^2 + 15*a^3 + 24*(a^3*cos(
d*x + c)^2 - a^3)*log(1/2*sin(d*x + c)) + 4*(a^3*cos(d*x + c)^4 - 8*a^3*cos
(d*x + c)^2 + 16*a^3)*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

giac [A] time = 1.43, size = 94, normalized size = 0.96

$$-\frac{2\,a^3\sin(dx+c)^3+9\,a^3\sin(dx+c)^2-12\,a^3\log(|\sin(dx+c)|)+12\,a^3\sin(dx+c)+\frac{3\left(6\,a^3\sin(dx+c)^2+6\,a^3\sin(dx+c)\right.}{\sin(dx+c)^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*(2*a^3*sin(d*x + c)^3 + 9*a^3*sin(d*x + c)^2 - 12*a^3*log(abs(sin(d*x
+ c))) + 12*a^3*sin(d*x + c) + 3*(6*a^3*sin(d*x + c)^2 + 6*a^3*sin(d*x + c)
 + a^3)/sin(d*x + c)^2)/d
```

maple [A] time = 0.24, size = 109, normalized size = 1.11

$$-\frac{8a^3\left(\cos^2(dx+c)\right)\sin(dx+c)}{3d}-\frac{16a^3\sin(dx+c)}{3d}+\frac{3a^3\left(\cos^2(dx+c)\right)}{2d}+\frac{2a^3\ln(\sin(dx+c))}{d}-\frac{3a^3\left(\cos^4(dx+\right.}{d\sin(dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -8/3/d*a^3*cos(d*x+c)^2*sin(d*x+c)-16/3*a^3*sin(d*x+c)/d+3/2/d*a^3*cos(d*x+
c)^2+2*a^3*ln(sin(d*x+c))/d-3/d*a^3/sin(d*x+c)*cos(d*x+c)^4-1/2/d*a^3*cot(d
*x+c)^2
```

maxima [A] time = 0.30, size = 80, normalized size = 0.82

$$-\frac{2\,a^3\sin(dx+c)^3+9\,a^3\sin(dx+c)^2-12\,a^3\log(\sin(dx+c))+12\,a^3\sin(dx+c)+\frac{3\left(6\,a^3\sin(dx+c)+a^3\right)}{\sin(dx+c)^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^3*sin(d*x + c)^3 + 9*a^3*sin(d*x + c)^2 - 12*a^3*log(sin(d*x + c)
) + 12*a^3*sin(d*x + c) + 3*(6*a^3*sin(d*x + c) + a^3)/sin(d*x + c)^2)/d
```

mupad [B] time = 6.76, size = 253, normalized size = 2.58

$$\frac{2\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-\frac{22\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\frac{49\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{2}+\frac{182\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{3}+\frac{51\,a^3\tan\left(\frac{c}{2}+\frac{d}{}\right.}{2}}{d\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+12\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (2*a^3*log(tan(c/2 + (d*x)/2)))/d - (a^3*tan(c/2 + (d*x)/2)^2)/(8*d) - ((3*
a^3*tan(c/2 + (d*x)/2)^2)/2 + 34*a^3*tan(c/2 + (d*x)/2)^3 + (51*a^3*tan(c/2
 + (d*x)/2)^4)/2 + (182*a^3*tan(c/2 + (d*x)/2)^5)/3 + (49*a^3*tan(c/2 + (d*
x)/2)^6)/2 + 22*a^3*tan(c/2 + (d*x)/2)^7 + a^3/2 + 6*a^3*tan(c/2 + (d*x)/2)
)/(d*(4*tan(c/2 + (d*x)/2)^2 + 12*tan(c/2 + (d*x)/2)^4 + 12*tan(c/2 + (d*x)
```

```
/2)^6 + 4*tan(c/2 + (d*x)/2)^8)) - (3*a^3*tan(c/2 + (d*x)/2))/(2*d) - (2*a^
3*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\sin(c+dx)\cot^3(c+dx)\,dx+\int 3\sin^2(c+dx)\cot^3(c+dx)\,dx+\int \sin^3(c+dx)\cot^3(c+dx)\,dx+\int \mathrm{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+a*sin(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*sin(c + d*x)*cot(c + d*x)**3, x) + Integral(3*sin(c + d*x)
**2*cot(c + d*x)**3, x) + Integral(sin(c + d*x)**3*cot(c + d*x)**3, x) + In
tegral(cot(c + d*x)**3, x))
```

3.29 $\int (a + a\sin(c + dx))^3 \tan^6(c + dx)\, dx$

Optimal. Leaf size=180

$$\frac{a^6\sin^5(c+dx)\cos(c+dx)}{5d(a-a\sin(c+dx))^3}-\frac{13a^5\sin^4(c+dx)\cos(c+dx)}{15d(a-a\sin(c+dx))^2}-\frac{136a^3\cos^3(c+dx)}{15d}+\frac{136a^3\cos(c+dx)}{5d}+\frac{23a^3\sin(c$$

```
[Out] -23/2*a^3*x+136/5*a^3*cos(d*x+c)/d-136/15*a^3*cos(d*x+c)^3/d+23/2*a^3*cos(d
*x+c)*sin(d*x+c)/d+1/5*a^6*cos(d*x+c)*sin(d*x+c)^5/d/(a-a*sin(d*x+c))^3-13/
15*a^5*cos(d*x+c)*sin(d*x+c)^4/d/(a-a*sin(d*x+c))^2+23/3*a^6*cos(d*x+c)*sin
(d*x+c)^3/d/(a^3-a^3*sin(d*x+c))
```

Rubi [A] time = 0.36, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2708, 2765, 2977, 2748, 2635, 8, 2633}

$$-\frac{136a^3\cos^3(c+dx)}{15d}+\frac{136a^3\cos(c+dx)}{5d}+\frac{23a^6\sin^3(c+dx)\cos(c+dx)}{3d\left(a^3-a^3\sin(c+dx)\right)}+\frac{a^6\sin^5(c+dx)\cos(c+dx)}{5d(a-a\sin(c+dx))^3}-\frac{13a^5\sin^4}{15d(a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^6,x]
```

```
[Out] (-23*a^3*x)/2 + (136*a^3*Cos[c + d*x])/(5*d) - (136*a^3*Cos[c + d*x]^3)/(15
*d) + (23*a^3*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (a^6*Cos[c + d*x]*Sin[c +
d*x]^5)/(5*d*(a - a*Sin[c + d*x])^3) - (13*a^5*Cos[c + d*x]*Sin[c + d*x]^4)
/(15*d*(a - a*Sin[c + d*x])^2) + (23*a^6*Cos[c + d*x]*Sin[c + d*x]^3)/(3*d*
(a^3 - a^3*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 2708

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_), x_Symbol] :> Dist[a^p, Int[Sin[e + f*x]^p/(a - b*Sin[e + f*x])^m, x], x
] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[
p, 2*m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int (a+a\sin(c+dx))^3\tan^6(c+dx)\,dx = a^6\int \frac{\sin^6(c+dx)}{(a-a\sin(c+dx))^3}\,dx$$
$$= \frac{a^6\cos(c+dx)\sin^5(c+dx)}{5d(a-a\sin(c+dx))^3} + \frac{1}{5}a^4\int \frac{\sin^4(c+dx)(-5a-8a\sin(c+dx)}{(a-a\sin(c+dx))^2}$$
$$= \frac{a^6\cos(c+dx)\sin^5(c+dx)}{5d(a-a\sin(c+dx))^3} - \frac{13a^5\cos(c+dx)\sin^4(c+dx)}{15d(a-a\sin(c+dx))^2} - \frac{1}{15}a^2\int$$
$$= \frac{a^6\cos(c+dx)\sin^5(c+dx)}{5d(a-a\sin(c+dx))^3} - \frac{13a^5\cos(c+dx)\sin^4(c+dx)}{15d(a-a\sin(c+dx))^2} + \frac{23a^4\cos}{3d(}$$
$$= \frac{a^6\cos(c+dx)\sin^5(c+dx)}{5d(a-a\sin(c+dx))^3} - \frac{13a^5\cos(c+dx)\sin^4(c+dx)}{15d(a-a\sin(c+dx))^2} + \frac{23a^4\cos}{3d(}$$
$$= \frac{23a^3\cos(c+dx)\sin(c+dx)}{2d} + \frac{a^6\cos(c+dx)\sin^5(c+dx)}{5d(a-a\sin(c+dx))^3} - \frac{13a^5\cos(}{15d(a}$$
$$= -\frac{23a^3x}{2} + \frac{136a^3\cos(c+dx)}{5d} - \frac{136a^3\cos^3(c+dx)}{15d} + \frac{23a^3\cos(c+dx)}{2d}$$

Mathematica [A] time = 5.12, size = 243, normalized size = 1.35

$$\frac{(a\sin(c+dx)+a)^3\left(-690(c+dx)+45\sin(2(c+dx))+405\cos(c+dx)-5\cos(3(c+dx))+\frac{1576\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c\right.}\right.}{60d\left(\sin\left(\frac{1}{2}(c+dx\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^6,x]
```

```
[Out] ((a + a*Sin[c + d*x])^3*(-690*(c + d*x) + 405*Cos[c + d*x] - 5*Cos[3*(c + d
*x)] + 12/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^4 - 112/(Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2])^2 + (24*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d
*x)/2])^5 - (224*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3
+ (1576*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) + 45*Sin[2*
(c + d*x)]))/(60*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)
```

fricas [A] time = 0.42, size = 289, normalized size = 1.61

$$-\frac{10\,a^3\cos(dx+c)^6 - 15\,a^3\cos(dx+c)^5 - 140\,a^3\cos(dx+c)^4 - 1380\,a^3dx + \left(345\,a^3dx - 839\,a^3\right)\cos(dx+c)^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] -1/30*(10*a^3*cos(d*x + c)^6 - 15*a^3*cos(d*x + c)^5 - 140*a^3*cos(d*x + c)
^4 - 1380*a^3*d*x + (345*a^3*d*x - 839*a^3)*cos(d*x + c)^3 + 6*a^3 + (1035*
a^3*d*x + 668*a^3)*cos(d*x + c)^2 - 6*(115*a^3*d*x - 233*a^3)*cos(d*x + c)
- (10*a^3*cos(d*x + c)^5 + 25*a^3*cos(d*x + c)^4 - 115*a^3*cos(d*x + c)^3 -
 1380*a^3*d*x - 6*a^3 + (345*a^3*d*x + 724*a^3)*cos(d*x + c)^2 - 6*(115*a^3
*d*x - 232*a^3)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3 + 3*d*cos(d*x
 + c)^2 - 2*d*cos(d*x + c) - (d*cos(d*x + c)^2 - 2*d*cos(d*x + c) - 4*d)*si
n(d*x + c) - 4*d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^6,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.38, size = 359, normalized size = 1.99

$$\frac{a^3\left(\frac{\sin^{10}(dx+c)}{5\cos(dx+c)^5}-\frac{\sin^{10}(dx+c)}{3\cos(dx+c)^3}+\frac{7\left(\sin^{10}(dx+c)\right)}{3\cos(dx+c)}+\frac{7\left(\frac{128}{35}+\sin^8(dx+c)+\frac{8\left(\sin^6(dx+c)\right)}{7}+\frac{48\left(\sin^4(dx+c)\right)}{35}+\frac{64\left(\sin^2(dx+c)\right)}{35}\right)\cos(dx+c)}{3}\right)+3a^3\left(\frac{\sin^9}{5\cos}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^3*tan(d*x+c)^6,x)
```

```
[Out] 1/d*(a^3*(1/5*sin(d*x+c)^10/cos(d*x+c)^5-1/3*sin(d*x+c)^10/cos(d*x+c)^3+7/3
*sin(d*x+c)^10/cos(d*x+c)+7/3*(128/35+sin(d*x+c)^8+8/7*sin(d*x+c)^6+48/35*s
in(d*x+c)^4+64/35*sin(d*x+c)^2)*cos(d*x+c))+3*a^3*(1/5*sin(d*x+c)^9/cos(d*x
+c)^5-4/15*sin(d*x+c)^9/cos(d*x+c)^3+8/5*sin(d*x+c)^9/cos(d*x+c)+8/5*(sin(d
*x+c)^7+7/6*sin(d*x+c)^5+35/24*sin(d*x+c)^3+35/16*sin(d*x+c))*cos(d*x+c)-7/
2*d*x-7/2*c)+3*a^3*(1/5*sin(d*x+c)^8/cos(d*x+c)^5-1/5*sin(d*x+c)^8/cos(d*x+
c)^3+sin(d*x+c)^8/cos(d*x+c)+(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*
x+c)^2)*cos(d*x+c))+a^3*(1/5*tan(d*x+c)^5-1/3*tan(d*x+c)^3+tan(d*x+c)-d*x-c
))
```

maxima [A] time = 0.40, size = 209, normalized size = 1.16

$$\frac{3\left(6\tan(dx+c)^5-20\tan(dx+c)^3-105\,dx-105\,c+\frac{15\tan(dx+c)}{\tan(dx+c)^2+1}+90\tan(dx+c)\right)a^3+2\left(3\tan(dx+c)^5-5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/30*(3*(6*tan(d*x + c)^5 - 20*tan(d*x + c)^3 - 105*d*x - 105*c + 15*tan(d*
x + c)/(tan(d*x + c)^2 + 1) + 90*tan(d*x + c))*a^3 + 2*(3*tan(d*x + c)^5 -
5*tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(d*x + c))*a^3 - 2*(5*cos(d*x + c)
^3 - (90*cos(d*x + c)^4 - 20*cos(d*x + c)^2 + 3)/cos(d*x + c)^5 - 60*cos(d*
x + c))*a^3 + 18*a^3*((15*cos(d*x + c)^4 - 5*cos(d*x + c)^2 + 1)/cos(d*x +
c)^5 + 5*cos(d*x + c)))/d
```

mupad [B] time = 11.05, size = 438, normalized size = 2.43

$$-\frac{23\,a^3\,x}{2}-\frac{\frac{23\,a^3\,(c+d\,x)}{2}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{115\,a^3\,(c+d\,x)}{2}-\frac{a^3\,(1725\,c+1725\,d\,x-4750)}{30}\right)-\frac{a^3\,(345\,c+345\,d\,x-1088)}{30}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^6*(a + a*sin(c + d*x))^3,x)
```

```
[Out] - (23*a^3*x)/2 - ((23*a^3*(c + d*x))/2 - tan(c/2 + (d*x)/2)*((115*a^3*(c +
d*x))/2 - (a^3*(1725*c + 1725*d*x - 4750))/30) - (a^3*(345*c + 345*d*x - 10
88))/30 + tan(c/2 + (d*x)/2)^10*((115*a^3*(c + d*x))/2 - (a^3*(1725*c + 172
5*d*x - 690))/30) - tan(c/2 + (d*x)/2)^9*((299*a^3*(c + d*x))/2 - (a^3*(448
5*c + 4485*d*x - 3450))/30) + tan(c/2 + (d*x)/2)^2*((299*a^3*(c + d*x))/2 -
 (a^3*(4485*c + 4485*d*x - 10694))/30) + tan(c/2 + (d*x)/2)^8*((575*a^3*(c
+ d*x))/2 - (a^3*(8625*c + 8625*d*x - 8740))/30) - tan(c/2 + (d*x)/2)^3*((5
75*a^3*(c + d*x))/2 - (a^3*(8625*c + 8625*d*x - 18460))/30) - tan(c/2 + (d*
x)/2)^7*(437*a^3*(c + d*x) - (a^3*(13110*c + 13110*d*x - 16100))/30) + tan(
c/2 + (d*x)/2)^4*(437*a^3*(c + d*x) - (a^3*(13110*c + 13110*d*x - 25244))/3
0) + tan(c/2 + (d*x)/2)^6*(529*a^3*(c + d*x) - (a^3*(15870*c + 15870*d*x -
23368))/30) - tan(c/2 + (d*x)/2)^5*(529*a^3*(c + d*x) - (a^3*(15870*c + 158
70*d*x - 26680))/30))/(d*(tan(c/2 + (d*x)/2) - 1)^5*(tan(c/2 + (d*x)/2)^2 +
 1)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**3*tan(d*x+c)**6,x)
```

```
[Out] Timed out
```

3.30 $\int (a+a\sin(c+dx))^3\tan^4(c+dx)\,dx$

Optimal. Leaf size=119

$$\frac{a^3\cos^3(c+dx)}{3d}-\frac{6a^3\cos(c+dx)}{d}-\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}-\frac{25a^3\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}+\frac{17a^3x}{2}$$

```
[Out] 17/2*a^3*x-6*a^3*cos(d*x+c)/d+1/3*a^3*cos(d*x+c)^3/d+2/3*a^3*cos(d*x+c)/d/(
1-sin(d*x+c))^2-25/3*a^3*cos(d*x+c)/d/(1-sin(d*x+c))-3/2*a^3*cos(d*x+c)*sin
(d*x+c)/d
```

Rubi [A] time = 0.19, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2709, 2650, 2648, 2638, 2635, 8, 2633}

$$\frac{a^3\cos^3(c+dx)}{3d}-\frac{6a^3\cos(c+dx)}{d}-\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}-\frac{25a^3\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}+\frac{17a^3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^4,x]
```

```
[Out] (17*a^3*x)/2 - (6*a^3*Cos[c + d*x])/d + (a^3*Cos[c + d*x]^3)/(3*d) + (2*a^3
*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x])^2) - (25*a^3*Cos[c + d*x])/(3*d*(1 -
 Sin[c + d*x])) - (3*a^3*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rubi steps

$$\begin{aligned}
\int (a+a\sin(c+dx))^3\tan^4(c+dx)\,dx &= a^4\int\left(\frac{7}{a}+\frac{2}{a(-1+\sin(c+dx))^2}+\frac{9}{a(-1+\sin(c+dx))}+\frac{5\sin(c+dx)}{a}\right.\\
&= 7a^3x+a^3\int\sin^3(c+dx)\,dx+\left(2a^3\right)\int\frac{1}{(-1+\sin(c+dx))^2}\,dx+\left(3a^3\right)\\
&= 7a^3x-\frac{5a^3\cos(c+dx)}{d}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}-\frac{9a^3\cos(c+dx)}{d(1-\sin(c+dx))}-\\
&= \frac{17a^3x}{2}-\frac{6a^3\cos(c+dx)}{d}+\frac{a^3\cos^3(c+dx)}{3d}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}-
\end{aligned}$$

Mathematica [A] time = 2.10, size = 177, normalized size = 1.49

$$\frac{(a\sin(c+dx)+a)^3\left(102(c+dx)-9\sin(2(c+dx))-69\cos(c+dx)+\cos(3(c+dx))-\frac{200\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}+\right.}{12d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^4,x]
```

```
[Out] ((a + a*Sin[c + d*x])^3*(102*(c + d*x) - 69*Cos[c + d*x] + Cos[3*(c + d*x)]
 + 8/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (16*Sin[(c + d*x)/2])/(Cos[(
c + d*x)/2] - Sin[(c + d*x)/2])^3 - (200*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2
] - Sin[(c + d*x)/2]) - 9*Sin[2*(c + d*x)]))/(12*d*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2])^6)
```

fricas [B] time = 0.46, size = 220, normalized size = 1.85

$$\frac{2\,a^3\cos(dx+c)^5+7\,a^3\cos(dx+c)^4-22\,a^3\cos(dx+c)^3-102\,a^3dx-4\,a^3+\left(51\,a^3dx+77\,a^3\right)\cos(dx+c)^2-\left(\right.}{6\left(d\cos(dx+c)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a^3*cos(d*x + c)^5 + 7*a^3*cos(d*x + c)^4 - 22*a^3*cos(d*x + c)^3 -
102*a^3*d*x - 4*a^3 + (51*a^3*d*x + 77*a^3)*cos(d*x + c)^2 - (51*a^3*d*x -
100*a^3)*cos(d*x + c) + (2*a^3*cos(d*x + c)^4 - 5*a^3*cos(d*x + c)^3 + 102*
a^3*d*x - 27*a^3*cos(d*x + c)^2 - 4*a^3 + (51*a^3*d*x - 104*a^3)*cos(d*x +
c))*sin(d*x + c))/(d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c) + 2*
d)*sin(d*x + c) - 2*d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.36, size = 266, normalized size = 2.24

$$a^{3}\left(\frac{\sin^{8}(dx+c)}{3\cos(dx+c)^{3}}-\frac{5\left(\sin^{8}(dx+c)\right)}{3\cos(dx+c)}-\frac{5\left(\frac{16}{5}+\sin^{6}(dx+c)+\frac{6\left(\sin^{4}(dx+c)\right)}{5}+\frac{8\left(\sin^{2}(dx+c)\right)}{5}\right)\cos(dx+c)}{3}\right)+3a^{3}\left(\frac{\sin^{7}(dx+c)}{3\cos(dx+c)^{3}}-\frac{4\left(\sin^{7}(dx+c)\right)}{3\cos(dx+c)}-\frac{4\left(\mathrm{s}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^3*tan(d*x+c)^4,x)
```

```
[Out] 1/d*(a^3*(1/3*sin(d*x+c)^8/cos(d*x+c)^3-5/3*sin(d*x+c)^8/cos(d*x+c)-5/3*(16
/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))+3*a^3*(1/3*s
in(d*x+c)^7/cos(d*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/3*(sin(d*x+c)^5+5/4*
sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c)+3*a^3*(1/3*sin(d*x+
c)^6/cos(d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^
2)*cos(d*x+c))+a^3*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c))
```

maxima [A] time = 0.42, size = 165, normalized size = 1.39

$$\frac{2\left(\cos\left(dx+c\right)^{3}-\frac{9\cos(dx+c)^{2}-1}{\cos(dx+c)^{3}}-9\cos\left(dx+c\right)\right)a^{3}+3\left(2\tan\left(dx+c\right)^{3}+15\,dx+15\,c-\frac{3\tan(dx+c)}{\tan(dx+c)^{2}+1}-12\tan\left(d\right.\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(cos(d*x + c)^3 - (9*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 - 9*cos(d*x
+ c))*a^3 + 3*(2*tan(d*x + c)^3 + 15*d*x + 15*c - 3*tan(d*x + c)/(tan(d*x +
 c)^2 + 1) - 12*tan(d*x + c))*a^3 + 2*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan
(d*x + c))*a^3 - 6*a^3*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x +
 c)))/d
```

mupad [B] time = 10.50, size = 371, normalized size = 3.12

$$\frac{17\,a^{3}\,x}{2}+\frac{\frac{17\,a^{3}\,(c+d\,x)}{2}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{51\,a^{3}\,(c+d\,x)}{2}-\frac{a^{3}\,(153\,c+153\,d\,x-378)}{6}\right)-\frac{a^{3}\,(51\,c+51\,d\,x-160)}{6}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}\left(\frac{51\,a^{3}\,(c+}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (17*a^3*x)/2 + ((17*a^3*(c + d*x))/2 - tan(c/2 + (d*x)/2)*((51*a^3*(c + d*x
))/2 - (a^3*(153*c + 153*d*x - 378))/6) - (a^3*(51*c + 51*d*x - 160))/6 + t
an(c/2 + (d*x)/2)^8*((51*a^3*(c + d*x))/2 - (a^3*(153*c + 153*d*x - 102))/6
) - tan(c/2 + (d*x)/2)^7*(51*a^3*(c + d*x) - (a^3*(306*c + 306*d*x - 306))/
6) + tan(c/2 + (d*x)/2)^2*(51*a^3*(c + d*x) - (a^3*(306*c + 306*d*x - 654))
/6) + tan(c/2 + (d*x)/2)^6*(85*a^3*(c + d*x) - (a^3*(510*c + 510*d*x - 578)
)/6) - tan(c/2 + (d*x)/2)^3*(85*a^3*(c + d*x) - (a^3*(510*c + 510*d*x - 102
2))/6) - tan(c/2 + (d*x)/2)^5*(102*a^3*(c + d*x) - (a^3*(612*c + 612*d*x -
918))/6) + tan(c/2 + (d*x)/2)^4*(102*a^3*(c + d*x) - (a^3*(612*c + 612*d*x
- 1002))/6))/(d*(tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2 + tan(c/2 + (d*x
)/2)^3 - 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\sin\left(c+dx\right)\tan^4\left(c+dx\right)dx+\int 3\sin^2\left(c+dx\right)\tan^4\left(c+dx\right)dx+\int \sin^3\left(c+dx\right)\tan^4\left(c+dx\right)dx+\int \mathrm{t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**3*tan(d*x+c)**4,x)
```

```
[Out] a**3*(Integral(3*sin(c + d*x)*tan(c + d*x)**4, x) + Integral(3*sin(c + d*x)
**2*tan(c + d*x)**4, x) + Integral(sin(c + d*x)**3*tan(c + d*x)**4, x) + In
tegral(tan(c + d*x)**4, x))
```

3.31 $\int (a+a\sin(c+dx))^3\tan^2(c+dx)\,dx$

Optimal. Leaf size=89

$$-\frac{a^3\cos^3(c+dx)}{3d}+\frac{5a^3\cos(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{11a^3x}{2}$$

```
[Out] -11/2*a^3*x+5*a^3*cos(d*x+c)/d-1/3*a^3*cos(d*x+c)^3/d+4*a^3*cos(d*x+c)/d/(1
-sin(d*x+c))+3/2*a^3*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A] time = 0.13, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2709, 2648, 2638, 2635, 8, 2633}

$$-\frac{a^3\cos^3(c+dx)}{3d}+\frac{5a^3\cos(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{11a^3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] (-11*a^3*x)/2 + (5*a^3*Cos[c + d*x])/d - (a^3*Cos[c + d*x]^3)/(3*d) + (4*a^
3*Cos[c + d*x])/(d*(1 - Sin[c + d*x])) + (3*a^3*Cos[c + d*x]*Sin[c + d*x])/
(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rubi steps

$$\int (a+a\sin(c+dx))^3\tan^2(c+dx)\,dx = a^2\int\left(-4a-\frac{4a}{-1+\sin(c+dx)}-4a\sin(c+dx)-3a\sin^2(c+dx)-a\sin\right.$$

$$= -4a^3x-a^3\int\sin^3(c+dx)\,dx-\left(3a^3\right)\int\sin^2(c+dx)\,dx-\left(4a^3\right)\int\frac{}{-1+}$$

$$= -4a^3x+\frac{4a^3\cos(c+dx)}{d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}+\frac{3a^3\cos(c+dx)\sin(c+}{2d}$$

$$= -\frac{11a^3x}{2}+\frac{5a^3\cos(c+dx)}{d}-\frac{a^3\cos^3(c+dx)}{3d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}+\frac{3a^3}{}$$

Mathematica [A] time = 0.48, size = 115, normalized size = 1.29

$$\frac{(a\sin(c+dx)+a)^3\left(-66(c+dx)+9\sin(2(c+dx))+57\cos(c+dx)-\cos(3(c+dx))+\frac{96\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}\right)}{12d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] ((a + a*Sin[c + d*x])^3*(-66*(c + d*x) + 57*Cos[c + d*x] - Cos[3*(c + d*x)]
 + (96*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) + 9*Sin[2*(c
 + d*x)]))/(12*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)
```

fricas [A] time = 0.43, size = 154, normalized size = 1.73

$$-\frac{2\,a^{3}\cos\left(dx+c\right)^{4}-7\,a^{3}\cos\left(dx+c\right)^{3}+33\,a^{3}dx-30\,a^{3}\cos\left(dx+c\right)^{2}-24\,a^{3}+3\left(11\,a^{3}dx-15\,a^{3}\right)\cos\left(dx+c\right)}{6\left(d\cos\left(dx+c\right)-d\sin\left(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*a^3*cos(d*x + c)^4 - 7*a^3*cos(d*x + c)^3 + 33*a^3*d*x - 30*a^3*cos
(d*x + c)^2 - 24*a^3 + 3*(11*a^3*d*x - 15*a^3)*cos(d*x + c) - (2*a^3*cos(d*
x + c)^3 + 33*a^3*d*x + 9*a^3*cos(d*x + c)^2 - 21*a^3*cos(d*x + c) + 24*a^3
)*sin(d*x + c))/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.31, size = 167, normalized size = 1.88

$$\frac{a^{3}\left(\frac{\sin^{6}(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}+\sin^{4}\left(dx+c\right)+\frac{4\left(\sin^{2}(dx+c)\right)}{3}\right)\cos\left(dx+c\right)\right)+3a^{3}\left(\frac{\sin^{5}(dx+c)}{\cos(dx+c)}+\left(\sin^{3}\left(dx+c\right)+\frac{3\sin(dx+c)}{2}\right)\cos\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^3*tan(d*x+c)^2,x)
```

```
[Out] 1/d*(a^3*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d
*x+c))+3*a^3*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x
+c)-3/2*d*x-3/2*c)+3*a^3*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+
c))+a^3*(tan(d*x+c)-d*x-c))
```

maxima [A] time = 0.44, size = 117, normalized size = 1.31

$$-\frac{2\left(\cos\left(dx+c\right)^{3}-\frac{3}{\cos(dx+c)}-6\,\cos\left(dx+c\right)\right)a^{3}+9\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^{2}+1}-2\,\tan\left(dx+c\right)\right)a^{3}+6\left(dx+c-t\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(2*(cos(d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*a^3 + 9*(3*d*x +
 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))*a^3 + 6*(d*x + c
 - tan(d*x + c))*a^3 - 18*a^3*(1/cos(d*x + c) + cos(d*x + c)))/d
```

mupad [B] time = 10.33, size = 288, normalized size = 3.24

$$-\frac{11\,a^3\,x}{2}-\frac{\frac{11\,a^3\,(c+d\,x)}{2}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{11\,a^3\,(c+d\,x)}{2}-\frac{a^3\,(33\,c+33\,d\,x-38)}{6}\right)-\frac{a^3\,(33\,c+33\,d\,x-104)}{6}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(\frac{11\,a^3\,(c+d\,x)}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + a*sin(c + d*x))^3,x)
```

```
[Out] - (11*a^3*x)/2 - ((11*a^3*(c + d*x))/2 - tan(c/2 + (d*x)/2)*((11*a^3*(c + d
*x))/2 - (a^3*(33*c + 33*d*x - 38))/6) - (a^3*(33*c + 33*d*x - 104))/6 + ta
n(c/2 + (d*x)/2)^6*((11*a^3*(c + d*x))/2 - (a^3*(33*c + 33*d*x - 66))/6) -
tan(c/2 + (d*x)/2)^5*((33*a^3*(c + d*x))/2 - (a^3*(99*c + 99*d*x - 66))/6)
- tan(c/2 + (d*x)/2)^3*((33*a^3*(c + d*x))/2 - (a^3*(99*c + 99*d*x - 120))/
6) + tan(c/2 + (d*x)/2)^4*((33*a^3*(c + d*x))/2 - (a^3*(99*c + 99*d*x - 192
))/6) + tan(c/2 + (d*x)/2)^2*((33*a^3*(c + d*x))/2 - (a^3*(99*c + 99*d*x -
246))/6))/(d*(tan(c/2 + (d*x)/2) - 1)*(tan(c/2 + (d*x)/2)^2 + 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\sin\left(c+dx\right)\tan^2\left(c+dx\right)dx+\int 3\sin^2\left(c+dx\right)\tan^2\left(c+dx\right)dx+\int \sin^3\left(c+dx\right)\tan^2\left(c+dx\right)dx+\int \mathrm{t}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**3*tan(d*x+c)**2,x)
```

```
[Out] a**3*(Integral(3*sin(c + d*x)*tan(c + d*x)**2, x) + Integral(3*sin(c + d*x)
**2*tan(c + d*x)**2, x) + Integral(sin(c + d*x)**3*tan(c + d*x)**2, x) + In
tegral(tan(c + d*x)**2, x))
```

3.32 $\int (a + a\sin(c + dx))^3\,dx$

Optimal. Leaf size=63

$$\frac{a^3\cos^3(c+dx)}{3d} - \frac{4a^3\cos(c+dx)}{d} - \frac{3a^3\sin(c+dx)\cos(c+dx)}{2d} + \frac{5a^3x}{2}$$

```
[Out] 5/2*a^3*x-4*a^3*cos(d*x+c)/d+1/3*a^3*cos(d*x+c)^3/d-3/2*a^3*cos(d*x+c)*sin(
d*x+c)/d
```

Rubi [A] time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2645, 2638, 2635, 8, 2633}

$$\frac{a^3\cos^3(c+dx)}{3d} - \frac{4a^3\cos(c+dx)}{d} - \frac{3a^3\sin(c+dx)\cos(c+dx)}{2d} + \frac{5a^3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (5*a^3*x)/2 - (4*a^3*Cos[c + d*x])/d + (a^3*Cos[c + d*x]^3)/(3*d) - (3*a^3*
Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2645

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(c+dx))^3\,dx &= \int \left(a^3+3a^3\sin(c+dx)+3a^3\sin^2(c+dx)+a^3\sin^3(c+dx)\right)dx\\ &= a^3x+a^3\int \sin^3(c+dx)\,dx+\left(3a^3\right)\int \sin(c+dx)\,dx+\left(3a^3\right)\int \sin^2(c+dx)\,dx\\ &= a^3x-\frac{3a^3\cos(c+dx)}{d}-\frac{3a^3\cos(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}\left(3a^3\right)\int 1\,dx-\frac{a^3\,\mathrm{Subst}\left(\int\right.}{}\\ &= \frac{5a^3x}{2}-\frac{4a^3\cos(c+dx)}{d}+\frac{a^3\cos^3(c+dx)}{3d}-\frac{3a^3\cos(c+dx)\sin(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.32, size = 44, normalized size = 0.70

$$\frac{a^3(-9\sin(2(c+dx))-45\cos(c+dx)+\cos(3(c+dx))+30c+30dx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(30*c + 30*d*x - 45*Cos[c + d*x] + Cos[3*(c + d*x)] - 9*Sin[2*(c + d*x
)]))/(12*d)
```

fricas [A] time = 0.40, size = 54, normalized size = 0.86

$$\frac{2\,a^3\cos\left(dx+c\right)^3+15\,a^3dx-9\,a^3\cos\left(dx+c\right)\sin\left(dx+c\right)-24\,a^3\cos\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a^3*cos(d*x + c)^3 + 15*a^3*d*x - 9*a^3*cos(d*x + c)*sin(d*x + c) -
24*a^3*cos(d*x + c))/d
```

giac [A] time = 0.37, size = 55, normalized size = 0.87

$$\frac{5}{2}\,a^3x+\frac{a^3\cos\left(3\,dx+3\,c\right)}{12\,d}-\frac{15\,a^3\cos\left(dx+c\right)}{4\,d}-\frac{3\,a^3\sin\left(2\,dx+2\,c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 5/2*a^3*x + 1/12*a^3*cos(3*d*x + 3*c)/d - 15/4*a^3*cos(d*x + c)/d - 3/4*a^3
*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.14, size = 74, normalized size = 1.17

$$\frac{-\frac{a^3\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3} + 3a^3\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right) - 3a^3\cos(dx+c) + a^3(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(-1/3*a^3*(2+sin(d*x+c)^2)*cos(d*x+c)+3*a^3*(-1/2*cos(d*x+c)*sin(d*x+c)
+1/2*d*x+1/2*c)-3*a^3*cos(d*x+c)+a^3*(d*x+c))
```

maxima [A] time = 0.29, size = 72, normalized size = 1.14

$$a^3x + \frac{\left(\cos(dx+c)^3 - 3\cos(dx+c)\right)a^3}{3d} + \frac{3(2dx+2c-\sin(2dx+2c))a^3}{4d} - \frac{3a^3\cos(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] a^3*x + 1/3*(cos(d*x + c)^3 - 3*cos(d*x + c))*a^3/d + 3/4*(2*d*x + 2*c - si
n(2*d*x + 2*c))*a^3/d - 3*a^3*cos(d*x + c)/d
```

mupad [B] time = 8.92, size = 156, normalized size = 2.48

$$\frac{5a^3x}{2} - \frac{\frac{5a^3(c+dx)}{2} - 3a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5 - \frac{a^3(15c+15dx-44)}{6} + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{15a^3(c+dx)}{2} - \frac{a^3(45c+45dx-36)}{6}\right) + \tan\left(\frac{c}{2}\right.}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3,x)
```

```
[Out] (5*a^3*x)/2 - ((5*a^3*(c + d*x))/2 - 3*a^3*tan(c/2 + (d*x)/2)^5 - (a^3*(15*
c + 15*d*x - 44))/6 + tan(c/2 + (d*x)/2)^4*((15*a^3*(c + d*x))/2 - (a^3*(45
*c + 45*d*x - 36))/6) + tan(c/2 + (d*x)/2)^2*((15*a^3*(c + d*x))/2 - (a^3*(
45*c + 45*d*x - 96))/6) + 3*a^3*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)^
2 + 1)^3)
```

sympy [A] time = 1.05, size = 121, normalized size = 1.92

$$\begin{cases} \frac{3a^3x\sin^2(c+dx)}{2} + \frac{3a^3x\cos^2(c+dx)}{2} + a^3x - \frac{a^3\sin^2(c+dx)\cos(c+dx)}{d} - \frac{3a^3\sin(c+dx)\cos(c+dx)}{2d} - \frac{2a^3\cos^3(c+dx)}{3d} - \frac{3a^3\cos(c+dx)}{d} & \text{fo} \\ x(a\sin(c)+a)^3 & \text{ot} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((3*a**3*x*sin(c + d*x)**2/2 + 3*a**3*x*cos(c + d*x)**2/2 + a**3*x
 - a**3*sin(c + d*x)**2*cos(c + d*x)/d - 3*a**3*sin(c + d*x)*cos(c + d*x)/(
2*d) - 2*a**3*cos(c + d*x)**3/(3*d) - 3*a**3*cos(c + d*x)/d, Ne(d, 0)), (x*
(a*sin(c) + a)**3, True))
```

3.33 $\int \cot^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=92

$$-\frac{a^3\cos^3(c+dx)}{3d}+\frac{3a^3\cos(c+dx)}{d}-\frac{a^3\cot(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{a^3x}{2}$$

```
[Out] 1/2*a^3*x-3*a^3*arctanh(cos(d*x+c))/d+3*a^3*cos(d*x+c)/d-1/3*a^3*cos(d*x+c)
^3/d-a^3*cot(d*x+c)/d+3/2*a^3*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A] time = 0.14, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2709, 3770, 3767, 8, 2638, 2635, 2633}

$$-\frac{a^3\cos^3(c+dx)}{3d}+\frac{3a^3\cos(c+dx)}{d}-\frac{a^3\cot(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{a^3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*x)/2 - (3*a^3*ArcTanh[Cos[c + d*x]])/d + (3*a^3*Cos[c + d*x])/d - (a^3
*Cos[c + d*x]^3)/(3*d) - (a^3*Cot[c + d*x])/d + (3*a^3*Cos[c + d*x]*Sin[c +
 d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\int \left(2a^5+3a^5\csc(c+dx)+a^5\csc^2(c+dx)-2a^5\sin(c+dx)-3a^5\sin^2(c$$
$$a^2$$
$$= 2a^3x+a^3\int\csc^2(c+dx)\,dx-a^3\int\sin^3(c+dx)\,dx-\left(2a^3\right)\int\sin(c+d$$
$$= 2a^3x-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{2a^3\cos(c+dx)}{d}+\frac{3a^3\cos(c+dx)\sin}{2d}$$
$$= \frac{a^3x}{2}-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{3a^3\cos(c+dx)}{d}-\frac{a^3\cos^3(c+dx)}{3d}-\frac{a^{}}{}$$

Mathematica [A] time = 1.09, size = 106, normalized size = 1.15

$$-\frac{a^3\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left((15-66\sin(c+dx))\cos(c+dx)+(2\sin(c+dx)+9)\cos(3(c+dx))-12\sin}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/48*(a^3*Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(Cos[c + d*x]*(15 - 66*Sin[c +
 d*x]) - 12*(c + d*x - 6*Log[Cos[(c + d*x)/2]] + 6*Log[Sin[(c + d*x)/2]])*S
in[c + d*x] + Cos[3*(c + d*x)]*(9 + 2*Sin[c + d*x])))/d
```

fricas [A] time = 0.45, size = 121, normalized size = 1.32

$$-\frac{9\,a^{3}\cos\left(dx+c\right)^{3}+9\,a^{3}\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-9\,a^{3}\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-3}{6\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/6*(9*a^3*cos(d*x + c)^3 + 9*a^3*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c)
 - 9*a^3*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*a^3*cos(d*x + c) + (
2*a^3*cos(d*x + c)^3 - 3*a^3*d*x - 18*a^3*cos(d*x + c))*sin(d*x + c))/(d*si
n(d*x + c))
```

giac [A] time = 1.39, size = 162, normalized size = 1.76

$$\frac{3\,(dx+c)a^{3}+18\,a^{3}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+3\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{3\left(6\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^{3}\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{2\left(9\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-1}{}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(3*(d*x + c)*a^3 + 18*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 3*a^3*tan(1/
2*d*x + 1/2*c) - 3*(6*a^3*tan(1/2*d*x + 1/2*c) + a^3)/tan(1/2*d*x + 1/2*c)
- 2*(9*a^3*tan(1/2*d*x + 1/2*c)^5 - 12*a^3*tan(1/2*d*x + 1/2*c)^4 - 36*a^3*
tan(1/2*d*x + 1/2*c)^2 - 9*a^3*tan(1/2*d*x + 1/2*c) - 16*a^3)/(tan(1/2*d*x
+ 1/2*c)^2 + 1)^3)/d
```

maple [A] time = 0.14, size = 105, normalized size = 1.14

$$-\frac{a^{3}\left(\cos^{3}\left(dx+c\right)\right)}{3d}+\frac{3a^{3}\cos\left(dx+c\right)\sin\left(dx+c\right)}{2d}+\frac{a^{3}x}{2}+\frac{a^{3}c}{2d}+\frac{3a^{3}\cos\left(dx+c\right)}{d}+\frac{3a^{3}\ln\left(\csc\left(dx+c\right)-\cot\left(dx+\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/3*a^3*cos(d*x+c)^3/d+3/2*a^3*cos(d*x+c)*sin(d*x+c)/d+1/2*a^3*x+1/2/d*a^3
*c+3*a^3*cos(d*x+c)/d+3/d*a^3*ln(csc(d*x+c)-cot(d*x+c))-a^3*cot(d*x+c)/d
```

maxima [A] time = 0.40, size = 93, normalized size = 1.01

$$-\frac{4\,a^{3}\cos\left(dx+c\right)^{3}-9\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^{3}+12\left(dx+c+\frac{1}{\tan\left(dx+c\right)}\right)a^{3}-18\,a^{3}\left(2\,\cos\left(dx+c\right)-\log\left(\right.\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/12*(4*a^3*cos(d*x + c)^3 - 9*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^3 + 12*(
d*x + c + 1/tan(d*x + c))*a^3 - 18*a^3*(2*cos(d*x + c) - log(cos(d*x + c) +
 1) + log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 6.77, size = 264, normalized size = 2.87

$$\frac{3\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{a^3\,\mathrm{atan}\left(\frac{a^6}{6\,a^6-a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{6\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{6\,a^6-a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}+\frac{-7\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+8\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (3*a^3*log(tan(c/2 + (d*x)/2)))/d + (a^3*atan(a^6/(6*a^6 - a^6*tan(c/2 + (d
*x)/2)) + (6*a^6*tan(c/2 + (d*x)/2))/(6*a^6 - a^6*tan(c/2 + (d*x)/2))))/d +
 (a^3*tan(c/2 + (d*x)/2))/(2*d) + (3*a^3*tan(c/2 + (d*x)/2)^2 + 24*a^3*tan(
c/2 + (d*x)/2)^3 - 3*a^3*tan(c/2 + (d*x)/2)^4 + 8*a^3*tan(c/2 + (d*x)/2)^5
- 7*a^3*tan(c/2 + (d*x)/2)^6 - a^3 + (32*a^3*tan(c/2 + (d*x)/2))/3)/(d*(2*t
an(c/2 + (d*x)/2) + 6*tan(c/2 + (d*x)/2)^3 + 6*tan(c/2 + (d*x)/2)^5 + 2*tan
(c/2 + (d*x)/2)^7))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\sin\left(c+dx\right)\cot^2\left(c+dx\right)dx+\int 3\sin^2\left(c+dx\right)\cot^2\left(c+dx\right)dx+\int \sin^3\left(c+dx\right)\cot^2\left(c+dx\right)dx+\int \mathrm{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*sin(c + d*x)*cot(c + d*x)**2, x) + Integral(3*sin(c + d*x)
**2*cot(c + d*x)**2, x) + Integral(sin(c + d*x)**3*cot(c + d*x)**2, x) + In
tegral(cot(c + d*x)**2, x))
```

3.34 $\int (a+a\sin(c+dx))^4\tan^5(c+dx)\,dx$

Optimal. Leaf size=129

$$\frac{a^6}{d(a-a\sin(c+dx))^2}-\frac{11a^5}{d(a-a\sin(c+dx))}-\frac{a^4\sin^4(c+dx)}{4d}-\frac{4a^4\sin^3(c+dx)}{3d}-\frac{9a^4\sin^2(c+dx)}{2d}-\frac{16a^4\sin(c+d}{d}$$

```
[Out] -25*a^4*ln(1-sin(d*x+c))/d-16*a^4*sin(d*x+c)/d-9/2*a^4*sin(d*x+c)^2/d-4/3*a
^4*sin(d*x+c)^3/d-1/4*a^4*sin(d*x+c)^4/d+a^6/d/(a-a*sin(d*x+c))^2-11*a^5/d/
(a-a*sin(d*x+c))
```

Rubi [A] time = 0.09, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 77}

$$-\frac{a^4\sin^4(c+dx)}{4d}-\frac{4a^4\sin^3(c+dx)}{3d}-\frac{9a^4\sin^2(c+dx)}{2d}+\frac{a^6}{d(a-a\sin(c+dx))^2}-\frac{11a^5}{d(a-a\sin(c+dx))}-\frac{16a^4\sin(c+}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^4*Tan[c + d*x]^5,x]
```

```
[Out] (-25*a^4*Log[1 - Sin[c + d*x]])/d - (16*a^4*Sin[c + d*x])/d - (9*a^4*Sin[c
+ d*x]^2)/(2*d) - (4*a^4*Sin[c + d*x]^3)/(3*d) - (a^4*Sin[c + d*x]^4)/(4*d)
 + a^6/(d*(a - a*Sin[c + d*x])^2) - (11*a^5)/(d*(a - a*Sin[c + d*x]))
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int (a + a\sin(c+dx))^4 \tan^5(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^5(a+x)}{(a-x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-16a^3 + \frac{2a^6}{(a-x)^3} - \frac{11a^5}{(a-x)^2} + \frac{25a^4}{a-x} - 9a^2x - 4ax^2 - x^3\right) dx, x, a\sin\right.}{d}$$

$$= -\frac{25a^4\log(1-\sin(c+dx))}{d} - \frac{16a^4\sin(c+dx)}{d} - \frac{9a^4\sin^2(c+dx)}{2d} - \frac{4a^4}{}$$

Mathematica [**A**] time = 0.45, size = 83, normalized size = 0.64

$$-\frac{a^4\left(3\sin^4(c+dx) + 16\sin^3(c+dx) + 54\sin^2(c+dx) + 192\sin(c+dx) + \frac{120-132\sin(c+dx)}{(\sin(c+dx)-1)^2} + 300\log(1-\sin(c+\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^4*Tan[c + d*x]^5,x]
```

```
[Out] -1/12*(a^4*(300*Log[1 - Sin[c + d*x]] + (120 - 132*Sin[c + d*x])/(-1 + Sin[
c + d*x])^2 + 192*Sin[c + d*x] + 54*Sin[c + d*x]^2 + 16*Sin[c + d*x]^3 + 3*
Sin[c + d*x]^4))/d
```

fricas [**A**] time = 0.45, size = 154, normalized size = 1.19

$$-\frac{24\,a^4\cos(dx+c)^6 - 272\,a^4\cos(dx+c)^4 - 2393\,a^4\cos(dx+c)^2 + 1906\,a^4 + 2400\left(a^4\cos(dx+c)^2 + 2\,a^4\sin(\right.}{96\left(d\cos(dx+c)^2 + 2\,d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/96*(24*a^4*cos(d*x + c)^6 - 272*a^4*cos(d*x + c)^4 - 2393*a^4*cos(d*x +
c)^2 + 1906*a^4 + 2400*(a^4*cos(d*x + c)^2 + 2*a^4*sin(d*x + c) - 2*a^4)*lo
g(-sin(d*x + c) + 1) - 10*(8*a^4*cos(d*x + c)^4 - 96*a^4*cos(d*x + c)^2 + 1
81*a^4)*sin(d*x + c))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2*d)
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.24, size = 387, normalized size = 3.00

$$-\frac{4a^4\left(\sin^6(dx+c)\right)}{d}-\frac{5a^4\left(\sin^5(dx+c)\right)}{d}-\frac{3a^4\left(\sin^8(dx+c)\right)}{4d}-\frac{5a^4\left(\sin^7(dx+c)\right)}{2d}-\frac{25a^4\left(\sin^3(dx+c)\right)}{3d}-\frac{25a^4 s}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^4*tan(d*x+c)^5,x)
```

```
[Out] -4/d*a^4*sin(d*x+c)^6-5/d*a^4*sin(d*x+c)^5-3/4/d*a^4*sin(d*x+c)^8-5/2/d*a^4
*sin(d*x+c)^7-25/3*a^4*sin(d*x+c)^3/d-25*a^4*sin(d*x+c)/d+25/d*a^4*ln(sec(d
*x+c)+tan(d*x+c))+1/4/d*a^4*sin(d*x+c)^10/cos(d*x+c)^4-3/4/d*a^4*sin(d*x+c)
^10/cos(d*x+c)^2+1/d*a^4*sin(d*x+c)^9/cos(d*x+c)^4-5/2/d*a^4*sin(d*x+c)^9/c
os(d*x+c)^2+3/2/d*a^4*sin(d*x+c)^8/cos(d*x+c)^4-3/d*a^4*sin(d*x+c)^8/cos(d*
x+c)^2+1/d*a^4*sin(d*x+c)^7/cos(d*x+c)^4-3/2/d*a^4*sin(d*x+c)^7/cos(d*x+c)^
2-6*a^4*sin(d*x+c)^4/d-12*a^4*sin(d*x+c)^2/d-25/d*a^4*ln(cos(d*x+c))+1/4/d*
a^4*tan(d*x+c)^4-1/2/d*a^4*tan(d*x+c)^2
```

maxima [A] time = 0.30, size = 109, normalized size = 0.84

$$-\frac{3\,a^4\sin(dx+c)^4+16\,a^4\sin(dx+c)^3+54\,a^4\sin(dx+c)^2+300\,a^4\log(\sin(dx+c)-1)+192\,a^4\sin(dx+c)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/12*(3*a^4*sin(d*x + c)^4 + 16*a^4*sin(d*x + c)^3 + 54*a^4*sin(d*x + c)^2
 + 300*a^4*log(sin(d*x + c) - 1) + 192*a^4*sin(d*x + c) - 12*(11*a^4*sin(d*
x + c) - 10*a^4)/(sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/d
```

mupad [B] time = 7.88, size = 379, normalized size = 2.94

$$\frac{25\,a^4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{50\,a^4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{d}-\frac{50\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}-150\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{1}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+10\tan\left(\frac{c}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5*(a + a*sin(c + d*x))^4,x)
```

```
[Out] (25*a^4*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (50*a^4*log(tan(c/2 + (d*x)/2) -
 1))/d - ((950*a^4*tan(c/2 + (d*x)/2)^3)/3 - 150*a^4*tan(c/2 + (d*x)/2)^2 -
```

```
 (1700*a^4*tan(c/2 + (d*x)/2)^4)/3 + (2180*a^4*tan(c/2 + (d*x)/2)^5)/3 - (2
452*a^4*tan(c/2 + (d*x)/2)^6)/3 + (2180*a^4*tan(c/2 + (d*x)/2)^7)/3 - (1700
*a^4*tan(c/2 + (d*x)/2)^8)/3 + (950*a^4*tan(c/2 + (d*x)/2)^9)/3 - 150*a^4*t
an(c/2 + (d*x)/2)^10 + 50*a^4*tan(c/2 + (d*x)/2)^11 + 50*a^4*tan(c/2 + (d*x
)/2))/(d*(10*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2) - 20*tan(c/2 + (d*
x)/2)^3 + 31*tan(c/2 + (d*x)/2)^4 - 40*tan(c/2 + (d*x)/2)^5 + 44*tan(c/2 +
(d*x)/2)^6 - 40*tan(c/2 + (d*x)/2)^7 + 31*tan(c/2 + (d*x)/2)^8 - 20*tan(c/2
 + (d*x)/2)^9 + 10*tan(c/2 + (d*x)/2)^10 - 4*tan(c/2 + (d*x)/2)^11 + tan(c/
2 + (d*x)/2)^12 + 1))
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**4*tan(d*x+c)**5,x)
```

```
[Out] Timed out
```

3.35 $\int (a+a\sin(c+dx))^4\tan^3(c+dx)\,dx$

Optimal. Leaf size=107

$$\frac{4a^5}{d(a-a\sin(c+dx))}+\frac{a^4\sin^4(c+dx)}{4d}+\frac{4a^4\sin^3(c+dx)}{3d}+\frac{4a^4\sin^2(c+dx)}{d}+\frac{12a^4\sin(c+dx)}{d}+\frac{16a^4\log(1-\sin(c$$

```
[Out] 16*a^4*ln(1-sin(d*x+c))/d+12*a^4*sin(d*x+c)/d+4*a^4*sin(d*x+c)^2/d+4/3*a^4*
sin(d*x+c)^3/d+1/4*a^4*sin(d*x+c)^4/d+4*a^5/d/(a-a*sin(d*x+c))
```

Rubi [A] time = 0.08, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 88}

$$\frac{a^4\sin^4(c+dx)}{4d}+\frac{4a^4\sin^3(c+dx)}{3d}+\frac{4a^4\sin^2(c+dx)}{d}+\frac{4a^5}{d(a-a\sin(c+dx))}+\frac{12a^4\sin(c+dx)}{d}+\frac{16a^4\log(1-\sin(c$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^4*Tan[c + d*x]^3,x]
```

```
[Out] (16*a^4*Log[1 - Sin[c + d*x]])/d + (12*a^4*Sin[c + d*x])/d + (4*a^4*Sin[c +
 d*x]^2)/d + (4*a^4*Sin[c + d*x]^3)/(3*d) + (a^4*Sin[c + d*x]^4)/(4*d) + (4
*a^5)/(d*(a - a*Sin[c + d*x]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int (a+a\sin(c+dx))^4\tan^3(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^3(a+x)^2}{(a-x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(12a^3+\frac{4a^5}{(a-x)^2}-\frac{16a^4}{a-x}+8a^2x+4ax^2+x^3\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{16a^4\log(1-\sin(c+dx))}{d}+\frac{12a^4\sin(c+dx)}{d}+\frac{4a^4\sin^2(c+dx)}{d}+\frac{4a^4\text{s}}{}$$

Mathematica [A] time = 0.15, size = 76, normalized size = 0.71

$$\frac{a^4\left(3\sin^4(c+dx)+16\sin^3(c+dx)+48\sin^2(c+dx)+144\sin(c+dx)+\frac{48}{1-\sin(c+dx)}+192\log(1-\sin(c+dx))\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^4*Tan[c + d*x]^3,x]
```

```
[Out] (a^4*(192*Log[1 - Sin[c + d*x]] + 48/(1 - Sin[c + d*x]) + 144*Sin[c + d*x]
+ 48*Sin[c + d*x]^2 + 16*Sin[c + d*x]^3 + 3*Sin[c + d*x]^4))/(12*d)
```

fricas [A] time = 0.46, size = 116, normalized size = 1.08

$$\frac{104\,a^4\cos(dx+c)^4-976\,a^4\cos(dx+c)^2+689\,a^4+1536\left(a^4\sin(dx+c)-a^4\right)\log(-\sin(dx+c)+1)+\left(24\,a^4\right.}{96\,(d\sin(dx+c)-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/96*(104*a^4*cos(d*x + c)^4 - 976*a^4*cos(d*x + c)^2 + 689*a^4 + 1536*(a^4
*sin(d*x + c) - a^4)*log(-sin(d*x + c) + 1) + (24*a^4*cos(d*x + c)^4 - 304*
a^4*cos(d*x + c)^2 - 1073*a^4)*sin(d*x + c))/(d*sin(d*x + c) - d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.20, size = 245, normalized size = 2.29

$$\frac{a^4 \left(\sin^8 (dx+c)\right)}{2d\cos (dx+c)^2}+\frac{a^4 \left(\sin^6 (dx+c)\right)}{2d}+\frac{15a^4 \left(\sin^4 (dx+c)\right)}{4d}+\frac{15a^4 \left(\sin^2 (dx+c)\right)}{2d}+\frac{16a^4 \ln (\cos (dx+c))}{d}+\frac{2a^4 \left(s\right.}{d\,\mathrm{co}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^4*tan(d*x+c)^3,x)
```

```
[Out] 1/2/d*a^4*sin(d*x+c)^8/cos(d*x+c)^2+1/2/d*a^4*sin(d*x+c)^6+15/4*a^4*sin(d*x
+c)^4/d+15/2*a^4*sin(d*x+c)^2/d+16/d*a^4*ln(cos(d*x+c))+2/d*a^4*sin(d*x+c)^
7/cos(d*x+c)^2+2/d*a^4*sin(d*x+c)^5+16/3*a^4*sin(d*x+c)^3/d+16*a^4*sin(d*x+
c)/d-16/d*a^4*ln(sec(d*x+c)+tan(d*x+c))+3/d*a^4*sin(d*x+c)^6/cos(d*x+c)^2+2
/d*a^4*sin(d*x+c)^5/cos(d*x+c)^2+1/2/d*a^4*tan(d*x+c)^2
```

maxima [A] time = 0.29, size = 85, normalized size = 0.79

$$\frac{3\,a^4 \sin (dx+c)^4 + 16\,a^4 \sin (dx+c)^3 + 48\,a^4 \sin (dx+c)^2 + 192\,a^4 \log (\sin (dx+c)-1) + 144\,a^4 \sin (dx+c) -}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(3*a^4*sin(d*x + c)^4 + 16*a^4*sin(d*x + c)^3 + 48*a^4*sin(d*x + c)^2
+ 192*a^4*log(sin(d*x + c) - 1) + 144*a^4*sin(d*x + c) - 48*a^4/(sin(d*x +
c) - 1))/d
```

mupad [B] time = 7.56, size = 320, normalized size = 2.99

$$\frac{32\,a^4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{d}+\frac{32\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9-32\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\frac{320\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{3}-\frac{340\,a^4\,\tan\left(\frac{c}{2}+\frac{d}{\cdot}\right.}{3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+10\,\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + a*sin(c + d*x))^4,x)
```

```
[Out] (32*a^4*log(tan(c/2 + (d*x)/2) - 1))/d + ((320*a^4*tan(c/2 + (d*x)/2)^3)/3
- 32*a^4*tan(c/2 + (d*x)/2)^2 - (340*a^4*tan(c/2 + (d*x)/2)^4)/3 + (424*a^4
*tan(c/2 + (d*x)/2)^5)/3 - (340*a^4*tan(c/2 + (d*x)/2)^6)/3 + (320*a^4*tan(
c/2 + (d*x)/2)^7)/3 - 32*a^4*tan(c/2 + (d*x)/2)^8 + 32*a^4*tan(c/2 + (d*x)/
2)^9 + 32*a^4*tan(c/2 + (d*x)/2))/(d*(5*tan(c/2 + (d*x)/2)^2 - 2*tan(c/2 +
(d*x)/2) - 8*tan(c/2 + (d*x)/2)^3 + 10*tan(c/2 + (d*x)/2)^4 - 12*tan(c/2 +
```

```
(d*x)/2)^5 + 10*tan(c/2 + (d*x)/2)^6 - 8*tan(c/2 + (d*x)/2)^7 + 5*tan(c/2 +
 (d*x)/2)^8 - 2*tan(c/2 + (d*x)/2)^9 + tan(c/2 + (d*x)/2)^10 + 1)) - (16*a^
4*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int 4\sin(c+dx)\tan^3(c+dx)\,dx + \int 6\sin^2(c+dx)\tan^3(c+dx)\,dx + \int 4\sin^3(c+dx)\tan^3(c+dx)\,dx + \int\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**4*tan(d*x+c)**3,x)
```

```
[Out] a**4*(Integral(4*sin(c + d*x)*tan(c + d*x)**3, x) + Integral(6*sin(c + d*x)
**2*tan(c + d*x)**3, x) + Integral(4*sin(c + d*x)**3*tan(c + d*x)**3, x) +
Integral(sin(c + d*x)**4*tan(c + d*x)**3, x) + Integral(tan(c + d*x)**3, x)
)
```

3.36 $\int (a + a\sin(c + dx))^4 \tan(c + dx)\,dx$

Optimal. Leaf size=88

$$-\frac{a^4\sin^4(c+dx)}{4d}-\frac{4a^4\sin^3(c+dx)}{3d}-\frac{7a^4\sin^2(c+dx)}{2d}-\frac{8a^4\sin(c+dx)}{d}-\frac{8a^4\log(1-\sin(c+dx))}{d}$$

```
[Out] -8*a^4*ln(1-sin(d*x+c))/d-8*a^4*sin(d*x+c)/d-7/2*a^4*sin(d*x+c)^2/d-4/3*a^4
*sin(d*x+c)^3/d-1/4*a^4*sin(d*x+c)^4/d
```

Rubi [A] time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2707, 77}

$$-\frac{a^4\sin^4(c+dx)}{4d}-\frac{4a^4\sin^3(c+dx)}{3d}-\frac{7a^4\sin^2(c+dx)}{2d}-\frac{8a^4\sin(c+dx)}{d}-\frac{8a^4\log(1-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^4*Tan[c + d*x],x]
```

```
[Out] (-8*a^4*Log[1 - Sin[c + d*x]])/d - (8*a^4*Sin[c + d*x])/d - (7*a^4*Sin[c +
d*x]^2)/(2*d) - (4*a^4*Sin[c + d*x]^3)/(3*d) - (a^4*Sin[c + d*x]^4)/(4*d)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int (a+a\sin(c+dx))^4\tan(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x(a+x)^3}{a-x}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-8a^3+\frac{8a^4}{a-x}-7a^2x-4ax^2-x^3\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{8a^4\log(1-\sin(c+dx))}{d}-\frac{8a^4\sin(c+dx)}{d}-\frac{7a^4\sin^2(c+dx)}{2d}-\frac{4a^4\sin}{3}$$

Mathematica [A] time = 0.07, size = 62, normalized size = 0.70

$$-\frac{a^4\left(3\sin^4(c+dx)+16\sin^3(c+dx)+42\sin^2(c+dx)+96\sin(c+dx)+96\log(1-\sin(c+dx))\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^4*Tan[c + d*x],x]
```

```
[Out] -1/12*(a^4*(96*Log[1 - Sin[c + d*x]] + 96*Sin[c + d*x] + 42*Sin[c + d*x]^2
+ 16*Sin[c + d*x]^3 + 3*Sin[c + d*x]^4))/d
```

fricas [A] time = 0.44, size = 74, normalized size = 0.84

$$-\frac{3\,a^4\cos(dx+c)^4-48\,a^4\cos(dx+c)^2+96\,a^4\log(-\sin(dx+c)+1)-16\left(a^4\cos(dx+c)^2-7\,a^4\right)\sin(dx+c)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/12*(3*a^4*cos(d*x + c)^4 - 48*a^4*cos(d*x + c)^2 + 96*a^4*log(-sin(d*x +
 c) + 1) - 16*(a^4*cos(d*x + c)^2 - 7*a^4)*sin(d*x + c))/d
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.16, size = 101, normalized size = 1.15

$$-\frac{a^4\left(\sin^4(dx+c)\right)}{4d}-\frac{7a^4\left(\sin^2(dx+c)\right)}{2d}-\frac{8a^4\ln(\cos(dx+c))}{d}-\frac{4a^4\left(\sin^3(dx+c)\right)}{3d}-\frac{8a^4\sin(dx+c)}{d}+\frac{8a^4\ln(\mathrm{sec}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^4*tan(d*x+c),x)
```

```
[Out] -1/4*a^4*sin(d*x+c)^4/d-7/2*a^4*sin(d*x+c)^2/d-8/d*a^4*ln(cos(d*x+c))-4/3*a
^4*sin(d*x+c)^3/d-8*a^4*sin(d*x+c)/d+8/d*a^4*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.29, size = 70, normalized size = 0.80

$$-\frac{3\,a^4\sin(dx+c)^4+16\,a^4\sin(dx+c)^3+42\,a^4\sin(dx+c)^2+96\,a^4\log(\sin(dx+c)-1)+96\,a^4\sin(dx+c)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/12*(3*a^4*sin(d*x + c)^4 + 16*a^4*sin(d*x + c)^3 + 42*a^4*sin(d*x + c)^2
 + 96*a^4*log(sin(d*x + c) - 1) + 96*a^4*sin(d*x + c))/d
```

mupad [B] time = 6.63, size = 131, normalized size = 1.49

$$\frac{8\,a^4\,\ln\left(\frac{1}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}\right)}{d}-\frac{28\,a^4\,\sin(c+d\,x)}{3\,d}-\frac{16\,a^4\,\ln\left(\frac{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{4\,a^4\cos(c+d\,x)^2}{d}-\frac{a^4\cos(c+d\,x)^4}{4\,d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + a*sin(c + d*x))^4,x)
```

```
[Out] (8*a^4*log(1/cos(c/2 + (d*x)/2)^2))/d - (28*a^4*sin(c + d*x))/(3*d) - (16*a
^4*log((cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2))/cos(c/2 + (d*x)/2)))/d + (
4*a^4*cos(c + d*x)^2)/d - (a^4*cos(c + d*x)^4)/(4*d) + (4*a^4*cos(c + d*x)^
2*sin(c + d*x))/(3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int 4\sin(c+dx)\tan(c+dx)\,dx+\int 6\sin^2(c+dx)\tan(c+dx)\,dx+\int 4\sin^3(c+dx)\tan(c+dx)\,dx+\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**4*tan(d*x+c),x)
```

```
[Out] a**4*(Integral(4*sin(c + d*x)*tan(c + d*x), x) + Integral(6*sin(c + d*x)**2
*tan(c + d*x), x) + Integral(4*sin(c + d*x)**3*tan(c + d*x), x) + Integral(
sin(c + d*x)**4*tan(c + d*x), x) + Integral(tan(c + d*x), x))
```

3.37 $\int \cot^3(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=102

$$-\frac{a^4\sin^4(c+dx)}{4d}-\frac{4a^4\sin^3(c+dx)}{3d}-\frac{5a^4\sin^2(c+dx)}{2d}-\frac{a^4\csc^2(c+dx)}{2d}-\frac{4a^4\csc(c+dx)}{d}+\frac{5a^4\log(\sin(c+dx))}{d}$$

```
[Out] -4*a^4*csc(d*x+c)/d-1/2*a^4*csc(d*x+c)^2/d+5*a^4*ln(sin(d*x+c))/d-5/2*a^4*s
in(d*x+c)^2/d-4/3*a^4*sin(d*x+c)^3/d-1/4*a^4*sin(d*x+c)^4/d
```

Rubi [A] time = 0.07, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 75}

$$-\frac{a^4\sin^4(c+dx)}{4d}-\frac{4a^4\sin^3(c+dx)}{3d}-\frac{5a^4\sin^2(c+dx)}{2d}-\frac{a^4\csc^2(c+dx)}{2d}-\frac{4a^4\csc(c+dx)}{d}+\frac{5a^4\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-4*a^4*Csc[c + d*x])/d - (a^4*Csc[c + d*x]^2)/(2*d) + (5*a^4*Log[Sin[c + d
*x]])/d - (5*a^4*Sin[c + d*x]^2)/(2*d) - (4*a^4*Sin[c + d*x]^3)/(3*d) - (a^
4*Sin[c + d*x]^4)/(4*d)
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \cot^3(c+dx)(a+a\sin(c+dx))^4\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)(a+x)^5}{x^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^6}{x^3}+\frac{4a^5}{x^2}+\frac{5a^4}{x}-5a^2x-4ax^2-x^3\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{4a^4\csc(c+dx)}{d}-\frac{a^4\csc^2(c+dx)}{2d}+\frac{5a^4\log(\sin(c+dx))}{d}-\frac{5a^4\sin^2(c+}{2d}$$

Mathematica [A] time = 0.13, size = 78, normalized size = 0.76

$$-\frac{a^4\sin^4(c+dx)\left(6\csc^6(c+dx)+48\csc^5(c+dx)+30\csc^2(c+dx)+16\csc(c+dx)+\csc^4(c+dx)(90-60\log(}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -1/12*(a^4*(3 + 16*Csc[c + d*x] + 30*Csc[c + d*x]^2 + 48*Csc[c + d*x]^5 + 6
*Csc[c + d*x]^6 + Csc[c + d*x]^4*(90 - 60*Log[Sin[c + d*x]]))*Sin[c + d*x]^
4)/d
```

fricas [A] time = 0.45, size = 131, normalized size = 1.28

$$-\frac{24\,a^4\cos(dx+c)^6-312\,a^4\cos(dx+c)^4+423\,a^4\cos(dx+c)^2-183\,a^4-480\left(a^4\cos(dx+c)^2-a^4\right)\log\left(\frac{1}{2}\sin\right.}{96\left(d\cos(dx+c)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/96*(24*a^4*cos(d*x + c)^6 - 312*a^4*cos(d*x + c)^4 + 423*a^4*cos(d*x + c
)^2 - 183*a^4 - 480*(a^4*cos(d*x + c)^2 - a^4)*log(1/2*sin(d*x + c)) - 128*
(a^4*cos(d*x + c)^4 - 2*a^4*cos(d*x + c)^2 + 4*a^4)*sin(d*x + c))/(d*cos(d*
x + c)^2 - d)
```

giac [A] time = 0.55, size = 96, normalized size = 0.94

$$-\frac{3\,a^4\sin(dx+c)^4+16\,a^4\sin(dx+c)^3+30\,a^4\sin(dx+c)^2-60\,a^4\log(|\sin(dx+c)|)+\frac{6\left(15\,a^4\sin(dx+c)^2+8\,a^4\sin(dx\right.}{\sin(dx+c)^2}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/12*(3*a^4*sin(d*x + c)^4 + 16*a^4*sin(d*x + c)^3 + 30*a^4*sin(d*x + c)^2
 - 60*a^4*log(abs(sin(d*x + c))) + 6*(15*a^4*sin(d*x + c)^2 + 8*a^4*sin(d*x
 + c) + a^4)/sin(d*x + c)^2)/d
```

maple [A] time = 0.24, size = 125, normalized size = 1.23

$$-\frac{a^4\left(\cos^4(dx+c)\right)}{4d}-\frac{8a^4\left(\cos^2(dx+c)\right)\sin(dx+c)}{3d}-\frac{16a^4\sin(dx+c)}{3d}+\frac{3a^4\left(\cos^2(dx+c)\right)}{d}+\frac{5a^4\ln(\sin(dx+c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+a*sin(d*x+c))^4,x)
```

```
[Out] -1/4/d*a^4*cos(d*x+c)^4-8/3/d*a^4*cos(d*x+c)^2*sin(d*x+c)-16/3*a^4*sin(d*x+
c)/d+3/d*a^4*cos(d*x+c)^2+5*a^4*ln(sin(d*x+c))/d-4/d*a^4/sin(d*x+c)*cos(d*x
+c)^4-1/2/d*a^4*cot(d*x+c)^2
```

maxima [A] time = 0.31, size = 82, normalized size = 0.80

$$-\frac{3\,a^4\sin(dx+c)^4+16\,a^4\sin(dx+c)^3+30\,a^4\sin(dx+c)^2-60\,a^4\log(\sin(dx+c))+\frac{6\left(8\,a^4\sin(dx+c)+a^4\right)}{\sin(dx+c)^2}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/12*(3*a^4*sin(d*x + c)^4 + 16*a^4*sin(d*x + c)^3 + 30*a^4*sin(d*x + c)^2
 - 60*a^4*log(sin(d*x + c)) + 6*(8*a^4*sin(d*x + c) + a^4)/sin(d*x + c)^2)/
d
```

mupad [B] time = 6.40, size = 298, normalized size = 2.92

$$\frac{5\,a^4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-\frac{8\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\frac{81\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{2}+\frac{224\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{3}+98\,a^4\tan\left(\frac{c}{2}\right.}{d\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+16\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + a*sin(c + d*x))^4,x)
```

```
[Out] (5*a^4*log(tan(c/2 + (d*x)/2)))/d - (a^4*tan(c/2 + (d*x)/2)^2)/(8*d) - (2*a
^4*tan(c/2 + (d*x)/2)^2 + 32*a^4*tan(c/2 + (d*x)/2)^3 + 43*a^4*tan(c/2 + (d
*x)/2)^4 + (272*a^4*tan(c/2 + (d*x)/2)^5)/3 + 98*a^4*tan(c/2 + (d*x)/2)^6 +
```

```
 (224*a^4*tan(c/2 + (d*x)/2)^7)/3 + (81*a^4*tan(c/2 + (d*x)/2)^8)/2 + 8*a^4
*tan(c/2 + (d*x)/2)^9 + a^4/2 + 8*a^4*tan(c/2 + (d*x)/2))/(d*(4*tan(c/2 + (
d*x)/2)^2 + 16*tan(c/2 + (d*x)/2)^4 + 24*tan(c/2 + (d*x)/2)^6 + 16*tan(c/2
+ (d*x)/2)^8 + 4*tan(c/2 + (d*x)/2)^10)) - (2*a^4*tan(c/2 + (d*x)/2))/d - (
5*a^4*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int 4\sin(c+dx)\cot^3(c+dx)\,dx + \int 6\sin^2(c+dx)\cot^3(c+dx)\,dx + \int 4\sin^3(c+dx)\cot^3(c+dx)\,dx + \int\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+a*sin(d*x+c))**4,x)
```

```
[Out] a**4*(Integral(4*sin(c + d*x)*cot(c + d*x)**3, x) + Integral(6*sin(c + d*x)
**2*cot(c + d*x)**3, x) + Integral(4*sin(c + d*x)**3*cot(c + d*x)**3, x) +
Integral(sin(c + d*x)**4*cot(c + d*x)**3, x) + Integral(cot(c + d*x)**3, x)
)
```

3.38 $\int (a + a\sin(c + dx))^4 \tan^4(c + dx)\, dx$

Optimal. Leaf size=143

$$\frac{4a^4\cos^3(c+dx)}{3d} - \frac{16a^4\cos(c+dx)}{d} - \frac{a^4\sin^3(c+dx)\cos(c+dx)}{4d} - \frac{35a^4\sin(c+dx)\cos(c+dx)}{8d} - \frac{56a^4\cos(c+d}{3d(1-\sin(c+d}$$

```
[Out] 163/8*a^4*x-16*a^4*cos(d*x+c)/d+4/3*a^4*cos(d*x+c)^3/d+4/3*a^4*cos(d*x+c)/d
/(1-sin(d*x+c))^2-56/3*a^4*cos(d*x+c)/d/(1-sin(d*x+c))-35/8*a^4*cos(d*x+c)*
sin(d*x+c)/d-1/4*a^4*cos(d*x+c)*sin(d*x+c)^3/d
```

Rubi [A] time = 0.20, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2709, 2650, 2648, 2638, 2635, 8, 2633}

$$\frac{4a^4\cos^3(c+dx)}{3d} - \frac{16a^4\cos(c+dx)}{d} - \frac{a^4\sin^3(c+dx)\cos(c+dx)}{4d} - \frac{35a^4\sin(c+dx)\cos(c+dx)}{8d} - \frac{56a^4\cos(c+d}{3d(1-\sin(c+d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^4*Tan[c + d*x]^4,x]
```

```
[Out] (163*a^4*x)/8 - (16*a^4*Cos[c + d*x])/d + (4*a^4*Cos[c + d*x]^3)/(3*d) + (4
*a^4*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x])^2) - (56*a^4*Cos[c + d*x])/(3*d*
(1 - Sin[c + d*x])) - (35*a^4*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (a^4*Cos[c
 + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(c+dx))^4\tan^4(c+dx)\,dx &= a^4\int \Big(16+\frac{4}{(-1+\sin(c+dx))^2}+\frac{20}{-1+\sin(c+dx)}+12\sin(c+dx)+8\\
&=16a^4x+a^4\int \sin^4(c+dx)\,dx+\left(4a^4\right)\int \frac{1}{(-1+\sin(c+dx))^2}\,dx+\left(4a^4\right)\\
&=16a^4x-\frac{12a^4\cos(c+dx)}{d}+\frac{4a^4\cos(c+dx)}{3d(1-\sin(c+dx))^2}-\frac{20a^4\cos(c+dx)}{d(1-\sin(c+dx))}-\\
&=20a^4x-\frac{16a^4\cos(c+dx)}{d}+\frac{4a^4\cos^3(c+dx)}{3d}+\frac{4a^4\cos(c+dx)}{3d(1-\sin(c+dx))^2}-\\
&=\frac{163a^4x}{8}-\frac{16a^4\cos(c+dx)}{d}+\frac{4a^4\cos^3(c+dx)}{3d}+\frac{4a^4\cos(c+dx)}{3d(1-\sin(c+dx))^2}-\end{aligned}$$

Mathematica [A] time = 1.65, size = 252, normalized size = 1.76

$$\frac{a^4\left(-11736c\sin\left(\frac{1}{2}(c+dx)\right)-11736dx\sin\left(\frac{1}{2}(c+dx)\right)-16488\sin\left(\frac{1}{2}(c+dx)\right)-3912c\sin\left(\frac{3}{2}(c+dx)\right)-3912d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^4*Tan[c + d*x]^4,x]
```

```
[Out] (a^4*(24*(209 + 489*c + 489*d*x)*Cos[(c + d*x)/2] - 24*(453 + 163*c + 163*d
*x)*Cos[(3*(c + d*x))/2] + 885*Cos[(5*(c + d*x))/2] - 129*Cos[(7*(c + d*x))
/2] - 23*Cos[(9*(c + d*x))/2] + 3*Cos[(11*(c + d*x))/2] - 16488*Sin[(c + d*
x)/2] - 11736*c*Sin[(c + d*x)/2] - 11736*d*x*Sin[(c + d*x)/2] + 3704*Sin[(3
*(c + d*x))/2] - 3912*c*Sin[(3*(c + d*x))/2] - 3912*d*x*Sin[(3*(c + d*x))/2
] + 885*Sin[(5*(c + d*x))/2] + 129*Sin[(7*(c + d*x))/2] - 23*Sin[(9*(c + d*
x))/2] - 3*Sin[(11*(c + d*x))/2]))/(384*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)
/2])^3)
```

fricas [A] time = 0.43, size = 247, normalized size = 1.73

$$-\frac{6\,a^4\cos\left(dx+c\right)^6-20\,a^4\cos\left(dx+c\right)^5-85\,a^4\cos\left(dx+c\right)^4+214\,a^4\cos\left(dx+c\right)^3+978\,a^4dx+32\,a^4-\left(489\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/24*(6*a^4*cos(d*x + c)^6 - 20*a^4*cos(d*x + c)^5 - 85*a^4*cos(d*x + c)^4
 + 214*a^4*cos(d*x + c)^3 + 978*a^4*d*x + 32*a^4 - (489*a^4*d*x + 721*a^4)*
cos(d*x + c)^2 + (489*a^4*d*x - 962*a^4)*cos(d*x + c) - (6*a^4*cos(d*x + c)
^5 + 26*a^4*cos(d*x + c)^4 - 59*a^4*cos(d*x + c)^3 + 978*a^4*d*x - 273*a^4*
cos(d*x + c)^2 - 32*a^4 + (489*a^4*d*x - 994*a^4)*cos(d*x + c))*sin(d*x + c
))/(d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c) + 2*d)*sin(d*x + c)
 - 2*d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.41, size = 360, normalized size = 2.52

$$a^4\left(\frac{\sin^9(dx+c)}{3\cos(dx+c)^3}-\frac{2(\sin^9(dx+c))}{\cos(dx+c)}-2\left(\sin^7(dx+c)+\frac{7(\sin^5(dx+c))}{6}+\frac{35(\sin^3(dx+c))}{24}+\frac{35\sin(dx+c)}{16}\right)\cos(dx+c)+\frac{35dx}{8}+\frac{35c}{8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^4*tan(d*x+c)^4,x)
```

```
[Out] 1/d*(a^4*(1/3*sin(d*x+c)^9/cos(d*x+c)^3-2*sin(d*x+c)^9/cos(d*x+c)-2*(sin(d*
x+c)^7+7/6*sin(d*x+c)^5+35/24*sin(d*x+c)^3+35/16*sin(d*x+c))*cos(d*x+c)+35/
8*d*x+35/8*c)+4*a^4*(1/3*sin(d*x+c)^8/cos(d*x+c)^3-5/3*sin(d*x+c)^8/cos(d*x
+c)-5/3*(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))+6
*a^4*(1/3*sin(d*x+c)^7/cos(d*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/3*(sin(d*
x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c)+4*a^4*(1
/3*sin(d*x+c)^6/cos(d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+4/3*
sin(d*x+c)^2)*cos(d*x+c))+a^4*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c))
```

maxima [A] time = 0.42, size = 238, normalized size = 1.66

$$32\left(\cos(dx+c)^3-\frac{9\cos(dx+c)^2-1}{\cos(dx+c)^3}-9\cos(dx+c)\right)a^4+\left(8\tan(dx+c)^3+105\,dx+105\,c-\frac{3\left(13\tan(dx+c)^3+11\tan(dx+\right.}{\tan(dx+c)^4+2\tan(dx+c)^2+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/24*(32*(cos(d*x + c)^3 - (9*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 - 9*cos(d*
x + c))*a^4 + (8*tan(d*x + c)^3 + 105*d*x + 105*c - 3*(13*tan(d*x + c)^3 +
11*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x + c)^2 + 1) - 72*tan(d*x + c))
*a^4 + 24*(2*tan(d*x + c)^3 + 15*d*x + 15*c - 3*tan(d*x + c)/(tan(d*x + c)^
2 + 1) - 12*tan(d*x + c))*a^4 + 8*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x
 + c))*a^4 - 32*a^4*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x + c)
))/d
```

mupad [B] time = 11.05, size = 437, normalized size = 3.06

$$\frac{163\,a^4\,x}{8}+\frac{\frac{163\,a^4\,(c+d\,x)}{8}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{489\,a^4\,(c+d\,x)}{8}-\frac{a^4\,(1467\,c+1467\,d\,x-3630)}{24}\right)-\frac{a^4\,(489\,c+489\,d\,x-1536)}{24}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + a*sin(c + d*x))^4,x)
```

```
[Out] (163*a^4*x)/8 + ((163*a^4*(c + d*x))/8 - tan(c/2 + (d*x)/2)*((489*a^4*(c +
d*x))/8 - (a^4*(1467*c + 1467*d*x - 3630))/24) - (a^4*(489*c + 489*d*x - 15
36))/24 + tan(c/2 + (d*x)/2)^10*((489*a^4*(c + d*x))/8 - (a^4*(1467*c + 146
7*d*x - 978))/24) - tan(c/2 + (d*x)/2)^9*((1141*a^4*(c + d*x))/8 - (a^4*(34
23*c + 3423*d*x - 2934))/24) + tan(c/2 + (d*x)/2)^2*((1141*a^4*(c + d*x))/8
 - (a^4*(3423*c + 3423*d*x - 7818))/24) + tan(c/2 + (d*x)/2)^8*((2119*a^4*(
c + d*x))/8 - (a^4*(6357*c + 6357*d*x - 6520))/24) - tan(c/2 + (d*x)/2)^3*(
(2119*a^4*(c + d*x))/8 - (a^4*(6357*c + 6357*d*x - 13448))/24) - tan(c/2 +
(d*x)/2)^7*((1467*a^4*(c + d*x))/4 - (a^4*(8802*c + 8802*d*x - 11736))/24)
+ tan(c/2 + (d*x)/2)^4*((1467*a^4*(c + d*x))/4 - (a^4*(8802*c + 8802*d*x -
15912))/24) + tan(c/2 + (d*x)/2)^6*((1793*a^4*(c + d*x))/4 - (a^4*(10758*c
+ 10758*d*x - 15364))/24) - tan(c/2 + (d*x)/2)^5*((1793*a^4*(c + d*x))/4 -
(a^4*(10758*c + 10758*d*x - 18428))/24))/(d*(tan(c/2 + (d*x)/2) - 1)^3*(tan
(c/2 + (d*x)/2)^2 + 1)^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**4*tan(d*x+c)**4,x)
```

```
[Out] Timed out
```

3.39 $\int (a + a\sin(c + dx))^4 \tan^2(c + dx)\, dx$

Optimal. Leaf size=113

$$-\frac{4a^4\cos^3(c+dx)}{3d}+\frac{12a^4\cos(c+dx)}{d}+\frac{a^4\sin^3(c+dx)\cos(c+dx)}{4d}+\frac{31a^4\sin(c+dx)\cos(c+dx)}{8d}+\frac{8a^4\cos(c+d}{d(1-\sin(c+d}$$

```
[Out] -95/8*a^4*x+12*a^4*cos(d*x+c)/d-4/3*a^4*cos(d*x+c)^3/d+8*a^4*cos(d*x+c)/d/(
1-sin(d*x+c))+31/8*a^4*cos(d*x+c)*sin(d*x+c)/d+1/4*a^4*cos(d*x+c)*sin(d*x+c
)^3/d
```

Rubi [A] time = 0.16, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2709, 2648, 2638, 2635, 8, 2633}

$$-\frac{4a^4\cos^3(c+dx)}{3d}+\frac{12a^4\cos(c+dx)}{d}+\frac{a^4\sin^3(c+dx)\cos(c+dx)}{4d}+\frac{31a^4\sin(c+dx)\cos(c+dx)}{8d}+\frac{8a^4\cos(c+d}{d(1-\sin(c+d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^4*Tan[c + d*x]^2,x]
```

```
[Out] (-95*a^4*x)/8 + (12*a^4*Cos[c + d*x])/d - (4*a^4*Cos[c + d*x]^3)/(3*d) + (8
*a^4*Cos[c + d*x])/(d*(1 - Sin[c + d*x])) + (31*a^4*Cos[c + d*x]*Sin[c + d*
x])/(8*d) + (a^4*Cos[c + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(c+dx))^4\tan^2(c+dx)\,dx &= a^2\int\left(-8a^2-\frac{8a^2}{-1+\sin(c+dx)}-8a^2\sin(c+dx)-7a^2\sin^2(c+dx)-\right.\\
&= -8a^4x-a^4\int\sin^4(c+dx)\,dx-\left(4a^4\right)\int\sin^3(c+dx)\,dx-\left(7a^4\right)\int\sin\\
&= -8a^4x+\frac{8a^4\cos(c+dx)}{d}+\frac{8a^4\cos(c+dx)}{d(1-\sin(c+dx))}+\frac{7a^4\cos(c+dx)\sin(c}{2d}\\
&= -\frac{23a^4x}{2}+\frac{12a^4\cos(c+dx)}{d}-\frac{4a^4\cos^3(c+dx)}{3d}+\frac{8a^4\cos(c+dx)}{d(1-\sin(c+dx))}+\\
&= -\frac{95a^4x}{8}+\frac{12a^4\cos(c+dx)}{d}-\frac{4a^4\cos^3(c+dx)}{3d}+\frac{8a^4\cos(c+dx)}{d(1-\sin(c+dx))}+\end{aligned}$$

Mathematica [A] time = 1.10, size = 125, normalized size = 1.11

$$\frac{(a\sin(c+dx)+a)^4\left(-1140(c+dx)+192\sin(2(c+dx))-3\sin(4(c+dx))+1056\cos(c+dx)-32\cos(3(c+d\right.}{96d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^4*Tan[c + d*x]^2,x]
```

```
[Out] ((a + a*Sin[c + d*x])^4*(-1140*(c + d*x) + 1056*Cos[c + d*x] - 32*Cos[3*(c
+ d*x)] + (1536*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) + 1
92*Sin[2*(c + d*x)] - 3*Sin[4*(c + d*x)]))/(96*d*(Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2])^8)
```

fricas [A] time = 0.44, size = 179, normalized size = 1.58

$$-\frac{6\,a^4\cos\left(dx+c\right)^5+32\,a^4\cos\left(dx+c\right)^4-73\,a^4\cos\left(dx+c\right)^3+285\,a^4dx-288\,a^4\cos\left(dx+c\right)^2-192\,a^4+3\,\big(95\,a}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(6*a^4*cos(d*x + c)^5 + 32*a^4*cos(d*x + c)^4 - 73*a^4*cos(d*x + c)^3
 + 285*a^4*d*x - 288*a^4*cos(d*x + c)^2 - 192*a^4 + 3*(95*a^4*d*x - 127*a^4
)*cos(d*x + c) + (6*a^4*cos(d*x + c)^4 - 26*a^4*cos(d*x + c)^3 - 285*a^4*d*
x - 99*a^4*cos(d*x + c)^2 + 189*a^4*cos(d*x + c) - 192*a^4)*sin(d*x + c))/(
d*cos(d*x + c) - d*sin(d*x + c) + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.31, size = 231, normalized size = 2.04

$$a^4\left(\frac{\sin^7(dx+c)}{\cos(dx+c)}+\left(\sin^5\left(dx+c\right)+\frac{5\left(\sin^3(dx+c)\right)}{4}+\frac{15\sin(dx+c)}{8}\right)\cos\left(dx+c\right)-\frac{15dx}{8}-\frac{15c}{8}\right)+4a^4\left(\frac{\sin^6(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}+\sin^4\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^4*tan(d*x+c)^2,x)
```

```
[Out] 1/d*(a^4*(sin(d*x+c)^7/cos(d*x+c)+(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d
*x+c))*cos(d*x+c)-15/8*d*x-15/8*c)+4*a^4*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(
d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+6*a^4*(sin(d*x+c)^5/cos(d*x+c)+(sin(
d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c)+4*a^4*(sin(d*x+c)^4/cos(
d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+a^4*(tan(d*x+c)-d*x-c))
```

maxima [A] time = 0.45, size = 181, normalized size = 1.60

$$-\frac{32\left(\cos\left(dx+c\right)^{3}-\frac{3}{\cos(dx+c)}-6\,\cos\left(dx+c\right)\right)a^{4}+3\left(15\,dx+15\,c-\frac{9\,\tan(dx+c)^{3}+7\,\tan(dx+c)}{\tan(dx+c)^{4}+2\,\tan(dx+c)^{2}+1}-8\,\tan\left(dx+c\right)\right)a^{4}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/24*(32*(cos(d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*a^4 + 3*(15*d*
x + 15*c - (9*tan(d*x + c)^3 + 7*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x
+ c)^2 + 1) - 8*tan(d*x + c))*a^4 + 72*(3*d*x + 3*c - tan(d*x + c)/(tan(d*x
 + c)^2 + 1) - 2*tan(d*x + c))*a^4 + 24*(d*x + c - tan(d*x + c))*a^4 - 96*a
^4*(1/cos(d*x + c) + cos(d*x + c)))/d
```

mupad [B] time = 10.26, size = 363, normalized size = 3.21

$$-\frac{95\,a^4\,x}{8}-\frac{\frac{95\,a^4\,(c+d\,x)}{8}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{95\,a^4\,(c+d\,x)}{8}-\frac{a^4\,(285\,c+285\,d\,x-326)}{24}\right)-\frac{a^4\,(285\,c+285\,d\,x-896)}{24}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\left(\frac{95\,a}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + a*sin(c + d*x))^4,x)
```

```
[Out] - (95*a^4*x)/8 - ((95*a^4*(c + d*x))/8 - tan(c/2 + (d*x)/2)*((95*a^4*(c + d
*x))/8 - (a^4*(285*c + 285*d*x - 326))/24) - (a^4*(285*c + 285*d*x - 896))/
24 + tan(c/2 + (d*x)/2)^8*((95*a^4*(c + d*x))/8 - (a^4*(285*c + 285*d*x - 5
70))/24) - tan(c/2 + (d*x)/2)^7*((95*a^4*(c + d*x))/2 - (a^4*(1140*c + 1140
*d*x - 570))/24) - tan(c/2 + (d*x)/2)^3*((95*a^4*(c + d*x))/2 - (a^4*(1140*
c + 1140*d*x - 1430))/24) + tan(c/2 + (d*x)/2)^6*((95*a^4*(c + d*x))/2 - (a
^4*(1140*c + 1140*d*x - 2154))/24) + tan(c/2 + (d*x)/2)^2*((95*a^4*(c + d*x
))/2 - (a^4*(1140*c + 1140*d*x - 3014))/24) - tan(c/2 + (d*x)/2)^5*((285*a^
4*(c + d*x))/4 - (a^4*(1710*c + 1710*d*x - 1770))/24) + tan(c/2 + (d*x)/2)^
4*((285*a^4*(c + d*x))/4 - (a^4*(1710*c + 1710*d*x - 3606))/24))/(d*(tan(c/
2 + (d*x)/2) - 1)*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int 4\sin\left(c+dx\right)\tan^2\left(c+dx\right)dx+\int 6\sin^2\left(c+dx\right)\tan^2\left(c+dx\right)dx+\int 4\sin^3\left(c+dx\right)\tan^2\left(c+dx\right)dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**4*tan(d*x+c)**2,x)
```

```
[Out] a**4*(Integral(4*sin(c + d*x)*tan(c + d*x)**2, x) + Integral(6*sin(c + d*x)
**2*tan(c + d*x)**2, x) + Integral(4*sin(c + d*x)**3*tan(c + d*x)**2, x) +
Integral(sin(c + d*x)**4*tan(c + d*x)**2, x) + Integral(tan(c + d*x)**2, x)
)
```

3.40 $\int (a + a\sin(c + dx))^4 \, dx$

Optimal. Leaf size=87

$$\frac{4a^4\cos^3(c+dx)}{3d} - \frac{8a^4\cos(c+dx)}{d} - \frac{a^4\sin^3(c+dx)\cos(c+dx)}{4d} - \frac{27a^4\sin(c+dx)\cos(c+dx)}{8d} + \frac{35a^4x}{8}$$

```
[Out] 35/8*a^4*x-8*a^4*cos(d*x+c)/d+4/3*a^4*cos(d*x+c)^3/d-27/8*a^4*cos(d*x+c)*si
n(d*x+c)/d-1/4*a^4*cos(d*x+c)*sin(d*x+c)^3/d
```

Rubi [A] time = 0.08, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2645, 2638, 2635, 8, 2633}

$$\frac{4a^4\cos^3(c+dx)}{3d} - \frac{8a^4\cos(c+dx)}{d} - \frac{a^4\sin^3(c+dx)\cos(c+dx)}{4d} - \frac{27a^4\sin(c+dx)\cos(c+dx)}{8d} + \frac{35a^4x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (35*a^4*x)/8 - (8*a^4*Cos[c + d*x])/d + (4*a^4*Cos[c + d*x]^3)/(3*d) - (27*
a^4*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (a^4*Cos[c + d*x]*Sin[c + d*x]^3)/(4
*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2645

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int (a+a\sin(c+dx))^4\,dx = \int \left(a^4+4a^4\sin(c+dx)+6a^4\sin^2(c+dx)+4a^4\sin^3(c+dx)+a^4\sin^4(c+dx)\right)dx$$
$$= a^4x + a^4\int \sin^4(c+dx)\,dx + \left(4a^4\right)\int \sin(c+dx)\,dx + \left(4a^4\right)\int \sin^3(c+dx)\,dx + \left(6a\right.$$
$$= a^4x - \frac{4a^4\cos(c+dx)}{d} - \frac{3a^4\cos(c+dx)\sin(c+dx)}{d} - \frac{a^4\cos(c+dx)\sin^3(c+dx)}{4d} +$$
$$= 4a^4x - \frac{8a^4\cos(c+dx)}{d} + \frac{4a^4\cos^3(c+dx)}{3d} - \frac{27a^4\cos(c+dx)\sin(c+dx)}{8d} - \frac{a^4\cos(c}{}$$
$$= \frac{35a^4x}{8} - \frac{8a^4\cos(c+dx)}{d} + \frac{4a^4\cos^3(c+dx)}{3d} - \frac{27a^4\cos(c+dx)\sin(c+dx)}{8d} - \frac{a^4\cos}{}$$

Mathematica [A] time = 0.40, size = 57, normalized size = 0.66

$$\frac{a^4(3(-56\sin(2(c+dx))+\sin(4(c+dx))+140c+140dx)-672\cos(c+dx)+32\cos(3(c+dx)))}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*(-672*Cos[c + d*x] + 32*Cos[3*(c + d*x)] + 3*(140*c + 140*d*x - 56*Sin
[2*(c + d*x)] + Sin[4*(c + d*x)])))/(96*d)
```

fricas [A] time = 0.42, size = 70, normalized size = 0.80

$$\frac{32\,a^4\cos(dx+c)^3+105\,a^4dx-192\,a^4\cos(dx+c)+3\left(2\,a^4\cos(dx+c)^3-29\,a^4\cos(dx+c)\right)\sin(dx+c)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/24*(32*a^4*cos(d*x + c)^3 + 105*a^4*d*x - 192*a^4*cos(d*x + c) + 3*(2*a^4
*cos(d*x + c)^3 - 29*a^4*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 1.53, size = 72, normalized size = 0.83

$$\frac{35}{8}\,a^4x+\frac{a^4\cos\left(3\,dx+3\,c\right)}{3\,d}-\frac{7\,a^4\cos\left(dx+c\right)}{d}+\frac{a^4\sin\left(4\,dx+4\,c\right)}{32\,d}-\frac{7\,a^4\sin\left(2\,dx+2\,c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 35/8*a^4*x + 1/3*a^4*cos(3*d*x + 3*c)/d - 7*a^4*cos(d*x + c)/d + 1/32*a^4*s
in(4*d*x + 4*c)/d - 7/4*a^4*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.20, size = 111, normalized size = 1.28

$$\frac{a^4\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-\frac{4a^4\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}+6a^4\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-4a^4}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(a^4*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)-4/3*
a^4*(2+sin(d*x+c)^2)*cos(d*x+c)+6*a^4*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1
/2*c)-4*a^4*cos(d*x+c)+a^4*(d*x+c))
```

maxima [A] time = 0.30, size = 108, normalized size = 1.24

$$a^4x+\frac{4\left(\cos\left(dx+c\right)^3-3\,\cos\left(dx+c\right)\right)a^4}{3\,d}+\frac{\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)-8\,\sin\left(2\,dx+2\,c\right)\right)a^4}{32\,d}+\frac{3\left(2\,dx+2\,c-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] a^4*x + 4/3*(cos(d*x + c)^3 - 3*cos(d*x + c))*a^4/d + 1/32*(12*d*x + 12*c +
 sin(4*d*x + 4*c) - 8*sin(2*d*x + 2*c))*a^4/d + 3/2*(2*d*x + 2*c - sin(2*d*
x + 2*c))*a^4/d - 4*a^4*cos(d*x + c)/d
```

mupad [B] time = 8.59, size = 237, normalized size = 2.72

$$\frac{35\,a^4\,x}{8}-\frac{\frac{35\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}-\frac{35\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}-\frac{27\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{a^4\,(105\,c+105\,d\,x)}{24}-\frac{a^4\,(105\,c+105\,d\,x-320)}{24}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^4,x)
```

```
[Out] (35*a^4*x)/8 - ((35*a^4*tan(c/2 + (d*x)/2)^3)/4 - (35*a^4*tan(c/2 + (d*x)/2
)^5)/4 - (27*a^4*tan(c/2 + (d*x)/2)^7)/4 + (a^4*(105*c + 105*d*x))/24 - (a^
4*(105*c + 105*d*x - 320))/24 + tan(c/2 + (d*x)/2)^6*((a^4*(105*c + 105*d*x
))/6 - (a^4*(420*c + 420*d*x - 192))/24) + tan(c/2 + (d*x)/2)^2*((a^4*(105*
c + 105*d*x))/6 - (a^4*(420*c + 420*d*x - 1088))/24) + tan(c/2 + (d*x)/2)^4
*((a^4*(105*c + 105*d*x))/4 - (a^4*(630*c + 630*d*x - 960))/24) + (27*a^4*t
an(c/2 + (d*x)/2))/4)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

sympy [A] time = 1.89, size = 224, normalized size = 2.57

$$\begin{cases} \frac{3a^4x\sin^4(c+dx)}{8} + \frac{3a^4x\sin^2(c+dx)\cos^2(c+dx)}{4} + 3a^4x\sin^2(c+dx) + \frac{3a^4x\cos^4(c+dx)}{8} + 3a^4x\cos^2(c+dx) + a^4x - \frac{5a^4\sin^3(c \\ x\left(a\sin(c)+a\right)^4 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((3*a**4*x*sin(c + d*x)**4/8 + 3*a**4*x*sin(c + d*x)**2*cos(c + d*
x)**2/4 + 3*a**4*x*sin(c + d*x)**2 + 3*a**4*x*cos(c + d*x)**4/8 + 3*a**4*x*
cos(c + d*x)**2 + a**4*x - 5*a**4*sin(c + d*x)**3*cos(c + d*x)/(8*d) - 4*a*
*4*sin(c + d*x)**2*cos(c + d*x)/d - 3*a**4*sin(c + d*x)*cos(c + d*x)**3/(8*
d) - 3*a**4*sin(c + d*x)*cos(c + d*x)/d - 8*a**4*cos(c + d*x)**3/(3*d) - 4*
a**4*cos(c + d*x)/d, Ne(d, 0)), (x*(a*sin(c) + a)**4, True))
```

3.41 $\int \cot^2(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=116

$$-\frac{4a^4\cos^3(c+dx)}{3d}+\frac{4a^4\cos(c+dx)}{d}-\frac{a^4\cot(c+dx)}{d}+\frac{a^4\sin^3(c+dx)\cos(c+dx)}{4d}+\frac{23a^4\sin(c+dx)\cos(c+dx)}{8d}$$

```
[Out] 17/8*a^4*x-4*a^4*arctanh(cos(d*x+c))/d+4*a^4*cos(d*x+c)/d-4/3*a^4*cos(d*x+c
)^3/d-a^4*cot(d*x+c)/d+23/8*a^4*cos(d*x+c)*sin(d*x+c)/d+1/4*a^4*cos(d*x+c)*
sin(d*x+c)^3/d
```

Rubi [A] time = 0.16, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2709, 3770, 3767, 8, 2635, 2633}

$$-\frac{4a^4\cos^3(c+dx)}{3d}+\frac{4a^4\cos(c+dx)}{d}-\frac{a^4\cot(c+dx)}{d}+\frac{a^4\sin^3(c+dx)\cos(c+dx)}{4d}+\frac{23a^4\sin(c+dx)\cos(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (17*a^4*x)/8 - (4*a^4*ArcTanh[Cos[c + d*x]])/d + (4*a^4*Cos[c + d*x])/d - (
4*a^4*Cos[c + d*x]^3)/(3*d) - (a^4*Cot[c + d*x])/d + (23*a^4*Cos[c + d*x]*S
in[c + d*x])/(8*d) + (a^4*Cos[c + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)(a+a\sin(c+dx))^4\,dx = \frac{\int \left(5a^6+4a^6\csc(c+dx)+a^6\csc^2(c+dx)-5a^6\sin^2(c+dx)-4a^6\sin^3(\right.}{a^2}$$

$$= 5a^4x + a^4\int \csc^2(c+dx)\,dx - a^4\int \sin^4(c+dx)\,dx + \left(4a^4\right)\int \csc(c+d$$

$$= 5a^4x - \frac{4a^4\tanh^{-1}(\cos(c+dx))}{d} + \frac{5a^4\cos(c+dx)\sin(c+dx)}{2d} + \frac{a^4\cos(}{}$$

$$= \frac{5a^4x}{2} - \frac{4a^4\tanh^{-1}(\cos(c+dx))}{d} + \frac{4a^4\cos(c+dx)}{d} - \frac{4a^4\cos^3(c+dx)}{3d} -$$

$$= \frac{17a^4x}{8} - \frac{4a^4\tanh^{-1}(\cos(c+dx))}{d} + \frac{4a^4\cos(c+dx)}{d} - \frac{4a^4\cos^3(c+dx)}{3d}$$

Mathematica [A] time = 1.60, size = 136, normalized size = 1.17

$$\frac{a^4\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(408c\sin(c+dx)+408dx\sin(c+dx)+320\sin(2(c+dx))-32\sin(4(c+dx))\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(-48*Cos[c + d*x] - 147*Cos[3*(c + d
*x)] + 3*Cos[5*(c + d*x)] + 408*c*Sin[c + d*x] + 408*d*x*Sin[c + d*x] - 768
```

```
*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] + 768*Log[Sin[(c + d*x)/2]]*Sin[c + d*x
] + 320*Sin[2*(c + d*x)] - 32*Sin[4*(c + d*x)]))/(384*d)
```

fricas [A] time = 0.45, size = 135, normalized size = 1.16

$$\frac{6\,a^4\cos\left(dx+c\right)^5 - 81\,a^4\cos\left(dx+c\right)^3 - 48\,a^4\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right) + 48\,a^4\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)\right.}{24\,d\sin\left(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/24*(6*a^4*cos(d*x + c)^5 - 81*a^4*cos(d*x + c)^3 - 48*a^4*log(1/2*cos(d*x
 + c) + 1/2)*sin(d*x + c) + 48*a^4*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c
) + 51*a^4*cos(d*x + c) - (32*a^4*cos(d*x + c)^3 - 51*a^4*d*x - 96*a^4*cos(
d*x + c))*sin(d*x + c))/(d*sin(d*x + c))
```

giac [A] time = 0.71, size = 194, normalized size = 1.67

$$\frac{51\left(dx+c\right)a^4 + 96\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right) + 12\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - \frac{12\left(8\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^4\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)} - \frac{2\left(69\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/24*(51*(d*x + c)*a^4 + 96*a^4*log(abs(tan(1/2*d*x + 1/2*c))) + 12*a^4*tan
(1/2*d*x + 1/2*c) - 12*(8*a^4*tan(1/2*d*x + 1/2*c) + a^4)/tan(1/2*d*x + 1/2
*c) - 2*(69*a^4*tan(1/2*d*x + 1/2*c)^7 + 93*a^4*tan(1/2*d*x + 1/2*c)^5 - 19
2*a^4*tan(1/2*d*x + 1/2*c)^4 - 93*a^4*tan(1/2*d*x + 1/2*c)^3 - 256*a^4*tan(
1/2*d*x + 1/2*c)^2 - 69*a^4*tan(1/2*d*x + 1/2*c) - 64*a^4)/(tan(1/2*d*x + 1
/2*c)^2 + 1)^4)/d
```

maple [A] time = 0.14, size = 127, normalized size = 1.09

$$-\frac{a^4\left(\cos^3\left(dx+c\right)\right)\sin\left(dx+c\right)}{4d}+\frac{25a^4\cos\left(dx+c\right)\sin\left(dx+c\right)}{8d}+\frac{17a^4x}{8}+\frac{17a^4c}{8d}-\frac{4a^4\left(\cos^3\left(dx+c\right)\right)}{3d}+\frac{4a^4\ln\left(\csc\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+a*sin(d*x+c))^4,x)
```

```
[Out] -1/4/d*a^4*cos(d*x+c)^3*sin(d*x+c)+25/8*a^4*cos(d*x+c)*sin(d*x+c)/d+17/8*a^
4*x+17/8/d*a^4*c-4/3*a^4*cos(d*x+c)^3/d+4/d*a^4*ln(csc(d*x+c)-cot(d*x+c))+4
*a^4*cos(d*x+c)/d-a^4*cot(d*x+c)/d
```

maxima [A] time = 0.40, size = 117, normalized size = 1.01

$$-\frac{128\,a^4\cos\left(dx+c\right)^3-3\left(4\,dx+4\,c-\sin\left(4\,dx+4\,c\right)\right)a^4-144\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^4+96\left(dx+c+\frac{1}{\tan\cdots}\right.}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/96*(128*a^4*cos(d*x + c)^3 - 3*(4*d*x + 4*c - sin(4*d*x + 4*c))*a^4 - 14
4*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^4 + 96*(d*x + c + 1/tan(d*x + c))*a^4
- 192*a^4*(2*cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)))
/d
```

mupad [B] time = 6.78, size = 295, normalized size = 2.54

$$\frac{4\,a^4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{17\,a^4\,\mathrm{atan}\left(\frac{289\,a^8}{16\left(34\,a^8-\frac{289\,a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}\right)}+\frac{34\,a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{34\,a^8-\frac{289\,a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}}\right)}{4\,d}+\frac{-\frac{25\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{2}-\frac{39\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{2}\cdots}{d\left(2\,\tan\left(\frac{c}{2}+\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + a*sin(c + d*x))^4,x)
```

```
[Out] (4*a^4*log(tan(c/2 + (d*x)/2)))/d + (17*a^4*atan((289*a^8)/(16*(34*a^8 - (2
89*a^8*tan(c/2 + (d*x)/2))/16)) + (34*a^8*tan(c/2 + (d*x)/2))/(34*a^8 - (28
9*a^8*tan(c/2 + (d*x)/2))/16)))/(4*d) + ((15*a^4*tan(c/2 + (d*x)/2)^2)/2 +
(128*a^4*tan(c/2 + (d*x)/2)^3)/3 + (19*a^4*tan(c/2 + (d*x)/2)^4)/2 + 32*a^4
*tan(c/2 + (d*x)/2)^5 - (39*a^4*tan(c/2 + (d*x)/2)^6)/2 - (25*a^4*tan(c/2 +
 (d*x)/2)^8)/2 - a^4 + (32*a^4*tan(c/2 + (d*x)/2))/3)/(d*(2*tan(c/2 + (d*x)
/2) + 8*tan(c/2 + (d*x)/2)^3 + 12*tan(c/2 + (d*x)/2)^5 + 8*tan(c/2 + (d*x)/
2)^7 + 2*tan(c/2 + (d*x)/2)^9)) + (a^4*tan(c/2 + (d*x)/2))/(2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int 4\sin\left(c+dx\right)\cot^2\left(c+dx\right)dx+\int 6\sin^2\left(c+dx\right)\cot^2\left(c+dx\right)dx+\int 4\sin^3\left(c+dx\right)\cot^2\left(c+dx\right)dx+\int\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+a*sin(d*x+c))**4,x)
```

```
[Out] a**4*(Integral(4*sin(c + d*x)*cot(c + d*x)**2, x) + Integral(6*sin(c + d*x)
**2*cot(c + d*x)**2, x) + Integral(4*sin(c + d*x)**3*cot(c + d*x)**2, x) +
Integral(sin(c + d*x)**4*cot(c + d*x)**2, x) + Integral(cot(c + d*x)**2, x)
)
```

3.42 $\int \cot^4(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=140

$$\frac{4a^4\cos^3(c+dx)}{3d}-\frac{a^4\cot^3(c+dx)}{3d}-\frac{5a^4\cot(c+dx)}{d}-\frac{a^4\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{19a^4\sin(c+dx)\cos(c+dx)}{8d}+\frac{2\ldots}{}$$

```
[Out] -61/8*a^4*x+2*a^4*arctanh(cos(d*x+c))/d+4/3*a^4*cos(d*x+c)^3/d-5*a^4*cot(d*
x+c)/d-1/3*a^4*cot(d*x+c)^3/d-2*a^4*cot(d*x+c)*csc(d*x+c)/d-19/8*a^4*cos(d*
x+c)*sin(d*x+c)/d-1/4*a^4*cos(d*x+c)*sin(d*x+c)^3/d
```

Rubi [A] time = 0.22, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2709, 3770, 3767, 8, 3768, 2638, 2635, 2633}

$$\frac{4a^4\cos^3(c+dx)}{3d}-\frac{a^4\cot^3(c+dx)}{3d}-\frac{5a^4\cot(c+dx)}{d}-\frac{a^4\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{19a^4\sin(c+dx)\cos(c+dx)}{8d}+\frac{2\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-61*a^4*x)/8 + (2*a^4*ArcTanh[Cos[c + d*x]])/d + (4*a^4*Cos[c + d*x]^3)/(3
*d) - (5*a^4*Cot[c + d*x])/d - (a^4*Cot[c + d*x]^3)/(3*d) - (2*a^4*Cot[c +
d*x]*Csc[c + d*x])/d - (19*a^4*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (a^4*Cos[
c + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)(a+a\sin(c+dx))^4\,dx = \frac{\int \left(-10a^8-4a^8\csc(c+dx)+4a^8\csc^2(c+dx)+4a^8\csc^3(c+dx)+a^8 c\right.}{}$$

$$= -10a^4x+a^4\int \csc^4(c+dx)\,dx+a^4\int \sin^4(c+dx)\,dx-\left(4a^4\right)\int \csc(c$$

$$= -10a^4x+\frac{4a^4\tanh^{-1}(\cos(c+dx))}{d}+\frac{4a^4\cos(c+dx)}{d}-\frac{2a^4\cot(c+dx)}{d}$$

$$= -8a^4x+\frac{2a^4\tanh^{-1}(\cos(c+dx))}{d}+\frac{4a^4\cos^3(c+dx)}{3d}-\frac{5a^4\cot(c+dx)}{d}$$

$$= -\frac{61a^4x}{8}+\frac{2a^4\tanh^{-1}(\cos(c+dx))}{d}+\frac{4a^4\cos^3(c+dx)}{3d}-\frac{5a^4\cot(c+d}{d}$$

Mathematica [A] time = 5.28, size = 209, normalized size = 1.49

$$\frac{a^4(\sin(c+dx)+1)^4\left(-732(c+dx)-120\sin(2(c+dx))+3\sin(4(c+dx))+96\cos(c+dx)+32\cos(3(c+dx))+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*(1 + Sin[c + d*x])^4*(-732*(c + d*x) + 96*Cos[c + d*x] + 32*Cos[3*(c +
 d*x)] - 224*Cot[(c + d*x)/2] - 48*Csc[(c + d*x)/2]^2 + 192*Log[Cos[(c + d*
x)/2]] - 192*Log[Sin[(c + d*x)/2]] + 48*Sec[(c + d*x)/2]^2 + 32*Csc[c + d*x
]^3*Sin[(c + d*x)/2]^4 - 2*Csc[(c + d*x)/2]^4*Sin[c + d*x] - 120*Sin[2*(c +
 d*x)] + 3*Sin[4*(c + d*x)] + 224*Tan[(c + d*x)/2]))/(96*d*(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2])^8)
```

fricas [A] time = 0.45, size = 219, normalized size = 1.56

$$-\frac{6\,a^4\cos(dx+c)^7-75\,a^4\cos(dx+c)^5+244\,a^4\cos(dx+c)^3-183\,a^4\cos(dx+c)-24\left(a^4\cos(dx+c)^2-a^4\right)\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/24*(6*a^4*cos(d*x + c)^7 - 75*a^4*cos(d*x + c)^5 + 244*a^4*cos(d*x + c)^
3 - 183*a^4*cos(d*x + c) - 24*(a^4*cos(d*x + c)^2 - a^4)*log(1/2*cos(d*x +
c) + 1/2)*sin(d*x + c) + 24*(a^4*cos(d*x + c)^2 - a^4)*log(-1/2*cos(d*x + c
) + 1/2)*sin(d*x + c) - (32*a^4*cos(d*x + c)^5 - 183*a^4*d*x*cos(d*x + c)^2
 - 32*a^4*cos(d*x + c)^3 + 183*a^4*d*x + 48*a^4*cos(d*x + c))*sin(d*x + c))
/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

giac [B] time = 1.29, size = 274, normalized size = 1.96

$$\frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+12\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-183\,(dx+c)a^4-48\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+57\,a^4\tan\left(\frac{1}{2}\,dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/24*(a^4*tan(1/2*d*x + 1/2*c)^3 + 12*a^4*tan(1/2*d*x + 1/2*c)^2 - 183*(d*x
 + c)*a^4 - 48*a^4*log(abs(tan(1/2*d*x + 1/2*c))) + 57*a^4*tan(1/2*d*x + 1/
2*c) + (88*a^4*tan(1/2*d*x + 1/2*c)^3 - 57*a^4*tan(1/2*d*x + 1/2*c)^2 - 12*
a^4*tan(1/2*d*x + 1/2*c) - a^4)/tan(1/2*d*x + 1/2*c)^3 + 2*(57*a^4*tan(1/2*
d*x + 1/2*c)^7 + 96*a^4*tan(1/2*d*x + 1/2*c)^6 + 81*a^4*tan(1/2*d*x + 1/2*c
)^5 + 96*a^4*tan(1/2*d*x + 1/2*c)^4 - 81*a^4*tan(1/2*d*x + 1/2*c)^3 + 32*a^
4*tan(1/2*d*x + 1/2*c)^2 - 57*a^4*tan(1/2*d*x + 1/2*c) + 32*a^4)/(tan(1/2*d
*x + 1/2*c)^2 + 1)^4)/d
```

maple [A] time = 0.23, size = 190, normalized size = 1.36

$$-\frac{23a^4\left(\cos^3(dx+c)\right)\sin(dx+c)}{4d}-\frac{69a^4\cos(dx+c)\sin(dx+c)}{8d}-\frac{61a^4x}{8}-\frac{61a^4c}{8d}-\frac{2a^4\left(\cos^3(dx+c)\right)}{3d}-\frac{2a^4\cos(}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+a*sin(d*x+c))^4,x)
```

```
[Out] -23/4/d*a^4*cos(d*x+c)^3*sin(d*x+c)-69/8*a^4*cos(d*x+c)*sin(d*x+c)/d-61/8*a
^4*x-61/8/d*a^4*c-2/3*a^4*cos(d*x+c)^3/d-2*a^4*cos(d*x+c)/d-2/d*a^4*ln(csc(
d*x+c)-cot(d*x+c))-6/d*a^4/sin(d*x+c)*cos(d*x+c)^5-2/d*a^4/sin(d*x+c)^2*cos
(d*x+c)^5-1/3*a^4*cot(d*x+c)^3/d+a^4*cot(d*x+c)/d
```

maxima [A] time = 0.40, size = 218, normalized size = 1.56

$$\frac{64\left(2\cos(dx+c)^3+6\cos(dx+c)-3\log(\cos(dx+c)+1)+3\log(\cos(dx+c)-1)\right)a^4+3(12dx+12c+s}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/96*(64*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) + 1) + 3*l
og(cos(d*x + c) - 1))*a^4 + 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d
*x + 2*c))*a^4 - 288*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3
+ tan(d*x + c)))*a^4 + 32*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c
)^3)*a^4 + 96*a^4*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3
*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 6.71, size = 384, normalized size = 2.74

$$\frac{a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{2d}+\frac{a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24d}-\frac{2a^4\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{61a^4\operatorname{atan}\left(\frac{3721a^8}{16\left(61a^8-\frac{3721a^8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{16}\right)}+\frac{61a^8\tan\left(\frac{c}{2}+\right.}{61a^8-\frac{3721a^8\tan}{16}}\right.}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + a*sin(c + d*x))^4,x)
```

```
[Out] (a^4*tan(c/2 + (d*x)/2)^2)/(2*d) + (a^4*tan(c/2 + (d*x)/2)^3)/(24*d) - (2*a
^4*log(tan(c/2 + (d*x)/2)))/d - (61*a^4*atan((3721*a^8)/(16*(61*a^8 - (3721
*a^8*tan(c/2 + (d*x)/2))/16)) + (61*a^8*tan(c/2 + (d*x)/2))/(61*a^8 - (3721
*a^8*tan(c/2 + (d*x)/2))/16)))/(4*d) + (19*a^4*tan(c/2 + (d*x)/2))/(8*d) -
((61*a^4*tan(c/2 + (d*x)/2)^2)/3 - (16*a^4*tan(c/2 + (d*x)/2)^3)/3 + 116*a^
4*tan(c/2 + (d*x)/2)^4 + (8*a^4*tan(c/2 + (d*x)/2)^5)/3 + (508*a^4*tan(c/2
+ (d*x)/2)^6)/3 - 48*a^4*tan(c/2 + (d*x)/2)^7 + (67*a^4*tan(c/2 + (d*x)/2)^
8)/3 - 60*a^4*tan(c/2 + (d*x)/2)^9 - 19*a^4*tan(c/2 + (d*x)/2)^10 + a^4/3 +
 4*a^4*tan(c/2 + (d*x)/2))/(d*(8*tan(c/2 + (d*x)/2)^3 + 32*tan(c/2 + (d*x)/
2)^5 + 48*tan(c/2 + (d*x)/2)^7 + 32*tan(c/2 + (d*x)/2)^9 + 8*tan(c/2 + (d*x
)/2)^11))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^4 \left(\int 4 \sin{(c + dx)} \cot^4{(c + dx)}\, dx + \int 6 \sin^2{(c + dx)} \cot^4{(c + dx)}\, dx + \int 4 \sin^3{(c + dx)} \cot^4{(c + dx)}\, dx + \int \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+a*sin(d*x+c))**4,x)
```

```
[Out] a**4*(Integral(4*sin(c + d*x)*cot(c + d*x)**4, x) + Integral(6*sin(c + d*x)
**2*cot(c + d*x)**4, x) + Integral(4*sin(c + d*x)**3*cot(c + d*x)**4, x) +
Integral(sin(c + d*x)**4*cot(c + d*x)**4, x) + Integral(cot(c + d*x)**4, x)
)
```

3.43 $\int \cot^6(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=198

$$-\frac{4a^4\cos^3(c+dx)}{3d}-\frac{4a^4\cos(c+dx)}{d}-\frac{a^4\cot^5(c+dx)}{5d}-\frac{5a^4\cot^3(c+dx)}{3d}+\frac{10a^4\cot(c+dx)}{d}+\frac{a^4\sin^3(c+dx)\cos(c}{4d}$$

```
[Out] 97/8*a^4*x+5/2*a^4*arctanh(cos(d*x+c))/d-4*a^4*cos(d*x+c)/d-4/3*a^4*cos(d*x
+c)^3/d+10*a^4*cot(d*x+c)/d-5/3*a^4*cot(d*x+c)^3/d-1/5*a^4*cot(d*x+c)^5/d+5
/2*a^4*cot(d*x+c)*csc(d*x+c)/d-a^4*cot(d*x+c)*csc(d*x+c)^3/d+15/8*a^4*cos(d
*x+c)*sin(d*x+c)/d+1/4*a^4*cos(d*x+c)*sin(d*x+c)^3/d
```

Rubi [A] time = 0.43, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {2709, 3767, 8, 3768, 3770, 2638, 2635, 2633}

$$-\frac{4a^4\cos^3(c+dx)}{3d}-\frac{4a^4\cos(c+dx)}{d}-\frac{a^4\cot^5(c+dx)}{5d}-\frac{5a^4\cot^3(c+dx)}{3d}+\frac{10a^4\cot(c+dx)}{d}+\frac{a^4\sin^3(c+dx)\cos(c}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (97*a^4*x)/8 + (5*a^4*ArcTanh[Cos[c + d*x]])/(2*d) - (4*a^4*Cos[c + d*x])/d
 - (4*a^4*Cos[c + d*x]^3)/(3*d) + (10*a^4*Cot[c + d*x])/d - (5*a^4*Cot[c +
d*x]^3)/(3*d) - (a^4*Cot[c + d*x]^5)/(5*d) + (5*a^4*Cot[c + d*x]*Csc[c + d*
x])/(2*d) - (a^4*Cot[c + d*x]*Csc[c + d*x]^3)/d + (15*a^4*Cos[c + d*x]*Sin[
c + d*x])/(8*d) + (a^4*Cos[c + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)(a+a\sin(c+dx))^4\,dx = \frac{\int \left(14a^{10} - 14a^{10}\csc^2(c+dx) - 8a^{10}\csc^3(c+dx) + 3a^{10}\csc^4(c+dx) + \right.}{}$$

$$= 14a^4x + a^4\int \csc^6(c+dx)\,dx - a^4\int \sin^4(c+dx)\,dx + \left(3a^4\right)\int \csc^4(c$$

$$= 14a^4x - \frac{8a^4\cos(c+dx)}{d} + \frac{4a^4\cot(c+dx)\csc(c+dx)}{d} - \frac{a^4\cot(c+dx)}{d}$$

$$= \frac{25a^4x}{2} + \frac{4a^4\tanh^{-1}(\cos(c+dx))}{d} - \frac{4a^4\cos(c+dx)}{d} - \frac{4a^4\cos^3(c+dx)}{3d}$$

$$= \frac{97a^4x}{8} + \frac{5a^4\tanh^{-1}(\cos(c+dx))}{2d} - \frac{4a^4\cos(c+dx)}{d} - \frac{4a^4\cos^3(c+dx)}{3d}$$

Mathematica [A] time = 1.58, size = 283, normalized size = 1.43

$$\frac{a^4(\sin(c+dx)+1)^4\left(5820(c+dx) + 480\sin(2(c+dx)) - 15\sin(4(c+dx)) - 2400\cos(c+dx) - 160\cos(3(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*(1 + Sin[c + d*x])^4*(5820*(c + d*x) - 2400*Cos[c + d*x] - 160*Cos[3*(
c + d*x)] + 2752*Cot[(c + d*x)/2] + 300*Csc[(c + d*x)/2]^2 - 30*Csc[(c + d*
x)/2]^4 + 1200*Log[Cos[(c + d*x)/2]] - 1200*Log[Sin[(c + d*x)/2]] - 300*Sec
[(c + d*x)/2]^2 + 30*Sec[(c + d*x)/2]^4 + 632*Csc[c + d*x]^3*Sin[(c + d*x)/
2]^4 + 96*Csc[c + d*x]^5*Sin[(c + d*x)/2]^6 - (79*Csc[(c + d*x)/2]^4*Sin[c
+ d*x])/2 - (3*Csc[(c + d*x)/2]^6*Sin[c + d*x])/2 + 480*Sin[2*(c + d*x)] -
15*Sin[4*(c + d*x)] - 2752*Tan[(c + d*x)/2]))/(480*d*(Cos[(c + d*x)/2] + Si
n[(c + d*x)/2])^8)
```

fricas [A] time = 0.47, size = 291, normalized size = 1.47

$$\frac{30\,a^4\cos(dx+c)^9 - 345\,a^4\cos(dx+c)^7 + 2231\,a^4\cos(dx+c)^5 - 3395\,a^4\cos(dx+c)^3 + 1455\,a^4\cos(dx+c)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/120*(30*a^4*cos(d*x + c)^9 - 345*a^4*cos(d*x + c)^7 + 2231*a^4*cos(d*x +
c)^5 - 3395*a^4*cos(d*x + c)^3 + 1455*a^4*cos(d*x + c) + 150*(a^4*cos(d*x +
```

```
 c)^4 - 2*a^4*cos(d*x + c)^2 + a^4)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c
) - 150*(a^4*cos(d*x + c)^4 - 2*a^4*cos(d*x + c)^2 + a^4)*log(-1/2*cos(d*x
+ c) + 1/2)*sin(d*x + c) - 5*(32*a^4*cos(d*x + c)^7 - 291*a^4*d*x*cos(d*x +
 c)^4 + 32*a^4*cos(d*x + c)^5 + 582*a^4*d*x*cos(d*x + c)^2 - 100*a^4*cos(d*
x + c)^3 - 291*a^4*d*x + 60*a^4*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c
)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

giac [A] time = 1.39, size = 339, normalized size = 1.71

$$\frac{3\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+30\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+85\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-240\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+5820\,(dx+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/480*(3*a^4*tan(1/2*d*x + 1/2*c)^5 + 30*a^4*tan(1/2*d*x + 1/2*c)^4 + 85*a^
4*tan(1/2*d*x + 1/2*c)^3 - 240*a^4*tan(1/2*d*x + 1/2*c)^2 + 5820*(d*x + c)*
a^4 - 1200*a^4*log(abs(tan(1/2*d*x + 1/2*c))) - 2670*a^4*tan(1/2*d*x + 1/2*
c) - 40*(45*a^4*tan(1/2*d*x + 1/2*c)^7 + 192*a^4*tan(1/2*d*x + 1/2*c)^6 + 6
9*a^4*tan(1/2*d*x + 1/2*c)^5 + 384*a^4*tan(1/2*d*x + 1/2*c)^4 - 69*a^4*tan(
1/2*d*x + 1/2*c)^3 + 320*a^4*tan(1/2*d*x + 1/2*c)^2 - 45*a^4*tan(1/2*d*x +
1/2*c) + 128*a^4)/(tan(1/2*d*x + 1/2*c)^2 + 1)^4 + (2740*a^4*tan(1/2*d*x +
1/2*c)^5 + 2670*a^4*tan(1/2*d*x + 1/2*c)^4 + 240*a^4*tan(1/2*d*x + 1/2*c)^3
 - 85*a^4*tan(1/2*d*x + 1/2*c)^2 - 30*a^4*tan(1/2*d*x + 1/2*c) - 3*a^4)/tan
(1/2*d*x + 1/2*c)^5)/d
```

maple [A] time = 0.23, size = 293, normalized size = 1.48

$$\frac{97a^4x}{8}+\frac{7a^4\sin\left(dx+c\right)\left(\cos^5\left(dx+c\right)\right)}{d}+\frac{97a^4c}{8d}-\frac{a^4\left(\cos^5\left(dx+c\right)\right)}{2d}-\frac{5a^4\left(\cos^3\left(dx+c\right)\right)}{6d}-\frac{a^4\left(\cos^7\left(dx+c\right)\right)}{d\sin\left(dx+c\right)^4}-\frac{2a^4}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+a*sin(d*x+c))^4,x)
```

```
[Out] 97/8*a^4*x+7/d*a^4*sin(d*x+c)*cos(d*x+c)^5+97/8/d*a^4*c-1/2/d*a^4*cos(d*x+c
)^5-5/6*a^4*cos(d*x+c)^3/d-1/d*a^4/sin(d*x+c)^4*cos(d*x+c)^7-2/d*a^4/sin(d*
x+c)^3*cos(d*x+c)^7+7/d*a^4/sin(d*x+c)*cos(d*x+c)^7+35/4/d*a^4*cos(d*x+c)^3
*sin(d*x+c)+105/8*a^4*cos(d*x+c)*sin(d*x+c)/d-1/2/d*a^4/sin(d*x+c)^2*cos(d*
x+c)^7-a^4*cot(d*x+c)/d-5/2*a^4*cos(d*x+c)/d-1/5*a^4*cot(d*x+c)^5/d+1/3*a^4
*cot(d*x+c)^3/d-5/2/d*a^4*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.41, size = 313, normalized size = 1.58

$$-\frac{40\left(4\,\cos\left(dx+c\right)^{3}-\frac{6\,\cos(dx+c)}{\cos(dx+c)^{2}-1}+24\,\cos\left(dx+c\right)-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx+c\right)-1\right)\right)a^{4}+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/120*(40*(4*cos(d*x + c)^3 - 6*cos(d*x + c)/(cos(d*x + c)^2 - 1) + 24*cos
(d*x + c) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a^4 + 15*(
15*d*x + 15*c + (15*tan(d*x + c)^4 + 25*tan(d*x + c)^2 + 8)/(tan(d*x + c)^5
 + 2*tan(d*x + c)^3 + tan(d*x + c)))*a^4 - 120*(15*d*x + 15*c + (15*tan(d*x
 + c)^4 + 10*tan(d*x + c)^2 - 2)/(tan(d*x + c)^5 + tan(d*x + c)^3))*a^4 + 8
*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(d*x + c)^5
)*a^4 + 30*a^4*(2*(9*cos(d*x + c)^3 - 7*cos(d*x + c))/(cos(d*x + c)^4 - 2*c
os(d*x + c)^2 + 1) - 16*cos(d*x + c) + 15*log(cos(d*x + c) + 1) - 15*log(co
s(d*x + c) - 1)))/d
```

mupad [B] time = 6.85, size = 454, normalized size = 2.29

$$\frac{17\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{96\,d}-\frac{a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2\,d}+\frac{a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{16\,d}+\frac{a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,d}-\frac{5\,a^4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}-\frac{97\,a^4\,\mathrm{atan}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6*(a + a*sin(c + d*x))^4,x)
```

```
[Out] (17*a^4*tan(c/2 + (d*x)/2)^3)/(96*d) - (a^4*tan(c/2 + (d*x)/2)^2)/(2*d) + (
a^4*tan(c/2 + (d*x)/2)^4)/(16*d) + (a^4*tan(c/2 + (d*x)/2)^5)/(160*d) - (5*
a^4*log(tan(c/2 + (d*x)/2)))/(2*d) - (97*a^4*atan((9409*a^8)/(16*((485*a^8)
/4 + (9409*a^8*tan(c/2 + (d*x)/2))/16)) - (485*a^8*tan(c/2 + (d*x)/2))/(4*(
(485*a^8)/4 + (9409*a^8*tan(c/2 + (d*x)/2))/16))))/(4*d) - ((97*a^4*tan(c/2
 + (d*x)/2)^2)/15 - 8*a^4*tan(c/2 + (d*x)/2)^3 - (2312*a^4*tan(c/2 + (d*x)/
2)^4)/15 + (868*a^4*tan(c/2 + (d*x)/2)^5)/3 - (3986*a^4*tan(c/2 + (d*x)/2)^
6)/5 + (2296*a^4*tan(c/2 + (d*x)/2)^7)/3 - (18437*a^4*tan(c/2 + (d*x)/2)^8)
/15 + 962*a^4*tan(c/2 + (d*x)/2)^9 - (1567*a^4*tan(c/2 + (d*x)/2)^10)/3 + 4
96*a^4*tan(c/2 + (d*x)/2)^11 - 58*a^4*tan(c/2 + (d*x)/2)^12 + a^4/5 + 2*a^4
*tan(c/2 + (d*x)/2))/(d*(32*tan(c/2 + (d*x)/2)^5 + 128*tan(c/2 + (d*x)/2)^7
```

```
 + 192*tan(c/2 + (d*x)/2)^9 + 128*tan(c/2 + (d*x)/2)^11 + 32*tan(c/2 + (d*x
)/2)^13)) - (89*a^4*tan(c/2 + (d*x)/2))/(16*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+a*sin(d*x+c))**4,x)
[Out] Timed out
```

3.44 $\int \frac{\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=130

$$\frac{\tan^8(c+dx)}{8ad}-\frac{35\tanh^{-1}(\sin(c+dx))}{128ad}-\frac{\tan^7(c+dx)\sec(c+dx)}{8ad}+\frac{7\tan^5(c+dx)\sec(c+dx)}{48ad}-\frac{35\tan^3(c+dx)\text{se}}{192ad}$$

```
[Out] -35/128*arctanh(sin(d*x+c))/a/d+35/128*sec(d*x+c)*tan(d*x+c)/a/d-35/192*sec
(d*x+c)*tan(d*x+c)^3/a/d+7/48*sec(d*x+c)*tan(d*x+c)^5/a/d-1/8*sec(d*x+c)*ta
n(d*x+c)^7/a/d+1/8*tan(d*x+c)^8/a/d
```

Rubi [A] time = 0.16, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2706, 2607, 30, 2611, 3770}

$$\frac{\tan^8(c+dx)}{8ad}-\frac{35\tanh^{-1}(\sin(c+dx))}{128ad}-\frac{\tan^7(c+dx)\sec(c+dx)}{8ad}+\frac{7\tan^5(c+dx)\sec(c+dx)}{48ad}-\frac{35\tan^3(c+dx)\text{se}}{192ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^7/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-35*ArcTanh[Sin[c + d*x]])/(128*a*d) + (35*Sec[c + d*x]*Tan[c + d*x])/(128
*a*d) - (35*Sec[c + d*x]*Tan[c + d*x]^3)/(192*a*d) + (7*Sec[c + d*x]*Tan[c
+ d*x]^5)/(48*a*d) - (Sec[c + d*x]*Tan[c + d*x]^7)/(8*a*d) + Tan[c + d*x]^8
/(8*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec^2(c+dx)\tan^7(c+dx)\,dx}{a} - \frac{\int \sec(c+dx)\tan^8(c+dx)\,dx}{a}\\
&= -\frac{\sec(c+dx)\tan^7(c+dx)}{8ad} + \frac{7\int \sec(c+dx)\tan^6(c+dx)\,dx}{8a} + \frac{\operatorname{Subst}\left(\int x^7\,dx, x, \tan(c\right.}{ad}\\
&= \frac{7\sec(c+dx)\tan^5(c+dx)}{48ad} - \frac{\sec(c+dx)\tan^7(c+dx)}{8ad} + \frac{\tan^8(c+dx)}{8ad} - \frac{35\int \sec(c+d}{}\\
&= -\frac{35\sec(c+dx)\tan^3(c+dx)}{192ad} + \frac{7\sec(c+dx)\tan^5(c+dx)}{48ad} - \frac{\sec(c+dx)\tan^7(c+dx)}{8ad}\\
&= \frac{35\sec(c+dx)\tan(c+dx)}{128ad} - \frac{35\sec(c+dx)\tan^3(c+dx)}{192ad} + \frac{7\sec(c+dx)\tan^5(c+dx)}{48ad}\\
&= -\frac{35\tanh^{-1}(\sin(c+dx))}{128ad} + \frac{35\sec(c+dx)\tan(c+dx)}{128ad} - \frac{35\sec(c+dx)\tan^3(c+dx)}{192ad} +
\end{aligned}$$

Mathematica [A] time = 0.98, size = 101, normalized size = 0.78

$$-\frac{\frac{279\sin^6(c+dx)+87\sin^5(c+dx)-424\sin^4(c+dx)-136\sin^3(c+dx)+249\sin^2(c+dx)+57\sin(c+dx)-48}{(\sin(c+dx)-1)^3(\sin(c+dx)+1)^4}+105\tanh^{-1}(\sin(c+dx))}{384ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^7/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/384*(105*ArcTanh[Sin[c + d*x]] + (-48 + 57*Sin[c + d*x] + 249*Sin[c + d*
x]^2 - 136*Sin[c + d*x]^3 - 424*Sin[c + d*x]^4 + 87*Sin[c + d*x]^5 + 279*Si
n[c + d*x]^6)/((-1 + Sin[c + d*x])^3*(1 + Sin[c + d*x])^4))/(a*d)
```

fricas [A] time = 0.45, size = 167, normalized size = 1.28

$$-\frac{558\cos(dx+c)^6 - 826\cos(dx+c)^4 + 476\cos(dx+c)^2 + 105\left(\cos(dx+c)^6\sin(dx+c) + \cos(dx+c)^6\right)\text{lo}}{768}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/768*(558*cos(d*x + c)^6 - 826*cos(d*x + c)^4 + 476*cos(d*x + c)^2 + 105*
(cos(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(sin(d*x + c) + 1) - 105*
(cos(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(-sin(d*x + c) + 1) - 2*(
87*cos(d*x + c)^4 - 38*cos(d*x + c)^2 + 8)*sin(d*x + c) - 112)/(a*d*cos(d*x
 + c)^6*sin(d*x + c) + a*d*cos(d*x + c)^6)
```

giac [A] time = 9.21, size = 136, normalized size = 1.05

$$-\frac{\frac{420\log(|\sin(dx+c)+1|)}{a} - \frac{420\log(|\sin(dx+c)-1|)}{a} + \frac{2\left(385\sin(dx+c)^3 - 807\sin(dx+c)^2 + 567\sin(dx+c) - 129\right)}{a(\sin(dx+c)-1)^3} - \frac{875\sin(dx+c)^4 + 1964\sin(dx+}{a(\text{s}}}{3072\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/3072*(420*log(abs(sin(d*x + c) + 1))/a - 420*log(abs(sin(d*x + c) - 1))/
a + 2*(385*sin(d*x + c)^3 - 807*sin(d*x + c)^2 + 567*sin(d*x + c) - 129)/(a
*(sin(d*x + c) - 1)^3) - (875*sin(d*x + c)^4 + 1964*sin(d*x + c)^3 + 1554*s
in(d*x + c)^2 + 396*sin(d*x + c) - 21)/(a*(sin(d*x + c) + 1)^4))/d
```

maple [A] time = 0.18, size = 162, normalized size = 1.25

$$-\frac{1}{96ad(\sin(dx+c)-1)^3} - \frac{9}{128ad(\sin(dx+c)-1)^2} - \frac{29}{128ad(\sin(dx+c)-1)} + \frac{35\ln(\sin(dx+c)-1)}{256ad} + \frac{}{64ad(1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^7/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/96/a/d/(sin(d*x+c)-1)^3-9/128/a/d/(sin(d*x+c)-1)^2-29/128/a/d/(sin(d*x+c
)-1)+35/256/a/d*ln(sin(d*x+c)-1)+1/64/a/d/(1+sin(d*x+c))^4-5/48/a/d/(1+sin(
d*x+c))^3+19/64/a/d/(1+sin(d*x+c))^2-1/2/a/d/(1+sin(d*x+c))-35/256*ln(1+sin
(d*x+c))/a/d
```

maxima [A] time = 0.31, size = 175, normalized size = 1.35

$$-\frac{\frac{2\left(279\sin(dx+c)^6 + 87\sin(dx+c)^5 - 424\sin(dx+c)^4 - 136\sin(dx+c)^3 + 249\sin(dx+c)^2 + 57\sin(dx+c) - 48\right)}{a\sin(dx+c)^7 + a\sin(dx+c)^6 - 3a\sin(dx+c)^5 - 3a\sin(dx+c)^4 + 3a\sin(dx+c)^3 + 3a\sin(dx+c)^2 - a\sin(dx+c) - a} + \frac{105\log(\sin(dx+c)+1)}{a} - \frac{105\log(\text{si}}{}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/768*(2*(279*sin(d*x + c)^6 + 87*sin(d*x + c)^5 - 424*sin(d*x + c)^4 - 13
6*sin(d*x + c)^3 + 249*sin(d*x + c)^2 + 57*sin(d*x + c) - 48)/(a*sin(d*x +
c)^7 + a*sin(d*x + c)^6 - 3*a*sin(d*x + c)^5 - 3*a*sin(d*x + c)^4 + 3*a*sin
(d*x + c)^3 + 3*a*sin(d*x + c)^2 - a*sin(d*x + c) - a) + 105*log(sin(d*x +
c) + 1)/a - 105*log(sin(d*x + c) - 1)/a)/d
```

mupad [B] time = 10.74, size = 388, normalized size = 2.98

$$\frac{\frac{35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{64}+\frac{35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{32}-\frac{245\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{96}-\frac{595\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{96}+\frac{791}{}}{d\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}-5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-12\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+9\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+30\,a\,\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^7/(a + a*sin(c + d*x)),x)
```

```
[Out] ((35*tan(c/2 + (d*x)/2))/64 + (35*tan(c/2 + (d*x)/2)^2)/32 - (245*tan(c/2 +
 (d*x)/2)^3)/96 - (595*tan(c/2 + (d*x)/2)^4)/96 + (791*tan(c/2 + (d*x)/2)^5
)/192 + (231*tan(c/2 + (d*x)/2)^6)/16 - (25*tan(c/2 + (d*x)/2)^7)/16 + (231
*tan(c/2 + (d*x)/2)^8)/16 + (791*tan(c/2 + (d*x)/2)^9)/192 - (595*tan(c/2 +
 (d*x)/2)^10)/96 - (245*tan(c/2 + (d*x)/2)^11)/96 + (35*tan(c/2 + (d*x)/2)^
12)/32 + (35*tan(c/2 + (d*x)/2)^13)/64)/(d*(a + 2*a*tan(c/2 + (d*x)/2) - 5*
a*tan(c/2 + (d*x)/2)^2 - 12*a*tan(c/2 + (d*x)/2)^3 + 9*a*tan(c/2 + (d*x)/2)
^4 + 30*a*tan(c/2 + (d*x)/2)^5 - 5*a*tan(c/2 + (d*x)/2)^6 - 40*a*tan(c/2 +
(d*x)/2)^7 - 5*a*tan(c/2 + (d*x)/2)^8 + 30*a*tan(c/2 + (d*x)/2)^9 + 9*a*tan
(c/2 + (d*x)/2)^10 - 12*a*tan(c/2 + (d*x)/2)^11 - 5*a*tan(c/2 + (d*x)/2)^12
 + 2*a*tan(c/2 + (d*x)/2)^13 + a*tan(c/2 + (d*x)/2)^14)) - (35*atanh(tan(c/
2 + (d*x)/2)))/(64*a*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^{7}(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**7/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**7/(sin(c + d*x) + 1), x)/a
```

3.45 $\int \frac{\tan^5(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=106

$$\frac{\tan^6(c+dx)}{6ad}+\frac{5\tanh^{-1}(\sin(c+dx))}{16ad}-\frac{\tan^5(c+dx)\sec(c+dx)}{6ad}+\frac{5\tan^3(c+dx)\sec(c+dx)}{24ad}-\frac{5\tan(c+dx)\sec(c}{16ad}$$

```
[Out] 5/16*arctanh(sin(d*x+c))/a/d-5/16*sec(d*x+c)*tan(d*x+c)/a/d+5/24*sec(d*x+c)
*tan(d*x+c)^3/a/d-1/6*sec(d*x+c)*tan(d*x+c)^5/a/d+1/6*tan(d*x+c)^6/a/d
```

Rubi [A] time = 0.14, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2706, 2607, 30, 2611, 3770}

$$\frac{\tan^6(c+dx)}{6ad}+\frac{5\tanh^{-1}(\sin(c+dx))}{16ad}-\frac{\tan^5(c+dx)\sec(c+dx)}{6ad}+\frac{5\tan^3(c+dx)\sec(c+dx)}{24ad}-\frac{5\tan(c+dx)\sec(c}{16ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*ArcTanh[Sin[c + d*x]])/(16*a*d) - (5*Sec[c + d*x]*Tan[c + d*x])/(16*a*d)
 + (5*Sec[c + d*x]*Tan[c + d*x]^3)/(24*a*d) - (Sec[c + d*x]*Tan[c + d*x]^5)
/(6*a*d) + Tan[c + d*x]^6/(6*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sec^2(c+dx)\tan^5(c+dx)\,dx}{a} - \frac{\int \sec(c+dx)\tan^6(c+dx)\,dx}{a}$$

$$= -\frac{\sec(c+dx)\tan^5(c+dx)}{6ad} + \frac{5\int \sec(c+dx)\tan^4(c+dx)\,dx}{6a} + \frac{\mathrm{Subst}\left(\int x^5\,dx, x, \tan(c\right.}{ad}$$

$$= \frac{5\sec(c+dx)\tan^3(c+dx)}{24ad} - \frac{\sec(c+dx)\tan^5(c+dx)}{6ad} + \frac{\tan^6(c+dx)}{6ad} - \frac{5\int \sec(c+dx}{}$$

$$= -\frac{5\sec(c+dx)\tan(c+dx)}{16ad} + \frac{5\sec(c+dx)\tan^3(c+dx)}{24ad} - \frac{\sec(c+dx)\tan^5(c+dx)}{6ad} +$$

$$= \frac{5\tanh^{-1}(\sin(c+dx))}{16ad} - \frac{5\sec(c+dx)\tan(c+dx)}{16ad} + \frac{5\sec(c+dx)\tan^3(c+dx)}{24ad} - \sec($$

Mathematica [A] time = 0.33, size = 84, normalized size = 0.79

$$\frac{-\frac{18}{1-\sin(c+dx)} + \frac{48}{\sin(c+dx)+1} + \frac{3}{(1-\sin(c+dx))^2} - \frac{21}{(\sin(c+dx)+1)^2} + \frac{4}{(\sin(c+dx)+1)^3} + 30\tanh^{-1}(\sin(c+dx))}{96ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + a*Sin[c + d*x]),x]
```

```
[Out] (30*ArcTanh[Sin[c + d*x]] + 3/(1 - Sin[c + d*x])^2 - 18/(1 - Sin[c + d*x])
+ 4/(1 + Sin[c + d*x])^3 - 21/(1 + Sin[c + d*x])^2 + 48/(1 + Sin[c + d*x]))
/(96*a*d)
```

fricas [A] time = 0.47, size = 147, normalized size = 1.39

$$\frac{66\cos(dx+c)^4 - 70\cos(dx+c)^2 + 15\left(\cos(dx+c)^4\sin(dx+c) + \cos(dx+c)^4\right)\log(\sin(dx+c)+1) - 15\left(\mathrm{co}\right.}{96\left(ad\cos(dx+c)^4\sin(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/96*(66*cos(d*x + c)^4 - 70*cos(d*x + c)^2 + 15*(cos(d*x + c)^4*sin(d*x +
c) + cos(d*x + c)^4)*log(sin(d*x + c) + 1) - 15*(cos(d*x + c)^4*sin(d*x + c
) + cos(d*x + c)^4)*log(-sin(d*x + c) + 1) - 2*(9*cos(d*x + c)^2 - 2)*sin(d
*x + c) + 20)/(a*d*cos(d*x + c)^4*sin(d*x + c) + a*d*cos(d*x + c)^4)
```

giac [A] time = 3.14, size = 116, normalized size = 1.09

$$\frac{\frac{30\,\log\left(|\sin(dx+c)+1|\right)}{a}-\frac{30\,\log\left(|\sin(dx+c)-1|\right)}{a}+\frac{3\left(15\,\sin(dx+c)^2-18\,\sin(dx+c)+5\right)}{a(\sin(dx+c)-1)^2}-\frac{55\,\sin(dx+c)^3+69\,\sin(dx+c)^2+15\,\sin(dx+c)-7}{a(\sin(dx+c)+1)^3}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/192*(30*log(abs(sin(d*x + c) + 1))/a - 30*log(abs(sin(d*x + c) - 1))/a +
3*(15*sin(d*x + c)^2 - 18*sin(d*x + c) + 5)/(a*(sin(d*x + c) - 1)^2) - (55*
sin(d*x + c)^3 + 69*sin(d*x + c)^2 + 15*sin(d*x + c) - 7)/(a*(sin(d*x + c)
+ 1)^3))/d
```

maple [A] time = 0.18, size = 126, normalized size = 1.19

$$\frac{1}{32ad\left(\sin\left(dx+c\right)-1\right)^{2}}+\frac{3}{16ad\left(\sin\left(dx+c\right)-1\right)}-\frac{5\ln\left(\sin\left(dx+c\right)-1\right)}{32ad}+\frac{1}{24ad\left(1+\sin\left(dx+c\right)\right)^{3}}-\frac{7}{32ad\left(1+\sin\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/32/a/d/(sin(d*x+c)-1)^2+3/16/a/d/(sin(d*x+c)-1)-5/32/a/d*ln(sin(d*x+c)-1)
+1/24/a/d/(1+sin(d*x+c))^3-7/32/a/d/(1+sin(d*x+c))^2+1/2/a/d/(1+sin(d*x+c))
+5/32*ln(1+sin(d*x+c))/a/d
```

maxima [A] time = 0.31, size = 130, normalized size = 1.23

$$\frac{\frac{2\left(33\,\sin(dx+c)^4+9\,\sin(dx+c)^3-31\,\sin(dx+c)^2-7\,\sin(dx+c)+8\right)}{a\sin(dx+c)^5+a\sin(dx+c)^4-2\,a\sin(dx+c)^3-2\,a\sin(dx+c)^2+a\sin(dx+c)+a}+\frac{15\,\log(\sin(dx+c)+1)}{a}-\frac{15\,\log(\sin(dx+c)-1)}{a}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/96*(2*(33*sin(d*x + c)^4 + 9*sin(d*x + c)^3 - 31*sin(d*x + c)^2 - 7*sin(d
*x + c) + 8)/(a*sin(d*x + c)^5 + a*sin(d*x + c)^4 - 2*a*sin(d*x + c)^3 - 2*
a*sin(d*x + c)^2 + a*sin(d*x + c) + a) + 15*log(sin(d*x + c) + 1)/a - 15*lo
g(sin(d*x + c) - 1)/a)/d
```

mupad [B] time = 10.43, size = 281, normalized size = 2.65

$$\frac{5\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,a\,d}-\frac{\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{8}+\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{4}-\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{3}-\frac{55\tan(}{1}}{d\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9-3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-8\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+2\,a\,\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + a*sin(c + d*x)),x)
```

```
[Out] (5*atanh(tan(c/2 + (d*x)/2)))/(8*a*d) - ((5*tan(c/2 + (d*x)/2))/8 + (5*tan(
c/2 + (d*x)/2)^2)/4 - (5*tan(c/2 + (d*x)/2)^3)/3 - (55*tan(c/2 + (d*x)/2)^4
)/12 + (3*tan(c/2 + (d*x)/2)^5)/4 - (55*tan(c/2 + (d*x)/2)^6)/12 - (5*tan(c
/2 + (d*x)/2)^7)/3 + (5*tan(c/2 + (d*x)/2)^8)/4 + (5*tan(c/2 + (d*x)/2)^9)/
8)/(d*(a + 2*a*tan(c/2 + (d*x)/2) - 3*a*tan(c/2 + (d*x)/2)^2 - 8*a*tan(c/2
+ (d*x)/2)^3 + 2*a*tan(c/2 + (d*x)/2)^4 + 12*a*tan(c/2 + (d*x)/2)^5 + 2*a*t
an(c/2 + (d*x)/2)^6 - 8*a*tan(c/2 + (d*x)/2)^7 - 3*a*tan(c/2 + (d*x)/2)^8 +
 2*a*tan(c/2 + (d*x)/2)^9 + a*tan(c/2 + (d*x)/2)^10))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^5(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**5/(sin(c + d*x) + 1), x)/a
```

3.46 $\int \frac{\tan^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=82

$$\frac{\tan^4(c+dx)}{4ad}-\frac{3\tanh^{-1}(\sin(c+dx))}{8ad}-\frac{\tan^3(c+dx)\sec(c+dx)}{4ad}+\frac{3\tan(c+dx)\sec(c+dx)}{8ad}$$

```
[Out] -3/8*arctanh(sin(d*x+c))/a/d+3/8*sec(d*x+c)*tan(d*x+c)/a/d-1/4*sec(d*x+c)*t
an(d*x+c)^3/a/d+1/4*tan(d*x+c)^4/a/d
```

Rubi [A] time = 0.12, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2706, 2607, 30, 2611, 3770}

$$\frac{\tan^4(c+dx)}{4ad}-\frac{3\tanh^{-1}(\sin(c+dx))}{8ad}-\frac{\tan^3(c+dx)\sec(c+dx)}{4ad}+\frac{3\tan(c+dx)\sec(c+dx)}{8ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*ArcTanh[Sin[c + d*x]])/(8*a*d) + (3*Sec[c + d*x]*Tan[c + d*x])/(8*a*d)
- (Sec[c + d*x]*Tan[c + d*x]^3)/(4*a*d) + Tan[c + d*x]^4/(4*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^3(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec^2(c+dx)\tan^3(c+dx)\,dx}{a} - \frac{\int \sec(c+dx)\tan^4(c+dx)\,dx}{a}\\ &= -\frac{\sec(c+dx)\tan^3(c+dx)}{4ad} + \frac{3\int \sec(c+dx)\tan^2(c+dx)\,dx}{4a} + \frac{\operatorname{Subst}\left(\int x^3\,dx, x, \tan(c\right.}{ad}\\ &= \frac{3\sec(c+dx)\tan(c+dx)}{8ad} - \frac{\sec(c+dx)\tan^3(c+dx)}{4ad} + \frac{\tan^4(c+dx)}{4ad} - \frac{3\int \sec(c+dx)}{8a}\\ &= -\frac{3\tanh^{-1}(\sin(c+dx))}{8ad} + \frac{3\sec(c+dx)\tan(c+dx)}{8ad} - \frac{\sec(c+dx)\tan^3(c+dx)}{4ad} + \frac{\tan^4}{4}\end{aligned}$$

Mathematica [A] time = 0.16, size = 54, normalized size = 0.66

$$-\frac{\frac{1}{\sin(c+dx)-1} + \frac{4}{\sin(c+dx)+1} - \frac{1}{(\sin(c+dx)+1)^2} + 3\tanh^{-1}(\sin(c+dx))}{8ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/8*(3*ArcTanh[Sin[c + d*x]] + (-1 + Sin[c + d*x])^(-1) - (1 + Sin[c + d*x
])^(-2) + 4/(1 + Sin[c + d*x]))/(a*d)
```

fricas [A] time = 0.42, size = 125, normalized size = 1.52

$$-\frac{10\cos(dx+c)^2 + 3\left(\cos(dx+c)^2\sin(dx+c) + \cos(dx+c)^2\right)\log(\sin(dx+c)+1) - 3\left(\cos(dx+c)^2\sin(dx\right.}{16\left(ad\cos(dx+c)^2\sin(dx+c) + ad\cos(dx+c)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(10*cos(d*x + c)^2 + 3*(cos(d*x + c)^2*sin(d*x + c) + cos(d*x + c)^2)
*log(sin(d*x + c) + 1) - 3*(cos(d*x + c)^2*sin(d*x + c) + cos(d*x + c)^2)*l
og(-sin(d*x + c) + 1) - 2*sin(d*x + c) - 6)/(a*d*cos(d*x + c)^2*sin(d*x + c
) + a*d*cos(d*x + c)^2)
```

giac [A] time = 0.77, size = 96, normalized size = 1.17

$$-\frac{\frac{6 \log(|\sin(dx+c)+1|)}{a} - \frac{6 \log(|\sin(dx+c)-1|)}{a} + \frac{2\,(3 \sin(dx+c)-1)}{a(\sin(dx+c)-1)} - \frac{9 \sin(dx+c)^2+2 \sin(dx+c)-3}{a(\sin(dx+c)+1)^2}}{32\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/32*(6*log(abs(sin(d*x + c) + 1))/a - 6*log(abs(sin(d*x + c) - 1))/a + 2*
(3*sin(d*x + c) - 1)/(a*(sin(d*x + c) - 1)) - (9*sin(d*x + c)^2 + 2*sin(d*x
 + c) - 3)/(a*(sin(d*x + c) + 1)^2))/d
```

maple [A] time = 0.17, size = 90, normalized size = 1.10

$$-\frac{1}{8ad\left(\sin\left(dx+c\right)-1\right)}+\frac{3\ln\left(\sin\left(dx+c\right)-1\right)}{16ad}+\frac{1}{8ad\left(1+\sin\left(dx+c\right)\right)^{2}}-\frac{1}{2ad\left(1+\sin\left(dx+c\right)\right)}-\frac{3\ln\left(1+\sin\left(dx\right.\right.}{16ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/8/a/d/(sin(d*x+c)-1)+3/16/a/d*ln(sin(d*x+c)-1)+1/8/a/d/(1+sin(d*x+c))^2-
1/2/a/d/(1+sin(d*x+c))-3/16*ln(1+sin(d*x+c))/a/d
```

maxima [A] time = 0.31, size = 89, normalized size = 1.09

$$-\frac{\frac{2\left(5 \sin(dx+c)^2+\sin(dx+c)-2\right)}{a \sin(dx+c)^3+a \sin(dx+c)^2-a \sin(dx+c)-a} + \frac{3 \log(\sin(dx+c)+1)}{a} - \frac{3 \log(\sin(dx+c)-1)}{a}}{16\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(2*(5*sin(d*x + c)^2 + sin(d*x + c) - 2)/(a*sin(d*x + c)^3 + a*sin(d*
x + c)^2 - a*sin(d*x + c) - a) + 3*log(sin(d*x + c) + 1)/a - 3*log(sin(d*x
+ c) - 1)/a)/d
```

mupad [B] time = 9.11, size = 172, normalized size = 2.10

$$\frac{\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{4}+\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{2}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{2}+\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{2}+\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}}{d\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+2\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-4\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + a*sin(c + d*x)),x)
```

```
[Out] ((3*tan(c/2 + (d*x)/2))/4 + (3*tan(c/2 + (d*x)/2)^2)/2 - tan(c/2 + (d*x)/2)
^3/2 + (3*tan(c/2 + (d*x)/2)^4)/2 + (3*tan(c/2 + (d*x)/2)^5)/4)/(d*(a + 2*a
*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^2 - 4*a*tan(c/2 + (d*x)/2)^3 - a
*tan(c/2 + (d*x)/2)^4 + 2*a*tan(c/2 + (d*x)/2)^5 + a*tan(c/2 + (d*x)/2)^6))
 - (3*atanh(tan(c/2 + (d*x)/2)))/(4*a*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^3(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**3/(sin(c + d*x) + 1), x)/a
```

3.47 $\int \frac{\tan(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=37

$$\frac{1}{2d(a\sin(c+dx)+a)}+\frac{\tanh^{-1}(\sin(c+dx))}{2ad}$$

```
[Out] 1/2*arctanh(sin(d*x+c))/a/d+1/2/d/(a+a*sin(d*x+c))
```

Rubi [A] time = 0.07, antiderivative size = 58, normalized size of antiderivative = 1.57, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2706, 2606, 30, 2611, 3770}

$$\frac{\sec^2(c+dx)}{2ad}+\frac{\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{\tan(c+dx)\sec(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + a*Sin[c + d*x]),x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(2*a*d) + Sec[c + d*x]^2/(2*a*d) - (Sec[c + d*x]*Tan[
c + d*x])/(2*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec^2(c+dx)\tan(c+dx)\,dx}{a}-\frac{\int \sec(c+dx)\tan^2(c+dx)\,dx}{a}\\ &= -\frac{\sec(c+dx)\tan(c+dx)}{2ad}+\frac{\int \sec(c+dx)\,dx}{2a}+\frac{\operatorname{Subst}(\int x\,dx,x,\sec(c+dx))}{ad}\\ &= \frac{\tanh^{-1}(\sin(c+dx))}{2ad}+\frac{\sec^2(c+dx)}{2ad}-\frac{\sec(c+dx)\tan(c+dx)}{2ad}\end{aligned}$$

Mathematica [A] time = 0.04, size = 28, normalized size = 0.76

$$\frac{\frac{1}{\sin(c+dx)+1}+\tanh^{-1}(\sin(c+dx))}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + a*Sin[c + d*x]),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]] + (1 + Sin[c + d*x])^(-1))/(2*a*d)
```

fricas [A] time = 0.42, size = 58, normalized size = 1.57

$$\frac{(\sin(dx+c)+1)\log(\sin(dx+c)+1)-(\sin(dx+c)+1)\log(-\sin(dx+c)+1)+2}{4\,(ad\sin(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*((sin(d*x + c) + 1)*log(sin(d*x + c) + 1) - (sin(d*x + c) + 1)*log(-sin
(d*x + c) + 1) + 2)/(a*d*sin(d*x + c) + a*d)
```

giac [A] time = 0.29, size = 58, normalized size = 1.57

$$\frac{\frac{\log(|\sin(dx+c)+1|)}{a} - \frac{\log(|\sin(dx+c)-1|)}{a} - \frac{\sin(dx+c)-1}{a(\sin(dx+c)+1)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(log(abs(sin(d*x + c) + 1))/a - log(abs(sin(d*x + c) - 1))/a - (sin(d*x
 + c) - 1)/(a*(sin(d*x + c) + 1)))/d
```

maple [A] time = 0.18, size = 54, normalized size = 1.46

$$-\frac{\ln(\sin(dx+c)-1)}{4ad} + \frac{1}{2ad\,(1+\sin(dx+c))} + \frac{\ln(1+\sin(dx+c))}{4ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/4/a/d*ln(sin(d*x+c)-1)+1/2/a/d/(1+sin(d*x+c))+1/4*ln(1+sin(d*x+c))/a/d
```

maxima [A] time = 0.35, size = 47, normalized size = 1.27

$$\frac{\frac{\log(\sin(dx+c)+1)}{a} - \frac{\log(\sin(dx+c)-1)}{a} + \frac{2}{a\sin(dx+c)+a}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(log(sin(d*x + c) + 1)/a - log(sin(d*x + c) - 1)/a + 2/(a*sin(d*x + c)
+ a))/d
```

mupad [B] time = 6.66, size = 61, normalized size = 1.65

$$\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + a*sin(c + d*x)),x)
```

```
[Out] atanh(tan(c/2 + (d*x)/2))/(a*d) - tan(c/2 + (d*x)/2)/(d*(a + 2*a*tan(c/2 +
(d*x)/2) + a*tan(c/2 + (d*x)/2)^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)/(sin(c + d*x) + 1), x)/a
```

3.48 $\int \frac{\cot(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=32

$$\frac{\log(\sin(c+dx))}{ad} - \frac{\log(\sin(c+dx)+1)}{ad}$$

```
[Out] ln(sin(d*x+c))/a/d-ln(1+sin(d*x+c))/a/d
```

Rubi [A] time = 0.04, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2707, 36, 29, 31}

$$\frac{\log(\sin(c+dx))}{ad} - \frac{\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + a*Sin[c + d*x]),x]
```

```
[Out] Log[Sin[c + d*x]]/(a*d) - Log[1 + Sin[c + d*x]]/(a*d)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, a\sin(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\log(\sin(c+dx))}{ad} - \frac{\log(1+\sin(c+dx))}{ad}$$

Mathematica [A] time = 0.02, size = 32, normalized size = 1.00

$$\frac{\log(\sin(c+dx))}{ad} - \frac{\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + a*Sin[c + d*x]),x]
[Out] Log[Sin[c + d*x]]/(a*d) - Log[1 + Sin[c + d*x]]/(a*d)
```

fricas [A] time = 0.42, size = 28, normalized size = 0.88

$$\frac{\log\left(\frac{1}{2}\sin\left(dx+c\right)\right) - \log\left(\sin\left(dx+c\right)+1\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] (log(1/2*sin(d*x + c)) - log(sin(d*x + c) + 1))/(a*d)
```

giac [A] time = 1.10, size = 33, normalized size = 1.03

$$-\frac{\frac{\log(|\sin(dx+c)+1|)}{a} - \frac{\log(|\sin(dx+c)|)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] -(log(abs(sin(d*x + c) + 1))/a - log(abs(sin(d*x + c)))/a)/d
```

maple [A] time = 0.12, size = 33, normalized size = 1.03

$$\frac{\ln\left(\sin\left(dx+c\right)\right)}{ad} - \frac{\ln\left(1+\sin\left(dx+c\right)\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] ln(sin(d*x+c))/a/d-ln(1+sin(d*x+c))/a/d
```

maxima [A] time = 0.30, size = 31, normalized size = 0.97

$$-\frac{\frac{\log(\sin(dx+c)+1)}{a} - \frac{\log(\sin(dx+c))}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(log(sin(d*x + c) + 1)/a - log(sin(d*x + c))/a)/d
```

mupad [B] time = 6.53, size = 32, normalized size = 1.00

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + a*sin(c + d*x)),x)
```

```
[Out] (log(tan(c/2 + (d*x)/2)) - 2*log(tan(c/2 + (d*x)/2) + 1))/(a*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot{\left(c+dx \right)}}{\sin{\left(c+dx \right)}+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)/(sin(c + d*x) + 1), x)/a
```

3.49 $\int \frac{\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=32

$$\frac{\csc(c+dx)}{ad}-\frac{\csc^2(c+dx)}{2ad}$$

```
[Out] csc(d*x+c)/a/d-1/2*csc(d*x+c)^2/a/d
```

Rubi [A] time = 0.07, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2706, 2606, 30, 8}

$$\frac{\csc(c+dx)}{ad}-\frac{\csc^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

```
[Out] Csc[c + d*x]/(a*d) - Csc[c + d*x]^2/(2*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cot(c+dx)\csc(c+dx)\,dx}{a} + \frac{\int \cot(c+dx)\csc^2(c+dx)\,dx}{a}$$
$$= \frac{\text{Subst}(\int 1\,dx, x, \csc(c+dx))}{ad} - \frac{\text{Subst}(\int x\,dx, x, \csc(c+dx))}{ad}$$
$$= \frac{\csc(c+dx)}{ad} - \frac{\csc^2(c+dx)}{2ad}$$

Mathematica [A] time = 0.03, size = 24, normalized size = 0.75

$$-\frac{(\csc(c+dx)-2)\csc(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + a*Sin[c + d*x]),x]
[Out] -1/2*((-2 + Csc[c + d*x])*Csc[c + d*x])/(a*d)
```

fricas [A] time = 0.42, size = 30, normalized size = 0.94

$$-\frac{2\,\sin(dx+c)-1}{2\left(ad\cos(dx+c)^2-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] -1/2*(2*sin(d*x + c) - 1)/(a*d*cos(d*x + c)^2 - a*d)
```

giac [A] time = 1.12, size = 26, normalized size = 0.81

$$\frac{2\,\sin(dx+c)-1}{2\,ad\sin(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] 1/2*(2*sin(d*x + c) - 1)/(a*d*sin(d*x + c)^2)
```

maple [A] time = 0.13, size = 30, normalized size = 0.94

$$-\frac{-\frac{1}{\sin(dx+c)}+\frac{1}{2\sin(dx+c)^2}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/a/d*(-1/sin(d*x+c)+1/2/sin(d*x+c)^2)
```

maxima [A] time = 0.30, size = 26, normalized size = 0.81

$$\frac{2 \sin(dx+c)-1}{2\, ad \sin(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*sin(d*x + c) - 1)/(a*d*sin(d*x + c)^2)
```

mupad [B] time = 6.58, size = 23, normalized size = 0.72

$$\frac{\sin(c+d\,x)-\frac{1}{2}}{a\,d\sin(c+d\,x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3/(a + a*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x) - 1/2)/(a*d*sin(c + d*x)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^3(c+dx)}{\sin(c+dx)+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**3/(sin(c + d*x) + 1), x)/a
```

3.50 $\int \frac{\cot^5(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=51

$$-\frac{\cot^4(c+dx)}{4ad}+\frac{\csc^3(c+dx)}{3ad}-\frac{\csc(c+dx)}{ad}$$

```
[Out] -1/4*cot(d*x+c)^4/a/d-csc(d*x+c)/a/d+1/3*csc(d*x+c)^3/a/d
```

Rubi [A] time = 0.09, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2706, 2607, 30, 2606}

$$-\frac{\cot^4(c+dx)}{4ad}+\frac{\csc^3(c+dx)}{3ad}-\frac{\csc(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + a*Sin[c + d*x]),x]
```

```
[Out] -Cot[c + d*x]^4/(4*a*d) - Csc[c + d*x]/(a*d) + Csc[c + d*x]^3/(3*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
```

```
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^5(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^3(c+dx)\csc(c+dx)\,dx}{a}+\frac{\int \cot^3(c+dx)\csc^2(c+dx)\,dx}{a}\\ &= -\frac{\text{Subst}\left(\int x^3\,dx,x,-\cot(c+dx)\right)}{ad}+\frac{\text{Subst}\left(\int \left(-1+x^2\right)\,dx,x,\csc(c+dx)\right)}{ad}\\ &= -\frac{\cot^4(c+dx)}{4ad}-\frac{\csc(c+dx)}{ad}+\frac{\csc^3(c+dx)}{3ad}\end{aligned}$$

Mathematica [A] time = 0.05, size = 30, normalized size = 0.59

$$-\frac{(\csc(c+dx)-1)^3(3\csc(c+dx)+5)}{12ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + a*Sin[c + d*x]),x]
[Out] -1/12*((-1 + Csc[c + d*x])^3*(5 + 3*Csc[c + d*x]))/(a*d)
```

fricas [A] time = 0.40, size = 63, normalized size = 1.24

$$-\frac{6\cos\left(dx+c\right)^2-4\left(3\cos\left(dx+c\right)^2-2\right)\sin\left(dx+c\right)-3}{12\left(ad\cos\left(dx+c\right)^4-2\,ad\cos\left(dx+c\right)^2+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] -1/12*(6*cos(d*x + c)^2 - 4*(3*cos(d*x + c)^2 - 2)*sin(d*x + c) - 3)/(a*d*c
os(d*x + c)^4 - 2*a*d*cos(d*x + c)^2 + a*d)
```

giac [A] time = 0.27, size = 46, normalized size = 0.90

$$-\frac{12\sin\left(dx+c\right)^3-6\sin\left(dx+c\right)^2-4\sin\left(dx+c\right)+3}{12\,ad\sin\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(12*sin(d*x + c)^3 - 6*sin(d*x + c)^2 - 4*sin(d*x + c) + 3)/(a*d*sin(
d*x + c)^4)
```

maple [A] time = 0.23, size = 49, normalized size = 0.96

$$\frac{-\frac{1}{\sin(dx+c)} + \frac{1}{2\sin(dx+c)^2} - \frac{1}{4\sin(dx+c)^4} + \frac{1}{3\sin(dx+c)^3}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(-1/sin(d*x+c)+1/2/sin(d*x+c)^2-1/4/sin(d*x+c)^4+1/3/sin(d*x+c)^3)
```

maxima [A] time = 0.31, size = 46, normalized size = 0.90

$$-\frac{12\,\sin\left(dx+c\right)^3 - 6\,\sin\left(dx+c\right)^2 - 4\,\sin\left(dx+c\right) + 3}{12\,ad\sin\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(12*sin(d*x + c)^3 - 6*sin(d*x + c)^2 - 4*sin(d*x + c) + 3)/(a*d*sin(
d*x + c)^4)
```

mupad [B] time = 6.56, size = 45, normalized size = 0.88

$$\frac{-\sin\left(c+d\,x\right)^3 + \frac{\sin(c+d\,x)^2}{2} + \frac{\sin(c+d\,x)}{3} - \frac{1}{4}}{a\,d\sin\left(c+d\,x\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5/(a + a*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)/3 + sin(c + d*x)^2/2 - sin(c + d*x)^3 - 1/4)/(a*d*sin(c + d*x
)^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^5\left(c+dx\right)}{\sin\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+a*sin(d*x+c)),x)
[Out] Integral(cot(c + d*x)**5/(sin(c + d*x) + 1), x)/a
```

3.51 $\int \frac{\cot^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=68

$$-\frac{\cot^6(c+dx)}{6ad}+\frac{\csc^5(c+dx)}{5ad}-\frac{2\csc^3(c+dx)}{3ad}+\frac{\csc(c+dx)}{ad}$$

```
[Out] -1/6*cot(d*x+c)^6/a/d+csc(d*x+c)/a/d-2/3*csc(d*x+c)^3/a/d+1/5*csc(d*x+c)^5/
a/d
```

Rubi [A] time = 0.09, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2706, 2607, 30, 2606, 194}

$$-\frac{\cot^6(c+dx)}{6ad}+\frac{\csc^5(c+dx)}{5ad}-\frac{2\csc^3(c+dx)}{3ad}+\frac{\csc(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7/(a + a*Sin[c + d*x]),x]
```

```
[Out] -Cot[c + d*x]^6/(6*a*d) + Csc[c + d*x]/(a*d) - (2*Csc[c + d*x]^3)/(3*a*d) +
 Csc[c + d*x]^5/(5*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^7(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^5(c+dx)\csc(c+dx)\,dx}{a}+\frac{\int \cot^5(c+dx)\csc^2(c+dx)\,dx}{a}\\
&= -\frac{\operatorname{Subst}\left(\int x^5\,dx, x, -\cot(c+dx)\right)}{ad}+\frac{\operatorname{Subst}\left(\int \left(-1+x^2\right)^2\,dx, x, \csc(c+dx)\right)}{ad}\\
&= -\frac{\cot^6(c+dx)}{6ad}+\frac{\operatorname{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, \csc(c+dx)\right)}{ad}\\
&= -\frac{\cot^6(c+dx)}{6ad}+\frac{\csc(c+dx)}{ad}-\frac{2\csc^3(c+dx)}{3ad}+\frac{\csc^5(c+dx)}{5ad}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 61, normalized size = 0.90

$$\frac{\csc^6(c+dx)(78\sin(c+dx)-5(7\sin(3(c+dx))-3\sin(5(c+dx))+5)-15\cos(4(c+dx)))}{240ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]^6*(-15*Cos[4*(c + d*x)] + 78*Sin[c + d*x] - 5*(5 + 7*Sin[3*(c
 + d*x)] - 3*Sin[5*(c + d*x)])))/(240*a*d)
```

fricas [A] time = 0.42, size = 96, normalized size = 1.41

$$\frac{15\cos(dx+c)^4-15\cos(dx+c)^2-2\left(15\cos(dx+c)^4-20\cos(dx+c)^2+8\right)\sin(dx+c)+5}{30\left(ad\cos(dx+c)^6-3\,ad\cos(dx+c)^4+3\,ad\cos(dx+c)^2-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

[Out] 1/30*(15*cos(d*x + c)^4 - 15*cos(d*x + c)^2 - 2*(15*cos(d*x + c)^4 - 20*cos(d*x + c)^2 + 8)*sin(d*x + c) + 5)/(a*d*cos(d*x + c)^6 - 3*a*d*cos(d*x + c)^4 + 3*a*d*cos(d*x + c)^2 - a*d)

giac [A] time = 0.97, size = 66, normalized size = 0.97

$$\frac{30 \sin(dx+c)^5 - 15 \sin(dx+c)^4 - 20 \sin(dx+c)^3 + 15 \sin(dx+c)^2 + 6 \sin(dx+c) - 5}{30\, ad \sin(dx+c)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="giac")

[Out] 1/30*(30*sin(d*x + c)^5 - 15*sin(d*x + c)^4 - 20*sin(d*x + c)^3 + 15*sin(d*x + c)^2 + 6*sin(d*x + c) - 5)/(a*d*sin(d*x + c)^6)

maple [A] time = 0.25, size = 67, normalized size = 0.99

$$\frac{-\frac{1}{6\sin(dx+c)^6} + \frac{1}{\sin(dx+c)} + \frac{1}{5\sin(dx+c)^5} - \frac{1}{2\sin(dx+c)^2} + \frac{1}{2\sin(dx+c)^4} - \frac{2}{3\sin(dx+c)^3}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^7/(a+a*sin(d*x+c)),x)

[Out] 1/d/a*(-1/6/sin(d*x+c)^6+1/sin(d*x+c)+1/5/sin(d*x+c)^5-1/2/sin(d*x+c)^2+1/2/sin(d*x+c)^4-2/3/sin(d*x+c)^3)

maxima [A] time = 0.30, size = 66, normalized size = 0.97

$$\frac{30 \sin(dx+c)^5 - 15 \sin(dx+c)^4 - 20 \sin(dx+c)^3 + 15 \sin(dx+c)^2 + 6 \sin(dx+c) - 5}{30\, ad \sin(dx+c)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] 1/30*(30*sin(d*x + c)^5 - 15*sin(d*x + c)^4 - 20*sin(d*x + c)^3 + 15*sin(d*x + c)^2 + 6*sin(d*x + c) - 5)/(a*d*sin(d*x + c)^6)

mupad [B] time = 6.79, size = 63, normalized size = 0.93

$$\frac{\sin(c+d\,x)^5 - \frac{\sin(c+d\,x)^4}{2} - \frac{2\sin(c+d\,x)^3}{3} + \frac{\sin(c+d\,x)^2}{2} + \frac{\sin(c+d\,x)}{5} - \frac{1}{6}}{a\,d\sin(c+d\,x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^7/(a + a*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)/5 + sin(c + d*x)^2/2 - (2*sin(c + d*x)^3)/3 - sin(c + d*x)^4/
2 + sin(c + d*x)^5 - 1/6)/(a*d*sin(c + d*x)^6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^7(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**7/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**7/(sin(c + d*x) + 1), x)/a
```

3.52 $\int \frac{\cot^9(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=84

$$-\frac{\cot^8(c+dx)}{8ad}+\frac{\csc^7(c+dx)}{7ad}-\frac{3\csc^5(c+dx)}{5ad}+\frac{\csc^3(c+dx)}{ad}-\frac{\csc(c+dx)}{ad}$$

```
[Out] -1/8*cot(d*x+c)^8/a/d-csc(d*x+c)/a/d+csc(d*x+c)^3/a/d-3/5*csc(d*x+c)^5/a/d+
1/7*csc(d*x+c)^7/a/d
```

Rubi [A] time = 0.10, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2706, 2607, 30, 2606, 194}

$$-\frac{\cot^8(c+dx)}{8ad}+\frac{\csc^7(c+dx)}{7ad}-\frac{3\csc^5(c+dx)}{5ad}+\frac{\csc^3(c+dx)}{ad}-\frac{\csc(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^9/(a + a*Sin[c + d*x]),x]
```

```
[Out] -Cot[c + d*x]^8/(8*a*d) - Csc[c + d*x]/(a*d) + Csc[c + d*x]^3/(a*d) - (3*Cs
c[c + d*x]^5)/(5*a*d) + Csc[c + d*x]^7/(7*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^9(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^7(c+dx)\csc(c+dx)\,dx}{a}+\frac{\int \cot^7(c+dx)\csc^2(c+dx)\,dx}{a}\\ &= -\frac{\operatorname{Subst}\left(\int x^7\,dx,x,-\cot(c+dx)\right)}{ad}+\frac{\operatorname{Subst}\left(\int \left(-1+x^2\right)^3\,dx,x,\csc(c+dx)\right)}{ad}\\ &= -\frac{\cot^8(c+dx)}{8ad}+\frac{\operatorname{Subst}\left(\int \left(-1+3x^2-3x^4+x^6\right)\,dx,x,\csc(c+dx)\right)}{ad}\\ &= -\frac{\cot^8(c+dx)}{8ad}-\frac{\csc(c+dx)}{ad}+\frac{\csc^3(c+dx)}{ad}-\frac{3\csc^5(c+dx)}{5ad}+\frac{\csc^7(c+dx)}{7ad}\end{aligned}$$

Mathematica [A] time = 0.21, size = 77, normalized size = 0.92

$$\frac{\csc^8(c+dx)(-513\sin(c+dx)+371\sin(3(c+dx))-105\sin(5(c+dx))+35\sin(7(c+dx))-245\cos(2(c+dx))}{2240ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^9/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]^8*(-245*Cos[2*(c + d*x)] - 35*Cos[6*(c + d*x)] - 513*Sin[c +
d*x] + 371*Sin[3*(c + d*x)] - 105*Sin[5*(c + d*x)] + 35*Sin[7*(c + d*x)]))/
(2240*a*d)
```

fricas [A] time = 0.43, size = 127, normalized size = 1.51

$$-\frac{140\cos(dx+c)^6-210\cos(dx+c)^4+140\cos(dx+c)^2-8\left(35\cos(dx+c)^6-70\cos(dx+c)^4+56\cos(dx}{280\left(ad\cos(dx+c)^8-4\,ad\cos(dx+c)^6+6\,ad\cos(dx+c)^4-4\,ad\cos(dx+c)^2+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/280*(140*cos(d*x + c)^6 - 210*cos(d*x + c)^4 + 140*cos(d*x + c)^2 - 8*(3
5*cos(d*x + c)^6 - 70*cos(d*x + c)^4 + 56*cos(d*x + c)^2 - 16)*sin(d*x + c)
 - 35)/(a*d*cos(d*x + c)^8 - 4*a*d*cos(d*x + c)^6 + 6*a*d*cos(d*x + c)^4 -
4*a*d*cos(d*x + c)^2 + a*d)
```

giac [A] time = 0.37, size = 86, normalized size = 1.02

$$-\frac{280\,\sin\left(dx+c\right)^{7}-140\,\sin\left(dx+c\right)^{6}-280\,\sin\left(dx+c\right)^{5}+210\,\sin\left(dx+c\right)^{4}+168\,\sin\left(dx+c\right)^{3}-140\,\sin\left(dx\right.}{280\,ad\sin\left(dx+c\right)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/280*(280*sin(d*x + c)^7 - 140*sin(d*x + c)^6 - 280*sin(d*x + c)^5 + 210*
sin(d*x + c)^4 + 168*sin(d*x + c)^3 - 140*sin(d*x + c)^2 - 40*sin(d*x + c)
+ 35)/(a*d*sin(d*x + c)^8)
```

maple [A] time = 0.28, size = 87, normalized size = 1.04

$$\frac{\frac{1}{2\sin\left(dx+c\right)^{6}}-\frac{1}{\sin\left(dx+c\right)}-\frac{3}{5\sin\left(dx+c\right)^{5}}+\frac{1}{7\sin\left(dx+c\right)^{7}}+\frac{1}{2\sin\left(dx+c\right)^{2}}-\frac{1}{8\sin\left(dx+c\right)^{8}}-\frac{3}{4\sin\left(dx+c\right)^{4}}+\frac{1}{\sin\left(dx+c\right)^{3}}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^9/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(1/2/sin(d*x+c)^6-1/sin(d*x+c)-3/5/sin(d*x+c)^5+1/7/sin(d*x+c)^7+1/2/
sin(d*x+c)^2-1/8/sin(d*x+c)^8-3/4/sin(d*x+c)^4+1/sin(d*x+c)^3)
```

maxima [A] time = 0.31, size = 86, normalized size = 1.02

$$-\frac{280\,\sin\left(dx+c\right)^{7}-140\,\sin\left(dx+c\right)^{6}-280\,\sin\left(dx+c\right)^{5}+210\,\sin\left(dx+c\right)^{4}+168\,\sin\left(dx+c\right)^{3}-140\,\sin\left(dx\right.}{280\,ad\sin\left(dx+c\right)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/280*(280*sin(d*x + c)^7 - 140*sin(d*x + c)^6 - 280*sin(d*x + c)^5 + 210*
sin(d*x + c)^4 + 168*sin(d*x + c)^3 - 140*sin(d*x + c)^2 - 40*sin(d*x + c)
+ 35)/(a*d*sin(d*x + c)^8)
```

mupad [B] time = 6.77, size = 83, normalized size = 0.99

$$\frac{-\sin\left(c+d\,x\right)^7+\frac{\sin(c+d\,x)^6}{2}+\sin\left(c+d\,x\right)^5-\frac{3\sin(c+d\,x)^4}{4}-\frac{3\sin(c+d\,x)^3}{5}+\frac{\sin(c+d\,x)^2}{2}+\frac{\sin(c+d\,x)}{7}-\frac{1}{8}}{a\,d\sin\left(c+d\,x\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^9/(a + a*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)/7 + sin(c + d*x)^2/2 - (3*sin(c + d*x)^3)/5 - (3*sin(c + d*x)
^4)/4 + sin(c + d*x)^5 + sin(c + d*x)^6/2 - sin(c + d*x)^7 - 1/8)/(a*d*sin(
c + d*x)^8)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^9\left(c+dx\right)}{\sin\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**9/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**9/(sin(c + d*x) + 1), x)/a
```

3.53 $\int \frac{\tan^6(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=84

$$\frac{\tan^7(c+dx)}{7ad}-\frac{\sec^7(c+dx)}{7ad}+\frac{3\sec^5(c+dx)}{5ad}-\frac{\sec^3(c+dx)}{ad}+\frac{\sec(c+dx)}{ad}$$

```
[Out] sec(d*x+c)/a/d-sec(d*x+c)^3/a/d+3/5*sec(d*x+c)^5/a/d-1/7*sec(d*x+c)^7/a/d+1
/7*tan(d*x+c)^7/a/d
```

Rubi [A] time = 0.10, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2706, 2607, 30, 2606, 194}

$$\frac{\tan^7(c+dx)}{7ad}-\frac{\sec^7(c+dx)}{7ad}+\frac{3\sec^5(c+dx)}{5ad}-\frac{\sec^3(c+dx)}{ad}+\frac{\sec(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^6/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sec[c + d*x]/(a*d) - Sec[c + d*x]^3/(a*d) + (3*Sec[c + d*x]^5)/(5*a*d) - Se
c[c + d*x]^7/(7*a*d) + Tan[c + d*x]^7/(7*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^6(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sec^2(c+dx)\tan^6(c+dx)\,dx}{a} - \frac{\int \sec(c+dx)\tan^7(c+dx)\,dx}{a}$$
$$= \frac{\text{Subst}\left(\int x^6\,dx, x, \tan(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int \left(-1+x^2\right)^3\,dx, x, \sec(c+dx)\right)}{ad}$$
$$= \frac{\tan^7(c+dx)}{7ad} - \frac{\text{Subst}\left(\int \left(-1+3x^2-3x^4+x^6\right)\,dx, x, \sec(c+dx)\right)}{ad}$$
$$= \frac{\sec(c+dx)}{ad} - \frac{\sec^3(c+dx)}{ad} + \frac{3\sec^5(c+dx)}{5ad} - \frac{\sec^7(c+dx)}{7ad} + \frac{\tan^7(c+dx)}{7ad}$$

Mathematica [A] time = 0.31, size = 146, normalized size = 1.74

$$\sec^5(c+dx)(2432\sin(c+dx) - 1905\sin(2(c+dx)) + 320\sin(3(c+dx)) - 1524\sin(4(c+dx)) + 960\sin(5(c+d$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^6/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Sec[c + d*x]^5*(2912 - 7620*Cos[c + d*x] + 3760*Cos[2*(c + d*x)] - 3810*Co
s[3*(c + d*x)] + 1440*Cos[4*(c + d*x)] - 762*Cos[5*(c + d*x)] + 80*Cos[6*(c
 + d*x)] + 2432*Sin[c + d*x] - 1905*Sin[2*(c + d*x)] + 320*Sin[3*(c + d*x)]
 - 1524*Sin[4*(c + d*x)] + 960*Sin[5*(c + d*x)] - 381*Sin[6*(c + d*x)]))/(1
7920*a*d*(1 + Sin[c + d*x]))
```

fricas [A] time = 0.41, size = 95, normalized size = 1.13

$$\frac{5\cos(dx+c)^6 + 15\cos(dx+c)^4 - 5\cos(dx+c)^2 + 2\left(15\cos(dx+c)^4 - 10\cos(dx+c)^2 + 3\right)\sin(dx+c) + 1}{35\left(ad\cos(dx+c)^5\sin(dx+c) + ad\cos(dx+c)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/35*(5*cos(d*x + c)^6 + 15*cos(d*x + c)^4 - 5*cos(d*x + c)^2 + 2*(15*cos(d
*x + c)^4 - 10*cos(d*x + c)^2 + 3)*sin(d*x + c) + 1)/(a*d*cos(d*x + c)^5*si
n(d*x + c) + a*d*cos(d*x + c)^5)
```

giac [B] time = 4.39, size = 172, normalized size = 2.05

$$-\frac{\frac{7\left(25\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}-120\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+210\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-140\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+33\right)}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^{5}}-\frac{175\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{6}+1260\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+3815\cdots}{\cdots}}{560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/560*(7*(25*tan(1/2*d*x + 1/2*c)^4 - 120*tan(1/2*d*x + 1/2*c)^3 + 210*tan
(1/2*d*x + 1/2*c)^2 - 140*tan(1/2*d*x + 1/2*c) + 33)/(a*(tan(1/2*d*x + 1/2*
c) - 1)^5) - (175*tan(1/2*d*x + 1/2*c)^6 + 1260*tan(1/2*d*x + 1/2*c)^5 + 38
15*tan(1/2*d*x + 1/2*c)^4 + 6020*tan(1/2*d*x + 1/2*c)^3 + 4641*tan(1/2*d*x
+ 1/2*c)^2 + 1792*tan(1/2*d*x + 1/2*c) + 281)/(a*(tan(1/2*d*x + 1/2*c) + 1)
^7))/d
```

maple [B] time = 0.18, size = 175, normalized size = 2.08

$$\frac{-\frac{1}{10\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{5}}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}+\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{5}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{6}}-\frac{\cdots}{10\left(\tan\left(\cdots\right.\right.}\cdots}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^6/(a+a*sin(d*x+c)),x)
```

```
[Out] 128/d/a*(-1/1280/(tan(1/2*d*x+1/2*c)-1)^5-1/512/(tan(1/2*d*x+1/2*c)-1)^4+1/
512/(tan(1/2*d*x+1/2*c)-1)^2-5/2048/(tan(1/2*d*x+1/2*c)-1)-1/448/(tan(1/2*d
*x+1/2*c)+1)^7+1/128/(tan(1/2*d*x+1/2*c)+1)^6-9/1280/(tan(1/2*d*x+1/2*c)+1)
^5-1/512/(tan(1/2*d*x+1/2*c)+1)^4+1/512/(tan(1/2*d*x+1/2*c)+1)^3+3/1024/(ta
n(1/2*d*x+1/2*c)+1)^2+5/2048/(tan(1/2*d*x+1/2*c)+1))
```

maxima [B] time = 0.38, size = 338, normalized size = 4.02

$$\frac{32\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1}-\frac{4\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{10\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{5\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{20\cdots}{\cdots}\cdots\right.}{35\left(a+\frac{2a\sin(dx+c)}{\cos(dx+c)+1}-\frac{4a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{10a\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{5a\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{20a\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{20a\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{5a\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 32/35*(2*sin(d*x + c)/(cos(d*x + c) + 1) - 4*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 - 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*sin(d*x + c)^4/(cos(d*x
+ c) + 1)^4 + 20*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 1)/((a + 2*a*sin(d*x
 + c)/(cos(d*x + c) + 1) - 4*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 10*a*s
in(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^
4 + 20*a*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 20*a*sin(d*x + c)^7/(cos(d*x
 + c) + 1)^7 - 5*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 10*a*sin(d*x + c)^
9/(cos(d*x + c) + 1)^9 + 4*a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 2*a*si
n(d*x + c)^11/(cos(d*x + c) + 1)^11 - a*sin(d*x + c)^12/(cos(d*x + c) + 1)^
12)*d)
```

mupad [B] time = 8.49, size = 99, normalized size = 1.18

$$-\frac{32\left(20\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{35\,a\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)^5\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^6/(a + a*sin(c + d*x)),x)
```

```
[Out] -(32*(2*tan(c/2 + (d*x)/2) - 4*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)
^3 + 5*tan(c/2 + (d*x)/2)^4 + 20*tan(c/2 + (d*x)/2)^5 + 1))/(35*a*d*(tan(c/
2 + (d*x)/2) - 1)^5*(tan(c/2 + (d*x)/2) + 1)^7)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^6(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**6/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**6/(sin(c + d*x) + 1), x)/a
```

3.54 $\int \frac{\tan^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=69

$$\frac{\tan^5(c+dx)}{5ad}-\frac{\sec^5(c+dx)}{5ad}+\frac{2\sec^3(c+dx)}{3ad}-\frac{\sec(c+dx)}{ad}$$

```
[Out] -sec(d*x+c)/a/d+2/3*sec(d*x+c)^3/a/d-1/5*sec(d*x+c)^5/a/d+1/5*tan(d*x+c)^5/
a/d
```

Rubi [A] time = 0.09, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2706, 2607, 30, 2606, 194}

$$\frac{\tan^5(c+dx)}{5ad}-\frac{\sec^5(c+dx)}{5ad}+\frac{2\sec^3(c+dx)}{3ad}-\frac{\sec(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Sec[c + d*x]/(a*d)) + (2*Sec[c + d*x]^3)/(3*a*d) - Sec[c + d*x]^5/(5*a*d)
 + Tan[c + d*x]^5/(5*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^4(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec^2(c+dx)\tan^4(c+dx)\,dx}{a} - \frac{\int \sec(c+dx)\tan^5(c+dx)\,dx}{a} \\ &= \frac{\operatorname{Subst}\left(\int x^4\,dx, x, \tan(c+dx)\right)}{ad} - \frac{\operatorname{Subst}\left(\int \left(-1+x^2\right)^2\,dx, x, \sec(c+dx)\right)}{ad} \\ &= \frac{\tan^5(c+dx)}{5ad} - \frac{\operatorname{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, \sec(c+dx)\right)}{ad} \\ &= -\frac{\sec(c+dx)}{ad} + \frac{2\sec^3(c+dx)}{3ad} - \frac{\sec^5(c+dx)}{5ad} + \frac{\tan^5(c+dx)}{5ad}\end{aligned}$$

Mathematica [A] time = 0.32, size = 106, normalized size = 1.54

$$-\frac{\sec^3(c+dx)(-64\sin(c+dx)-178\sin(2(c+dx))+192\sin(3(c+dx))-89\sin(4(c+dx))-534\cos(c+dx)+}{960ad(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/960*(Sec[c + d*x]^3*(200 - 534*Cos[c + d*x] + 288*Cos[2*(c + d*x)] - 178
*Cos[3*(c + d*x)] + 24*Cos[4*(c + d*x)] - 64*Sin[c + d*x] - 178*Sin[2*(c +
d*x)] + 192*Sin[3*(c + d*x)] - 89*Sin[4*(c + d*x)]))/(a*d*(1 + Sin[c + d*x]
))
```

fricas [A] time = 0.42, size = 75, normalized size = 1.09

$$-\frac{3\cos(dx+c)^4+6\cos(dx+c)^2+4\left(3\cos(dx+c)^2-1\right)\sin(dx+c)-1}{15\left(ad\cos(dx+c)^3\sin(dx+c)+ad\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/15*(3*cos(d*x + c)^4 + 6*cos(d*x + c)^2 + 4*(3*cos(d*x + c)^2 - 1)*sin(d
*x + c) - 1)/(a*d*cos(d*x + c)^3*sin(d*x + c) + a*d*cos(d*x + c)^3)
```

giac [A] time = 6.78, size = 120, normalized size = 1.74

$$\frac{\frac{5\left(9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-24\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+11\right)}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^{3}}-\frac{45\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}+240\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+490\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+320\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+73}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^{5}}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/120*(5*(9*tan(1/2*d*x + 1/2*c)^2 - 24*tan(1/2*d*x + 1/2*c) + 11)/(a*(tan(
1/2*d*x + 1/2*c) - 1)^3) - (45*tan(1/2*d*x + 1/2*c)^4 + 240*tan(1/2*d*x + 1
/2*c)^3 + 490*tan(1/2*d*x + 1/2*c)^2 + 320*tan(1/2*d*x + 1/2*c) + 73)/(a*(t
an(1/2*d*x + 1/2*c) + 1)^5))/d
```

maple [B] time = 0.17, size = 130, normalized size = 1.88

$$\frac{-\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{3}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}-\frac{1}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\right.\right.}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

```
[Out] 32/d/a*(-1/192/(tan(1/2*d*x+1/2*c)-1)^3-1/128/(tan(1/2*d*x+1/2*c)-1)^2+3/25
6/(tan(1/2*d*x+1/2*c)-1)-1/80/(tan(1/2*d*x+1/2*c)+1)^5+1/32/(tan(1/2*d*x+1/
2*c)+1)^4-1/96/(tan(1/2*d*x+1/2*c)+1)^3-1/64/(tan(1/2*d*x+1/2*c)+1)^2-3/256
/(tan(1/2*d*x+1/2*c)+1))
```

maxima [B] time = 0.32, size = 214, normalized size = 3.10

$$-\frac{16\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{6\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+1\right)}{15\left(a+\frac{2a\sin(dx+c)}{\cos(dx+c)+1}-\frac{2a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{6a\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{6a\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{2a\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{2a\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{a\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -16/15*(2*sin(d*x + c)/(cos(d*x + c) + 1) - 2*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - 6*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1)/((a + 2*a*sin(d*x + c)/
(cos(d*x + c) + 1) - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 6*a*sin(d*x
+ c)^3/(cos(d*x + c) + 1)^3 + 6*a*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 2*a
*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 2*a*sin(d*x + c)^7/(cos(d*x + c) + 1
)^7 - a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*d)
```

mupad [B] time = 6.73, size = 73, normalized size = 1.06

$$\frac{16\left(-6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{15\,a\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)^3\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4/(a + a*sin(c + d*x)),x)
```

```
[Out] (16*(2*tan(c/2 + (d*x)/2) - 2*tan(c/2 + (d*x)/2)^2 - 6*tan(c/2 + (d*x)/2)^3
 + 1))/(15*a*d*(tan(c/2 + (d*x)/2) - 1)^3*(tan(c/2 + (d*x)/2) + 1)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^4(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**4/(sin(c + d*x) + 1), x)/a
```

3.55 $\int \frac{\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=50

$$\frac{\tan^3(c+dx)}{3ad}-\frac{\sec^3(c+dx)}{3ad}+\frac{\sec(c+dx)}{ad}$$

```
[Out] sec(d*x+c)/a/d-1/3*sec(d*x+c)^3/a/d+1/3*tan(d*x+c)^3/a/d
```

Rubi [A] time = 0.09, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2706, 2607, 30, 2606}

$$\frac{\tan^3(c+dx)}{3ad}-\frac{\sec^3(c+dx)}{3ad}+\frac{\sec(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sec[c + d*x]/(a*d) - Sec[c + d*x]^3/(3*a*d) + Tan[c + d*x]^3/(3*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
```

```
- Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec^2(c+dx)\tan^2(c+dx)\,dx}{a} - \frac{\int \sec(c+dx)\tan^3(c+dx)\,dx}{a}\\ &= \frac{\text{Subst}\left(\int x^2\,dx, x, \tan(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int \left(-1+x^2\right)\,dx, x, \sec(c+dx)\right)}{ad}\\ &= \frac{\sec(c+dx)}{ad} - \frac{\sec^3(c+dx)}{3ad} + \frac{\tan^3(c+dx)}{3ad}\end{aligned}$$

Mathematica [B] time = 0.16, size = 106, normalized size = 2.12

$$\frac{8\sin(c+dx)-5\sin(2(c+dx))-10\cos(c+dx)+2\cos(2(c+dx))+6}{12ad(\sin(c+dx)+1)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + a*Sin[c + d*x]),x]
[Out] (6 - 10*Cos[c + d*x] + 2*Cos[2*(c + d*x)] + 8*Sin[c + d*x] - 5*Sin[2*(c + d
*x)])/(12*a*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2])*(1 + Sin[c + d*x]))
```

fricas [A] time = 0.42, size = 47, normalized size = 0.94

$$\frac{\cos\left(dx+c\right)^2+2\,\sin\left(dx+c\right)+1}{3\left(ad\cos\left(dx+c\right)\sin\left(dx+c\right)+ad\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] 1/3*(cos(d*x + c)^2 + 2*sin(d*x + c) + 1)/(a*d*cos(d*x + c)*sin(d*x + c) +
a*d*cos(d*x + c))
```

giac [A] time = 0.66, size = 68, normalized size = 1.36

$$-\frac{\frac{3}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}-\frac{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(3/(a*(tan(1/2*d*x + 1/2*c) - 1)) - (3*tan(1/2*d*x + 1/2*c)^2 + 12*tan
(1/2*d*x + 1/2*c) + 5)/(a*(tan(1/2*d*x + 1/2*c) + 1)^3))/d
```

maple [A] time = 0.14, size = 70, normalized size = 1.40

$$\frac{-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{8}{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+16}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] 8/d/a*(-1/16/(tan(1/2*d*x+1/2*c)-1)-1/12/(tan(1/2*d*x+1/2*c)+1)^3+1/8/(tan(
1/2*d*x+1/2*c)+1)^2+1/16/(tan(1/2*d*x+1/2*c)+1))
```

maxima [A] time = 0.30, size = 90, normalized size = 1.80

$$\frac{4\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{3\left(a+\frac{2a\sin(dx+c)}{\cos(dx+c)+1}-\frac{2a\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{a\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 4/3*(2*sin(d*x + c)/(cos(d*x + c) + 1) + 1)/((a + 2*a*sin(d*x + c)/(cos(d*x
 + c) + 1) - 2*a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - a*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4)*d)
```

mupad [B] time = 6.40, size = 47, normalized size = 0.94

$$-\frac{4\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{3\,a\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,{\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + a*sin(c + d*x)),x)
```

```
[Out] -(4*(2*tan(c/2 + (d*x)/2) + 1))/(3*a*d*(tan(c/2 + (d*x)/2) - 1)*(tan(c/2 +
(d*x)/2) + 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**2/(sin(c + d*x) + 1), x)/a
```

3.56 $\int \frac{1}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=23

$$-\frac{\cos(c+dx)}{d(a\sin(c+dx)+a)}$$

```
[Out] -cos(d*x+c)/d/(a+a*sin(d*x+c))
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2648}

$$-\frac{\cos(c+dx)}{d(a\sin(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(-1),x]
```

```
[Out] -(Cos[c + d*x]/(d*(a + a*Sin[c + d*x])))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{a+a\sin(c+dx)}\,dx = -\frac{\cos(c+dx)}{d(a+a\sin(c+dx))}$$

Mathematica [B] time = 0.04, size = 48, normalized size = 2.09

$$\frac{2\sin\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d(a\sin(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(-1),x]
```

```
[Out] (2*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/(d*(a + a*Sin[c
+ d*x]))
```

fricas [A] time = 0.42, size = 42, normalized size = 1.83

$$-\frac{\cos(dx+c)-\sin(dx+c)+1}{ad\cos(dx+c)+ad\sin(dx+c)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(cos(d*x + c) - sin(d*x + c) + 1)/(a*d*cos(d*x + c) + a*d*sin(d*x + c) + a
*d)
```

giac [A] time = 0.34, size = 21, normalized size = 0.91

$$-\frac{2}{ad\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -2/(a*d*(tan(1/2*d*x + 1/2*c) + 1))
```

maple [A] time = 0.07, size = 22, normalized size = 0.96

$$-\frac{2}{ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(d*x+c)),x)
```

```
[Out] -2/a/d/(tan(1/2*d*x+1/2*c)+1)
```

maxima [A] time = 0.33, size = 27, normalized size = 1.17

$$-\frac{2}{\left(a+\frac{a\sin(dx+c)}{\cos(dx+c)+1}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -2/((a + a*sin(d*x + c)/(cos(d*x + c) + 1))*d)
```

mupad [B] time = 6.43, size = 21, normalized size = 0.91

$$-\frac{2}{a\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*sin(c + d*x)),x)
[Out] -2/(a*d*(tan(c/2 + (d*x)/2) + 1))
```

sympy [A] time = 0.83, size = 27, normalized size = 1.17

$$\begin{cases} -\frac{2}{ad\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} & \text{for } d \neq 0 \\ \frac{x}{a\sin(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c)),x)
[Out] Piecewise((-2/(a*d*tan(c/2 + d*x/2) + a*d), Ne(d, 0)), (x/(a*sin(c) + a), T
rue))
```

3.57 $\int \frac{\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=29

$$\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\cot(c+dx)}{ad}$$

```
[Out] arctanh(cos(d*x+c))/a/d-cot(d*x+c)/a/d
```

Rubi [A] time = 0.05, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2706, 3767, 8, 3770}

$$\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

```
[Out] ArcTanh[Cos[c + d*x]]/(a*d) - Cot[c + d*x]/(a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \csc(c+dx)\,dx}{a} + \frac{\int \csc^2(c+dx)\,dx}{a}$$
$$= \frac{\tanh^{-1}(\cos(c+dx))}{ad} - \frac{\text{Subst}(\int 1\,dx, x, \cot(c+dx))}{ad}$$
$$= \frac{\tanh^{-1}(\cos(c+dx))}{ad} - \frac{\cot(c+dx)}{ad}$$

Mathematica [B] time = 0.23, size = 69, normalized size = 2.38

$$-\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(\cos(c+dx)+\sin(c+dx)\left(\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/2*(Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(Cos[c + d*x] + (-Log[Cos[(c + d*x)
/2]] + Log[Sin[(c + d*x)/2]])*Sin[c + d*x]))/(a*d)
```

fricas [B] time = 0.41, size = 62, normalized size = 2.14

$$\frac{\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-2\cos(dx+c)}{2\,ad\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - log(-1/2*cos(d*x + c) + 1/2
)*sin(d*x + c) - 2*cos(d*x + c))/(a*d*sin(d*x + c))
```

giac [B] time = 4.00, size = 65, normalized size = 2.24

$$-\frac{\frac{2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}-\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}-\frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*log(abs(tan(1/2*d*x + 1/2*c)))/a - tan(1/2*d*x + 1/2*c)/a - (2*tan(
1/2*d*x + 1/2*c) - 1)/(a*tan(1/2*d*x + 1/2*c)))/d
```

maple [A] time = 0.18, size = 56, normalized size = 1.93

$$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}-\frac{1}{2ad\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/2/a/d*tan(1/2*d*x+1/2*c)-1/2/a/d/tan(1/2*d*x+1/2*c)-1/a/d*ln(tan(1/2*d*x+
1/2*c))
```

maxima [B] time = 0.30, size = 70, normalized size = 2.41

$$-\frac{\frac{2\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\cos(dx+c)+1}{a\sin(dx+c)}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*log(sin(d*x + c)/(cos(d*x + c) + 1))/a + (cos(d*x + c) + 1)/(a*sin(
d*x + c)) - sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

mupad [B] time = 6.64, size = 25, normalized size = 0.86

$$-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)+\cot\left(c+d\,x\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + a*sin(c + d*x)),x)
```

```
[Out] -(log(tan(c/2 + (d*x)/2)) + cot(c + d*x))/(a*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^{2}\left(c+dx\right)}{\sin\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+a*sin(d*x+c)),x)
[Out] Integral(cot(c + d*x)**2/(sin(c + d*x) + 1), x)/a
```

3.58 $\int \frac{\cot^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=58

$$-\frac{\cot^3(c+dx)}{3ad}-\frac{\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

```
[Out] -1/2*arctanh(cos(d*x+c))/a/d-1/3*cot(d*x+c)^3/a/d+1/2*cot(d*x+c)*csc(d*x+c)
/a/d
```

Rubi [A] time = 0.09, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2706, 2607, 30, 2611, 3770}

$$-\frac{\cot^3(c+dx)}{3ad}-\frac{\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + a*Sin[c + d*x]),x]
```

```
[Out] -ArcTanh[Cos[c + d*x]]/(2*a*d) - Cot[c + d*x]^3/(3*a*d) + (Cot[c + d*x]*Csc
[c + d*x])/(2*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^4(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^2(c+dx)\csc(c+dx)\,dx}{a}+\frac{\int \cot^2(c+dx)\csc^2(c+dx)\,dx}{a}\\ &= \frac{\cot(c+dx)\csc(c+dx)}{2ad}+\frac{\int \csc(c+dx)\,dx}{2a}+\frac{\text{Subst}\left(\int x^2\,dx,x,-\cot(c+dx)\right)}{ad}\\ &= -\frac{\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{\cot^3(c+dx)}{3ad}+\frac{\cot(c+dx)\csc(c+dx)}{2ad}\end{aligned}$$

Mathematica [B] time = 0.51, size = 124, normalized size = 2.14

$$-\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^2\left(\cos(3(c+dx))+(3-6\sin(c+dx))\cos(\right.}{96ad(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/96*(Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(Csc[(c + d*x)/2] + Sec[(c + d*x)/
2])^2*(Cos[3*(c + d*x)] + Cos[c + d*x]*(3 - 6*Sin[c + d*x]) + 6*(Log[Cos[(c
 + d*x)/2]] - Log[Sin[(c + d*x)/2]])*Sin[c + d*x]^3))/(a*d*(1 + Sin[c + d*x
]))
```

fricas [B] time = 0.43, size = 111, normalized size = 1.91

$$\frac{4\cos(dx+c)^3-3\left(\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+3\left(\cos(dx+c)^2-1\right)\log\left(-\frac{1}{2}\cos(\right.}{12\left(ad\cos(dx+c)^2-ad\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(4*cos(d*x + c)^3 - 3*(cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2)
*sin(d*x + c) + 3*(cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x
 + c) - 6*cos(d*x + c)*sin(d*x + c))/((a*d*cos(d*x + c)^2 - a*d)*sin(d*x +
c))
```

giac [B] time = 0.23, size = 127, normalized size = 2.19

$$\frac{\frac{12\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}+\frac{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-3a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^3}-\frac{22\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(12*log(abs(tan(1/2*d*x + 1/2*c)))/a + (a^2*tan(1/2*d*x + 1/2*c)^3 - 3
*a^2*tan(1/2*d*x + 1/2*c)^2 - 3*a^2*tan(1/2*d*x + 1/2*c))/a^3 - (22*tan(1/2
*d*x + 1/2*c)^3 - 3*tan(1/2*d*x + 1/2*c)^2 - 3*tan(1/2*d*x + 1/2*c) + 1)/(a
*tan(1/2*d*x + 1/2*c)^3))/d
```

maple [B] time = 0.20, size = 132, normalized size = 2.28

$$\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{24ad}-\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{1}{8ad\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}+\frac{1}{8ad\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\frac{1}{24ad\tan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/24/a/d*tan(1/2*d*x+1/2*c)^3-1/8/a/d*tan(1/2*d*x+1/2*c)^2-1/8/a/d*tan(1/2*
d*x+1/2*c)+1/8/a/d/tan(1/2*d*x+1/2*c)+1/2/a/d*ln(tan(1/2*d*x+1/2*c))+1/8/a/
d/tan(1/2*d*x+1/2*c)^2-1/24/a/d/tan(1/2*d*x+1/2*c)^3
```

maxima [B] time = 0.30, size = 155, normalized size = 2.67

$$-\frac{\frac{\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a}-\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)^3}{a\sin(dx+c)^3}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/24*((3*sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a - 12*log(sin(d*x + c)/(cos(
d*x + c) + 1))/a - (3*sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 - 1)*(cos(d*x + c) + 1)^3/(a*sin(d*x + c)^3))/d
```

mupad [B] time = 6.63, size = 115, normalized size = 1.98

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a\,d}+\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.\right.}{8\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4/(a + a*sin(c + d*x)),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(24*a*d) - tan(c/2 + (d*x)/2)^2/(8*a*d) + log(tan(c/2
+ (d*x)/2))/(2*a*d) - tan(c/2 + (d*x)/2)/(8*a*d) + (cot(c/2 + (d*x)/2)^3*(t
an(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^2 - 1/3))/(8*a*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^4(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**4/(sin(c + d*x) + 1), x)/a
```

3.59 $\int \frac{\cot^6(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=82

$$-\frac{\cot^5(c+dx)}{5ad}+\frac{3\tanh^{-1}(\cos(c+dx))}{8ad}+\frac{\cot^3(c+dx)\csc(c+dx)}{4ad}-\frac{3\cot(c+dx)\csc(c+dx)}{8ad}$$

```
[Out] 3/8*arctanh(cos(d*x+c))/a/d-1/5*cot(d*x+c)^5/a/d-3/8*cot(d*x+c)*csc(d*x+c)/
a/d+1/4*cot(d*x+c)^3*csc(d*x+c)/a/d
```

Rubi [A] time = 0.11, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2706, 2607, 30, 2611, 3770}

$$-\frac{\cot^5(c+dx)}{5ad}+\frac{3\tanh^{-1}(\cos(c+dx))}{8ad}+\frac{\cot^3(c+dx)\csc(c+dx)}{4ad}-\frac{3\cot(c+dx)\csc(c+dx)}{8ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(8*a*d) - Cot[c + d*x]^5/(5*a*d) - (3*Cot[c + d*x
]*Csc[c + d*x])/(8*a*d) + (Cot[c + d*x]^3*Csc[c + d*x])/(4*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cot^4(c+dx)\csc(c+dx)\,dx}{a} + \frac{\int \cot^4(c+dx)\csc^2(c+dx)\,dx}{a}$$
$$= \frac{\cot^3(c+dx)\csc(c+dx)}{4ad} + \frac{3\int \cot^2(c+dx)\csc(c+dx)\,dx}{4a} + \frac{\text{Subst}\left(\int x^4\,dx, x, -\cot(\right.}{ad}$$
$$= -\frac{\cot^5(c+dx)}{5ad} - \frac{3\cot(c+dx)\csc(c+dx)}{8ad} + \frac{\cot^3(c+dx)\csc(c+dx)}{4ad} - \frac{3\int \csc(c+d}{8a}$$
$$= \frac{3\tanh^{-1}(\cos(c+dx))}{8ad} - \frac{\cot^5(c+dx)}{5ad} - \frac{3\cot(c+dx)\csc(c+dx)}{8ad} + \frac{\cot^3(c+dx)\csc(}{4ad}$$

Mathematica [B] time = 0.76, size = 189, normalized size = 2.30

$$-\frac{\csc^5(c+dx)\left(20\sin(2(c+dx)) - 50\sin(4(c+dx)) + 80\cos(c+dx) + 40\cos(3(c+dx)) + 8\cos(5(c+dx)) + 1\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/640*(Csc[c + d*x]^5*(80*Cos[c + d*x] + 40*Cos[3*(c + d*x)] + 8*Cos[5*(c
+ d*x)] - 150*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] + 150*Log[Sin[(c + d*x)/2]
]*Sin[c + d*x] + 20*Sin[2*(c + d*x)] + 75*Log[Cos[(c + d*x)/2]]*Sin[3*(c +
d*x)] - 75*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 50*Sin[4*(c + d*x)] - 1
5*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] + 15*Log[Sin[(c + d*x)/2]]*Sin[5*(
c + d*x)]))/(a*d)
```

fricas [B] time = 0.43, size = 155, normalized size = 1.89

$$-\frac{16\cos(dx+c)^5 - 15\left(\cos(dx+c)^4 - 2\cos(dx+c)^2 + 1\right)\log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right)\sin(dx+c) + 15\left(\cos(dx\right.}{80\left(ad\cos(dx+c)^4 - 2a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/80*(16*cos(d*x + c)^5 - 15*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(1
/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 15*(cos(d*x + c)^4 - 2*cos(d*x + c)^2
 + 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 10*(5*cos(d*x + c)^3 - 3*
cos(d*x + c))*sin(d*x + c))/((a*d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^2 + a
*d)*sin(d*x + c))
```

giac [B] time = 1.96, size = 187, normalized size = 2.28

$$-\frac{\frac{120 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}-\frac{2\,a^4 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-5\,a^4 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-10\,a^4 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+40\,a^4 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+20\,a^4 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^5}-\frac{274}{}}{320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/320*(120*log(abs(tan(1/2*d*x + 1/2*c)))/a - (2*a^4*tan(1/2*d*x + 1/2*c)^
5 - 5*a^4*tan(1/2*d*x + 1/2*c)^4 - 10*a^4*tan(1/2*d*x + 1/2*c)^3 + 40*a^4*t
an(1/2*d*x + 1/2*c)^2 + 20*a^4*tan(1/2*d*x + 1/2*c))/a^5 - (274*tan(1/2*d*x
 + 1/2*c)^5 - 20*tan(1/2*d*x + 1/2*c)^4 - 40*tan(1/2*d*x + 1/2*c)^3 + 10*ta
n(1/2*d*x + 1/2*c)^2 + 5*tan(1/2*d*x + 1/2*c) - 2)/(a*tan(1/2*d*x + 1/2*c)^
5))/d
```

maple [B] time = 0.24, size = 208, normalized size = 2.54

$$\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{160ad}-\frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad}-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{32ad}+\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16ad}-\frac{1}{16ad\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\right.\right.}{8ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/160/a/d*tan(1/2*d*x+1/2*c)^5-1/64/a/d*tan(1/2*d*x+1/2*c)^4-1/32/a/d*tan(1
/2*d*x+1/2*c)^3+1/8/a/d*tan(1/2*d*x+1/2*c)^2+1/16/a/d*tan(1/2*d*x+1/2*c)-1/
16/a/d/tan(1/2*d*x+1/2*c)-3/8/a/d*ln(tan(1/2*d*x+1/2*c))-1/160/a/d/tan(1/2*
d*x+1/2*c)^5-1/8/a/d/tan(1/2*d*x+1/2*c)^2+1/64/a/d/tan(1/2*d*x+1/2*c)^4+1/3
2/a/d/tan(1/2*d*x+1/2*c)^3
```

maxima [B] time = 0.50, size = 234, normalized size = 2.85

$$\frac{\frac{\frac{20\sin(dx+c)}{\cos(dx+c)+1}+\frac{40\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{5\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{2\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a}-\frac{120\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{40\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\cdots\right.}{a\sin(dx+c)^5}}{320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/320*((20*sin(d*x + c)/(cos(d*x + c) + 1) + 40*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 - 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 5*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4 + 2*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a - 120*log(sin(d*x
 + c)/(cos(d*x + c) + 1))/a + (5*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 - 40*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 2
0*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 2)*(cos(d*x + c) + 1)^5/(a*sin(d*x
+ c)^5))/d
```

mupad [B] time = 6.66, size = 183, normalized size = 2.23

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{32\,a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,a\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,a\,d}-\frac{3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,a\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,a\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right.\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6/(a + a*sin(c + d*x)),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a*d) - tan(c/2 + (d*x)/2)^3/(32*a*d) - tan(c/2 + (d
*x)/2)^4/(64*a*d) + tan(c/2 + (d*x)/2)^5/(160*a*d) - (3*log(tan(c/2 + (d*x)
/2)))/(8*a*d) + tan(c/2 + (d*x)/2)/(16*a*d) - (cot(c/2 + (d*x)/2)^5*(4*tan(
c/2 + (d*x)/2)^3 - tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)/2 + 2*tan(c/2
+ (d*x)/2)^4 + 1/5))/(32*a*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^6(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**6/(sin(c + d*x) + 1), x)/a
```

3.60 $\int \frac{\cot^8(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=106

$$-\frac{\cot^7(c+dx)}{7ad}-\frac{5\tanh^{-1}(\cos(c+dx))}{16ad}+\frac{\cot^5(c+dx)\csc(c+dx)}{6ad}-\frac{5\cot^3(c+dx)\csc(c+dx)}{24ad}+\frac{5\cot(c+dx)\csc(c+dx)}{16ad}$$

```
[Out] -5/16*arctanh(cos(d*x+c))/a/d-1/7*cot(d*x+c)^7/a/d+5/16*cot(d*x+c)*csc(d*x+
c)/a/d-5/24*cot(d*x+c)^3*csc(d*x+c)/a/d+1/6*cot(d*x+c)^5*csc(d*x+c)/a/d
```

Rubi [A] time = 0.13, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2706, 2607, 30, 2611, 3770}

$$-\frac{\cot^7(c+dx)}{7ad}-\frac{5\tanh^{-1}(\cos(c+dx))}{16ad}+\frac{\cot^5(c+dx)\csc(c+dx)}{6ad}-\frac{5\cot^3(c+dx)\csc(c+dx)}{24ad}+\frac{5\cot(c+dx)\csc(c+dx)}{16ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^8/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-5*ArcTanh[Cos[c + d*x]])/(16*a*d) - Cot[c + d*x]^7/(7*a*d) + (5*Cot[c + d
*x]*Csc[c + d*x])/(16*a*d) - (5*Cot[c + d*x]^3*Csc[c + d*x])/(24*a*d) + (Co
t[c + d*x]^5*Csc[c + d*x])/(6*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^8(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^6(c+dx)\csc(c+dx)\,dx}{a}+\frac{\int \cot^6(c+dx)\csc^2(c+dx)\,dx}{a}\\
&= \frac{\cot^5(c+dx)\csc(c+dx)}{6ad}+\frac{5\int \cot^4(c+dx)\csc(c+dx)\,dx}{6a}+\frac{\text{Subst}\left(\int x^6\,dx,x,-\cot(\right.}{ad}\\
&= -\frac{\cot^7(c+dx)}{7ad}-\frac{5\cot^3(c+dx)\csc(c+dx)}{24ad}+\frac{\cot^5(c+dx)\csc(c+dx)}{6ad}-\frac{5\int \cot^2(c+}{}\\
&= -\frac{\cot^7(c+dx)}{7ad}+\frac{5\cot(c+dx)\csc(c+dx)}{16ad}-\frac{5\cot^3(c+dx)\csc(c+dx)}{24ad}+\frac{\cot^5(c+dx)}{6}\\
&= -\frac{5\tanh^{-1}(\cos(c+dx))}{16ad}-\frac{\cot^7(c+dx)}{7ad}+\frac{5\cot(c+dx)\csc(c+dx)}{16ad}-\frac{5\cot^3(c+dx)\,c}{24ad}
\end{aligned}$$

Mathematica [B] time = 0.92, size = 284, normalized size = 2.68

$$-\frac{\csc^5(c+dx)\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^2\left(-1190\sin(2(c+dx))+392\sin(4(c+dx))-462\sin(6(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^8/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/86016*(Csc[c + d*x]^5*(Csc[(c + d*x)/2] + Sec[(c + d*x)/2])^2*(1680*Cos[
c + d*x] + 1008*Cos[3*(c + d*x)] + 336*Cos[5*(c + d*x)] + 48*Cos[7*(c + d*x
)] + 3675*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] - 3675*Log[Sin[(c + d*x)/2]]*S
in[c + d*x] - 1190*Sin[2*(c + d*x)] - 2205*Log[Cos[(c + d*x)/2]]*Sin[3*(c +
 d*x)] + 2205*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] + 392*Sin[4*(c + d*x)]
 + 735*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] - 735*Log[Sin[(c + d*x)/2]]*S
```

```
in[5*(c + d*x)] - 462*Sin[6*(c + d*x)] - 105*Log[Cos[(c + d*x)/2]]*Sin[7*(c
 + d*x)] + 105*Log[Sin[(c + d*x)/2]]*Sin[7*(c + d*x)]))/(a*d*(1 + Sin[c + d
*x]))
```

fricas [B] time = 0.44, size = 198, normalized size = 1.87

$$\frac{96\cos(dx+c)^7-105\left(\cos(dx+c)^6-3\cos(dx+c)^4+3\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c}{672\left(ad\,\mathrm{co}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/672*(96*cos(d*x + c)^7 - 105*(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d
*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 105*(cos(d*x + c)
^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2)*
sin(d*x + c) - 14*(33*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 15*cos(d*x + c))
*sin(d*x + c))/((a*d*cos(d*x + c)^6 - 3*a*d*cos(d*x + c)^4 + 3*a*d*cos(d*x
+ c)^2 - a*d)*sin(d*x + c))
```

giac [B] time = 0.32, size = 244, normalized size = 2.30

$$\frac{\frac{840\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}+\frac{3a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-7a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-21a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+63a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+63a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-315a^6\,\mathrm{ta}}{a^7}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2688*(840*log(abs(tan(1/2*d*x + 1/2*c)))/a + (3*a^6*tan(1/2*d*x + 1/2*c)^
7 - 7*a^6*tan(1/2*d*x + 1/2*c)^6 - 21*a^6*tan(1/2*d*x + 1/2*c)^5 + 63*a^6*t
an(1/2*d*x + 1/2*c)^4 + 63*a^6*tan(1/2*d*x + 1/2*c)^3 - 315*a^6*tan(1/2*d*x
 + 1/2*c)^2 - 105*a^6*tan(1/2*d*x + 1/2*c))/a^7 - (2178*tan(1/2*d*x + 1/2*c
)^7 - 105*tan(1/2*d*x + 1/2*c)^6 - 315*tan(1/2*d*x + 1/2*c)^5 + 63*tan(1/2*
d*x + 1/2*c)^4 + 63*tan(1/2*d*x + 1/2*c)^3 - 21*tan(1/2*d*x + 1/2*c)^2 - 7*
tan(1/2*d*x + 1/2*c) + 3)/(a*tan(1/2*d*x + 1/2*c)^7))/d
```

maple [B] time = 0.32, size = 284, normalized size = 2.68

$$\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{896ad}-\frac{\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)}{384ad}-\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{128ad}+\frac{3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128ad}+\frac{3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128ad}-\frac{15\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128ad}-\frac{5\tan\left(\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^8/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/896/a/d*tan(1/2*d*x+1/2*c)^7-1/384/a/d*tan(1/2*d*x+1/2*c)^6-1/128/a/d*tan
(1/2*d*x+1/2*c)^5+3/128/a/d*tan(1/2*d*x+1/2*c)^4+3/128/a/d*tan(1/2*d*x+1/2*
c)^3-15/128/a/d*tan(1/2*d*x+1/2*c)^2-5/128/a/d*tan(1/2*d*x+1/2*c)+1/384/a/d
/tan(1/2*d*x+1/2*c)^6+5/128/a/d/tan(1/2*d*x+1/2*c)+5/16/a/d*ln(tan(1/2*d*x+
1/2*c))+1/128/a/d/tan(1/2*d*x+1/2*c)^5-1/896/a/d/tan(1/2*d*x+1/2*c)^7+15/12
8/a/d/tan(1/2*d*x+1/2*c)^2-3/128/a/d/tan(1/2*d*x+1/2*c)^4-3/128/a/d/tan(1/2
*d*x+1/2*c)^3
```

maxima [B] time = 0.33, size = 315, normalized size = 2.97

$$-\frac{\frac{\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{315\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{63\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{63\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{7\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{3\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a}-\frac{840\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\left(\frac{7\,\sin(dx+c)}{\cos(dx+c)+1}\right.\cdots}{\cdots}}{2688\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2688*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 315*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 - 63*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 63*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 7*sin(d*x +
c)^6/(cos(d*x + c) + 1)^6 - 3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a - 840*
log(sin(d*x + c)/(cos(d*x + c) + 1))/a - (7*sin(d*x + c)/(cos(d*x + c) + 1)
 + 21*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 63*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 - 63*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 315*sin(d*x + c)^5/(cos(
d*x + c) + 1)^5 + 105*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 3)*(cos(d*x + c
) + 1)^7/(a*sin(d*x + c)^7))/d
```

mupad [B] time = 8.22, size = 387, normalized size = 3.65

$$\frac{3\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-7\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}+7\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-21\cos\left(\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^8/(a + a*sin(c + d*x)),x)
```

```
[Out] (3*sin(c/2 + (d*x)/2)^14 - 3*cos(c/2 + (d*x)/2)^14 - 7*cos(c/2 + (d*x)/2)*s
in(c/2 + (d*x)/2)^13 + 7*cos(c/2 + (d*x)/2)^13*sin(c/2 + (d*x)/2) - 21*cos(
c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^12 + 63*cos(c/2 + (d*x)/2)^3*sin(c/2 +
```

```
(d*x)/2)^11 + 63*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^10 - 315*cos(c/2 +
 (d*x)/2)^5*sin(c/2 + (d*x)/2)^9 - 105*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)
/2)^8 + 105*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^6 + 315*cos(c/2 + (d*x)
/2)^9*sin(c/2 + (d*x)/2)^5 - 63*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^4
- 63*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)^3 + 21*cos(c/2 + (d*x)/2)^12*
sin(c/2 + (d*x)/2)^2 + 840*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c
/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^7)/(2688*a*d*cos(c/2 + (d*x)/2)^7*sin(c/
2 + (d*x)/2)^7)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^8(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**8/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**8/(sin(c + d*x) + 1), x)/a
```

3.61 $\int \frac{\tan^7(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=189

$$\frac{a^3}{80d(a\sin(c+dx)+a)^5}-\frac{5a^2}{64d(a\sin(c+dx)+a)^4}+\frac{21}{256d\left(a^2-a^2\sin(c+dx)\right)}+\frac{35}{256d\left(a^2\sin(c+dx)+a^2\right)}-\frac{7\tan}{}$$

```
[Out] -7/128*arctanh(sin(d*x+c))/a^2/d+1/192*a/d/(a-a*sin(d*x+c))^3-1/32/d/(a-a*s
in(d*x+c))^2+1/80*a^3/d/(a+a*sin(d*x+c))^5-5/64*a^2/d/(a+a*sin(d*x+c))^4+19
/96*a/d/(a+a*sin(d*x+c))^3-1/4/d/(a+a*sin(d*x+c))^2+21/256/d/(a^2-a^2*sin(d
*x+c))+35/256/d/(a^2+a^2*sin(d*x+c))
```

Rubi [A] time = 0.15, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2707, 88, 206}

$$\frac{a^3}{80d(a\sin(c+dx)+a)^5}-\frac{5a^2}{64d(a\sin(c+dx)+a)^4}+\frac{21}{256d\left(a^2-a^2\sin(c+dx)\right)}+\frac{35}{256d\left(a^2\sin(c+dx)+a^2\right)}-\frac{7\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^7/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-7*ArcTanh[Sin[c + d*x]])/(128*a^2*d) + a/(192*d*(a - a*Sin[c + d*x])^3) -
 1/(32*d*(a - a*Sin[c + d*x])^2) + a^3/(80*d*(a + a*Sin[c + d*x])^5) - (5*a
^2)/(64*d*(a + a*Sin[c + d*x])^4) + (19*a)/(96*d*(a + a*Sin[c + d*x])^3) -
1/(4*d*(a + a*Sin[c + d*x])^2) + 21/(256*d*(a^2 - a^2*Sin[c + d*x])) + 35/(
256*d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tan^7(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^7}{(a-x)^4(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a}{64(a-x)^4} - \frac{1}{16(a-x)^3} + \frac{21}{256a(a-x)^2} - \frac{a^3}{16(a+x)^6} + \frac{5a^2}{16(a+x)^5} - \frac{19a}{32(a+x)^4} + \frac{1}{2(a+x)^3} - \frac{25}{}\right.\right.}{d}$$

$$= \frac{a}{192d(a-a\sin(c+dx))^3} - \frac{1}{32d(a-a\sin(c+dx))^2} + \frac{a^3}{80d(a+a\sin(c+dx))^5} - \frac{}{64d}$$

$$= -\frac{7\tanh^{-1}(\sin(c+dx))}{128a^2d} + \frac{a}{192d(a-a\sin(c+dx))^3} - \frac{1}{32d(a-a\sin(c+dx))^2} + \frac{}{80d}$$

Mathematica [A] time = 1.64, size = 112, normalized size = 0.59

$$-\frac{210\tanh^{-1}(\sin(c+dx)) - \frac{2\left(105\sin^7(c+dx) - 750\sin^6(c+dx) - 815\sin^5(c+dx) + 560\sin^4(c+dx) + 1039\sin^3(c+dx) + 78\sin^2(c+dx) - 393\sin(c+\right.}{(\sin(c+dx)-1)^3(\sin(c+dx)+1)^5}}{3840a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^7/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/3840*(210*ArcTanh[Sin[c + d*x]] - (2*(-144 - 393*Sin[c + d*x] + 78*Sin[c
 + d*x]^2 + 1039*Sin[c + d*x]^3 + 560*Sin[c + d*x]^4 - 815*Sin[c + d*x]^5 -
 750*Sin[c + d*x]^6 + 105*Sin[c + d*x]^7))/((-1 + Sin[c + d*x])^3*(1 + Sin[
c + d*x])^5))/(a^2*d)
```

fricas [A] time = 0.49, size = 218, normalized size = 1.15

$$\frac{1500\cos(dx+c)^6 - 3380\cos(dx+c)^4 + 2104\cos(dx+c)^2 - 105\left(\cos(dx+c)^8 - 2\cos(dx+c)^6\sin(dx+c) -\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3840*(1500*cos(d*x + c)^6 - 3380*cos(d*x + c)^4 + 2104*cos(d*x + c)^2 - 1
05*(cos(d*x + c)^8 - 2*cos(d*x + c)^6*sin(d*x + c) - 2*cos(d*x + c)^6)*log(
sin(d*x + c) + 1) + 105*(cos(d*x + c)^8 - 2*cos(d*x + c)^6*sin(d*x + c) - 2
*cos(d*x + c)^6)*log(-sin(d*x + c) + 1) - 2*(105*cos(d*x + c)^6 + 500*cos(d
*x + c)^4 - 276*cos(d*x + c)^2 + 64)*sin(d*x + c) - 512)/(a^2*d*cos(d*x + c
)^8 - 2*a^2*d*cos(d*x + c)^6*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^6)
```

giac [A] time = 71.38, size = 146, normalized size = 0.77

$$-\frac{\frac{420 \log(|\sin(dx+c)+1|)}{a^2}-\frac{420 \log(|\sin(dx+c)-1|)}{a^2}+\frac{10\left(77 \sin(dx+c)^3-105 \sin(dx+c)^2+27 \sin(dx+c)+9\right)}{a^2(\sin(dx+c)-1)^3}-\frac{959 \sin(dx+c)^5+6895 \sin(dx+c)^4 \cdots}{\cdots}}{15360\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/15360*(420*log(abs(sin(d*x + c) + 1))/a^2 - 420*log(abs(sin(d*x + c) - 1
))/a^2 + 10*(77*sin(d*x + c)^3 - 105*sin(d*x + c)^2 + 27*sin(d*x + c) + 9)/
(a^2*(sin(d*x + c) - 1)^3) - (959*sin(d*x + c)^5 + 6895*sin(d*x + c)^4 + 14
150*sin(d*x + c)^3 + 13710*sin(d*x + c)^2 + 6555*sin(d*x + c) + 1251)/(a^2*
(sin(d*x + c) + 1)^5))/d
```

maple [A] time = 0.25, size = 180, normalized size = 0.95

$$-\frac{1}{192a^2d\left(\sin\left(dx+c\right)-1\right)^3}-\frac{1}{32a^2d\left(\sin\left(dx+c\right)-1\right)^2}-\frac{21}{256a^2d\left(\sin\left(dx+c\right)-1\right)}+\frac{7\ln\left(\sin\left(dx+c\right)-1\right)}{256a^2d}+\frac{\cdots}{80a^2d \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^7/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/192/a^2/d/(sin(d*x+c)-1)^3-1/32/a^2/d/(sin(d*x+c)-1)^2-21/256/a^2/d/(sin
(d*x+c)-1)+7/256/a^2/d*ln(sin(d*x+c)-1)+1/80/a^2/d/(1+sin(d*x+c))^5-5/64/a^
2/d/(1+sin(d*x+c))^4+19/96/a^2/d/(1+sin(d*x+c))^3-1/4/a^2/d/(1+sin(d*x+c))^
2+35/256/a^2/d/(1+sin(d*x+c))-7/256*ln(1+sin(d*x+c))/a^2/d
```

maxima [A] time = 0.31, size = 202, normalized size = 1.07

$$\frac{\frac{2\left(105 \sin(dx+c)^7-750 \sin(dx+c)^6-815 \sin(dx+c)^5+560 \sin(dx+c)^4+1039 \sin(dx+c)^3+78 \sin(dx+c)^2-393 \sin(dx+c)-144\right)}{a^2 \sin(dx+c)^8+2 a^2 \sin(dx+c)^7-2 a^2 \sin(dx+c)^6-6 a^2 \sin(dx+c)^5+6 a^2 \sin(dx+c)^3+2 a^2 \sin(dx+c)^2-2 a^2 \sin(dx+c)-a^2}-\frac{105 \log(\sin(dx+c)+\cdots}{a^2}\cdots}{3840\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3840*(2*(105*sin(d*x + c)^7 - 750*sin(d*x + c)^6 - 815*sin(d*x + c)^5 + 5
60*sin(d*x + c)^4 + 1039*sin(d*x + c)^3 + 78*sin(d*x + c)^2 - 393*sin(d*x +
 c) - 144)/(a^2*sin(d*x + c)^8 + 2*a^2*sin(d*x + c)^7 - 2*a^2*sin(d*x + c)^
6 - 6*a^2*sin(d*x + c)^5 + 6*a^2*sin(d*x + c)^3 + 2*a^2*sin(d*x + c)^2 - 2*
a^2*sin(d*x + c) - a^2) - 105*log(sin(d*x + c) + 1)/a^2 + 105*log(sin(d*x +
 c) - 1)/a^2)/d
```

mupad [B] time = 10.46, size = 444, normalized size = 2.35

$$\frac{\frac{7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}}{64}+\frac{7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}}{16}+\frac{7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{192}-\frac{49\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{24}-\frac{693\tan\left(\frac{c}{2}+\right.}{320}}{d\left(a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}+4a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}-20a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}-20a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+36a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^7/(a + a*sin(c + d*x))^2,x)
```

```
[Out] ((7*tan(c/2 + (d*x)/2))/64 + (7*tan(c/2 + (d*x)/2)^2)/16 + (7*tan(c/2 + (d*
x)/2)^3)/192 - (49*tan(c/2 + (d*x)/2)^4)/24 - (693*tan(c/2 + (d*x)/2)^5)/32
0 + (791*tan(c/2 + (d*x)/2)^6)/240 + (1207*tan(c/2 + (d*x)/2)^7)/192 + (123
*tan(c/2 + (d*x)/2)^8)/4 + (1207*tan(c/2 + (d*x)/2)^9)/192 + (791*tan(c/2 +
 (d*x)/2)^10)/240 - (693*tan(c/2 + (d*x)/2)^11)/320 - (49*tan(c/2 + (d*x)/2
)^12)/24 + (7*tan(c/2 + (d*x)/2)^13)/192 + (7*tan(c/2 + (d*x)/2)^14)/16 + (
7*tan(c/2 + (d*x)/2)^15)/64)/(d*(36*a^2*tan(c/2 + (d*x)/2)^5 - 20*a^2*tan(c
/2 + (d*x)/2)^4 - 20*a^2*tan(c/2 + (d*x)/2)^3 + 64*a^2*tan(c/2 + (d*x)/2)^6
 - 20*a^2*tan(c/2 + (d*x)/2)^7 - 90*a^2*tan(c/2 + (d*x)/2)^8 - 20*a^2*tan(c
/2 + (d*x)/2)^9 + 64*a^2*tan(c/2 + (d*x)/2)^10 + 36*a^2*tan(c/2 + (d*x)/2)^
11 - 20*a^2*tan(c/2 + (d*x)/2)^12 - 20*a^2*tan(c/2 + (d*x)/2)^13 + 4*a^2*ta
n(c/2 + (d*x)/2)^15 + a^2*tan(c/2 + (d*x)/2)^16 + a^2 + 4*a^2*tan(c/2 + (d*
x)/2))) - (7*atanh(tan(c/2 + (d*x)/2)))/(64*a^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^7(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**7/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**7/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2
```

3.62 $\int \frac{\tan^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=146

$$\frac{a^2}{32d(a\sin(c+dx)+a)^4}-\frac{5}{64d\left(a^2-a^2\sin(c+dx)\right)}-\frac{5}{32d\left(a^2\sin(c+dx)+a^2\right)}+\frac{5\tanh^{-1}(\sin(c+dx))}{64a^2d}-\frac{}{48d(a\sin}$$

```
[Out] 5/64*arctanh(sin(d*x+c))/a^2/d+1/64/d/(a-a*sin(d*x+c))^2+1/32*a^2/d/(a+a*si
n(d*x+c))^4-7/48*a/d/(a+a*sin(d*x+c))^3+1/4/d/(a+a*sin(d*x+c))^2-5/64/d/(a^
2-a^2*sin(d*x+c))-5/32/d/(a^2+a^2*sin(d*x+c))
```

Rubi [A] time = 0.11, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2707, 88, 206}

$$\frac{a^2}{32d(a\sin(c+dx)+a)^4}-\frac{5}{64d\left(a^2-a^2\sin(c+dx)\right)}-\frac{5}{32d\left(a^2\sin(c+dx)+a^2\right)}+\frac{5\tanh^{-1}(\sin(c+dx))}{64a^2d}-\frac{}{48d(a\sin}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (5*ArcTanh[Sin[c + d*x]])/(64*a^2*d) + 1/(64*d*(a - a*Sin[c + d*x])^2) + a^
2/(32*d*(a + a*Sin[c + d*x])^4) - (7*a)/(48*d*(a + a*Sin[c + d*x])^3) + 1/(
4*d*(a + a*Sin[c + d*x])^2) - 5/(64*d*(a^2 - a^2*Sin[c + d*x])) - 5/(32*d*(
a^2 + a^2*Sin[c + d*x]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
```

```
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(a-x)^3(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{32(a-x)^3} - \frac{5}{64a(a-x)^2} - \frac{a^2}{8(a+x)^5} + \frac{7a}{16(a+x)^4} - \frac{1}{2(a+x)^3} + \frac{5}{32a(a+x)^2} + \frac{5}{64a\left(a^2-x^2\right)}\right) dx\right.}{d}$$

$$= \frac{1}{64d(a-a\sin(c+dx))^2} + \frac{a^2}{32d(a+a\sin(c+dx))^4} - \frac{7a}{48d(a+a\sin(c+dx))^3} + \frac{}{4d(a}$$

$$= \frac{5\tanh^{-1}(\sin(c+dx))}{64a^2d} + \frac{1}{64d(a-a\sin(c+dx))^2} + \frac{a^2}{32d(a+a\sin(c+dx))^4} - \frac{}{48d(a}$$

Mathematica [A] time = 0.45, size = 91, normalized size = 0.62

$$\frac{\frac{-15\sin^5(c+dx)+66\sin^4(c+dx)+74\sin^3(c+dx)-14\sin^2(c+dx)-47\sin(c+dx)-16}{(\sin(c+dx)-1)^2(\sin(c+dx)+1)^4} + 15\tanh^{-1}(\sin(c+dx))}{192a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (15*ArcTanh[Sin[c + d*x]] + (-16 - 47*Sin[c + d*x] - 14*Sin[c + d*x]^2 + 74
*Sin[c + d*x]^3 + 66*Sin[c + d*x]^4 - 15*Sin[c + d*x]^5)/((-1 + Sin[c + d*x
])^2*(1 + Sin[c + d*x])^4))/(192*a^2*d)
```

fricas [A] time = 0.46, size = 198, normalized size = 1.36

$$-\frac{132\cos(dx+c)^4 - 236\cos(dx+c)^2 - 15\left(\cos(dx+c)^6 - 2\cos(dx+c)^4\sin(dx+c) - 2\cos(dx+c)^4\right)\log(s}{384\left(a^2d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/384*(132*cos(d*x + c)^4 - 236*cos(d*x + c)^2 - 15*(cos(d*x + c)^6 - 2*co
s(d*x + c)^4*sin(d*x + c) - 2*cos(d*x + c)^4)*log(sin(d*x + c) + 1) + 15*(c
```

```
os(d*x + c)^6 - 2*cos(d*x + c)^4*sin(d*x + c) - 2*cos(d*x + c)^4)*log(-sin(
d*x + c) + 1) - 2*(15*cos(d*x + c)^4 + 44*cos(d*x + c)^2 - 12)*sin(d*x + c)
 + 72)/(a^2*d*cos(d*x + c)^6 - 2*a^2*d*cos(d*x + c)^4*sin(d*x + c) - 2*a^2*
d*cos(d*x + c)^4)
```

giac [A] time = 19.85, size = 126, normalized size = 0.86

$$\frac{\frac{60 \log(|\sin(dx+c)+1|)}{a^2} - \frac{60 \log(|\sin(dx+c)-1|)}{a^2} + \frac{6\left(15 \sin(dx+c)^2-10 \sin(dx+c)-1\right)}{a^2(\sin(dx+c)-1)^2} - \frac{125 \sin(dx+c)^4+740 \sin(dx+c)^3+1086 \sin(dx+c)^2+676}{a^2(\sin(dx+c)+1)^4}}{1536\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1536*(60*log(abs(sin(d*x + c) + 1))/a^2 - 60*log(abs(sin(d*x + c) - 1))/a
^2 + 6*(15*sin(d*x + c)^2 - 10*sin(d*x + c) - 1)/(a^2*(sin(d*x + c) - 1)^2)
 - (125*sin(d*x + c)^4 + 740*sin(d*x + c)^3 + 1086*sin(d*x + c)^2 + 676*sin
(d*x + c) + 157)/(a^2*(sin(d*x + c) + 1)^4))/d
```

maple [A] time = 0.24, size = 144, normalized size = 0.99

$$\frac{1}{64a^2d\left(\sin\left(dx+c\right)-1\right)^2}+\frac{5}{64a^2d\left(\sin\left(dx+c\right)-1\right)}-\frac{5\ln\left(\sin\left(dx+c\right)-1\right)}{128a^2d}+\frac{1}{32a^2d\left(1+\sin\left(dx+c\right)\right)^4}-\frac{}{48a^2d\left(1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/64/a^2/d/(sin(d*x+c)-1)^2+5/64/a^2/d/(sin(d*x+c)-1)-5/128/a^2/d*ln(sin(d*
x+c)-1)+1/32/a^2/d/(1+sin(d*x+c))^4-7/48/a^2/d/(1+sin(d*x+c))^3+1/4/a^2/d/(
1+sin(d*x+c))^2-5/32/a^2/d/(1+sin(d*x+c))+5/128*ln(1+sin(d*x+c))/a^2/d
```

maxima [A] time = 0.32, size = 167, normalized size = 1.14

$$-\frac{\frac{2\left(15 \sin(dx+c)^5-66 \sin(dx+c)^4-74 \sin(dx+c)^3+14 \sin(dx+c)^2+47 \sin(dx+c)+16\right)}{a^2 \sin(dx+c)^6+2\,a^2 \sin(dx+c)^5-a^2 \sin(dx+c)^4-4\,a^2 \sin(dx+c)^3-a^2 \sin(dx+c)^2+2\,a^2 \sin(dx+c)+a^2} - \frac{15 \log(\sin(dx+c)+1)}{a^2} + \frac{15 \log(\sin(dx+c)-1}{a^2}}{384\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/384*(2*(15*sin(d*x + c)^5 - 66*sin(d*x + c)^4 - 74*sin(d*x + c)^3 + 14*s
in(d*x + c)^2 + 47*sin(d*x + c) + 16)/(a^2*sin(d*x + c)^6 + 2*a^2*sin(d*x +
 c)^5 - a^2*sin(d*x + c)^4 - 4*a^2*sin(d*x + c)^3 - a^2*sin(d*x + c)^2 + 2*
a^2*sin(d*x + c) + a^2) - 15*log(sin(d*x + c) + 1)/a^2 + 15*log(sin(d*x + c
) - 1)/a^2)/d
```

mupad [B] time = 10.50, size = 361, normalized size = 2.47

$$\frac{5\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{32\,a^2\,d}-\frac{\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{32}+\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{8}+\frac{35\tan(}{9}}{d\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-12\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + a*sin(c + d*x))^2,x)
```

```
[Out] (5*atanh(tan(c/2 + (d*x)/2)))/(32*a^2*d) - ((5*tan(c/2 + (d*x)/2))/32 + (5*
tan(c/2 + (d*x)/2)^2)/8 + (35*tan(c/2 + (d*x)/2)^3)/96 - (5*tan(c/2 + (d*x)
/2)^4)/3 - (121*tan(c/2 + (d*x)/2)^5)/48 - (119*tan(c/2 + (d*x)/2)^6)/12 -
(121*tan(c/2 + (d*x)/2)^7)/48 - (5*tan(c/2 + (d*x)/2)^8)/3 + (35*tan(c/2 +
(d*x)/2)^9)/96 + (5*tan(c/2 + (d*x)/2)^10)/8 + (5*tan(c/2 + (d*x)/2)^11)/32
)/(d*(2*a^2*tan(c/2 + (d*x)/2)^2 - 12*a^2*tan(c/2 + (d*x)/2)^3 - 17*a^2*tan
(c/2 + (d*x)/2)^4 + 8*a^2*tan(c/2 + (d*x)/2)^5 + 28*a^2*tan(c/2 + (d*x)/2)^
6 + 8*a^2*tan(c/2 + (d*x)/2)^7 - 17*a^2*tan(c/2 + (d*x)/2)^8 - 12*a^2*tan(c
/2 + (d*x)/2)^9 + 2*a^2*tan(c/2 + (d*x)/2)^10 + 4*a^2*tan(c/2 + (d*x)/2)^11
 + a^2*tan(c/2 + (d*x)/2)^12 + a^2 + 4*a^2*tan(c/2 + (d*x)/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\tan^5(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**5/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2
```

3.63 $\int \frac{\tan^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=104

$$\frac{1}{16d\left(a^2-a^2\sin(c+dx)\right)}+\frac{3}{16d\left(a^2\sin(c+dx)+a^2\right)}-\frac{\tanh^{-1}(\sin(c+dx))}{8a^2d}+\frac{a}{12d(a\sin(c+dx)+a)^3}-\frac{}{4d(a\sin(c}$$

```
[Out] -1/8*arctanh(sin(d*x+c))/a^2/d+1/12*a/d/(a+a*sin(d*x+c))^3-1/4/d/(a+a*sin(d
*x+c))^2+1/16/d/(a^2-a^2*sin(d*x+c))+3/16/d/(a^2+a^2*sin(d*x+c))
```

Rubi [A] time = 0.09, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2707, 88, 206}

$$\frac{1}{16d\left(a^2-a^2\sin(c+dx)\right)}+\frac{3}{16d\left(a^2\sin(c+dx)+a^2\right)}-\frac{\tanh^{-1}(\sin(c+dx))}{8a^2d}+\frac{a}{12d(a\sin(c+dx)+a)^3}-\frac{}{4d(a\sin(c}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -ArcTanh[Sin[c + d*x]]/(8*a^2*d) + a/(12*d*(a + a*Sin[c + d*x])^3) - 1/(4*d
*(a + a*Sin[c + d*x])^2) + 1/(16*d*(a^2 - a^2*Sin[c + d*x])) + 3/(16*d*(a^2
 + a^2*Sin[c + d*x]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
```

```
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{(a-x)^2(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{16a(a-x)^2} - \frac{a}{4(a+x)^4} + \frac{1}{2(a+x)^3} - \frac{3}{16a(a+x)^2} - \frac{1}{8a\left(a^2-x^2\right)}\right) dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a}{12d(a+a\sin(c+dx))^3} - \frac{1}{4d(a+a\sin(c+dx))^2} + \frac{1}{16d\left(a^2-a^2\sin(c+dx)\right)} + \frac{}{16d\,(}$$

$$= -\frac{\tanh^{-1}(\sin(c+dx))}{8a^2d} + \frac{a}{12d(a+a\sin(c+dx))^3} - \frac{1}{4d(a+a\sin(c+dx))^2} + \frac{}{16d\left(a^2\right.}$$

Mathematica [A] time = 0.32, size = 70, normalized size = 0.67

$$-\frac{-\frac{3}{1-\sin(c+dx)} - \frac{9}{\sin(c+dx)+1} + \frac{12}{(\sin(c+dx)+1)^2} - \frac{4}{(\sin(c+dx)+1)^3} + 6\tanh^{-1}(\sin(c+dx))}{48a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/48*(6*ArcTanh[Sin[c + d*x]] - 3/(1 - Sin[c + d*x]) - 4/(1 + Sin[c + d*x]
)^3 + 12/(1 + Sin[c + d*x])^2 - 9/(1 + Sin[c + d*x]))/(a^2*d)
```

fricas [A] time = 0.43, size = 178, normalized size = 1.71

$$\frac{12\cos(dx+c)^2 - 3\left(\cos(dx+c)^4 - 2\cos(dx+c)^2\sin(dx+c) - 2\cos(dx+c)^2\right)\log(\sin(dx+c)+1) + 3\left(\cos\right.}{48\left(a^2d\cos(dx+c)^4 - 2a^2d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/48*(12*cos(d*x + c)^2 - 3*(cos(d*x + c)^4 - 2*cos(d*x + c)^2*sin(d*x + c)
 - 2*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + 3*(cos(d*x + c)^4 - 2*cos(d*x
+ c)^2*sin(d*x + c) - 2*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(3*cos(d
```

```
*x + c)^2 + 4)*sin(d*x + c) - 16)/(a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x +
 c)^2*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^2)
```

giac [A] time = 2.06, size = 102, normalized size = 0.98

$$-\frac{\frac{6\log\left(\left|\sin\left(dx+c\right)+1\right|\right)}{a^2}-\frac{6\log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{a^2}+\frac{6\sin\left(dx+c\right)}{a^2\left(\sin\left(dx+c\right)-1\right)}-\frac{11\sin\left(dx+c\right)^3+51\sin\left(dx+c\right)^2+45\sin\left(dx+c\right)+13}{a^2\left(\sin\left(dx+c\right)+1\right)^3}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/96*(6*log(abs(sin(d*x + c) + 1))/a^2 - 6*log(abs(sin(d*x + c) - 1))/a^2
+ 6*sin(d*x + c)/(a^2*(sin(d*x + c) - 1)) - (11*sin(d*x + c)^3 + 51*sin(d*x
 + c)^2 + 45*sin(d*x + c) + 13)/(a^2*(sin(d*x + c) + 1)^3))/d
```

maple [A] time = 0.24, size = 108, normalized size = 1.04

$$-\frac{1}{16a^2d\left(\sin\left(dx+c\right)-1\right)}+\frac{\ln\left(\sin\left(dx+c\right)-1\right)}{16a^2d}+\frac{1}{12a^2d\left(1+\sin\left(dx+c\right)\right)^3}-\frac{1}{4a^2d\left(1+\sin\left(dx+c\right)\right)^2}+\frac{}{16a^2d\,(1+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/16/a^2/d/(sin(d*x+c)-1)+1/16/a^2/d*ln(sin(d*x+c)-1)+1/12/a^2/d/(1+sin(d*
x+c))^3-1/4/a^2/d/(1+sin(d*x+c))^2+3/16/a^2/d/(1+sin(d*x+c))-1/16*ln(1+sin(
d*x+c))/a^2/d
```

maxima [A] time = 0.30, size = 110, normalized size = 1.06

$$\frac{\frac{2\left(3\sin\left(dx+c\right)^3-6\sin\left(dx+c\right)^2-7\sin\left(dx+c\right)-2\right)}{a^2\sin\left(dx+c\right)^4+2\,a^2\sin\left(dx+c\right)^3-2\,a^2\sin\left(dx+c\right)-a^2}-\frac{3\log\left(\sin\left(dx+c\right)+1\right)}{a^2}+\frac{3\log\left(\sin\left(dx+c\right)-1\right)}{a^2}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/48*(2*(3*sin(d*x + c)^3 - 6*sin(d*x + c)^2 - 7*sin(d*x + c) - 2)/(a^2*sin
(d*x + c)^4 + 2*a^2*sin(d*x + c)^3 - 2*a^2*sin(d*x + c) - a^2) - 3*log(sin(
d*x + c) + 1)/a^2 + 3*log(sin(d*x + c) - 1)/a^2)/d
```

mupad [B] time = 10.05, size = 240, normalized size = 2.31

$$\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\frac{13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12}+\frac{10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}+\frac{13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12}+\mathrm{t}}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-10\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\,a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + a*sin(c + d*x))^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)/4 + tan(c/2 + (d*x)/2)^2 + (13*tan(c/2 + (d*x)/2)^3)/12
 + (10*tan(c/2 + (d*x)/2)^4)/3 + (13*tan(c/2 + (d*x)/2)^5)/12 + tan(c/2 + (
d*x)/2)^6 + tan(c/2 + (d*x)/2)^7/4)/(d*(4*a^2*tan(c/2 + (d*x)/2)^2 - 4*a^2*
tan(c/2 + (d*x)/2)^3 - 10*a^2*tan(c/2 + (d*x)/2)^4 - 4*a^2*tan(c/2 + (d*x)/
2)^5 + 4*a^2*tan(c/2 + (d*x)/2)^6 + 4*a^2*tan(c/2 + (d*x)/2)^7 + a^2*tan(c/
2 + (d*x)/2)^8 + a^2 + 4*a^2*tan(c/2 + (d*x)/2))) - atanh(tan(c/2 + (d*x)/2
))/(4*a^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^3(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**3/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2
```

3.64 $\int \frac{\tan(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=60

$$-\frac{1}{4d\left(a^2\sin(c+dx)+a^2\right)}+\frac{\tanh^{-1}(\sin(c+dx))}{4a^2d}+\frac{1}{4d(a\sin(c+dx)+a)^2}$$

```
[Out] 1/4*arctanh(sin(d*x+c))/a^2/d+1/4/d/(a+a*sin(d*x+c))^2-1/4/d/(a^2+a^2*sin(d
*x+c))
```

Rubi [A] time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2707, 77, 206}

$$-\frac{1}{4d\left(a^2\sin(c+dx)+a^2\right)}+\frac{\tanh^{-1}(\sin(c+dx))}{4a^2d}+\frac{1}{4d(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(4*a^2*d) + 1/(4*d*(a + a*Sin[c + d*x])^2) - 1/(4*d*(
a^2 + a^2*Sin[c + d*x]))
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
```

```
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tan(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(a-x)(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{2(a+x)^3}+\frac{1}{4a(a+x)^2}+\frac{1}{4a(a^2-x^2)}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{1}{4d(a+a\sin(c+dx))^2}-\frac{1}{4d\left(a^2+a^2\sin(c+dx)\right)}+\frac{\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, a\sin(c+d\right.}{4ad}$$
$$= \frac{\tanh^{-1}(\sin(c+dx))}{4a^2d}+\frac{1}{4d(a+a\sin(c+dx))^2}-\frac{1}{4d\left(a^2+a^2\sin(c+dx)\right)}$$

Mathematica [A] time = 0.08, size = 36, normalized size = 0.60

$$\frac{\tanh^{-1}(\sin(c+dx))-\frac{\sin(c+dx)}{(\sin(c+dx)+1)^2}}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + a*Sin[c + d*x])^2,x]
[Out] (ArcTanh[Sin[c + d*x]] - Sin[c + d*x]/(1 + Sin[c + d*x])^2)/(4*a^2*d)
```

fricas [A] time = 0.41, size = 104, normalized size = 1.73

$$\frac{\left(\cos(dx+c)^2-2\sin(dx+c)-2\right)\log(\sin(dx+c)+1)-\left(\cos(dx+c)^2-2\sin(dx+c)-2\right)\log(-\sin(dx+c)}{8\left(a^2d\cos(dx+c)^2-2a^2d\sin(dx+c)-2a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
[Out] 1/8*((cos(d*x + c)^2 - 2*sin(d*x + c) - 2)*log(sin(d*x + c) + 1) - (cos(d*x
 + c)^2 - 2*sin(d*x + c) - 2)*log(-sin(d*x + c) + 1) + 2*sin(d*x + c))/(a^2
*d*cos(d*x + c)^2 - 2*a^2*d*sin(d*x + c) - 2*a^2*d)
```

giac [A] time = 1.00, size = 90, normalized size = 1.50

$$\frac{\frac{\log\left(\left|\frac{1}{\sin(dx+c)}+\sin(dx+c)+2\right|\right)}{a^2}-\frac{\log\left(\left|\frac{1}{\sin(dx+c)}+\sin(dx+c)-2\right|\right)}{a^2}-\frac{\frac{1}{\sin(dx+c)}+\sin(dx+c)+6}{a^2\left(\frac{1}{\sin(dx+c)}+\sin(dx+c)+2\right)}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(log(abs(1/sin(d*x + c) + sin(d*x + c) + 2))/a^2 - log(abs(1/sin(d*x +
 c) + sin(d*x + c) - 2))/a^2 - (1/sin(d*x + c) + sin(d*x + c) + 6)/(a^2*(1/
sin(d*x + c) + sin(d*x + c) + 2)))/d
```

maple [A] time = 0.24, size = 72, normalized size = 1.20

$$-\frac{\ln\left(\sin\left(dx+c\right)-1\right)}{8a^2d}+\frac{1}{4a^2d\left(1+\sin\left(dx+c\right)\right)^2}-\frac{1}{4a^2d\left(1+\sin\left(dx+c\right)\right)}+\frac{\ln\left(1+\sin\left(dx+c\right)\right)}{8a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/8/a^2/d*ln(sin(d*x+c)-1)+1/4/a^2/d/(1+sin(d*x+c))^2-1/4/a^2/d/(1+sin(d*x
+c))+1/8*ln(1+sin(d*x+c))/a^2/d
```

maxima [A] time = 0.30, size = 70, normalized size = 1.17

$$-\frac{\frac{2\sin(dx+c)}{a^2\sin(dx+c)^2+2\,a^2\sin(dx+c)+a^2}-\frac{\log(\sin(dx+c)+1)}{a^2}+\frac{\log(\sin(dx+c)-1)}{a^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/8*(2*sin(d*x + c)/(a^2*sin(d*x + c)^2 + 2*a^2*sin(d*x + c) + a^2) - log(
sin(d*x + c) + 1)/a^2 + log(sin(d*x + c) - 1)/a^2)/d
```

mupad [B] time = 7.64, size = 116, normalized size = 1.93

$$\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,a^2\,d}-\frac{\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{2}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}}{d\,\left(a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+4\,a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+6\,a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+4\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] atanh(tan(c/2 + (d*x)/2))/(2*a^2*d) - (tan(c/2 + (d*x)/2)/2 + tan(c/2 + (d*
x)/2)^3/2)/(d*(6*a^2*tan(c/2 + (d*x)/2)^2 + 4*a^2*tan(c/2 + (d*x)/2)^3 + a^
2*tan(c/2 + (d*x)/2)^4 + a^2 + 4*a^2*tan(c/2 + (d*x)/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2
```

3.65 $\int \frac{\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=52

$$\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)}+\frac{\log(\sin(c+dx))}{a^2d}-\frac{\log(\sin(c+dx)+1)}{a^2d}$$

```
[Out] ln(sin(d*x+c))/a^2/d-ln(1+sin(d*x+c))/a^2/d+1/d/(a^2+a^2*sin(d*x+c))
```

Rubi [A] time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2707, 44}

$$\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)}+\frac{\log(\sin(c+dx))}{a^2d}-\frac{\log(\sin(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] Log[Sin[c + d*x]]/(a^2*d) - Log[1 + Sin[c + d*x]]/(a^2*d) + 1/(d*(a^2 + a^2
*Sin[c + d*x]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2x} - \frac{1}{a(a+x)^2} - \frac{1}{a^2(a+x)}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\log(\sin(c+dx))}{a^2d} - \frac{\log(1+\sin(c+dx))}{a^2d} + \frac{1}{d\left(a^2+a^2\sin(c+dx)\right)}$$

Mathematica [A] time = 0.06, size = 36, normalized size = 0.69

$$\frac{\frac{1}{\sin(c+dx)+1} + \log(\sin(c+dx)) - \log(\sin(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Log[Sin[c + d*x]] - Log[1 + Sin[c + d*x]] + (1 + Sin[c + d*x])^(-1))/(a^2*
d)
```

fricas [A] time = 0.41, size = 59, normalized size = 1.13

$$\frac{(\sin(dx+c)+1)\log\left(\frac{1}{2}\sin(dx+c)\right) - (\sin(dx+c)+1)\log(\sin(dx+c)+1)+1}{a^2d\sin(dx+c)+a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] ((sin(d*x + c) + 1)*log(1/2*sin(d*x + c)) - (sin(d*x + c) + 1)*log(sin(d*x
+ c) + 1) + 1)/(a^2*d*sin(d*x + c) + a^2*d)
```

giac [A] time = 0.39, size = 45, normalized size = 0.87

$$\frac{a\left(\frac{\log\left(\left|-\frac{a}{a\sin(dx+c)+a}+1\right|\right)}{a^3} + \frac{1}{(a\sin(dx+c)+a)a^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] a*(log(abs(-a/(a*sin(d*x + c) + a) + 1))/a^3 + 1/((a*sin(d*x + c) + a)*a^2)
)/d
```

maple [A] time = 0.15, size = 50, normalized size = 0.96

$$\frac{\ln(\sin(dx+c))}{a^2d}+\frac{1}{a^2d(1+\sin(dx+c))}-\frac{\ln(1+\sin(dx+c))}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] ln(sin(d*x+c))/a^2/d+1/a^2/d/(1+sin(d*x+c))-ln(1+sin(d*x+c))/a^2/d
```

maxima [A] time = 0.30, size = 46, normalized size = 0.88

$$\frac{\frac{1}{a^2\sin(dx+c)+a^2}-\frac{\log(\sin(dx+c)+1)}{a^2}+\frac{\log(\sin(dx+c))}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (1/(a^2*sin(d*x + c) + a^2) - log(sin(d*x + c) + 1)/a^2 + log(sin(d*x + c))
/a^2)/d
```

mupad [B] time = 6.59, size = 87, normalized size = 1.67

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}-\frac{2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{a^2\,d}-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a^2*d) - (2*log(tan(c/2 + (d*x)/2) + 1))/(a^2*d) -
 (2*tan(c/2 + (d*x)/2))/(d*(a^2*tan(c/2 + (d*x)/2)^2 + a^2 + 2*a^2*tan(c/2
+ (d*x)/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\cot(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c))**2,x)

[Out] Integral(cot(c + d*x)/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2
```

3.66 $\int \frac{\cot^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=65

$$-\frac{\csc^2(c+dx)}{2a^2d}+\frac{2\csc(c+dx)}{a^2d}+\frac{2\log(\sin(c+dx))}{a^2d}-\frac{2\log(\sin(c+dx)+1)}{a^2d}$$

```
[Out] 2*csc(d*x+c)/a^2/d-1/2*csc(d*x+c)^2/a^2/d+2*ln(sin(d*x+c))/a^2/d-2*ln(1+sin
(d*x+c))/a^2/d
```

Rubi [A] time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 77}

$$-\frac{\csc^2(c+dx)}{2a^2d}+\frac{2\csc(c+dx)}{a^2d}+\frac{2\log(\sin(c+dx))}{a^2d}-\frac{2\log(\sin(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*Csc[c + d*x])/(a^2*d) - Csc[c + d*x]^2/(2*a^2*d) + (2*Log[Sin[c + d*x]])
/(a^2*d) - (2*Log[1 + Sin[c + d*x]])/(a^2*d)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{a-x}{x^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{x^3}-\frac{2}{ax^2}+\frac{2}{a^2x}-\frac{2}{a^2(a+x)}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{2\csc(c+dx)}{a^2d}-\frac{\csc^2(c+dx)}{2a^2d}+\frac{2\log(\sin(c+dx))}{a^2d}-\frac{2\log(1+\sin(c+dx))}{a^2d}$$

Mathematica [A] time = 0.07, size = 49, normalized size = 0.75

$$\frac{-\csc^2(c+dx)+4\csc(c+dx)+4\log(\sin(c+dx))-4\log(\sin(c+dx)+1)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (4*Csc[c + d*x] - Csc[c + d*x]^2 + 4*Log[Sin[c + d*x]] - 4*Log[1 + Sin[c + d*x]])/(2*a^2*d)
```

fricas [A] time = 0.42, size = 76, normalized size = 1.17

$$\frac{4\left(\cos\left(dx+c\right)^2-1\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)-4\left(\cos\left(dx+c\right)^2-1\right)\log\left(\sin\left(dx+c\right)+1\right)-4\sin\left(dx+c\right)+1}{2\left(a^2d\cos\left(dx+c\right)^2-a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*(cos(d*x + c)^2 - 1)*log(1/2*sin(d*x + c)) - 4*(cos(d*x + c)^2 - 1)*log(sin(d*x + c) + 1) - 4*sin(d*x + c) + 1)/(a^2*d*cos(d*x + c)^2 - a^2*d)
```

giac [A] time = 0.49, size = 115, normalized size = 1.77

$$-\frac{\frac{32\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2}-\frac{16\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2}+\frac{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-8a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^4}+\frac{24\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/8*(32*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 16*log(abs(tan(1/2*d*x +
1/2*c)))/a^2 + (a^2*tan(1/2*d*x + 1/2*c)^2 - 8*a^2*tan(1/2*d*x + 1/2*c))/a^
4 + (24*tan(1/2*d*x + 1/2*c)^2 - 8*tan(1/2*d*x + 1/2*c) + 1)/(a^2*tan(1/2*d
*x + 1/2*c)^2))/d
```

maple [A] time = 0.32, size = 66, normalized size = 1.02

$$-\frac{1}{2a^2d\sin(dx+c)^2}+\frac{2}{a^2d\sin(dx+c)}+\frac{2\ln(\sin(dx+c))}{a^2d}-\frac{2\ln(1+\sin(dx+c))}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/2/a^2/d/sin(d*x+c)^2+2/a^2/d/sin(d*x+c)+2*ln(sin(d*x+c))/a^2/d-2*ln(1+si
n(d*x+c))/a^2/d
```

maxima [A] time = 0.30, size = 55, normalized size = 0.85

$$-\frac{\frac{4\log(\sin(dx+c)+1)}{a^2}-\frac{4\log(\sin(dx+c))}{a^2}-\frac{4\sin(dx+c)-1}{a^2\sin(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(4*log(sin(d*x + c) + 1)/a^2 - 4*log(sin(d*x + c))/a^2 - (4*sin(d*x +
c) - 1)/(a^2*sin(d*x + c)^2))/d
```

mupad [B] time = 6.53, size = 103, normalized size = 1.58

$$\frac{2\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^2d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8a^2d}-\frac{4\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{a^2d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^2d}+\frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{1}{8}\right)}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3/(a + a*sin(c + d*x))^2,x)
```

```
[Out] (2*log(tan(c/2 + (d*x)/2)))/(a^2*d) - tan(c/2 + (d*x)/2)^2/(8*a^2*d) - (4*l
og(tan(c/2 + (d*x)/2) + 1))/(a^2*d) + tan(c/2 + (d*x)/2)/(a^2*d) + (cot(c/2
 + (d*x)/2)^2*(tan(c/2 + (d*x)/2) - 1/8))/(a^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\cot^3(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**3/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2
```

3.67 $\int \frac{\cot^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=55

$$-\frac{\csc^4(c+dx)}{4a^2d}+\frac{2\csc^3(c+dx)}{3a^2d}-\frac{\csc^2(c+dx)}{2a^2d}$$

```
[Out] -1/2*csc(d*x+c)^2/a^2/d+2/3*csc(d*x+c)^3/a^2/d-1/4*csc(d*x+c)^4/a^2/d
```

Rubi [A] time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 43}

$$-\frac{\csc^4(c+dx)}{4a^2d}+\frac{2\csc^3(c+dx)}{3a^2d}-\frac{\csc^2(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -Csc[c + d*x]^2/(2*a^2*d) + (2*Csc[c + d*x]^3)/(3*a^2*d) - Csc[c + d*x]^4/(
4*a^2*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2}{x^5}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^2}{x^5}-\frac{2a}{x^4}+\frac{1}{x^3}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{\csc^2(c+dx)}{2a^2d}+\frac{2\csc^3(c+dx)}{3a^2d}-\frac{\csc^4(c+dx)}{4a^2d}$$

Mathematica [A] time = 0.07, size = 38, normalized size = 0.69

$$\frac{\csc^4(c+dx)(8\sin(c+dx)+3\cos(2(c+dx))-6)}{12a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Csc[c + d*x]^4*(-6 + 3*Cos[2*(c + d*x)] + 8*Sin[c + d*x]))/(12*a^2*d)
```

fricas [A] time = 0.39, size = 57, normalized size = 1.04

$$\frac{6\cos(dx+c)^2+8\sin(dx+c)-9}{12\left(a^2d\cos(dx+c)^4-2a^2d\cos(dx+c)^2+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(6*cos(d*x + c)^2 + 8*sin(d*x + c) - 9)/(a^2*d*cos(d*x + c)^4 - 2*a^2*
d*cos(d*x + c)^2 + a^2*d)
```

giac [A] time = 0.60, size = 36, normalized size = 0.65

$$-\frac{6\sin(dx+c)^2-8\sin(dx+c)+3}{12a^2d\sin(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/12*(6*sin(d*x + c)^2 - 8*sin(d*x + c) + 3)/(a^2*d*sin(d*x + c)^4)
```

maple [A] time = 0.25, size = 39, normalized size = 0.71

$$\frac{-\frac{1}{2\sin(dx+c)^2}-\frac{1}{4\sin(dx+c)^4}+\frac{2}{3\sin(dx+c)^3}}{d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d/a^2*(-1/2/sin(d*x+c)^2-1/4/sin(d*x+c)^4+2/3/sin(d*x+c)^3)
```

maxima [A] time = 0.30, size = 36, normalized size = 0.65

$$-\frac{6\,\sin\left(dx+c\right)^2-8\,\sin\left(dx+c\right)+3}{12\,a^2d\sin\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/12*(6*sin(d*x + c)^2 - 8*sin(d*x + c) + 3)/(a^2*d*sin(d*x + c)^4)
```

mupad [B] time = 6.33, size = 36, normalized size = 0.65

$$-\frac{\frac{\sin(c+d\,x)^2}{2}-\frac{2\,\sin(c+d\,x)}{3}+\frac{1}{4}}{a^2\,d\sin\left(c+d\,x\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5/(a + a*sin(c + d*x))^2,x)
```

```
[Out] -(sin(c + d*x)^2/2 - (2*sin(c + d*x))/3 + 1/4)/(a^2*d*sin(c + d*x)^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^5(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**5/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2
```

3.68 $\int \frac{\cot^7(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=73

$$-\frac{\csc^6(c+dx)}{6a^2d}+\frac{2\csc^5(c+dx)}{5a^2d}-\frac{2\csc^3(c+dx)}{3a^2d}+\frac{\csc^2(c+dx)}{2a^2d}$$

```
[Out] 1/2*csc(d*x+c)^2/a^2/d-2/3*csc(d*x+c)^3/a^2/d+2/5*csc(d*x+c)^5/a^2/d-1/6*cs
c(d*x+c)^6/a^2/d
```

Rubi [A] time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 75}

$$-\frac{\csc^6(c+dx)}{6a^2d}+\frac{2\csc^5(c+dx)}{5a^2d}-\frac{2\csc^3(c+dx)}{3a^2d}+\frac{\csc^2(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] Csc[c + d*x]^2/(2*a^2*d) - (2*Csc[c + d*x]^3)/(3*a^2*d) + (2*Csc[c + d*x]^5
)/(5*a^2*d) - Csc[c + d*x]^6/(6*a^2*d)
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^7(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)}{x^7}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^4}{x^7} - \frac{2a^3}{x^6} + \frac{2a}{x^4} - \frac{1}{x^3}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\csc^2(c+dx)}{2a^2d} - \frac{2\csc^3(c+dx)}{3a^2d} + \frac{2\csc^5(c+dx)}{5a^2d} - \frac{\csc^6(c+dx)}{6a^2d}$$

Mathematica [A] time = 0.07, size = 73, normalized size = 1.00

$$-\frac{\csc^6(c+dx)}{6a^2d} + \frac{2\csc^5(c+dx)}{5a^2d} - \frac{2\csc^3(c+dx)}{3a^2d} + \frac{\csc^2(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] Csc[c + d*x]^2/(2*a^2*d) - (2*Csc[c + d*x]^3)/(3*a^2*d) + (2*Csc[c + d*x]^5
)/(5*a^2*d) - Csc[c + d*x]^6/(6*a^2*d)
```

fricas [A] time = 0.44, size = 94, normalized size = 1.29

$$-\frac{15\cos(dx+c)^4 - 30\cos(dx+c)^2 + 4\left(5\cos(dx+c)^2 - 2\right)\sin(dx+c) + 10}{30\left(a^2d\cos(dx+c)^6 - 3a^2d\cos(dx+c)^4 + 3a^2d\cos(dx+c)^2 - a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/30*(15*cos(d*x + c)^4 - 30*cos(d*x + c)^2 + 4*(5*cos(d*x + c)^2 - 2)*sin
(d*x + c) + 10)/(a^2*d*cos(d*x + c)^6 - 3*a^2*d*cos(d*x + c)^4 + 3*a^2*d*co
s(d*x + c)^2 - a^2*d)
```

giac [A] time = 0.61, size = 46, normalized size = 0.63

$$\frac{15\sin(dx+c)^4 - 20\sin(dx+c)^3 + 12\sin(dx+c) - 5}{30a^2d\sin(dx+c)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/30*(15*sin(d*x + c)^4 - 20*sin(d*x + c)^3 + 12*sin(d*x + c) - 5)/(a^2*d*s
in(d*x + c)^6)
```

maple [A] time = 0.29, size = 49, normalized size = 0.67

$$\frac{-\frac{1}{6\sin(dx+c)^6}+\frac{2}{5\sin(dx+c)^5}+\frac{1}{2\sin(dx+c)^2}-\frac{2}{3\sin(dx+c)^3}}{d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^7/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d/a^2*(-1/6/sin(d*x+c)^6+2/5/sin(d*x+c)^5+1/2/sin(d*x+c)^2-2/3/sin(d*x+c)
^3)
```

maxima [A] time = 0.31, size = 46, normalized size = 0.63

$$\frac{15\,\sin\left(dx+c\right)^4-20\,\sin\left(dx+c\right)^3+12\,\sin\left(dx+c\right)-5}{30\,a^2d\sin\left(dx+c\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/30*(15*sin(d*x + c)^4 - 20*sin(d*x + c)^3 + 12*sin(d*x + c) - 5)/(a^2*d*s
in(d*x + c)^6)
```

mupad [B] time = 6.37, size = 46, normalized size = 0.63

$$\frac{15\sin\left(c+d\,x\right)^4-20\sin\left(c+d\,x\right)^3+12\,\sin\left(c+d\,x\right)-5}{30\,a^2\,d\sin\left(c+d\,x\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^7/(a + a*sin(c + d*x))^2,x)
```

```
[Out] (12*sin(c + d*x) - 20*sin(c + d*x)^3 + 15*sin(c + d*x)^4 - 5)/(30*a^2*d*sin
(c + d*x)^6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\cot^7(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**7/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**7/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2
```

3.69 $\int \frac{\cot^9(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=127

$$-\frac{\csc^8(c+dx)}{8a^2d}+\frac{2\csc^7(c+dx)}{7a^2d}+\frac{\csc^6(c+dx)}{6a^2d}-\frac{4\csc^5(c+dx)}{5a^2d}+\frac{\csc^4(c+dx)}{4a^2d}+\frac{2\csc^3(c+dx)}{3a^2d}-\frac{\csc^2(c+dx)}{2a^2d}$$

```
[Out] -1/2*csc(d*x+c)^2/a^2/d+2/3*csc(d*x+c)^3/a^2/d+1/4*csc(d*x+c)^4/a^2/d-4/5*c
sc(d*x+c)^5/a^2/d+1/6*csc(d*x+c)^6/a^2/d+2/7*csc(d*x+c)^7/a^2/d-1/8*csc(d*x
+c)^8/a^2/d
```

Rubi [A] time = 0.07, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 88}

$$-\frac{\csc^8(c+dx)}{8a^2d}+\frac{2\csc^7(c+dx)}{7a^2d}+\frac{\csc^6(c+dx)}{6a^2d}-\frac{4\csc^5(c+dx)}{5a^2d}+\frac{\csc^4(c+dx)}{4a^2d}+\frac{2\csc^3(c+dx)}{3a^2d}-\frac{\csc^2(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^9/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -Csc[c + d*x]^2/(2*a^2*d) + (2*Csc[c + d*x]^3)/(3*a^2*d) + Csc[c + d*x]^4/(
4*a^2*d) - (4*Csc[c + d*x]^5)/(5*a^2*d) + Csc[c + d*x]^6/(6*a^2*d) + (2*Csc
[c + d*x]^7)/(7*a^2*d) - Csc[c + d*x]^8/(8*a^2*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^9(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^4(a+x)^2}{x^9}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^6}{x^9}-\frac{2a^5}{x^8}-\frac{a^4}{x^7}+\frac{4a^3}{x^6}-\frac{a^2}{x^5}-\frac{2a}{x^4}+\frac{1}{x^3}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{\csc^2(c+dx)}{2a^2d}+\frac{2\csc^3(c+dx)}{3a^2d}+\frac{\csc^4(c+dx)}{4a^2d}-\frac{4\csc^5(c+dx)}{5a^2d}+\frac{\csc^6(c+dx)}{6a^2d}+\frac{2}{}$$

Mathematica [A] time = 0.15, size = 78, normalized size = 0.61

$$\frac{\csc^2(c+dx)\left(-105\csc^6(c+dx)+240\csc^5(c+dx)+140\csc^4(c+dx)-672\csc^3(c+dx)+210\csc^2(c+dx)+56\right.}{840a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^9/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Csc[c + d*x]^2*(-420 + 560*Csc[c + d*x] + 210*Csc[c + d*x]^2 - 672*Csc[c +
 d*x]^3 + 140*Csc[c + d*x]^4 + 240*Csc[c + d*x]^5 - 105*Csc[c + d*x]^6))/(8
40*a^2*d)
```

fricas [A] time = 0.43, size = 127, normalized size = 1.00

$$\frac{420\cos(dx+c)^6-1050\cos(dx+c)^4+700\cos(dx+c)^2+16\left(35\cos(dx+c)^4-28\cos(dx+c)^2+8\right)\sin(dx}{840\left(a^2d\cos(dx+c)^8-4a^2d\cos(dx+c)^6+6a^2d\cos(dx+c)^4-4a^2d\cos(dx+c)^2+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^9/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/840*(420*cos(d*x + c)^6 - 1050*cos(d*x + c)^4 + 700*cos(d*x + c)^2 + 16*(
35*cos(d*x + c)^4 - 28*cos(d*x + c)^2 + 8)*sin(d*x + c) - 175)/(a^2*d*cos(d
*x + c)^8 - 4*a^2*d*cos(d*x + c)^6 + 6*a^2*d*cos(d*x + c)^4 - 4*a^2*d*cos(d
*x + c)^2 + a^2*d)
```

giac [A] time = 0.77, size = 76, normalized size = 0.60

$$-\frac{420\sin(dx+c)^6-560\sin(dx+c)^5-210\sin(dx+c)^4+672\sin(dx+c)^3-140\sin(dx+c)^2-240\sin(dx}{840a^2d\sin(dx+c)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^9/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/840*(420*sin(d*x + c)^6 - 560*sin(d*x + c)^5 - 210*sin(d*x + c)^4 + 672*
sin(d*x + c)^3 - 140*sin(d*x + c)^2 - 240*sin(d*x + c) + 105)/(a^2*d*sin(d*
x + c)^8)
```

maple [A] time = 0.32, size = 79, normalized size = 0.62

$$\frac{\frac{1}{6\sin(dx+c)^6} - \frac{4}{5\sin(dx+c)^5} + \frac{2}{7\sin(dx+c)^7} - \frac{1}{2\sin(dx+c)^2} - \frac{1}{8\sin(dx+c)^8} + \frac{1}{4\sin(dx+c)^4} + \frac{2}{3\sin(dx+c)^3}}{d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^9/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d/a^2*(1/6/sin(d*x+c)^6-4/5/sin(d*x+c)^5+2/7/sin(d*x+c)^7-1/2/sin(d*x+c)^
2-1/8/sin(d*x+c)^8+1/4/sin(d*x+c)^4+2/3/sin(d*x+c)^3)
```

maxima [A] time = 0.33, size = 76, normalized size = 0.60

$$-\frac{420\,\sin\left(dx+c\right)^6 - 560\,\sin\left(dx+c\right)^5 - 210\,\sin\left(dx+c\right)^4 + 672\,\sin\left(dx+c\right)^3 - 140\,\sin\left(dx+c\right)^2 - 240\,\sin\left(d\right.}{840\,a^2 d\sin\left(dx+c\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^9/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/840*(420*sin(d*x + c)^6 - 560*sin(d*x + c)^5 - 210*sin(d*x + c)^4 + 672*
sin(d*x + c)^3 - 140*sin(d*x + c)^2 - 240*sin(d*x + c) + 105)/(a^2*d*sin(d*
x + c)^8)
```

mupad [B] time = 6.54, size = 76, normalized size = 0.60

$$\frac{-420\sin\left(c+d\,x\right)^6 + 560\sin\left(c+d\,x\right)^5 + 210\sin\left(c+d\,x\right)^4 - 672\sin\left(c+d\,x\right)^3 + 140\sin\left(c+d\,x\right)^2 + 240\,\sin\left(c\right.}{840\,a^2\,d\sin\left(c+d\,x\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^9/(a + a*sin(c + d*x))^2,x)
```

```
[Out] (240*sin(c + d*x) + 140*sin(c + d*x)^2 - 672*sin(c + d*x)^3 + 210*sin(c + d
*x)^4 + 560*sin(c + d*x)^5 - 420*sin(c + d*x)^6 - 105)/(840*a^2*d*sin(c + d
*x)^8)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^9(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**9/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**9/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2
```

3.70 $\int \frac{\cot^{11}(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=145

$$-\frac{\csc^{10}(c+dx)}{10a^2d}+\frac{2\csc^9(c+dx)}{9a^2d}+\frac{\csc^8(c+dx)}{4a^2d}-\frac{6\csc^7(c+dx)}{7a^2d}+\frac{6\csc^5(c+dx)}{5a^2d}-\frac{\csc^4(c+dx)}{2a^2d}-\frac{2\csc^3(c+dx)}{3a^2d}+\frac{\mathrm{cs}}{}$$

```
[Out] 1/2*csc(d*x+c)^2/a^2/d-2/3*csc(d*x+c)^3/a^2/d-1/2*csc(d*x+c)^4/a^2/d+6/5*cs
c(d*x+c)^5/a^2/d-6/7*csc(d*x+c)^7/a^2/d+1/4*csc(d*x+c)^8/a^2/d+2/9*csc(d*x+
c)^9/a^2/d-1/10*csc(d*x+c)^10/a^2/d
```

Rubi [**A**] time = 0.08, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 88}

$$-\frac{\csc^{10}(c+dx)}{10a^2d}+\frac{2\csc^9(c+dx)}{9a^2d}+\frac{\csc^8(c+dx)}{4a^2d}-\frac{6\csc^7(c+dx)}{7a^2d}+\frac{6\csc^5(c+dx)}{5a^2d}-\frac{\csc^4(c+dx)}{2a^2d}-\frac{2\csc^3(c+dx)}{3a^2d}+\frac{\mathrm{cs}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^11/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] Csc[c + d*x]^2/(2*a^2*d) - (2*Csc[c + d*x]^3)/(3*a^2*d) - Csc[c + d*x]^4/(2
*a^2*d) + (6*Csc[c + d*x]^5)/(5*a^2*d) - (6*Csc[c + d*x]^7)/(7*a^2*d) + Csc
[c + d*x]^8/(4*a^2*d) + (2*Csc[c + d*x]^9)/(9*a^2*d) - Csc[c + d*x]^10/(10*
a^2*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^{11}(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^5(a+x)^3}{x^{11}}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^8}{x^{11}} - \frac{2a^7}{x^{10}} - \frac{2a^6}{x^9} + \frac{6a^5}{x^8} - \frac{6a^3}{x^6} + \frac{2a^2}{x^5} + \frac{2a}{x^4} - \frac{1}{x^3}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\csc^2(c+dx)}{2a^2d} - \frac{2\csc^3(c+dx)}{3a^2d} - \frac{\csc^4(c+dx)}{2a^2d} + \frac{6\csc^5(c+dx)}{5a^2d} - \frac{6\csc^7(c+dx)}{7a^2d} + \frac{\text{cs}}{}$$

Mathematica [A] time = 0.21, size = 88, normalized size = 0.61

$$\frac{\csc^2(c+dx)\left(-126\csc^8(c+dx) + 280\csc^7(c+dx) + 315\csc^6(c+dx) - 1080\csc^5(c+dx) + 1512\csc^3(c+dx) - \right.}{1260a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^11/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Csc[c + d*x]^2*(630 - 840*Csc[c + d*x] - 630*Csc[c + d*x]^2 + 1512*Csc[c +
 d*x]^3 - 1080*Csc[c + d*x]^5 + 315*Csc[c + d*x]^6 + 280*Csc[c + d*x]^7 - 1
26*Csc[c + d*x]^8))/(1260*a^2*d)
```

fricas [A] time = 0.44, size = 162, normalized size = 1.12

$$-\frac{630\cos(dx+c)^8 - 1890\cos(dx+c)^6 + 1890\cos(dx+c)^4 - 945\cos(dx+c)^2 + 8\left(105\cos(dx+c)^6 - 126\text{ c}\right.}{1260\left(a^2d\cos(dx+c)^{10} - 5a^2d\cos(dx+c)^8 + 10a^2d\cos(dx+c)^6 - 10a^2d\cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^11/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/1260*(630*cos(d*x + c)^8 - 1890*cos(d*x + c)^6 + 1890*cos(d*x + c)^4 - 9
45*cos(d*x + c)^2 + 8*(105*cos(d*x + c)^6 - 126*cos(d*x + c)^4 + 72*cos(d*x
 + c)^2 - 16)*sin(d*x + c) + 189)/(a^2*d*cos(d*x + c)^10 - 5*a^2*d*cos(d*x
+ c)^8 + 10*a^2*d*cos(d*x + c)^6 - 10*a^2*d*cos(d*x + c)^4 + 5*a^2*d*cos(d*
x + c)^2 - a^2*d)
```

giac [A] time = 1.75, size = 86, normalized size = 0.59

$$\frac{630\sin(dx+c)^8 - 840\sin(dx+c)^7 - 630\sin(dx+c)^6 + 1512\sin(dx+c)^5 - 1080\sin(dx+c)^3 + 315\sin(dx}{1260\,a^2d\sin(dx+c)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^11/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1260*(630*sin(d*x + c)^8 - 840*sin(d*x + c)^7 - 630*sin(d*x + c)^6 + 1512
*sin(d*x + c)^5 - 1080*sin(d*x + c)^3 + 315*sin(d*x + c)^2 + 280*sin(d*x +
c) - 126)/(a^2*d*sin(d*x + c)^10)
```

maple [A] time = 0.38, size = 89, normalized size = 0.61

$$\frac{\frac{6}{5\sin(dx+c)^5}-\frac{6}{7\sin(dx+c)^7}+\frac{1}{2\sin(dx+c)^2}+\frac{2}{9\sin(dx+c)^9}+\frac{1}{4\sin(dx+c)^8}-\frac{1}{2\sin(dx+c)^4}-\frac{2}{3\sin(dx+c)^3}-\frac{1}{10\sin(dx+c)^{10}}}{d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^11/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d/a^2*(6/5/sin(d*x+c)^5-6/7/sin(d*x+c)^7+1/2/sin(d*x+c)^2+2/9/sin(d*x+c)^
9+1/4/sin(d*x+c)^8-1/2/sin(d*x+c)^4-2/3/sin(d*x+c)^3-1/10/sin(d*x+c)^10)
```

maxima [A] time = 0.31, size = 86, normalized size = 0.59

$$\frac{630\,\sin(dx+c)^8-840\,\sin(dx+c)^7-630\,\sin(dx+c)^6+1512\,\sin(dx+c)^5-1080\,\sin(dx+c)^3+315\,\sin(d}{1260\,a^2 d\sin(dx+c)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^11/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/1260*(630*sin(d*x + c)^8 - 840*sin(d*x + c)^7 - 630*sin(d*x + c)^6 + 1512
*sin(d*x + c)^5 - 1080*sin(d*x + c)^3 + 315*sin(d*x + c)^2 + 280*sin(d*x +
c) - 126)/(a^2*d*sin(d*x + c)^10)
```

mupad [B] time = 6.63, size = 85, normalized size = 0.59

$$\frac{\frac{\sin(c+d\,x)^8}{2}-\frac{2\sin(c+d\,x)^7}{3}-\frac{\sin(c+d\,x)^6}{2}+\frac{6\sin(c+d\,x)^5}{5}-\frac{6\sin(c+d\,x)^3}{7}+\frac{\sin(c+d\,x)^2}{4}+\frac{2\,\sin(c+d\,x)}{9}-\frac{1}{10}}{a^2\,d\sin(c+d\,x)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^11/(a + a*sin(c + d*x))^2,x)
```

```
[Out] ((2*sin(c + d*x))/9 + sin(c + d*x)^2/4 - (6*sin(c + d*x)^3)/7 + (6*sin(c +
d*x)^5)/5 - sin(c + d*x)^6/2 - (2*sin(c + d*x)^7)/3 + sin(c + d*x)^8/2 - 1/
10)/(a^2*d*sin(c + d*x)^10)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**11/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.71 $\int \frac{\cot^{13}(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=199

$$-\frac{\csc^{12}(c+dx)}{12a^2d}+\frac{2\csc^{11}(c+dx)}{11a^2d}+\frac{3\csc^{10}(c+dx)}{10a^2d}-\frac{8\csc^9(c+dx)}{9a^2d}-\frac{\csc^8(c+dx)}{4a^2d}+\frac{12\csc^7(c+dx)}{7a^2d}-\frac{\csc^6(c+dx)}{3a^2d}$$

```
[Out] -1/2*csc(d*x+c)^2/a^2/d+2/3*csc(d*x+c)^3/a^2/d+3/4*csc(d*x+c)^4/a^2/d-8/5*c
sc(d*x+c)^5/a^2/d-1/3*csc(d*x+c)^6/a^2/d+12/7*csc(d*x+c)^7/a^2/d-1/4*csc(d*
x+c)^8/a^2/d-8/9*csc(d*x+c)^9/a^2/d+3/10*csc(d*x+c)^10/a^2/d+2/11*csc(d*x+c
)^11/a^2/d-1/12*csc(d*x+c)^12/a^2/d
```

Rubi [A] time = 0.10, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 88}

$$-\frac{\csc^{12}(c+dx)}{12a^2d}+\frac{2\csc^{11}(c+dx)}{11a^2d}+\frac{3\csc^{10}(c+dx)}{10a^2d}-\frac{8\csc^9(c+dx)}{9a^2d}-\frac{\csc^8(c+dx)}{4a^2d}+\frac{12\csc^7(c+dx)}{7a^2d}-\frac{\csc^6(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^13/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -Csc[c + d*x]^2/(2*a^2*d) + (2*Csc[c + d*x]^3)/(3*a^2*d) + (3*Csc[c + d*x]^
4)/(4*a^2*d) - (8*Csc[c + d*x]^5)/(5*a^2*d) - Csc[c + d*x]^6/(3*a^2*d) + (1
2*Csc[c + d*x]^7)/(7*a^2*d) - Csc[c + d*x]^8/(4*a^2*d) - (8*Csc[c + d*x]^9)
/(9*a^2*d) + (3*Csc[c + d*x]^10)/(10*a^2*d) + (2*Csc[c + d*x]^11)/(11*a^2*d
) - Csc[c + d*x]^12/(12*a^2*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^{13}(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^6(a+x)^4}{x^{13}}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^{10}}{x^{13}} - \frac{2a^9}{x^{12}} - \frac{3a^8}{x^{11}} + \frac{8a^7}{x^{10}} + \frac{2a^6}{x^9} - \frac{12a^5}{x^8} + \frac{2a^4}{x^7} + \frac{8a^3}{x^6} - \frac{3a^2}{x^5} - \frac{2a}{x^4} + \frac{1}{x^3}\right) dx, x, a\sin\right.}{d}$$

$$= -\frac{\csc^2(c+dx)}{2a^2d} + \frac{2\csc^3(c+dx)}{3a^2d} + \frac{3\csc^4(c+dx)}{4a^2d} - \frac{8\csc^5(c+dx)}{5a^2d} - \frac{\csc^6(c+dx)}{3a^2d} +$$

Mathematica [A] time = 0.34, size = 118, normalized size = 0.59

$$-\frac{\csc^2(c+dx)\left(1155\csc^{10}(c+dx) - 2520\csc^9(c+dx) - 4158\csc^8(c+dx) + 12320\csc^7(c+dx) + 3465\csc^6(c+\right.}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^13/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/13860*(Csc[c + d*x]^2*(6930 - 9240*Csc[c + d*x] - 10395*Csc[c + d*x]^2 +
 22176*Csc[c + d*x]^3 + 4620*Csc[c + d*x]^4 - 23760*Csc[c + d*x]^5 + 3465*C
sc[c + d*x]^6 + 12320*Csc[c + d*x]^7 - 4158*Csc[c + d*x]^8 - 2520*Csc[c + d
*x]^9 + 1155*Csc[c + d*x]^10))/(a^2*d)
```

fricas [A] time = 0.47, size = 195, normalized size = 0.98

$$\frac{6930\cos(dx+c)^{10} - 24255\cos(dx+c)^8 + 32340\cos(dx+c)^6 - 24255\cos(dx+c)^4 + 9702\cos(dx+c)^2 + 8}{13860\left(a^2d\cos(dx+c)^{12} - 6a^2d\cos(dx+c)^{10} + 15a^2d\cos(dx+c)^8 - 2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^13/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/13860*(6930*cos(d*x + c)^10 - 24255*cos(d*x + c)^8 + 32340*cos(d*x + c)^6
 - 24255*cos(d*x + c)^4 + 9702*cos(d*x + c)^2 + 8*(1155*cos(d*x + c)^8 - 18
48*cos(d*x + c)^6 + 1584*cos(d*x + c)^4 - 704*cos(d*x + c)^2 + 128)*sin(d*x
 + c) - 1617)/(a^2*d*cos(d*x + c)^12 - 6*a^2*d*cos(d*x + c)^10 + 15*a^2*d*c
os(d*x + c)^8 - 20*a^2*d*cos(d*x + c)^6 + 15*a^2*d*cos(d*x + c)^4 - 6*a^2*d
*cos(d*x + c)^2 + a^2*d)
```

giac [A] time = 1.33, size = 116, normalized size = 0.58

$$-\frac{6930\sin(dx+c)^{10} - 9240\sin(dx+c)^9 - 10395\sin(dx+c)^8 + 22176\sin(dx+c)^7 + 4620\sin(dx+c)^6 - 23}{13860\,a^2d\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^13/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/13860*(6930*sin(d*x + c)^10 - 9240*sin(d*x + c)^9 - 10395*sin(d*x + c)^8
 + 22176*sin(d*x + c)^7 + 4620*sin(d*x + c)^6 - 23760*sin(d*x + c)^5 + 3465
*sin(d*x + c)^4 + 12320*sin(d*x + c)^3 - 4158*sin(d*x + c)^2 - 2520*sin(d*x
 + c) + 1155)/(a^2*d*sin(d*x + c)^12)
```

maple [A] time = 0.42, size = 119, normalized size = 0.60

$$\frac{-\frac{1}{3\sin(dx+c)^6}-\frac{8}{5\sin(dx+c)^5}-\frac{1}{12\sin(dx+c)^{12}}+\frac{12}{7\sin(dx+c)^7}-\frac{1}{2\sin(dx+c)^2}-\frac{8}{9\sin(dx+c)^9}-\frac{1}{4\sin(dx+c)^8}+\frac{3}{4\sin(dx+c)^4}+\frac{2}{11\sin(dx+c)^{11}}}{d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^13/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d/a^2*(-1/3/sin(d*x+c)^6-8/5/sin(d*x+c)^5-1/12/sin(d*x+c)^12+12/7/sin(d*x
+c)^7-1/2/sin(d*x+c)^2-8/9/sin(d*x+c)^9-1/4/sin(d*x+c)^8+3/4/sin(d*x+c)^4+2
/11/sin(d*x+c)^11+2/3/sin(d*x+c)^3+3/10/sin(d*x+c)^10)
```

maxima [A] time = 0.30, size = 116, normalized size = 0.58

$$-\frac{6930\sin(dx+c)^{10}-9240\sin(dx+c)^{9}-10395\sin(dx+c)^{8}+22176\sin(dx+c)^{7}+4620\sin(dx+c)^{6}-2}{13860\,a^2d\,s}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^13/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/13860*(6930*sin(d*x + c)^10 - 9240*sin(d*x + c)^9 - 10395*sin(d*x + c)^8
 + 22176*sin(d*x + c)^7 + 4620*sin(d*x + c)^6 - 23760*sin(d*x + c)^5 + 3465
*sin(d*x + c)^4 + 12320*sin(d*x + c)^3 - 4158*sin(d*x + c)^2 - 2520*sin(d*x
 + c) + 1155)/(a^2*d*sin(d*x + c)^12)
```

mupad [B] time = 6.85, size = 116, normalized size = 0.58

$$-\frac{\frac{\sin(c+d\,x)^{10}}{2}-\frac{2\sin(c+d\,x)^9}{3}-\frac{3\sin(c+d\,x)^8}{4}+\frac{8\sin(c+d\,x)^7}{5}+\frac{\sin(c+d\,x)^6}{3}-\frac{12\sin(c+d\,x)^5}{7}+\frac{\sin(c+d\,x)^4}{4}+\frac{8\sin(c+d\,x)^3}{9}-\frac{3\sin(c+d\,x}{10}}{a^2\,d\sin(c+d\,x)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^13/(a + a*sin(c + d*x))^2,x)
```

```
[Out] -((8*sin(c + d*x)^3)/9 - (3*sin(c + d*x)^2)/10 - (2*sin(c + d*x))/11 + sin(
c + d*x)^4/4 - (12*sin(c + d*x)^5)/7 + sin(c + d*x)^6/3 + (8*sin(c + d*x)^7
```

```
)/5 - (3*sin(c + d*x)^8)/4 - (2*sin(c + d*x)^9)/3 + sin(c + d*x)^10/2 + 1/1
2)/(a^2*d*sin(c + d*x)^12)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**13/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.72 $\int \frac{\tan^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=171

$$-\frac{1}{32d\left(a^3-a^3\sin(c+dx)\right)}-\frac{5}{128d\left(a^3\sin(c+dx)+a^3\right)}+\frac{\tanh^{-1}(\sin(c+dx))}{128a^3d}+\frac{a^2}{40d(a\sin(c+dx)+a)^5}-\frac{}{64d(a\,\mathrm{s}}$$

```
[Out] 1/128*arctanh(sin(d*x+c))/a^3/d+1/128/a/d/(a-a*sin(d*x+c))^2+1/40*a^2/d/(a+
a*sin(d*x+c))^5-7/64*a/d/(a+a*sin(d*x+c))^4+1/6/d/(a+a*sin(d*x+c))^3-5/64/a
/d/(a+a*sin(d*x+c))^2-1/32/d/(a^3-a^3*sin(d*x+c))-5/128/d/(a^3+a^3*sin(d*x+
c))
```

Rubi [A] time = 0.12, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2707, 88, 206}

$$\frac{a^2}{40d(a\sin(c+dx)+a)^5}-\frac{1}{32d\left(a^3-a^3\sin(c+dx)\right)}-\frac{5}{128d\left(a^3\sin(c+dx)+a^3\right)}+\frac{\tanh^{-1}(\sin(c+dx))}{128a^3d}-\frac{}{64d(a\sin}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(128*a^3*d) + 1/(128*a*d*(a - a*Sin[c + d*x])^2) + a^
2/(40*d*(a + a*Sin[c + d*x])^5) - (7*a)/(64*d*(a + a*Sin[c + d*x])^4) + 1/(
6*d*(a + a*Sin[c + d*x])^3) - 5/(64*a*d*(a + a*Sin[c + d*x])^2) - 1/(32*d*(
a^3 - a^3*Sin[c + d*x])) - 5/(128*d*(a^3 + a^3*Sin[c + d*x]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
```

```
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(a-x)^3(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{64a(a-x)^3} - \frac{1}{32a^2(a-x)^2} - \frac{a^2}{8(a+x)^6} + \frac{7a}{16(a+x)^5} - \frac{1}{2(a+x)^4} + \frac{5}{32a(a+x)^3} + \frac{5}{128a^2(a+x)^2}\right.\right.}{d}$$

$$= \frac{1}{128ad(a-a\sin(c+dx))^2} + \frac{a^2}{40d(a+a\sin(c+dx))^5} - \frac{7a}{64d(a+a\sin(c+dx))^4} + \frac{}{6d}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{128a^3d} + \frac{1}{128ad(a-a\sin(c+dx))^2} + \frac{a^2}{40d(a+a\sin(c+dx))^5} - \frac{}{64d(a}$$

Mathematica [A] time = 0.68, size = 102, normalized size = 0.60

$$\frac{15\tanh^{-1}(\sin(c+dx)) - \frac{15\sin^6(c+dx)+45\sin^5(c+dx)-620\sin^4(c+dx)-540\sin^3(c+dx)+157\sin^2(c+dx)+351\sin(c+dx)+112}{(\sin(c+dx)-1)^2(\sin(c+dx)+1)^5}}{1920a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (15*ArcTanh[Sin[c + d*x]] - (112 + 351*Sin[c + d*x] + 157*Sin[c + d*x]^2 -
540*Sin[c + d*x]^3 - 620*Sin[c + d*x]^4 + 45*Sin[c + d*x]^5 + 15*Sin[c + d*
x]^6)/((-1 + Sin[c + d*x])^2*(1 + Sin[c + d*x])^5))/(1920*a^3*d)
```

fricas [A] time = 0.46, size = 248, normalized size = 1.45

$$-\frac{30\cos(dx+c)^6 + 1150\cos(dx+c)^4 - 2076\cos(dx+c)^2 - 15\left(3\cos(dx+c)^6 - 4\cos(dx+c)^4 + \left(\cos(dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/3840*(30*cos(d*x + c)^6 + 1150*cos(d*x + c)^4 - 2076*cos(d*x + c)^2 - 15
*(3*cos(d*x + c)^6 - 4*cos(d*x + c)^4 + (cos(d*x + c)^6 - 4*cos(d*x + c)^4)
```

```
*sin(d*x + c))*log(sin(d*x + c) + 1) + 15*(3*cos(d*x + c)^6 - 4*cos(d*x + c
)^4 + (cos(d*x + c)^6 - 4*cos(d*x + c)^4)*sin(d*x + c))*log(-sin(d*x + c) +
 1) - 18*(5*cos(d*x + c)^4 + 50*cos(d*x + c)^2 - 16)*sin(d*x + c) + 672)/(3
*a^3*d*cos(d*x + c)^6 - 4*a^3*d*cos(d*x + c)^4 + (a^3*d*cos(d*x + c)^6 - 4*
a^3*d*cos(d*x + c)^4)*sin(d*x + c))
```

giac [A] time = 24.62, size = 136, normalized size = 0.80

$$\frac{\frac{60\log(|\sin(dx+c)+1|)}{a^3} - \frac{60\log(|\sin(dx+c)-1|)}{a^3} + \frac{30\left(3\sin(dx+c)^2+10\sin(dx+c)-9\right)}{a^3(\sin(dx+c)-1)^2} - \frac{137\sin(dx+c)^5+1285\sin(dx+c)^4+4970\sin(dx+c)^3+601}{a^3(\sin(dx+c)+1)^5}}{15360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/15360*(60*log(abs(sin(d*x + c) + 1))/a^3 - 60*log(abs(sin(d*x + c) - 1))/
a^3 + 30*(3*sin(d*x + c)^2 + 10*sin(d*x + c) - 9)/(a^3*(sin(d*x + c) - 1)^2
) - (137*sin(d*x + c)^5 + 1285*sin(d*x + c)^4 + 4970*sin(d*x + c)^3 + 6010*
sin(d*x + c)^2 + 3245*sin(d*x + c) + 673)/(a^3*(sin(d*x + c) + 1)^5))/d
```

maple [A] time = 0.26, size = 162, normalized size = 0.95

$$\frac{1}{128a^3d\left(\sin\left(dx+c\right)-1\right)^2}+\frac{1}{32a^3d\left(\sin\left(dx+c\right)-1\right)}-\frac{\ln\left(\sin\left(dx+c\right)-1\right)}{256a^3d}+\frac{1}{40a^3d\left(1+\sin\left(dx+c\right)\right)^5}-\frac{}{64a^3d\,(1+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/128/a^3/d/(sin(d*x+c)-1)^2+1/32/a^3/d/(sin(d*x+c)-1)-1/256/a^3/d*ln(sin(d
*x+c)-1)+1/40/a^3/d/(1+sin(d*x+c))^5-7/64/a^3/d/(1+sin(d*x+c))^4+1/6/a^3/d/
(1+sin(d*x+c))^3-5/64/a^3/d/(1+sin(d*x+c))^2-5/128/a^3/d/(1+sin(d*x+c))+1/2
56*ln(1+sin(d*x+c))/a^3/d
```

maxima [A] time = 0.32, size = 188, normalized size = 1.10

$$-\frac{\frac{2\left(15\sin(dx+c)^6+45\sin(dx+c)^5-620\sin(dx+c)^4-540\sin(dx+c)^3+157\sin(dx+c)^2+351\sin(dx+c)+112\right)}{a^3\sin(dx+c)^7+3a^3\sin(dx+c)^6+a^3\sin(dx+c)^5-5a^3\sin(dx+c)^4-5a^3\sin(dx+c)^3+a^3\sin(dx+c)^2+3a^3\sin(dx+c)+a^3} - \frac{15\log(\sin(dx+c)+1)}{a^3} + 15}{3840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/3840*(2*(15*sin(d*x + c)^6 + 45*sin(d*x + c)^5 - 620*sin(d*x + c)^4 - 54
0*sin(d*x + c)^3 + 157*sin(d*x + c)^2 + 351*sin(d*x + c) + 112)/(a^3*sin(d*
```

```
x + c)^7 + 3*a^3*sin(d*x + c)^6 + a^3*sin(d*x + c)^5 - 5*a^3*sin(d*x + c)^4
 - 5*a^3*sin(d*x + c)^3 + a^3*sin(d*x + c)^2 + 3*a^3*sin(d*x + c) + a^3) -
15*log(sin(d*x + c) + 1)/a^3 + 15*log(sin(d*x + c) - 1)/a^3)/d
```

mupad [B] time = 10.05, size = 418, normalized size = 2.44

$$\frac{-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{64}-\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{32}-\frac{17\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{96}+\frac{\tan\left(\frac{c}{2}+\right.}{32}}{d\left(a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+6\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}+11\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-4\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}-39\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + a*sin(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^4/32 - (3*tan(c/2 + (d*x)/2)^2)/32 - (17*tan(c/2 + (d*x
)/2)^3)/96 - tan(c/2 + (d*x)/2)/64 + (527*tan(c/2 + (d*x)/2)^5)/960 + (901*
tan(c/2 + (d*x)/2)^6)/80 + (711*tan(c/2 + (d*x)/2)^7)/80 + (901*tan(c/2 + (
d*x)/2)^8)/80 + (527*tan(c/2 + (d*x)/2)^9)/960 + tan(c/2 + (d*x)/2)^10/32 -
 (17*tan(c/2 + (d*x)/2)^11)/96 - (3*tan(c/2 + (d*x)/2)^12)/32 - tan(c/2 + (
d*x)/2)^13/64)/(d*(11*a^3*tan(c/2 + (d*x)/2)^2 - 4*a^3*tan(c/2 + (d*x)/2)^3
 - 39*a^3*tan(c/2 + (d*x)/2)^4 - 38*a^3*tan(c/2 + (d*x)/2)^5 + 27*a^3*tan(c
/2 + (d*x)/2)^6 + 72*a^3*tan(c/2 + (d*x)/2)^7 + 27*a^3*tan(c/2 + (d*x)/2)^8
 - 38*a^3*tan(c/2 + (d*x)/2)^9 - 39*a^3*tan(c/2 + (d*x)/2)^10 - 4*a^3*tan(c
/2 + (d*x)/2)^11 + 11*a^3*tan(c/2 + (d*x)/2)^12 + 6*a^3*tan(c/2 + (d*x)/2)^
13 + a^3*tan(c/2 + (d*x)/2)^14 + a^3 + 6*a^3*tan(c/2 + (d*x)/2))) + atanh(t
an(c/2 + (d*x)/2))/(64*a^3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^5(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**5/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d
*x) + 1), x)/a**3
```

3.73 $\int \frac{\tan^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=126

$$\frac{1}{32d\left(a^3-a^3\sin(c+dx)\right)}+\frac{1}{16d\left(a^3\sin(c+dx)+a^3\right)}-\frac{\tanh^{-1}(\sin(c+dx))}{32a^3d}+\frac{a}{16d(a\sin(c+dx)+a)^4}-\frac{}{6d(a\sin(c}$$

```
[Out] -1/32*arctanh(sin(d*x+c))/a^3/d+1/16*a/d/(a+a*sin(d*x+c))^4-1/6/d/(a+a*sin(
d*x+c))^3+3/32/a/d/(a+a*sin(d*x+c))^2+1/32/d/(a^3-a^3*sin(d*x+c))+1/16/d/(a
^3+a^3*sin(d*x+c))
```

Rubi [A] time = 0.09, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2707, 88, 206}

$$\frac{1}{32d\left(a^3-a^3\sin(c+dx)\right)}+\frac{1}{16d\left(a^3\sin(c+dx)+a^3\right)}-\frac{\tanh^{-1}(\sin(c+dx))}{32a^3d}+\frac{a}{16d(a\sin(c+dx)+a)^4}-\frac{}{6d(a\sin(c}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -ArcTanh[Sin[c + d*x]]/(32*a^3*d) + a/(16*d*(a + a*Sin[c + d*x])^4) - 1/(6*
d*(a + a*Sin[c + d*x])^3) + 3/(32*a*d*(a + a*Sin[c + d*x])^2) + 1/(32*d*(a^
3 - a^3*Sin[c + d*x])) + 1/(16*d*(a^3 + a^3*Sin[c + d*x]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
```

```
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{(a-x)^2(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{32a^2(a-x)^2} - \frac{a}{4(a+x)^5} + \frac{1}{2(a+x)^4} - \frac{3}{16a(a+x)^3} - \frac{1}{16a^2(a+x)^2} - \frac{1}{32a^2\left(a^2-x^2\right)}\right) dx, x, a\sin\ldots\right.}{d}$$

$$= \frac{a}{16d(a+a\sin(c+dx))^4} - \frac{1}{6d(a+a\sin(c+dx))^3} + \frac{3}{32ad(a+a\sin(c+dx))^2} + \frac{\ldots}{32d\,(\ldots}$$

$$= -\frac{\tanh^{-1}(\sin(c+dx))}{32a^3d} + \frac{a}{16d(a+a\sin(c+dx))^4} - \frac{1}{6d(a+a\sin(c+dx))^3} + \frac{\ldots}{32ad(a\ldots}$$

Mathematica [A] time = 0.36, size = 82, normalized size = 0.65

$$-\frac{-\frac{3}{1-\sin(c+dx)} - \frac{6}{\sin(c+dx)+1} - \frac{9}{(\sin(c+dx)+1)^2} + \frac{16}{(\sin(c+dx)+1)^3} - \frac{6}{(\sin(c+dx)+1)^4} + 3\tanh^{-1}(\sin(c+dx))}{96a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/96*(3*ArcTanh[Sin[c + d*x]] - 3/(1 - Sin[c + d*x]) - 6/(1 + Sin[c + d*x]
)^4 + 16/(1 + Sin[c + d*x])^3 - 9/(1 + Sin[c + d*x])^2 - 6/(1 + Sin[c + d*x
]))/(a^3*d)
```

fricas [A] time = 0.45, size = 226, normalized size = 1.79

$$\frac{6\cos(dx+c)^4 + 38\cos(dx+c)^2 - 3\left(3\cos(dx+c)^4 - 4\cos(dx+c)^2 + \left(\cos(dx+c)^4 - 4\cos(dx+c)^2\right)\sin(\ldots\right.}{192\left(3a^3d\cos(\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/192*(6*cos(d*x + c)^4 + 38*cos(d*x + c)^2 - 3*(3*cos(d*x + c)^4 - 4*cos(d
*x + c)^2 + (cos(d*x + c)^4 - 4*cos(d*x + c)^2)*sin(d*x + c))*log(sin(d*x +
 c) + 1) + 3*(3*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + (cos(d*x + c)^4 - 4*cos
```

```
(d*x + c)^2)*sin(d*x + c))*log(-sin(d*x + c) + 1) - 18*(cos(d*x + c)^2 + 2)
*sin(d*x + c) - 60)/(3*a^3*d*cos(d*x + c)^4 - 4*a^3*d*cos(d*x + c)^2 + (a^3
*d*cos(d*x + c)^4 - 4*a^3*d*cos(d*x + c)^2)*sin(d*x + c))
```

giac [A] time = 1.91, size = 114, normalized size = 0.90

$$-\frac{\frac{12\,\log\left(\left|\sin\left(dx+c\right)+1\right|\right)}{a^{3}}-\frac{12\,\log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{a^{3}}+\frac{12\,\left(\sin\left(dx+c\right)+1\right)}{a^{3}\left(\sin\left(dx+c\right)-1\right)}-\frac{25\,\sin\left(dx+c\right)^{4}+148\,\sin\left(dx+c\right)^{3}+366\,\sin\left(dx+c\right)^{2}+260\,\sin\left(dx+c\right)+65}{a^{3}\left(\sin\left(dx+c\right)+1\right)^{4}}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/768*(12*log(abs(sin(d*x + c) + 1))/a^3 - 12*log(abs(sin(d*x + c) - 1))/a
^3 + 12*(sin(d*x + c) + 1)/(a^3*(sin(d*x + c) - 1)) - (25*sin(d*x + c)^4 +
148*sin(d*x + c)^3 + 366*sin(d*x + c)^2 + 260*sin(d*x + c) + 65)/(a^3*(sin(
d*x + c) + 1)^4))/d
```

maple [A] time = 0.26, size = 126, normalized size = 1.00

$$-\frac{1}{32a^{3}d\left(\sin\left(dx+c\right)-1\right)}+\frac{\ln\left(\sin\left(dx+c\right)-1\right)}{64a^{3}d}+\frac{1}{16a^{3}d\left(1+\sin\left(dx+c\right)\right)^{4}}-\frac{1}{6a^{3}d\left(1+\sin\left(dx+c\right)\right)^{3}}+\frac{}{32a^{3}d\left(1+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/32/a^3/d/(sin(d*x+c)-1)+1/64/a^3/d*ln(sin(d*x+c)-1)+1/16/a^3/d/(1+sin(d*
x+c))^4-1/6/a^3/d/(1+sin(d*x+c))^3+3/32/a^3/d/(1+sin(d*x+c))^2+1/16/a^3/d/(
1+sin(d*x+c))-1/64*ln(1+sin(d*x+c))/a^3/d
```

maxima [A] time = 0.30, size = 146, normalized size = 1.16

$$\frac{\frac{2\left(3\,\sin\left(dx+c\right)^{4}+9\,\sin\left(dx+c\right)^{3}-25\,\sin\left(dx+c\right)^{2}-27\,\sin\left(dx+c\right)-8\right)}{a^{3}\sin\left(dx+c\right)^{5}+3\,a^{3}\sin\left(dx+c\right)^{4}+2\,a^{3}\sin\left(dx+c\right)^{3}-2\,a^{3}\sin\left(dx+c\right)^{2}-3\,a^{3}\sin\left(dx+c\right)-a^{3}}-\frac{3\,\log\left(\sin\left(dx+c\right)+1\right)}{a^{3}}+\frac{3\,\log\left(\sin\left(dx+c\right)-1\right)}{a^{3}}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/192*(2*(3*sin(d*x + c)^4 + 9*sin(d*x + c)^3 - 25*sin(d*x + c)^2 - 27*sin(
d*x + c) - 8)/(a^3*sin(d*x + c)^5 + 3*a^3*sin(d*x + c)^4 + 2*a^3*sin(d*x +
c)^3 - 2*a^3*sin(d*x + c)^2 - 3*a^3*sin(d*x + c) - a^3) - 3*log(sin(d*x + c
) + 1)/a^3 + 3*log(sin(d*x + c) - 1)/a^3)/d
```

mupad [B] time = 9.93, size = 302, normalized size = 2.40

$$\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{16}+\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{8}+\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{6}+\frac{37\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{8}+\frac{101\tan\left(\frac{c}{2}+\right.}{24}}{d\left(a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+6\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+13\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+8\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-14\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-28\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + a*sin(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)/16 + (3*tan(c/2 + (d*x)/2)^2)/8 + (5*tan(c/2 + (d*x)/2)
^3)/6 + (37*tan(c/2 + (d*x)/2)^4)/8 + (101*tan(c/2 + (d*x)/2)^5)/24 + (37*t
an(c/2 + (d*x)/2)^6)/8 + (5*tan(c/2 + (d*x)/2)^7)/6 + (3*tan(c/2 + (d*x)/2)
^8)/8 + tan(c/2 + (d*x)/2)^9/16)/(d*(13*a^3*tan(c/2 + (d*x)/2)^2 + 8*a^3*ta
n(c/2 + (d*x)/2)^3 - 14*a^3*tan(c/2 + (d*x)/2)^4 - 28*a^3*tan(c/2 + (d*x)/2
)^5 - 14*a^3*tan(c/2 + (d*x)/2)^6 + 8*a^3*tan(c/2 + (d*x)/2)^7 + 13*a^3*tan
(c/2 + (d*x)/2)^8 + 6*a^3*tan(c/2 + (d*x)/2)^9 + a^3*tan(c/2 + (d*x)/2)^10
+ a^3 + 6*a^3*tan(c/2 + (d*x)/2))) - atanh(tan(c/2 + (d*x)/2))/(16*a^3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^3(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**3/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d
*x) + 1), x)/a**3
```

3.74 $\int \frac{\tan(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=82

$$-\frac{1}{8d\left(a^3\sin(c+dx)+a^3\right)}+\frac{\tanh^{-1}(\sin(c+dx))}{8a^3d}-\frac{1}{8ad(a\sin(c+dx)+a)^2}+\frac{1}{6d(a\sin(c+dx)+a)^3}$$

```
[Out] 1/8*arctanh(sin(d*x+c))/a^3/d+1/6/d/(a+a*sin(d*x+c))^3-1/8/a/d/(a+a*sin(d*x
+c))^2-1/8/d/(a^3+a^3*sin(d*x+c))
```

Rubi [A] time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2707, 77, 206}

$$-\frac{1}{8d\left(a^3\sin(c+dx)+a^3\right)}+\frac{\tanh^{-1}(\sin(c+dx))}{8a^3d}-\frac{1}{8ad(a\sin(c+dx)+a)^2}+\frac{1}{6d(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(8*a^3*d) + 1/(6*d*(a + a*Sin[c + d*x])^3) - 1/(8*a*d
*(a + a*Sin[c + d*x])^2) - 1/(8*d*(a^3 + a^3*Sin[c + d*x]))
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
```

```
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tan(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(a-x)(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{2(a+x)^4}+\frac{1}{4a(a+x)^3}+\frac{1}{8a^2(a+x)^2}+\frac{1}{8a^2\left(a^2-x^2\right)}\right)dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{1}{6d(a+a\sin(c+dx))^3}-\frac{1}{8ad(a+a\sin(c+dx))^2}-\frac{1}{8d\left(a^3+a^3\sin(c+dx)\right)}+\frac{\text{Subst}}{}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{8a^3d}+\frac{1}{6d(a+a\sin(c+dx))^3}-\frac{1}{8ad(a+a\sin(c+dx))^2}-\frac{}{8d\left(a^3+a\right.}$$

Mathematica [A] time = 0.15, size = 52, normalized size = 0.63

$$\frac{3\tanh^{-1}(\sin(c+dx))-\frac{3\sin^2(c+dx)+9\sin(c+dx)+2}{(\sin(c+dx)+1)^3}}{24a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]] - (2 + 9*Sin[c + d*x] + 3*Sin[c + d*x]^2)/(1 + Sin
[c + d*x])^3)/(24*a^3*d)
```

fricas [B] time = 0.44, size = 154, normalized size = 1.88

$$-\frac{6\cos(dx+c)^2-3\left(3\cos(dx+c)^2+\left(\cos(dx+c)^2-4\right)\sin(dx+c)-4\right)\log(\sin(dx+c)+1)+3\left(3\cos(dx\right.}{48\left(3a^3d\cos(dx+c)^2-4a^3d+\left(a^3d\cos(dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/48*(6*cos(d*x + c)^2 - 3*(3*cos(d*x + c)^2 + (cos(d*x + c)^2 - 4)*sin(d*
x + c) - 4)*log(sin(d*x + c) + 1) + 3*(3*cos(d*x + c)^2 + (cos(d*x + c)^2 -
 4)*sin(d*x + c) - 4)*log(-sin(d*x + c) + 1) - 18*sin(d*x + c) - 10)/(3*a^3
```

```
*d*cos(d*x + c)^2 - 4*a^3*d + (a^3*d*cos(d*x + c)^2 - 4*a^3*d)*sin(d*x + c)
)
```

giac [A] time = 0.79, size = 81, normalized size = 0.99

$$\frac{\frac{6\log\left(|\sin(dx+c)+1|\right)}{a^3}-\frac{6\log\left(|\sin(dx+c)-1|\right)}{a^3}-\frac{11\sin(dx+c)^3+45\sin(dx+c)^2+69\sin(dx+c)+19}{a^3(\sin(dx+c)+1)^3}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/96*(6*log(abs(sin(d*x + c) + 1))/a^3 - 6*log(abs(sin(d*x + c) - 1))/a^3 -
 (11*sin(d*x + c)^3 + 45*sin(d*x + c)^2 + 69*sin(d*x + c) + 19)/(a^3*(sin(d
*x + c) + 1)^3))/d
```

maple [A] time = 0.24, size = 90, normalized size = 1.10

$$-\frac{\ln\left(\sin\left(dx+c\right)-1\right)}{16a^3d}+\frac{1}{6a^3d\left(1+\sin\left(dx+c\right)\right)^3}-\frac{1}{8a^3d\left(1+\sin\left(dx+c\right)\right)^2}-\frac{1}{8a^3d\left(1+\sin\left(dx+c\right)\right)}+\frac{\ln\left(1+\sin\left(d\right.\right.}{16a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/16/a^3/d*ln(sin(d*x+c)-1)+1/6/a^3/d/(1+sin(d*x+c))^3-1/8/a^3/d/(1+sin(d*
x+c))^2-1/8/a^3/d/(1+sin(d*x+c))+1/16*ln(1+sin(d*x+c))/a^3/d
```

maxima [A] time = 0.30, size = 98, normalized size = 1.20

$$-\frac{\frac{2\left(3\sin(dx+c)^2+9\sin(dx+c)+2\right)}{a^3\sin(dx+c)^3+3a^3\sin(dx+c)^2+3a^3\sin(dx+c)+a^3}-\frac{3\log(\sin(dx+c)+1)}{a^3}+\frac{3\log(\sin(dx+c)-1)}{a^3}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/48*(2*(3*sin(d*x + c)^2 + 9*sin(d*x + c) + 2)/(a^3*sin(d*x + c)^3 + 3*a^
3*sin(d*x + c)^2 + 3*a^3*sin(d*x + c) + a^3) - 3*log(sin(d*x + c) + 1)/a^3
+ 3*log(sin(d*x + c) - 1)/a^3)/d
```

mupad [B] time = 8.82, size = 186, normalized size = 2.27

$$\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,a^3\,d}+\frac{-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{2}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}-}{d\,\left(a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+15\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+20\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] atanh(tan(c/2 + (d*x)/2))/(4*a^3*d) + (tan(c/2 + (d*x)/2)^2/2 - tan(c/2 + (
d*x)/2)/4 + tan(c/2 + (d*x)/2)^3/6 + tan(c/2 + (d*x)/2)^4/2 - tan(c/2 + (d*
x)/2)^5/4)/(d*(15*a^3*tan(c/2 + (d*x)/2)^2 + 20*a^3*tan(c/2 + (d*x)/2)^3 +
15*a^3*tan(c/2 + (d*x)/2)^4 + 6*a^3*tan(c/2 + (d*x)/2)^5 + a^3*tan(c/2 + (d
*x)/2)^6 + a^3 + 6*a^3*tan(c/2 + (d*x)/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d*x)
 + 1), x)/a**3
```

3.75 $\int \frac{\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=74

$$\frac{1}{d\left(a^3\sin(c+dx)+a^3\right)}+\frac{\log(\sin(c+dx))}{a^3d}-\frac{\log(\sin(c+dx)+1)}{a^3d}+\frac{1}{2ad(a\sin(c+dx)+a)^2}$$

```
[Out] ln(sin(d*x+c))/a^3/d-ln(1+sin(d*x+c))/a^3/d+1/2/a/d/(a+a*sin(d*x+c))^2+1/d/
(a^3+a^3*sin(d*x+c))
```

Rubi [A] time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2707, 44}

$$\frac{1}{d\left(a^3\sin(c+dx)+a^3\right)}+\frac{\log(\sin(c+dx))}{a^3d}-\frac{\log(\sin(c+dx)+1)}{a^3d}+\frac{1}{2ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] Log[Sin[c + d*x]]/(a^3*d) - Log[1 + Sin[c + d*x]]/(a^3*d) + 1/(2*a*d*(a + a
*Sin[c + d*x])^2) + 1/(d*(a^3 + a^3*Sin[c + d*x]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^3x} - \frac{1}{a(a+x)^3} - \frac{1}{a^2(a+x)^2} - \frac{1}{a^3(a+x)}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\log(\sin(c+dx))}{a^3d} - \frac{\log(1+\sin(c+dx))}{a^3d} + \frac{1}{2ad(a+a\sin(c+dx))^2} + \frac{1}{d\left(a^3+a^3\sin(\right.}$$

Mathematica [A] time = 0.18, size = 52, normalized size = 0.70

$$\frac{\frac{2\sin(c+dx)+3}{(\sin(c+dx)+1)^2} + 2\log(\sin(c+dx)) - 2\log(\sin(c+dx)+1)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (2*Log[Sin[c + d*x]] - 2*Log[1 + Sin[c + d*x]] + (3 + 2*Sin[c + d*x])/(1 +
Sin[c + d*x])^2)/(2*a^3*d)
```

fricas [A] time = 0.45, size = 104, normalized size = 1.41

$$\frac{2\left(\cos(dx+c)^2 - 2\sin(dx+c) - 2\right)\log\left(\frac{1}{2}\sin(dx+c)\right) - 2\left(\cos(dx+c)^2 - 2\sin(dx+c) - 2\right)\log(\sin(dx+c}{2\left(a^3d\cos(dx+c)^2 - 2a^3d\sin(dx+c) - 2a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(cos(d*x + c)^2 - 2*sin(d*x + c) - 2)*log(1/2*sin(d*x + c)) - 2*(cos
(d*x + c)^2 - 2*sin(d*x + c) - 2)*log(sin(d*x + c) + 1) - 2*sin(d*x + c) -
3)/(a^3*d*cos(d*x + c)^2 - 2*a^3*d*sin(d*x + c) - 2*a^3*d)
```

giac [A] time = 0.64, size = 59, normalized size = 0.80

$$-\frac{\frac{2\log(|\sin(dx+c)+1|)}{a^3} - \frac{2\log(|\sin(dx+c)|)}{a^3} - \frac{2\sin(dx+c)+3}{a^3(\sin(dx+c)+1)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*log(abs(sin(d*x + c) + 1))/a^3 - 2*log(abs(sin(d*x + c)))/a^3 - (2*
sin(d*x + c) + 3)/(a^3*(sin(d*x + c) + 1)^2))/d
```

maple [A] time = 0.17, size = 68, normalized size = 0.92

$$\frac{\ln\left(\sin\left(dx+c\right)\right)}{a^{3}d}+\frac{1}{2a^{3}d\left(1+\sin\left(dx+c\right)\right)^{2}}+\frac{1}{a^{3}d\left(1+\sin\left(dx+c\right)\right)}-\frac{\ln\left(1+\sin\left(dx+c\right)\right)}{a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] ln(sin(d*x+c))/a^3/d+1/2/a^3/d/(1+sin(d*x+c))^2+1/a^3/d/(1+sin(d*x+c))-ln(1
+sin(d*x+c))/a^3/d
```

maxima [A] time = 0.30, size = 72, normalized size = 0.97

$$\frac{\frac{2\sin\left(dx+c\right)+3}{a^{3}\sin\left(dx+c\right)^{2}+2a^{3}\sin\left(dx+c\right)+a^{3}}-\frac{2\log\left(\sin\left(dx+c\right)+1\right)}{a^{3}}+\frac{2\log\left(\sin\left(dx+c\right)\right)}{a^{3}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((2*sin(d*x + c) + 3)/(a^3*sin(d*x + c)^2 + 2*a^3*sin(d*x + c) + a^3) -
 2*log(sin(d*x + c) + 1)/a^3 + 2*log(sin(d*x + c))/a^3)/d
```

mupad [B] time = 6.65, size = 148, normalized size = 2.00

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}-\frac{4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+6\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+4\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a^3\right)}-\frac{2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a^3*d) - (4*tan(c/2 + (d*x)/2) + 6*tan(c/2 + (d*x)
/2)^2 + 4*tan(c/2 + (d*x)/2)^3)/(d*(6*a^3*tan(c/2 + (d*x)/2)^2 + 4*a^3*tan(
c/2 + (d*x)/2)^3 + a^3*tan(c/2 + (d*x)/2)^4 + a^3 + 4*a^3*tan(c/2 + (d*x)/2
))) - (2*log(tan(c/2 + (d*x)/2) + 1))/(a^3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot\left(c+dx\right)}{\sin^{3}\left(c+dx\right)+3\sin^{2}\left(c+dx\right)+3\sin\left(c+dx\right)+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d*x)
 + 1), x)/a**3
```

3.76 $\int \frac{\cot^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=86

$$\frac{2}{d\left(a^3\sin(c+dx)+a^3\right)}-\frac{\csc^2(c+dx)}{2a^3d}+\frac{3\csc(c+dx)}{a^3d}+\frac{5\log(\sin(c+dx))}{a^3d}-\frac{5\log(\sin(c+dx)+1)}{a^3d}$$

```
[Out] 3*csc(d*x+c)/a^3/d-1/2*csc(d*x+c)^2/a^3/d+5*ln(sin(d*x+c))/a^3/d-5*ln(1+sin
(d*x+c))/a^3/d+2/d/(a^3+a^3*sin(d*x+c))
```

Rubi [A] time = 0.07, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 77}

$$\frac{2}{d\left(a^3\sin(c+dx)+a^3\right)}-\frac{\csc^2(c+dx)}{2a^3d}+\frac{3\csc(c+dx)}{a^3d}+\frac{5\log(\sin(c+dx))}{a^3d}-\frac{5\log(\sin(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (3*Csc[c + d*x])/(a^3*d) - Csc[c + d*x]^2/(2*a^3*d) + (5*Log[Sin[c + d*x]])
/(a^3*d) - (5*Log[1 + Sin[c + d*x]])/(a^3*d) + 2/(d*(a^3 + a^3*Sin[c + d*x]
))
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{a-x}{x^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^3} - \frac{3}{a^2x^2} + \frac{5}{a^3x} - \frac{2}{a^2(a+x)^2} - \frac{5}{a^3(a+x)}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{3\csc(c+dx)}{a^3d} - \frac{\csc^2(c+dx)}{2a^3d} + \frac{5\log(\sin(c+dx))}{a^3d} - \frac{5\log(1+\sin(c+dx))}{a^3d} + \frac{}{d\left(a^3\right.}$$

Mathematica [A] time = 0.19, size = 61, normalized size = 0.71

$$\frac{\frac{4}{\sin(c+dx)+1} - \csc^2(c+dx) + 6\csc(c+dx) + 10\log(\sin(c+dx)) - 10\log(\sin(c+dx)+1)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (6*Csc[c + d*x] - Csc[c + d*x]^2 + 10*Log[Sin[c + d*x]] - 10*Log[1 + Sin[c
+ d*x]] + 4/(1 + Sin[c + d*x]))/(2*a^3*d)
```

fricas [A] time = 0.45, size = 147, normalized size = 1.71

$$\frac{10\cos(dx+c)^2 + 10\left(\cos(dx+c)^2 + \left(\cos(dx+c)^2 - 1\right)\sin(dx+c) - 1\right)\log\left(\frac{1}{2}\sin(dx+c)\right) - 10\left(\cos(dx+c)^2\right.}{2\left(a^3d\cos(dx+c)^2 - a^3d + \left(a^3d\cos(dx+c)^2 - \right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(10*cos(d*x + c)^2 + 10*(cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x
+ c) - 1)*log(1/2*sin(d*x + c)) - 10*(cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)
*sin(d*x + c) - 1)*log(sin(d*x + c) + 1) - 5*sin(d*x + c) - 9)/(a^3*d*cos(d
*x + c)^2 - a^3*d + (a^3*d*cos(d*x + c)^2 - a^3*d)*sin(d*x + c))
```

giac [A] time = 0.48, size = 154, normalized size = 1.79

$$-\frac{\frac{80\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3} - \frac{40\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3} - \frac{30\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 + 40\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 53\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + 10\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - 1}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)^2 a^3} + \cdots}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(80*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 40*log(abs(tan(1/2*d*x +
1/2*c)))/a^3 - (30*tan(1/2*d*x + 1/2*c)^4 + 40*tan(1/2*d*x + 1/2*c)^3 + 53*
tan(1/2*d*x + 1/2*c)^2 + 10*tan(1/2*d*x + 1/2*c) - 1)/((tan(1/2*d*x + 1/2*c
)^2 + tan(1/2*d*x + 1/2*c))^2*a^3) + (a^3*tan(1/2*d*x + 1/2*c)^2 - 12*a^3*t
an(1/2*d*x + 1/2*c))/a^6)/d
```

maple [A] time = 0.47, size = 84, normalized size = 0.98

$$-\frac{1}{2a^3d\sin(dx+c)^2}+\frac{3}{a^3d\sin(dx+c)}+\frac{5\ln(\sin(dx+c))}{a^3d}+\frac{2}{a^3d(1+\sin(dx+c))}-\frac{5\ln(1+\sin(dx+c))}{a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/2/a^3/d/sin(d*x+c)^2+3/a^3/d/sin(d*x+c)+5*ln(sin(d*x+c))/a^3/d+2/a^3/d/(
1+sin(d*x+c))-5*ln(1+sin(d*x+c))/a^3/d
```

maxima [A] time = 0.34, size = 80, normalized size = 0.93

$$\frac{\frac{10\sin(dx+c)^2+5\sin(dx+c)-1}{a^3\sin(dx+c)^3+a^3\sin(dx+c)^2}-\frac{10\log(\sin(dx+c)+1)}{a^3}+\frac{10\log(\sin(dx+c))}{a^3}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((10*sin(d*x + c)^2 + 5*sin(d*x + c) - 1)/(a^3*sin(d*x + c)^3 + a^3*sin
(d*x + c)^2) - 10*log(sin(d*x + c) + 1)/a^3 + 10*log(sin(d*x + c))/a^3)/d
```

mupad [B] time = 6.70, size = 169, normalized size = 1.97

$$\frac{5\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^3d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8a^3d}-\frac{10\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{a^3d}+\frac{-10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+\frac{23\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{2}+5\tan}{d\left(4a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+8a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3/(a + a*sin(c + d*x))^3,x)
```

```
[Out] (5*log(tan(c/2 + (d*x)/2)))/(a^3*d) - tan(c/2 + (d*x)/2)^2/(8*a^3*d) - (10*
log(tan(c/2 + (d*x)/2) + 1))/(a^3*d) + (5*tan(c/2 + (d*x)/2) + (23*tan(c/2
```

```
+ (d*x)/2)^2)/2 - 10*tan(c/2 + (d*x)/2)^3 - 1/2)/(d*(4*a^3*tan(c/2 + (d*x)/
2)^2 + 8*a^3*tan(c/2 + (d*x)/2)^3 + 4*a^3*tan(c/2 + (d*x)/2)^4)) + (3*tan(c
/2 + (d*x)/2))/(2*a^3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^3(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)**3/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d
*x) + 1), x)/a**3
```

3.77 $\int \frac{\cot^5(c+dx)}{(a+a\sin(c+dx))^3} \, dx$

Optimal. Leaf size=96

$$-\frac{\csc^4(c+dx)}{4a^3d}+\frac{\csc^3(c+dx)}{a^3d}-\frac{2\csc^2(c+dx)}{a^3d}+\frac{4\csc(c+dx)}{a^3d}+\frac{4\log(\sin(c+dx))}{a^3d}-\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

```
[Out] 4*csc(d*x+c)/a^3/d-2*csc(d*x+c)^2/a^3/d+csc(d*x+c)^3/a^3/d-1/4*csc(d*x+c)^4
/a^3/d+4*ln(sin(d*x+c))/a^3/d-4*ln(1+sin(d*x+c))/a^3/d
```

Rubi [A] time = 0.07, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 88}

$$-\frac{\csc^4(c+dx)}{4a^3d}+\frac{\csc^3(c+dx)}{a^3d}-\frac{2\csc^2(c+dx)}{a^3d}+\frac{4\csc(c+dx)}{a^3d}+\frac{4\log(\sin(c+dx))}{a^3d}-\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (4*Csc[c + d*x])/(a^3*d) - (2*Csc[c + d*x]^2)/(a^3*d) + Csc[c + d*x]^3/(a^3
*d) - Csc[c + d*x]^4/(4*a^3*d) + (4*Log[Sin[c + d*x]])/(a^3*d) - (4*Log[1 +
 Sin[c + d*x]])/(a^3*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2}{x^5(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a}{x^5} - \frac{3}{x^4} + \frac{4}{ax^3} - \frac{4}{a^2x^2} + \frac{4}{a^3x} - \frac{4}{a^3(a+x)}\right) dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{4\csc(c+dx)}{a^3d} - \frac{2\csc^2(c+dx)}{a^3d} + \frac{\csc^3(c+dx)}{a^3d} - \frac{\csc^4(c+dx)}{4a^3d} + \frac{4\log(\sin(c+dx))}{a^3d} -$$

Mathematica [A] time = 0.31, size = 69, normalized size = 0.72

$$\frac{-\csc^4(c+dx) + 4\csc^3(c+dx) - 8\csc^2(c+dx) + 16\csc(c+dx) + 16\log(\sin(c+dx)) - 16\log(\sin(c+dx)+1)}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (16*Csc[c + d*x] - 8*Csc[c + d*x]^2 + 4*Csc[c + d*x]^3 - Csc[c + d*x]^4 + 1
6*Log[Sin[c + d*x]] - 16*Log[1 + Sin[c + d*x]])/(4*a^3*d)
```

fricas [A] time = 0.43, size = 131, normalized size = 1.36

$$\frac{8\cos(dx+c)^2 + 16\left(\cos(dx+c)^4 - 2\cos(dx+c)^2 + 1\right)\log\left(\frac{1}{2}\sin(dx+c)\right) - 16\left(\cos(dx+c)^4 - 2\cos(dx+c\right.}{4\left(a^3d\cos(dx+c)^4 - 2a^3d\cos(dx+c)^2 + a^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(8*cos(d*x + c)^2 + 16*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(1/2*
sin(d*x + c)) - 16*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(sin(d*x + c)
 + 1) - 4*(4*cos(d*x + c)^2 - 5)*sin(d*x + c) - 9)/(a^3*d*cos(d*x + c)^4 -
2*a^3*d*cos(d*x + c)^2 + a^3*d)
```

giac [A] time = 0.96, size = 174, normalized size = 1.81

$$-\frac{\frac{1536\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3} - \frac{768\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3} + \frac{1600\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 - 456\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 108\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - 24\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.}{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4}}{192d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/192*(1536*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 768*log(abs(tan(1/2*d
*x + 1/2*c)))/a^3 + (1600*tan(1/2*d*x + 1/2*c)^4 - 456*tan(1/2*d*x + 1/2*c)
^3 + 108*tan(1/2*d*x + 1/2*c)^2 - 24*tan(1/2*d*x + 1/2*c) + 3)/(a^3*tan(1/2
*d*x + 1/2*c)^4) + 3*(a^9*tan(1/2*d*x + 1/2*c)^4 - 8*a^9*tan(1/2*d*x + 1/2*
c)^3 + 36*a^9*tan(1/2*d*x + 1/2*c)^2 - 152*a^9*tan(1/2*d*x + 1/2*c))/a^12)/
d
```

maple [A] time = 0.34, size = 97, normalized size = 1.01

$$-\frac{1}{4a^3d\sin(dx+c)^4}+\frac{1}{a^3d\sin(dx+c)^3}-\frac{2}{a^3d\sin(dx+c)^2}+\frac{4}{a^3d\sin(dx+c)}+\frac{4\ln(\sin(dx+c))}{a^3d}-\frac{4\ln(1+\sin(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/4/a^3/d/sin(d*x+c)^4+1/a^3/d/sin(d*x+c)^3-2/a^3/d/sin(d*x+c)^2+4/a^3/d/s
in(d*x+c)+4*ln(sin(d*x+c))/a^3/d-4*ln(1+sin(d*x+c))/a^3/d
```

maxima [A] time = 0.37, size = 75, normalized size = 0.78

$$-\frac{\frac{16\log(\sin(dx+c)+1)}{a^3}-\frac{16\log(\sin(dx+c))}{a^3}-\frac{16\sin(dx+c)^3-8\sin(dx+c)^2+4\sin(dx+c)-1}{a^3\sin(dx+c)^4}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*(16*log(sin(d*x + c) + 1)/a^3 - 16*log(sin(d*x + c))/a^3 - (16*sin(d*x
 + c)^3 - 8*sin(d*x + c)^2 + 4*sin(d*x + c) - 1)/(a^3*sin(d*x + c)^4))/d
```

mupad [B] time = 6.72, size = 171, normalized size = 1.78

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{8a^3d}-\frac{9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{16a^3d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{64a^3d}+\frac{4\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^3d}-\frac{8\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{a^3d}+\frac{19\tan\left(\frac{c}{2}+\frac{dx}{2}\right.}{8a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5/(a + a*sin(c + d*x))^3,x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(8*a^3*d) - (9*tan(c/2 + (d*x)/2)^2)/(16*a^3*d) - tan(
c/2 + (d*x)/2)^4/(64*a^3*d) + (4*log(tan(c/2 + (d*x)/2)))/(a^3*d) - (8*log(
tan(c/2 + (d*x)/2) + 1))/(a^3*d) + (19*tan(c/2 + (d*x)/2))/(8*a^3*d) + (cot
```

```
(c/2 + (d*x)/2)^4*(2*tan(c/2 + (d*x)/2) - 9*tan(c/2 + (d*x)/2)^2 + 38*tan(c
/2 + (d*x)/2)^3 - 1/4))/(16*a^3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^5(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)**5/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d
*x) + 1), x)/a**3
```

3.78 $\int \frac{\cot^7(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=73

$$-\frac{\csc^6(c+dx)}{6a^3d}+\frac{3\csc^5(c+dx)}{5a^3d}-\frac{3\csc^4(c+dx)}{4a^3d}+\frac{\csc^3(c+dx)}{3a^3d}$$

```
[Out] 1/3*csc(d*x+c)^3/a^3/d-3/4*csc(d*x+c)^4/a^3/d+3/5*csc(d*x+c)^5/a^3/d-1/6*cs
c(d*x+c)^6/a^3/d
```

Rubi [A] time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 43}

$$-\frac{\csc^6(c+dx)}{6a^3d}+\frac{3\csc^5(c+dx)}{5a^3d}-\frac{3\csc^4(c+dx)}{4a^3d}+\frac{\csc^3(c+dx)}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] Csc[c + d*x]^3/(3*a^3*d) - (3*Csc[c + d*x]^4)/(4*a^3*d) + (3*Csc[c + d*x]^5
)/(5*a^3*d) - Csc[c + d*x]^6/(6*a^3*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^7(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{(a-x)^3}{x^7}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\operatorname{Subst}\left(\int \left(\frac{a^3}{x^7}-\frac{3a^2}{x^6}+\frac{3a}{x^5}-\frac{1}{x^4}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\csc^3(c+dx)}{3a^3d}-\frac{3\csc^4(c+dx)}{4a^3d}+\frac{3\csc^5(c+dx)}{5a^3d}-\frac{\csc^6(c+dx)}{6a^3d}$$

Mathematica [**A**] time = 0.10, size = 48, normalized size = 0.66

$$\frac{\csc^3(c+dx)\left(-10\csc^3(c+dx)+36\csc^2(c+dx)-45\csc(c+dx)+20\right)}{60a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Csc[c + d*x]^3*(20 - 45*Csc[c + d*x] + 36*Csc[c + d*x]^2 - 10*Csc[c + d*x]
^3))/(60*a^3*d)
```

fricas [**A**] time = 0.41, size = 84, normalized size = 1.15

$$-\frac{45\cos(dx+c)^2-4\left(5\cos(dx+c)^2-14\right)\sin(dx+c)-55}{60\left(a^3d\cos(dx+c)^6-3a^3d\cos(dx+c)^4+3a^3d\cos(dx+c)^2-a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/60*(45*cos(d*x + c)^2 - 4*(5*cos(d*x + c)^2 - 14)*sin(d*x + c) - 55)/(a^
3*d*cos(d*x + c)^6 - 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^2 - a^3*
d)
```

giac [**A**] time = 0.81, size = 46, normalized size = 0.63

$$\frac{20\sin(dx+c)^3-45\sin(dx+c)^2+36\sin(dx+c)-10}{60a^3d\sin(dx+c)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(20*sin(d*x + c)^3 - 45*sin(d*x + c)^2 + 36*sin(d*x + c) - 10)/(a^3*d*
sin(d*x + c)^6)
```

maple [A] time = 0.33, size = 49, normalized size = 0.67

$$\frac{-\frac{1}{6\sin(dx+c)^6}+\frac{3}{5\sin(dx+c)^5}-\frac{3}{4\sin(dx+c)^4}+\frac{1}{3\sin(dx+c)^3}}{d\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^7/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d/a^3*(-1/6/sin(d*x+c)^6+3/5/sin(d*x+c)^5-3/4/sin(d*x+c)^4+1/3/sin(d*x+c)
^3)
```

maxima [A] time = 0.30, size = 46, normalized size = 0.63

$$\frac{20\sin(dx+c)^3-45\sin(dx+c)^2+36\sin(dx+c)-10}{60\,a^3d\sin(dx+c)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(20*sin(d*x + c)^3 - 45*sin(d*x + c)^2 + 36*sin(d*x + c) - 10)/(a^3*d*
sin(d*x + c)^6)
```

mupad [B] time = 6.69, size = 46, normalized size = 0.63

$$\frac{20\sin(c+d\,x)^3-45\sin(c+d\,x)^2+36\sin(c+d\,x)-10}{60\,a^3\,d\sin(c+d\,x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^7/(a + a*sin(c + d*x))^3,x)
```

```
[Out] (36*sin(c + d*x) - 45*sin(c + d*x)^2 + 20*sin(c + d*x)^3 - 10)/(60*a^3*d*si
n(c + d*x)^6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^7(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**7/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)**7/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d
*x) + 1), x)/a**3
```

3.79 $\int \frac{\cot^9(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=109

$$-\frac{\csc^8(c+dx)}{8a^3d}+\frac{3\csc^7(c+dx)}{7a^3d}-\frac{\csc^6(c+dx)}{3a^3d}-\frac{2\csc^5(c+dx)}{5a^3d}+\frac{3\csc^4(c+dx)}{4a^3d}-\frac{\csc^3(c+dx)}{3a^3d}$$

```
[Out] -1/3*csc(d*x+c)^3/a^3/d+3/4*csc(d*x+c)^4/a^3/d-2/5*csc(d*x+c)^5/a^3/d-1/3*c
sc(d*x+c)^6/a^3/d+3/7*csc(d*x+c)^7/a^3/d-1/8*csc(d*x+c)^8/a^3/d
```

Rubi [A] time = 0.07, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 75}

$$-\frac{\csc^8(c+dx)}{8a^3d}+\frac{3\csc^7(c+dx)}{7a^3d}-\frac{\csc^6(c+dx)}{3a^3d}-\frac{2\csc^5(c+dx)}{5a^3d}+\frac{3\csc^4(c+dx)}{4a^3d}-\frac{\csc^3(c+dx)}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^9/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -Csc[c + d*x]^3/(3*a^3*d) + (3*Csc[c + d*x]^4)/(4*a^3*d) - (2*Csc[c + d*x]^
5)/(5*a^3*d) - Csc[c + d*x]^6/(3*a^3*d) + (3*Csc[c + d*x]^7)/(7*a^3*d) - Cs
c[c + d*x]^8/(8*a^3*d)
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^9(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^4(a+x)}{x^9}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^5}{x^9}-\frac{3a^4}{x^8}+\frac{2a^3}{x^7}+\frac{2a^2}{x^6}-\frac{3a}{x^5}+\frac{1}{x^4}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{\csc^3(c+dx)}{3a^3d}+\frac{3\csc^4(c+dx)}{4a^3d}-\frac{2\csc^5(c+dx)}{5a^3d}-\frac{\csc^6(c+dx)}{3a^3d}+\frac{3\csc^7(c+dx)}{7a^3d}-$$

Mathematica [A] time = 0.08, size = 68, normalized size = 0.62

$$-\frac{\csc^3(c+dx)\left(105\csc^5(c+dx)-360\csc^4(c+dx)+280\csc^3(c+dx)+336\csc^2(c+dx)-630\csc(c+dx)+28\right.}{840a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^9/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/840*(Csc[c + d*x]^3*(280 - 630*Csc[c + d*x] + 336*Csc[c + d*x]^2 + 280*C
sc[c + d*x]^3 - 360*Csc[c + d*x]^4 + 105*Csc[c + d*x]^5))/(a^3*d)
```

fricas [A] time = 0.42, size = 117, normalized size = 1.07

$$\frac{630\cos(dx+c)^4-980\cos(dx+c)^2-8\left(35\cos(dx+c)^4-112\cos(dx+c)^2+32\right)\sin(dx+c)+245}{840\left(a^3d\cos(dx+c)^8-4a^3d\cos(dx+c)^6+6a^3d\cos(dx+c)^4-4a^3d\cos(dx+c)^2+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^9/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/840*(630*cos(d*x + c)^4 - 980*cos(d*x + c)^2 - 8*(35*cos(d*x + c)^4 - 112
*cos(d*x + c)^2 + 32)*sin(d*x + c) + 245)/(a^3*d*cos(d*x + c)^8 - 4*a^3*d*c
os(d*x + c)^6 + 6*a^3*d*cos(d*x + c)^4 - 4*a^3*d*cos(d*x + c)^2 + a^3*d)
```

giac [A] time = 2.04, size = 66, normalized size = 0.61

$$-\frac{280\sin(dx+c)^5-630\sin(dx+c)^4+336\sin(dx+c)^3+280\sin(dx+c)^2-360\sin(dx+c)+105}{840a^3d\sin(dx+c)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^9/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/840*(280*sin(d*x + c)^5 - 630*sin(d*x + c)^4 + 336*sin(d*x + c)^3 + 280*
sin(d*x + c)^2 - 360*sin(d*x + c) + 105)/(a^3*d*sin(d*x + c)^8)
```

maple [A] time = 0.36, size = 69, normalized size = 0.63

$$\frac{-\frac{1}{3\sin(dx+c)^6}-\frac{2}{5\sin(dx+c)^5}+\frac{3}{7\sin(dx+c)^7}-\frac{1}{8\sin(dx+c)^8}+\frac{3}{4\sin(dx+c)^4}-\frac{1}{3\sin(dx+c)^3}}{d\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^9/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d/a^3*(-1/3/sin(d*x+c)^6-2/5/sin(d*x+c)^5+3/7/sin(d*x+c)^7-1/8/sin(d*x+c)
^8+3/4/sin(d*x+c)^4-1/3/sin(d*x+c)^3)
```

maxima [A] time = 0.33, size = 66, normalized size = 0.61

$$-\frac{280\,\sin\left(dx+c\right)^5-630\,\sin\left(dx+c\right)^4+336\,\sin\left(dx+c\right)^3+280\,\sin\left(dx+c\right)^2-360\,\sin\left(dx+c\right)+105}{840\,a^3d\sin\left(dx+c\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^9/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/840*(280*sin(d*x + c)^5 - 630*sin(d*x + c)^4 + 336*sin(d*x + c)^3 + 280*
sin(d*x + c)^2 - 360*sin(d*x + c) + 105)/(a^3*d*sin(d*x + c)^8)
```

mupad [B] time = 6.63, size = 66, normalized size = 0.61

$$-\frac{280\sin\left(c+d\,x\right)^5-630\sin\left(c+d\,x\right)^4+336\sin\left(c+d\,x\right)^3+280\sin\left(c+d\,x\right)^2-360\,\sin\left(c+d\,x\right)+105}{840\,a^3\,d\sin\left(c+d\,x\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^9/(a + a*sin(c + d*x))^3,x)
```

```
[Out] -(280*sin(c + d*x)^2 - 360*sin(c + d*x) + 336*sin(c + d*x)^3 - 630*sin(c +
d*x)^4 + 280*sin(c + d*x)^5 + 105)/(840*a^3*d*sin(c + d*x)^8)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^9(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**9/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)**9/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d
*x) + 1), x)/a**3
```

3.80 $\int \frac{\cot^{11}(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=145

$$-\frac{\csc^{10}(c+dx)}{10a^3d}+\frac{\csc^9(c+dx)}{3a^3d}-\frac{\csc^8(c+dx)}{8a^3d}-\frac{5\csc^7(c+dx)}{7a^3d}+\frac{5\csc^6(c+dx)}{6a^3d}+\frac{\csc^5(c+dx)}{5a^3d}-\frac{3\csc^4(c+dx)}{4a^3d}+\csc$$

```
[Out] 1/3*csc(d*x+c)^3/a^3/d-3/4*csc(d*x+c)^4/a^3/d+1/5*csc(d*x+c)^5/a^3/d+5/6*cs
c(d*x+c)^6/a^3/d-5/7*csc(d*x+c)^7/a^3/d-1/8*csc(d*x+c)^8/a^3/d+1/3*csc(d*x+
c)^9/a^3/d-1/10*csc(d*x+c)^10/a^3/d
```

Rubi [A] time = 0.08, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 88}

$$-\frac{\csc^{10}(c+dx)}{10a^3d}+\frac{\csc^9(c+dx)}{3a^3d}-\frac{\csc^8(c+dx)}{8a^3d}-\frac{5\csc^7(c+dx)}{7a^3d}+\frac{5\csc^6(c+dx)}{6a^3d}+\frac{\csc^5(c+dx)}{5a^3d}-\frac{3\csc^4(c+dx)}{4a^3d}+\csc$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^11/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] Csc[c + d*x]^3/(3*a^3*d) - (3*Csc[c + d*x]^4)/(4*a^3*d) + Csc[c + d*x]^5/(5
*a^3*d) + (5*Csc[c + d*x]^6)/(6*a^3*d) - (5*Csc[c + d*x]^7)/(7*a^3*d) - Csc
[c + d*x]^8/(8*a^3*d) + Csc[c + d*x]^9/(3*a^3*d) - Csc[c + d*x]^10/(10*a^3*
d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^{11}(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^5(a+x)^2}{x^{11}}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^7}{x^{11}} - \frac{3a^6}{x^{10}} + \frac{a^5}{x^9} + \frac{5a^4}{x^8} - \frac{5a^3}{x^7} - \frac{a^2}{x^6} + \frac{3a}{x^5} - \frac{1}{x^4}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\csc^3(c+dx)}{3a^3d} - \frac{3\csc^4(c+dx)}{4a^3d} + \frac{\csc^5(c+dx)}{5a^3d} + \frac{5\csc^6(c+dx)}{6a^3d} - \frac{5\csc^7(c+dx)}{7a^3d} - \frac{\text{cs}}{}$$

Mathematica [A] time = 0.11, size = 88, normalized size = 0.61

$$\frac{\csc^3(c+dx)\left(-84\csc^7(c+dx) + 280\csc^6(c+dx) - 105\csc^5(c+dx) - 600\csc^4(c+dx) + 700\csc^3(c+dx) + 168\right.}{840a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^11/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Csc[c + d*x]^3*(280 - 630*Csc[c + d*x] + 168*Csc[c + d*x]^2 + 700*Csc[c +
d*x]^3 - 600*Csc[c + d*x]^4 - 105*Csc[c + d*x]^5 + 280*Csc[c + d*x]^6 - 84*
Csc[c + d*x]^7))/(840*a^3*d)
```

fricas [A] time = 0.44, size = 152, normalized size = 1.05

$$-\frac{630\cos(dx+c)^6 - 1190\cos(dx+c)^4 + 595\cos(dx+c)^2 - 8\left(35\cos(dx+c)^6 - 126\cos(dx+c)^4 + 72\cos(\right.}{840\left(a^3d\cos(dx+c)^{10} - 5a^3d\cos(dx+c)^8 + 10a^3d\cos(dx+c)^6 - 10a^3d\cos(dx+c)^4 + 5a^3d\,\text{c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^11/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/840*(630*cos(d*x + c)^6 - 1190*cos(d*x + c)^4 + 595*cos(d*x + c)^2 - 8*(
35*cos(d*x + c)^6 - 126*cos(d*x + c)^4 + 72*cos(d*x + c)^2 - 16)*sin(d*x +
c) - 119)/(a^3*d*cos(d*x + c)^10 - 5*a^3*d*cos(d*x + c)^8 + 10*a^3*d*cos(d*
x + c)^6 - 10*a^3*d*cos(d*x + c)^4 + 5*a^3*d*cos(d*x + c)^2 - a^3*d)
```

giac [A] time = 2.48, size = 86, normalized size = 0.59

$$\frac{280\sin(dx+c)^7 - 630\sin(dx+c)^6 + 168\sin(dx+c)^5 + 700\sin(dx+c)^4 - 600\sin(dx+c)^3 - 105\sin(dx+}{840\,a^3d\sin(dx+c)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^11/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/840*(280*sin(d*x + c)^7 - 630*sin(d*x + c)^6 + 168*sin(d*x + c)^5 + 700*s
in(d*x + c)^4 - 600*sin(d*x + c)^3 - 105*sin(d*x + c)^2 + 280*sin(d*x + c)
- 84)/(a^3*d*sin(d*x + c)^10)
```

maple [A] time = 0.38, size = 89, normalized size = 0.61

$$\frac{\frac{5}{6\sin(dx+c)^{6}}+\frac{1}{5\sin(dx+c)^{5}}-\frac{5}{7\sin(dx+c)^{7}}+\frac{1}{3\sin(dx+c)^{9}}-\frac{1}{8\sin(dx+c)^{8}}-\frac{3}{4\sin(dx+c)^{4}}+\frac{1}{3\sin(dx+c)^{3}}-\frac{1}{10\sin(dx+c)^{10}}}{d\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^11/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d/a^3*(5/6/sin(d*x+c)^6+1/5/sin(d*x+c)^5-5/7/sin(d*x+c)^7+1/3/sin(d*x+c)^
9-1/8/sin(d*x+c)^8-3/4/sin(d*x+c)^4+1/3/sin(d*x+c)^3-1/10/sin(d*x+c)^10)
```

maxima [A] time = 0.31, size = 86, normalized size = 0.59

$$\frac{280\,\sin\left(dx+c\right)^{7}-630\,\sin\left(dx+c\right)^{6}+168\,\sin\left(dx+c\right)^{5}+700\,\sin\left(dx+c\right)^{4}-600\,\sin\left(dx+c\right)^{3}-105\,\sin\left(dx\right.}{840\,a^{3}d\sin\left(dx+c\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^11/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/840*(280*sin(d*x + c)^7 - 630*sin(d*x + c)^6 + 168*sin(d*x + c)^5 + 700*s
in(d*x + c)^4 - 600*sin(d*x + c)^3 - 105*sin(d*x + c)^2 + 280*sin(d*x + c)
- 84)/(a^3*d*sin(d*x + c)^10)
```

mupad [B] time = 6.81, size = 86, normalized size = 0.59

$$\frac{280\sin\left(c+d\,x\right)^{7}-630\sin\left(c+d\,x\right)^{6}+168\sin\left(c+d\,x\right)^{5}+700\sin\left(c+d\,x\right)^{4}-600\sin\left(c+d\,x\right)^{3}-105\sin\left(c+\right.}{840\,a^{3}\,d\sin\left(c+d\,x\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^11/(a + a*sin(c + d*x))^3,x)
```

```
[Out] (280*sin(c + d*x) - 105*sin(c + d*x)^2 - 600*sin(c + d*x)^3 + 700*sin(c + d
*x)^4 + 168*sin(c + d*x)^5 - 630*sin(c + d*x)^6 + 280*sin(c + d*x)^7 - 84)/
(840*a^3*d*sin(c + d*x)^10)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**11/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.81 $\int \frac{\cot^{13}(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=145

$$-\frac{\csc^{12}(c+dx)}{12a^3d}+\frac{3\csc^{11}(c+dx)}{11a^3d}-\frac{8\csc^9(c+dx)}{9a^3d}+\frac{3\csc^8(c+dx)}{4a^3d}+\frac{6\csc^7(c+dx)}{7a^3d}-\frac{4\csc^6(c+dx)}{3a^3d}+\frac{3\csc^4(c+dx)}{4a^3d}$$

```
[Out] -1/3*csc(d*x+c)^3/a^3/d+3/4*csc(d*x+c)^4/a^3/d-4/3*csc(d*x+c)^6/a^3/d+6/7*c
sc(d*x+c)^7/a^3/d+3/4*csc(d*x+c)^8/a^3/d-8/9*csc(d*x+c)^9/a^3/d+3/11*csc(d*
x+c)^11/a^3/d-1/12*csc(d*x+c)^12/a^3/d
```

Rubi [A] time = 0.08, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 88}

$$-\frac{\csc^{12}(c+dx)}{12a^3d}+\frac{3\csc^{11}(c+dx)}{11a^3d}-\frac{8\csc^9(c+dx)}{9a^3d}+\frac{3\csc^8(c+dx)}{4a^3d}+\frac{6\csc^7(c+dx)}{7a^3d}-\frac{4\csc^6(c+dx)}{3a^3d}+\frac{3\csc^4(c+dx)}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^13/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -Csc[c + d*x]^3/(3*a^3*d) + (3*Csc[c + d*x]^4)/(4*a^3*d) - (4*Csc[c + d*x]^
6)/(3*a^3*d) + (6*Csc[c + d*x]^7)/(7*a^3*d) + (3*Csc[c + d*x]^8)/(4*a^3*d)
 - (8*Csc[c + d*x]^9)/(9*a^3*d) + (3*Csc[c + d*x]^11)/(11*a^3*d) - Csc[c + d
*x]^12/(12*a^3*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^{13}(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^6(a+x)^3}{x^{13}}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^9}{x^{13}} - \frac{3a^8}{x^{12}} + \frac{8a^6}{x^{10}} - \frac{6a^5}{x^9} - \frac{6a^4}{x^8} + \frac{8a^3}{x^7} - \frac{3a}{x^5} + \frac{1}{x^4}\right) dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{\csc^3(c+dx)}{3a^3d} + \frac{3\csc^4(c+dx)}{4a^3d} - \frac{4\csc^6(c+dx)}{3a^3d} + \frac{6\csc^7(c+dx)}{7a^3d} + \frac{3\csc^8(c+dx)}{4a^3d}$$

Mathematica [A] time = 0.12, size = 88, normalized size = 0.61

$$\frac{\csc^3(c+dx)\left(-231\csc^9(c+dx) + 756\csc^8(c+dx) - 2464\csc^6(c+dx) + 2079\csc^5(c+dx) + 2376\csc^4(c+dx)\right.}{2772a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^13/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Csc[c + d*x]^3*(-924 + 2079*Csc[c + d*x] - 3696*Csc[c + d*x]^3 + 2376*Csc[
c + d*x]^4 + 2079*Csc[c + d*x]^5 - 2464*Csc[c + d*x]^6 + 756*Csc[c + d*x]^8
 - 231*Csc[c + d*x]^9))/(2772*a^3*d)
```

fricas [A] time = 0.46, size = 185, normalized size = 1.28

$$\frac{2079\cos(dx+c)^8 - 4620\cos(dx+c)^6 + 3465\cos(dx+c)^4 - 1386\cos(dx+c)^2 - 4\left(231\cos(dx+c)^8 - 924\right.}{2772\left(a^3d\cos(dx+c)^{12} - 6a^3d\cos(dx+c)^{10} + 15a^3d\cos(dx+c)^8 - 20a^3d\cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^13/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2772*(2079*cos(d*x + c)^8 - 4620*cos(d*x + c)^6 + 3465*cos(d*x + c)^4 - 1
386*cos(d*x + c)^2 - 4*(231*cos(d*x + c)^8 - 924*cos(d*x + c)^6 + 792*cos(d
*x + c)^4 - 352*cos(d*x + c)^2 + 64)*sin(d*x + c) + 231)/(a^3*d*cos(d*x + c
)^12 - 6*a^3*d*cos(d*x + c)^10 + 15*a^3*d*cos(d*x + c)^8 - 20*a^3*d*cos(d*x
 + c)^6 + 15*a^3*d*cos(d*x + c)^4 - 6*a^3*d*cos(d*x + c)^2 + a^3*d)
```

giac [A] time = 6.36, size = 86, normalized size = 0.59

$$-\frac{924\sin(dx+c)^9 - 2079\sin(dx+c)^8 + 3696\sin(dx+c)^6 - 2376\sin(dx+c)^5 - 2079\sin(dx+c)^4 + 2464\,\mathrm{s}}{2772\,a^3d\sin(dx+c)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^13/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2772*(924*sin(d*x + c)^9 - 2079*sin(d*x + c)^8 + 3696*sin(d*x + c)^6 - 2
376*sin(d*x + c)^5 - 2079*sin(d*x + c)^4 + 2464*sin(d*x + c)^3 - 756*sin(d*
x + c) + 231)/(a^3*d*sin(d*x + c)^12)
```

maple [A] time = 0.43, size = 89, normalized size = 0.61

$$\frac{-\frac{4}{3\sin(dx+c)^6}-\frac{1}{12\sin(dx+c)^{12}}+\frac{6}{7\sin(dx+c)^7}-\frac{8}{9\sin(dx+c)^9}+\frac{3}{4\sin(dx+c)^8}+\frac{3}{4\sin(dx+c)^4}+\frac{3}{11\sin(dx+c)^{11}}-\frac{1}{3\sin(dx+c)^3}}{d\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^13/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d/a^3*(-4/3/sin(d*x+c)^6-1/12/sin(d*x+c)^12+6/7/sin(d*x+c)^7-8/9/sin(d*x+
c)^9+3/4/sin(d*x+c)^8+3/4/sin(d*x+c)^4+3/11/sin(d*x+c)^11-1/3/sin(d*x+c)^3)
```

maxima [A] time = 0.30, size = 86, normalized size = 0.59

$$-\frac{924\,\sin(dx+c)^9-2079\,\sin(dx+c)^8+3696\,\sin(dx+c)^6-2376\,\sin(dx+c)^5-2079\,\sin(dx+c)^4+2464}{2772\,a^3d\sin(dx+c)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^13/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2772*(924*sin(d*x + c)^9 - 2079*sin(d*x + c)^8 + 3696*sin(d*x + c)^6 - 2
376*sin(d*x + c)^5 - 2079*sin(d*x + c)^4 + 2464*sin(d*x + c)^3 - 756*sin(d*
x + c) + 231)/(a^3*d*sin(d*x + c)^12)
```

mupad [B] time = 6.82, size = 85, normalized size = 0.59

$$\frac{-\frac{\sin(c+d\,x)^9}{3}+\frac{3\sin(c+d\,x)^8}{4}-\frac{4\sin(c+d\,x)^6}{3}+\frac{6\sin(c+d\,x)^5}{7}+\frac{3\sin(c+d\,x)^4}{4}-\frac{8\sin(c+d\,x)^3}{9}+\frac{3\,\sin(c+d\,x)}{11}-\frac{1}{12}}{a^3\,d\sin(c+d\,x)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^13/(a + a*sin(c + d*x))^3,x)
```

```
[Out] ((3*sin(c + d*x))/11 - (8*sin(c + d*x)^3)/9 + (3*sin(c + d*x)^4)/4 + (6*sin
(c + d*x)^5)/7 - (4*sin(c + d*x)^6)/3 + (3*sin(c + d*x)^8)/4 - sin(c + d*x)
^9/3 - 1/12)/(a^3*d*sin(c + d*x)^12)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**13/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.82 $\int \frac{\tan^5(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=195

$$-\frac{3}{256d\left(a^4-a^4\sin(c+dx)\right)}-\frac{1}{256d\left(a^4\sin(c+dx)+a^4\right)}-\frac{\tanh^{-1}(\sin(c+dx))}{128a^4d}+\frac{a^2}{48d(a\sin(c+dx)+a)^6}+\frac{}{256d\,(}$$

```
[Out] -1/128*arctanh(sin(d*x+c))/a^4/d+1/48*a^2/d/(a+a*sin(d*x+c))^6-7/80*a/d/(a+
a*sin(d*x+c))^5+1/8/d/(a+a*sin(d*x+c))^4-5/96/a/d/(a+a*sin(d*x+c))^3+1/256/
d/(a^2-a^2*sin(d*x+c))^2-5/256/d/(a^2+a^2*sin(d*x+c))^2-3/256/d/(a^4-a^4*si
n(d*x+c))-1/256/d/(a^4+a^4*sin(d*x+c))
```

Rubi [A] time = 0.13, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2707, 88, 206}

$$\frac{a^2}{48d(a\sin(c+dx)+a)^6}-\frac{3}{256d\left(a^4-a^4\sin(c+dx)\right)}-\frac{1}{256d\left(a^4\sin(c+dx)+a^4\right)}+\frac{1}{256d\left(a^2-a^2\sin(c+dx)\right)^2}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -ArcTanh[Sin[c + d*x]]/(128*a^4*d) + a^2/(48*d*(a + a*Sin[c + d*x])^6) - (7
*a)/(80*d*(a + a*Sin[c + d*x])^5) + 1/(8*d*(a + a*Sin[c + d*x])^4) - 5/(96*
a*d*(a + a*Sin[c + d*x])^3) + 1/(256*d*(a^2 - a^2*Sin[c + d*x])^2) - 5/(256
*d*(a^2 + a^2*Sin[c + d*x])^2) - 3/(256*d*(a^4 - a^4*Sin[c + d*x])) - 1/(25
6*d*(a^4 + a^4*Sin[c + d*x]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(a-x)^3(a+x)^7}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{128a^2(a-x)^3} - \frac{3}{256a^3(a-x)^2} - \frac{a^2}{8(a+x)^7} + \frac{7a}{16(a+x)^6} - \frac{1}{2(a+x)^5} + \frac{5}{32a(a+x)^4} + \frac{5}{128a^2(a+x)\cdots}\right.\right.}{d}$$

$$= \frac{a^2}{48d(a+a\sin(c+dx))^6} - \frac{7a}{80d(a+a\sin(c+dx))^5} + \frac{1}{8d(a+a\sin(c+dx))^4} - \frac{\cdots}{96ad(\cdots}$$

$$= -\frac{\tanh^{-1}(\sin(c+dx))}{128a^4d} + \frac{a^2}{48d(a+a\sin(c+dx))^6} - \frac{7a}{80d(a+a\sin(c+dx))^5} + \frac{\cdots}{8d(a+\cdots}$$

Mathematica [A] time = 1.45, size = 112, normalized size = 0.57

$$-\frac{30\tanh^{-1}(\sin(c+dx)) - \frac{2\left(15\sin^7(c+dx)+60\sin^6(c+dx)+65\sin^5(c+dx)+440\sin^4(c+dx)+257\sin^3(c+dx)-132\sin^2(c+dx)-177\sin(c+dx)-4\cdots\right.}{(\sin(c+dx)-1)^2(\sin(c+dx)+1)^6}}{3840a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -1/3840*(30*ArcTanh[Sin[c + d*x]] - (2*(-48 - 177*Sin[c + d*x] - 132*Sin[c
+ d*x]^2 + 257*Sin[c + d*x]^3 + 440*Sin[c + d*x]^4 + 65*Sin[c + d*x]^5 + 60
*Sin[c + d*x]^6 + 15*Sin[c + d*x]^7))/((-1 + Sin[c + d*x])^2*(1 + Sin[c + d
*x])^6))/(a^4*d)
```

fricas [A] time = 0.47, size = 290, normalized size = 1.49

$$-\frac{120\cos(dx+c)^6 - 1240\cos(dx+c)^4 + 1856\cos(dx+c)^2 + 15\left(\cos(dx+c)^8 - 8\cos(dx+c)^6 + 8\cos(dx+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3840*(120*cos(d*x + c)^6 - 1240*cos(d*x + c)^4 + 1856*cos(d*x + c)^2 + 1
5*(cos(d*x + c)^8 - 8*cos(d*x + c)^6 + 8*cos(d*x + c)^4 - 4*(cos(d*x + c)^6
 - 2*cos(d*x + c)^4)*sin(d*x + c))*log(sin(d*x + c) + 1) - 15*(cos(d*x + c)
^8 - 8*cos(d*x + c)^6 + 8*cos(d*x + c)^4 - 4*(cos(d*x + c)^6 - 2*cos(d*x +
c)^4)*sin(d*x + c))*log(-sin(d*x + c) + 1) + 2*(15*cos(d*x + c)^6 - 110*cos
(d*x + c)^4 + 432*cos(d*x + c)^2 - 160)*sin(d*x + c) - 640)/(a^4*d*cos(d*x
+ c)^8 - 8*a^4*d*cos(d*x + c)^6 + 8*a^4*d*cos(d*x + c)^4 - 4*(a^4*d*cos(d*x
 + c)^6 - 2*a^4*d*cos(d*x + c)^4)*sin(d*x + c))
```

giac [A] time = 7.59, size = 146, normalized size = 0.75

$$-\frac{\frac{60 \log(|\sin(dx+c)+1|)}{a^4} - \frac{60 \log(|\sin(dx+c)-1|)}{a^4} + \frac{30\left(3 \sin(dx+c)^2-12 \sin(dx+c)+7\right)}{a^4(\sin(dx+c)-1)^2} - \frac{147 \sin(dx+c)^6+822 \sin(dx+c)^5+1605 \sin(dx+c)^4+34}{a^4(\sin(dx+}}{15360\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/15360*(60*log(abs(sin(d*x + c) + 1))/a^4 - 60*log(abs(sin(d*x + c) - 1))
/a^4 + 30*(3*sin(d*x + c)^2 - 12*sin(d*x + c) + 7)/(a^4*(sin(d*x + c) - 1)^
2) - (147*sin(d*x + c)^6 + 822*sin(d*x + c)^5 + 1605*sin(d*x + c)^4 + 340*s
in(d*x + c)^3 - 675*sin(d*x + c)^2 - 522*sin(d*x + c) - 117)/(a^4*(sin(d*x
+ c) + 1)^6))/d
```

maple [A] time = 0.24, size = 180, normalized size = 0.92

$$\frac{1}{256a^4d\,(\sin(dx+c)-1)^2}+\frac{3}{256a^4d\,(\sin(dx+c)-1)}+\frac{\ln(\sin(dx+c)-1)}{256a^4d}+\frac{1}{48a^4d\,(1+\sin(dx+c))^6}-\frac{}{80a^4d\,(1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+a*sin(d*x+c))^4,x)
```

```
[Out] 1/256/a^4/d/(sin(d*x+c)-1)^2+3/256/a^4/d/(sin(d*x+c)-1)+1/256/a^4/d*ln(sin(
d*x+c)-1)+1/48/a^4/d/(1+sin(d*x+c))^6-7/80/a^4/d/(1+sin(d*x+c))^5+1/8/a^4/d
/(1+sin(d*x+c))^4-5/96/a^4/d/(1+sin(d*x+c))^3-5/256/a^4/d/(1+sin(d*x+c))^2-
1/256/a^4/d/(1+sin(d*x+c))-1/256*ln(1+sin(d*x+c))/a^4/d
```

maxima [A] time = 0.53, size = 213, normalized size = 1.09

$$\frac{\frac{2\left(15 \sin(dx+c)^7+60 \sin(dx+c)^6+65 \sin(dx+c)^5+440 \sin(dx+c)^4+257 \sin(dx+c)^3-132 \sin(dx+c)^2-177 \sin(dx+c)-48\right)}{a^4 \sin(dx+c)^8+4\,a^4 \sin(dx+c)^7+4\,a^4 \sin(dx+c)^6-4\,a^4 \sin(dx+c)^5-10\,a^4 \sin(dx+c)^4-4\,a^4 \sin(dx+c)^3+4\,a^4 \sin(dx+c)^2+4\,a^4 \sin(dx+c)+a^4} - \frac{15 \log(\mathrm{si}}{}}{3840\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3840*(2*(15*sin(d*x + c)^7 + 60*sin(d*x + c)^6 + 65*sin(d*x + c)^5 + 440*
sin(d*x + c)^4 + 257*sin(d*x + c)^3 - 132*sin(d*x + c)^2 - 177*sin(d*x + c)
 - 48)/(a^4*sin(d*x + c)^8 + 4*a^4*sin(d*x + c)^7 + 4*a^4*sin(d*x + c)^6 -
4*a^4*sin(d*x + c)^5 - 10*a^4*sin(d*x + c)^4 - 4*a^4*sin(d*x + c)^3 + 4*a^4
*sin(d*x + c)^2 + 4*a^4*sin(d*x + c) + a^4) - 15*log(sin(d*x + c) + 1)/a^4
+ 15*log(sin(d*x + c) - 1)/a^4)/d
```

mupad [B] time = 10.66, size = 476, normalized size = 2.44

$$\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{64}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}}{8}+\frac{73\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{192}+\frac{5}{}\cdots}{d\left(a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}+8\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}+24\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+24\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}-36\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + a*sin(c + d*x))^4,x)
```

```
[Out] (tan(c/2 + (d*x)/2)/64 + tan(c/2 + (d*x)/2)^2/8 + (73*tan(c/2 + (d*x)/2)^3)
/192 + (5*tan(c/2 + (d*x)/2)^4)/12 - (139*tan(c/2 + (d*x)/2)^5)/320 + (1073
*tan(c/2 + (d*x)/2)^6)/120 + (10277*tan(c/2 + (d*x)/2)^7)/960 + (237*tan(c/
2 + (d*x)/2)^8)/10 + (10277*tan(c/2 + (d*x)/2)^9)/960 + (1073*tan(c/2 + (d*
x)/2)^10)/120 - (139*tan(c/2 + (d*x)/2)^11)/320 + (5*tan(c/2 + (d*x)/2)^12)
/12 + (73*tan(c/2 + (d*x)/2)^13)/192 + tan(c/2 + (d*x)/2)^14/8 + tan(c/2 +
(d*x)/2)^15/64)/(d*(24*a^4*tan(c/2 + (d*x)/2)^2 + 24*a^4*tan(c/2 + (d*x)/2)
^3 - 36*a^4*tan(c/2 + (d*x)/2)^4 - 120*a^4*tan(c/2 + (d*x)/2)^5 - 88*a^4*ta
n(c/2 + (d*x)/2)^6 + 88*a^4*tan(c/2 + (d*x)/2)^7 + 198*a^4*tan(c/2 + (d*x)/
2)^8 + 88*a^4*tan(c/2 + (d*x)/2)^9 - 88*a^4*tan(c/2 + (d*x)/2)^10 - 120*a^4
*tan(c/2 + (d*x)/2)^11 - 36*a^4*tan(c/2 + (d*x)/2)^12 + 24*a^4*tan(c/2 + (d
*x)/2)^13 + 24*a^4*tan(c/2 + (d*x)/2)^14 + 8*a^4*tan(c/2 + (d*x)/2)^15 + a^
4*tan(c/2 + (d*x)/2)^16 + a^4 + 8*a^4*tan(c/2 + (d*x)/2))) - atanh(tan(c/2
+ (d*x)/2))/(64*a^4*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^{5}{\left(c+dx \right)}}{\sin^{4}{\left(c+dx \right)}+4\sin^{3}{\left(c+dx \right)}+6\sin^{2}{\left(c+dx \right)}+4\sin{\left(c+dx \right)}+1}\, dx}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Integral(tan(c + d*x)**5/(sin(c + d*x)**4 + 4*sin(c + d*x)**3 + 6*sin(c + d
*x)**2 + 4*sin(c + d*x) + 1), x)/a**4
```

3.83 $\int \frac{\tan^3(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=132

$$\frac{1}{64d\left(a^4-a^4\sin(c+dx)\right)}+\frac{1}{64d\left(a^4\sin(c+dx)+a^4\right)}+\frac{1}{32d\left(a^2\sin(c+dx)+a^2\right)^2}+\frac{a}{20d(a\sin(c+dx)+a)^5}-\frac{}{8d(}$$

```
[Out] 1/20*a/d/(a+a*sin(d*x+c))^5-1/8/d/(a+a*sin(d*x+c))^4+1/16/a/d/(a+a*sin(d*x+
c))^3+1/32/d/(a^2+a^2*sin(d*x+c))^2+1/64/d/(a^4-a^4*sin(d*x+c))+1/64/d/(a^4
+a^4*sin(d*x+c))
```

Rubi [A] time = 0.09, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 88}

$$\frac{1}{64d\left(a^4-a^4\sin(c+dx)\right)}+\frac{1}{64d\left(a^4\sin(c+dx)+a^4\right)}+\frac{1}{32d\left(a^2\sin(c+dx)+a^2\right)^2}+\frac{a}{20d(a\sin(c+dx)+a)^5}-\frac{}{8d(}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] a/(20*d*(a + a*Sin[c + d*x])^5) - 1/(8*d*(a + a*Sin[c + d*x])^4) + 1/(16*a*
d*(a + a*Sin[c + d*x])^3) + 1/(32*d*(a^2 + a^2*Sin[c + d*x])^2) + 1/(64*d*(
a^4 - a^4*Sin[c + d*x])) + 1/(64*d*(a^4 + a^4*Sin[c + d*x]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{(a-x)^2(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{64a^3(a-x)^2} - \frac{a}{4(a+x)^6} + \frac{1}{2(a+x)^5} - \frac{3}{16a(a+x)^4} - \frac{1}{16a^2(a+x)^3} - \frac{1}{64a^3(a+x)^2}\right) dx, x, a\sin\right.}{d}$$
$$= \frac{a}{20d(a+a\sin(c+dx))^5} - \frac{1}{8d(a+a\sin(c+dx))^4} + \frac{1}{16ad(a+a\sin(c+dx))^3} + \frac{}{32d\,(}$$

Mathematica [A] time = 0.10, size = 50, normalized size = 0.38

$$-\frac{5\sin^2(c+dx)+4\sin(c+dx)+1}{20a^4d(\sin(c+dx)-1)(\sin(c+dx)+1)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -1/20*(1 + 4*Sin[c + d*x] + 5*Sin[c + d*x]^2)/(a^4*d*(-1 + Sin[c + d*x])*(1
 + Sin[c + d*x])^5)
```

fricas [A] time = 0.45, size = 102, normalized size = 0.77

$$-\frac{5\cos(dx+c)^2 - 4\sin(dx+c) - 6}{20\left(a^4d\cos(dx+c)^6 - 8a^4d\cos(dx+c)^4 + 8a^4d\cos(dx+c)^2 - 4\left(a^4d\cos(dx+c)^4 - 2a^4d\cos(dx+c)^2\right)\sin\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/20*(5*cos(d*x + c)^2 - 4*sin(d*x + c) - 6)/(a^4*d*cos(d*x + c)^6 - 8*a^4
*d*cos(d*x + c)^4 + 8*a^4*d*cos(d*x + c)^2 - 4*(a^4*d*cos(d*x + c)^4 - 2*a^
4*d*cos(d*x + c)^2)*sin(d*x + c))
```

giac [A] time = 2.34, size = 76, normalized size = 0.58

$$-\frac{\frac{5}{a^4(\sin(dx+c)-1)} - \frac{5\sin(dx+c)^4+30\sin(dx+c)^3+80\sin(dx+c)^2+50\sin(dx+c)+11}{a^4(\sin(dx+c)+1)^5}}{320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/320*(5/(a^4*(sin(d*x + c) - 1)) - (5*sin(d*x + c)^4 + 30*sin(d*x + c)^3
+ 80*sin(d*x + c)^2 + 50*sin(d*x + c) + 11)/(a^4*(sin(d*x + c) + 1)^5))/d
```

maple [A] time = 0.24, size = 81, normalized size = 0.61

$$\frac{-\frac{1}{64(\sin(dx+c)-1)}+\frac{1}{20(1+\sin(dx+c))^5}-\frac{1}{8(1+\sin(dx+c))^4}+\frac{1}{16(1+\sin(dx+c))^3}+\frac{1}{32(1+\sin(dx+c))^2}+\frac{1}{64+64\sin(dx+c)}}{d\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+a*sin(d*x+c))^4,x)
```

```
[Out] 1/d/a^4*(-1/64/(sin(d*x+c)-1)+1/20/(1+sin(d*x+c))^5-1/8/(1+sin(d*x+c))^4+1/
16/(1+sin(d*x+c))^3+1/32/(1+sin(d*x+c))^2+1/64/(1+sin(d*x+c)))
```

maxima [A] time = 0.37, size = 95, normalized size = 0.72

$$-\frac{5\,\sin\left(dx+c\right)^2+4\,\sin\left(dx+c\right)+1}{20\,\left(a^4\sin\left(dx+c\right)^6+4\,a^4\sin\left(dx+c\right)^5+5\,a^4\sin\left(dx+c\right)^4-5\,a^4\sin\left(dx+c\right)^2-4\,a^4\sin\left(dx+c\right)-a^4\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/20*(5*sin(d*x + c)^2 + 4*sin(d*x + c) + 1)/((a^4*sin(d*x + c)^6 + 4*a^4*
sin(d*x + c)^5 + 5*a^4*sin(d*x + c)^4 - 5*a^4*sin(d*x + c)^2 - 4*a^4*sin(d*
x + c) - a^4)*d)
```

mupad [B] time = 7.51, size = 172, normalized size = 1.30

$$\frac{4\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{32\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{5}+\frac{56\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{5}+\frac{32\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{5}+4\,\mathrm{co}}{a^4\,d\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)^2\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + a*sin(c + d*x))^4,x)
```

```
[Out] (4*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^8 + (32*cos(c/2 + (d*x)/2)^5*sin
(c/2 + (d*x)/2)^7)/5 + (56*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6)/5 + (
32*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^5)/5 + 4*cos(c/2 + (d*x)/2)^8*si
n(c/2 + (d*x)/2)^4)/(a^4*d*(cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2))^2*(cos
(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2))^10)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^3(c+dx)}{\sin^4(c+dx)+4\sin^3(c+dx)+6\sin^2(c+dx)+4\sin(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Integral(tan(c + d*x)**3/(sin(c + d*x)**4 + 4*sin(c + d*x)**3 + 6*sin(c + d
*x)**2 + 4*sin(c + d*x) + 1), x)/a**4
```

3.84 $\int \frac{\tan(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=105

$$-\frac{1}{16d\left(a^4\sin(c+dx)+a^4\right)}+\frac{\tanh^{-1}(\sin(c+dx))}{16a^4d}-\frac{1}{16d\left(a^2\sin(c+dx)+a^2\right)^2}-\frac{1}{12ad(a\sin(c+dx)+a)^3}+\frac{}{8d(a\,\mathrm{si}}$$

```
[Out] 1/16*arctanh(sin(d*x+c))/a^4/d+1/8/d/(a+a*sin(d*x+c))^4-1/12/a/d/(a+a*sin(d
*x+c))^3-1/16/d/(a^2+a^2*sin(d*x+c))^2-1/16/d/(a^4+a^4*sin(d*x+c))
```

Rubi [A] time = 0.07, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2707, 77, 206}

$$-\frac{1}{16d\left(a^4\sin(c+dx)+a^4\right)}-\frac{1}{16d\left(a^2\sin(c+dx)+a^2\right)^2}+\frac{\tanh^{-1}(\sin(c+dx))}{16a^4d}-\frac{1}{12ad(a\sin(c+dx)+a)^3}+\frac{}{8d(a\,\mathrm{si}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(16*a^4*d) + 1/(8*d*(a + a*Sin[c + d*x])^4) - 1/(12*a
*d*(a + a*Sin[c + d*x])^3) - 1/(16*d*(a^2 + a^2*Sin[c + d*x])^2) - 1/(16*d*
(a^4 + a^4*Sin[c + d*x]))
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
```

```
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tan(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(a-x)(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{2(a+x)^5}+\frac{1}{4a(a+x)^4}+\frac{1}{8a^2(a+x)^3}+\frac{1}{16a^3(a+x)^2}+\frac{1}{16a^3\left(a^2-x^2\right)}\right)dx, x, a\sin(c+dx$$

$$= \frac{1}{8d(a+a\sin(c+dx))^4}-\frac{1}{12ad(a+a\sin(c+dx))^3}-\frac{1}{16d\left(a^2+a^2\sin(c+dx)\right)^2}-$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{16a^4d}+\frac{1}{8d(a+a\sin(c+dx))^4}-\frac{1}{12ad(a+a\sin(c+dx))^3}-\frac{}{16d\left(a^2\right.}$$

Mathematica [A] time = 0.26, size = 62, normalized size = 0.59

$$\frac{3\tanh^{-1}(\sin(c+dx))-\frac{3\sin^3(c+dx)+12\sin^2(c+dx)+19\sin(c+dx)+4}{(\sin(c+dx)+1)^4}}{48a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]] - (4 + 19*Sin[c + d*x] + 12*Sin[c + d*x]^2 + 3*Sin
[c + d*x]^3)/(1 + Sin[c + d*x])^4)/(48*a^4*d)
```

fricas [B] time = 0.44, size = 198, normalized size = 1.89

$$\frac{24\cos(dx+c)^2+3\left(\cos(dx+c)^4-8\cos(dx+c)^2-4\left(\cos(dx+c)^2-2\right)\sin(dx+c)+8\right)\log(\sin(dx+c)+1}{96\left(a^4d\cos(dx+c)^4-8a^4d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/96*(24*cos(d*x + c)^2 + 3*(cos(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*(cos(d*x
 + c)^2 - 2)*sin(d*x + c) + 8)*log(sin(d*x + c) + 1) - 3*(cos(d*x + c)^4 -
```

```
8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)*log(-sin(d*x +
c) + 1) + 2*(3*cos(d*x + c)^2 - 22)*sin(d*x + c) - 32)/(a^4*d*cos(d*x + c)^
4 - 8*a^4*d*cos(d*x + c)^2 + 8*a^4*d - 4*(a^4*d*cos(d*x + c)^2 - 2*a^4*d)*s
in(d*x + c))
```

giac [A] time = 0.72, size = 91, normalized size = 0.87

$$\frac{\frac{12 \log(|\sin(dx+c)+1|)}{a^4} - \frac{12 \log(|\sin(dx+c)-1|)}{a^4} - \frac{25 \sin(dx+c)^4+124 \sin(dx+c)^3+246 \sin(dx+c)^2+252 \sin(dx+c)+57}{a^4(\sin(dx+c)+1)^4}}{384\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/384*(12*log(abs(sin(d*x + c) + 1))/a^4 - 12*log(abs(sin(d*x + c) - 1))/a^
4 - (25*sin(d*x + c)^4 + 124*sin(d*x + c)^3 + 246*sin(d*x + c)^2 + 252*sin(
d*x + c) + 57)/(a^4*(sin(d*x + c) + 1)^4))/d
```

maple [A] time = 0.25, size = 108, normalized size = 1.03

$$-\frac{\ln(\sin(dx+c)-1)}{32a^4d}+\frac{1}{8a^4d\,(1+\sin(dx+c))^4}-\frac{1}{12a^4d\,(1+\sin(dx+c))^3}-\frac{1}{16a^4d\,(1+\sin(dx+c))^2}-\frac{}{16a^4d\,(1+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+a*sin(d*x+c))^4,x)
```

```
[Out] -1/32/a^4/d*ln(sin(d*x+c)-1)+1/8/a^4/d/(1+sin(d*x+c))^4-1/12/a^4/d/(1+sin(d
*x+c))^3-1/16/a^4/d/(1+sin(d*x+c))^2-1/16/a^4/d/(1+sin(d*x+c))+1/32*ln(1+si
n(d*x+c))/a^4/d
```

maxima [A] time = 0.33, size = 121, normalized size = 1.15

$$-\frac{\frac{2\left(3 \sin(dx+c)^3+12 \sin(dx+c)^2+19 \sin(dx+c)+4\right)}{a^4 \sin(dx+c)^4+4\,a^4 \sin(dx+c)^3+6\,a^4 \sin(dx+c)^2+4\,a^4 \sin(dx+c)+a^4} - \frac{3 \log(\sin(dx+c)+1)}{a^4} + \frac{3 \log(\sin(dx+c)-1)}{a^4}}{96\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/96*(2*(3*sin(d*x + c)^3 + 12*sin(d*x + c)^2 + 19*sin(d*x + c) + 4)/(a^4*
sin(d*x + c)^4 + 4*a^4*sin(d*x + c)^3 + 6*a^4*sin(d*x + c)^2 + 4*a^4*sin(d*
x + c) + a^4) - 3*log(sin(d*x + c) + 1)/a^4 + 3*log(sin(d*x + c) - 1)/a^4)/
d
```

mupad [B] time = 10.07, size = 240, normalized size = 2.29

$$\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,a^4\,d}+\frac{-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{8}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\frac{43\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{24}+\frac{10\tan}{}}{d\,\left(a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+8\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+28\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+56\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+7\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + a*sin(c + d*x))^4,x)
```

```
[Out] atanh(tan(c/2 + (d*x)/2))/(8*a^4*d) + (tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*
x)/2)/8 + (43*tan(c/2 + (d*x)/2)^3)/24 + (10*tan(c/2 + (d*x)/2)^4)/3 + (43*
tan(c/2 + (d*x)/2)^5)/24 + tan(c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^7/8)/(
d*(28*a^4*tan(c/2 + (d*x)/2)^2 + 56*a^4*tan(c/2 + (d*x)/2)^3 + 70*a^4*tan(c
/2 + (d*x)/2)^4 + 56*a^4*tan(c/2 + (d*x)/2)^5 + 28*a^4*tan(c/2 + (d*x)/2)^6
 + 8*a^4*tan(c/2 + (d*x)/2)^7 + a^4*tan(c/2 + (d*x)/2)^8 + a^4 + 8*a^4*tan(
c/2 + (d*x)/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan{\left(c+dx \right)}}{\sin^{4}{\left(c+dx \right)}+4\sin^{3}{\left(c+dx \right)}+6\sin^{2}{\left(c+dx \right)}+4\sin{\left(c+dx \right)}+1}\, dx}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Integral(tan(c + d*x)/(sin(c + d*x)**4 + 4*sin(c + d*x)**3 + 6*sin(c + d*x)
**2 + 4*sin(c + d*x) + 1), x)/a**4
```

3.85 $\int \frac{\cot^3(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=106

$$\frac{5}{d\left(a^4\sin(c+dx)+a^4\right)}-\frac{\csc^2(c+dx)}{2a^4d}+\frac{4\csc(c+dx)}{a^4d}+\frac{9\log(\sin(c+dx))}{a^4d}-\frac{9\log(\sin(c+dx)+1)}{a^4d}+\frac{1}{d\left(a^2\sin(c+\right.}$$

```
[Out] 4*csc(d*x+c)/a^4/d-1/2*csc(d*x+c)^2/a^4/d+9*ln(sin(d*x+c))/a^4/d-9*ln(1+sin
(d*x+c))/a^4/d+1/d/(a^2+a^2*sin(d*x+c))^2+5/d/(a^4+a^4*sin(d*x+c))
```

Rubi [A] time = 0.08, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 77}

$$\frac{5}{d\left(a^4\sin(c+dx)+a^4\right)}+\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)^2}-\frac{\csc^2(c+dx)}{2a^4d}+\frac{4\csc(c+dx)}{a^4d}+\frac{9\log(\sin(c+dx))}{a^4d}-\frac{9\log(\sin(c}{a^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (4*Csc[c + d*x])/(a^4*d) - Csc[c + d*x]^2/(2*a^4*d) + (9*Log[Sin[c + d*x]])
/(a^4*d) - (9*Log[1 + Sin[c + d*x]])/(a^4*d) + 1/(d*(a^2 + a^2*Sin[c + d*x]
)^2) + 5/(d*(a^4 + a^4*Sin[c + d*x]))
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{a-x}{x^3(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2x^3} - \frac{4}{a^3x^2} + \frac{9}{a^4x} - \frac{2}{a^2(a+x)^3} - \frac{5}{a^3(a+x)^2} - \frac{9}{a^4(a+x)}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{4\csc(c+dx)}{a^4d} - \frac{\csc^2(c+dx)}{2a^4d} + \frac{9\log(\sin(c+dx))}{a^4d} - \frac{9\log(1+\sin(c+dx))}{a^4d} + \frac{}{d\left(a^2\right.}$$

Mathematica [A] time = 0.80, size = 73, normalized size = 0.69

$$\frac{\frac{10}{\sin(c+dx)+1} + \frac{2}{(\sin(c+dx)+1)^2} - \csc^2(c+dx) + 8\csc(c+dx) + 18\log(\sin(c+dx)) - 18\log(\sin(c+dx)+1)}{2a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (8*Csc[c + d*x] - Csc[c + d*x]^2 + 18*Log[Sin[c + d*x]] - 18*Log[1 + Sin[c
+ d*x]] + 2/(1 + Sin[c + d*x])^2 + 10/(1 + Sin[c + d*x]))/(2*a^4*d)
```

fricas [A] time = 0.45, size = 196, normalized size = 1.85

$$-\frac{27\cos(dx+c)^2 - 18\left(\cos(dx+c)^4 - 3\cos(dx+c)^2 - 2\left(\cos(dx+c)^2 - 1\right)\sin(dx+c) + 2\right)\log\left(\frac{1}{2}\sin(dx+c\right.}{2\left(a^4d\cos(dx+c)^4 - 3a^4d\,\mathrm{c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/2*(27*cos(d*x + c)^2 - 18*(cos(d*x + c)^4 - 3*cos(d*x + c)^2 - 2*(cos(d*
x + c)^2 - 1)*sin(d*x + c) + 2)*log(1/2*sin(d*x + c)) + 18*(cos(d*x + c)^4
- 3*cos(d*x + c)^2 - 2*(cos(d*x + c)^2 - 1)*sin(d*x + c) + 2)*log(sin(d*x +
 c) + 1) + 6*(3*cos(d*x + c)^2 - 4)*sin(d*x + c) - 26)/(a^4*d*cos(d*x + c)^
4 - 3*a^4*d*cos(d*x + c)^2 + 2*a^4*d - 2*(a^4*d*cos(d*x + c)^2 - a^4*d)*sin
(d*x + c))
```

giac [A] time = 1.18, size = 185, normalized size = 1.75

$$-\frac{\frac{144\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^4} - \frac{72\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^4} + \frac{108\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - 16\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1}{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2} + \frac{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - 16a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.}{a^8}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/8*(144*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 72*log(abs(tan(1/2*d*x +
 1/2*c)))/a^4 + (108*tan(1/2*d*x + 1/2*c)^2 - 16*tan(1/2*d*x + 1/2*c) + 1)/
(a^4*tan(1/2*d*x + 1/2*c)^2) + (a^4*tan(1/2*d*x + 1/2*c)^2 - 16*a^4*tan(1/2
*d*x + 1/2*c))/a^8 - 4*(75*tan(1/2*d*x + 1/2*c)^4 + 272*tan(1/2*d*x + 1/2*c
)^3 + 402*tan(1/2*d*x + 1/2*c)^2 + 272*tan(1/2*d*x + 1/2*c) + 75)/(a^4*(tan
(1/2*d*x + 1/2*c) + 1)^4))/d
```

maple [A] time = 0.34, size = 101, normalized size = 0.95

$$-\frac{1}{2a^4d\sin(dx+c)^2}+\frac{4}{a^4d\sin(dx+c)}+\frac{9\ln(\sin(dx+c))}{a^4d}+\frac{1}{a^4d(1+\sin(dx+c))^2}+\frac{5}{a^4d(1+\sin(dx+c))}-\frac{9\ln(\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+a*sin(d*x+c))^4,x)
```

```
[Out] -1/2/a^4/d/sin(d*x+c)^2+4/a^4/d/sin(d*x+c)+9*ln(sin(d*x+c))/a^4/d+1/a^4/d/(
1+sin(d*x+c))^2+5/a^4/d/(1+sin(d*x+c))-9*ln(1+sin(d*x+c))/a^4/d
```

maxima [A] time = 0.33, size = 103, normalized size = 0.97

$$\frac{\frac{18\sin(dx+c)^3+27\sin(dx+c)^2+6\sin(dx+c)-1}{a^4\sin(dx+c)^4+2a^4\sin(dx+c)^3+a^4\sin(dx+c)^2}-\frac{18\log(\sin(dx+c)+1)}{a^4}+\frac{18\log(\sin(dx+c))}{a^4}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/2*((18*sin(d*x + c)^3 + 27*sin(d*x + c)^2 + 6*sin(d*x + c) - 1)/(a^4*sin(
d*x + c)^4 + 2*a^4*sin(d*x + c)^3 + a^4*sin(d*x + c)^2) - 18*log(sin(d*x +
c) + 1)/a^4 + 18*log(sin(d*x + c))/a^4)/d
```

mupad [B] time = 6.61, size = 228, normalized size = 2.15

$$\frac{9\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^4\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a^4\,d}-\frac{48\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\frac{129\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{2}+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-29\tan\left(\frac{c}{2}+\cdots\right.}{d\left(4\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+16\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+24\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+16\,a^4\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3/(a + a*sin(c + d*x))^4,x)
```

```
[Out] (9*log(tan(c/2 + (d*x)/2)))/(a^4*d) - tan(c/2 + (d*x)/2)^2/(8*a^4*d) - (10*
tan(c/2 + (d*x)/2)^3 - 29*tan(c/2 + (d*x)/2)^2 - 6*tan(c/2 + (d*x)/2) + (12
9*tan(c/2 + (d*x)/2)^4)/2 + 48*tan(c/2 + (d*x)/2)^5 + 1/2)/(d*(4*a^4*tan(c/
2 + (d*x)/2)^2 + 16*a^4*tan(c/2 + (d*x)/2)^3 + 24*a^4*tan(c/2 + (d*x)/2)^4
+ 16*a^4*tan(c/2 + (d*x)/2)^5 + 4*a^4*tan(c/2 + (d*x)/2)^6)) - (18*log(tan(
c/2 + (d*x)/2) + 1))/(a^4*d) + (2*tan(c/2 + (d*x)/2))/(a^4*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^3(c+dx)}{\sin^4(c+dx)+4\sin^3(c+dx)+6\sin^2(c+dx)+4\sin(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Integral(cot(c + d*x)**3/(sin(c + d*x)**4 + 4*sin(c + d*x)**3 + 6*sin(c + d
*x)**2 + 4*sin(c + d*x) + 1), x)/a**4
```

3.86 $\int \frac{\cot^7(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=135

$$-\frac{\csc^6(c+dx)}{6a^4d}+\frac{4\csc^5(c+dx)}{5a^4d}-\frac{7\csc^4(c+dx)}{4a^4d}+\frac{8\csc^3(c+dx)}{3a^4d}-\frac{4\csc^2(c+dx)}{a^4d}+\frac{8\csc(c+dx)}{a^4d}+\frac{8\log(\sin(c+dx$$

```
[Out] 8*csc(d*x+c)/a^4/d-4*csc(d*x+c)^2/a^4/d+8/3*csc(d*x+c)^3/a^4/d-7/4*csc(d*x+
c)^4/a^4/d+4/5*csc(d*x+c)^5/a^4/d-1/6*csc(d*x+c)^6/a^4/d+8*ln(sin(d*x+c))/a
^4/d-8*ln(1+sin(d*x+c))/a^4/d
```

Rubi [A] time = 0.09, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 88}

$$-\frac{\csc^6(c+dx)}{6a^4d}+\frac{4\csc^5(c+dx)}{5a^4d}-\frac{7\csc^4(c+dx)}{4a^4d}+\frac{8\csc^3(c+dx)}{3a^4d}-\frac{4\csc^2(c+dx)}{a^4d}+\frac{8\csc(c+dx)}{a^4d}+\frac{8\log(\sin(c+dx$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (8*Csc[c + d*x])/(a^4*d) - (4*Csc[c + d*x]^2)/(a^4*d) + (8*Csc[c + d*x]^3)/
(3*a^4*d) - (7*Csc[c + d*x]^4)/(4*a^4*d) + (4*Csc[c + d*x]^5)/(5*a^4*d) - C
sc[c + d*x]^6/(6*a^4*d) + (8*Log[Sin[c + d*x]])/(a^4*d) - (8*Log[1 + Sin[c
+ d*x]])/(a^4*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^7(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^3}{x^7(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^2}{x^7} - \frac{4a}{x^6} + \frac{7}{x^5} - \frac{8}{ax^4} + \frac{8}{a^2x^3} - \frac{8}{a^3x^2} + \frac{8}{a^4x} - \frac{8}{a^4(a+x)}\right)dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{8\csc(c+dx)}{a^4d} - \frac{4\csc^2(c+dx)}{a^4d} + \frac{8\csc^3(c+dx)}{3a^4d} - \frac{7\csc^4(c+dx)}{4a^4d} + \frac{4\csc^5(c+dx)}{5a^4d} -$$

Mathematica [A] time = 0.16, size = 89, normalized size = 0.66

$$\frac{-10\csc^6(c+dx) + 48\csc^5(c+dx) - 105\csc^4(c+dx) + 160\csc^3(c+dx) - 240\csc^2(c+dx) + 480\csc(c+dx) + 4}{60a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (480*Csc[c + d*x] - 240*Csc[c + d*x]^2 + 160*Csc[c + d*x]^3 - 105*Csc[c + d
*x]^4 + 48*Csc[c + d*x]^5 - 10*Csc[c + d*x]^6 + 480*Log[Sin[c + d*x]] - 480
*Log[1 + Sin[c + d*x]])/(60*a^4*d)
```

fricas [A] time = 0.45, size = 186, normalized size = 1.38

$$\frac{240\cos(dx+c)^4 - 585\cos(dx+c)^2 + 480\left(\cos(dx+c)^6 - 3\cos(dx+c)^4 + 3\cos(dx+c)^2 - 1\right)\log\left(\frac{1}{2}\sin(dx\right.}{60\left(a^4d\cos(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/60*(240*cos(d*x + c)^4 - 585*cos(d*x + c)^2 + 480*(cos(d*x + c)^6 - 3*cos
(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(1/2*sin(d*x + c)) - 480*(cos(d*x +
c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(sin(d*x + c) + 1) - 16*
(30*cos(d*x + c)^4 - 70*cos(d*x + c)^2 + 43)*sin(d*x + c) + 355)/(a^4*d*cos
(d*x + c)^6 - 3*a^4*d*cos(d*x + c)^4 + 3*a^4*d*cos(d*x + c)^2 - a^4*d)
```

giac [A] time = 1.12, size = 232, normalized size = 1.72

$$-\frac{\frac{30720\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^4} - \frac{15360\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^4} + \frac{37632\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6 - 10080\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 + 2835\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 - 880\tan(}{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/1920*(30720*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 15360*log(abs(tan(1
/2*d*x + 1/2*c)))/a^4 + (37632*tan(1/2*d*x + 1/2*c)^6 - 10080*tan(1/2*d*x +
 1/2*c)^5 + 2835*tan(1/2*d*x + 1/2*c)^4 - 880*tan(1/2*d*x + 1/2*c)^3 + 240*
tan(1/2*d*x + 1/2*c)^2 - 48*tan(1/2*d*x + 1/2*c) + 5)/(a^4*tan(1/2*d*x + 1/
2*c)^6) + (5*a^20*tan(1/2*d*x + 1/2*c)^6 - 48*a^20*tan(1/2*d*x + 1/2*c)^5 +
 240*a^20*tan(1/2*d*x + 1/2*c)^4 - 880*a^20*tan(1/2*d*x + 1/2*c)^3 + 2835*a
^20*tan(1/2*d*x + 1/2*c)^2 - 10080*a^20*tan(1/2*d*x + 1/2*c))/a^24)/d
```

maple [A] time = 0.33, size = 130, normalized size = 0.96

$$-\frac{1}{6a^4d\sin(dx+c)^6}+\frac{4}{5a^4d\sin(dx+c)^5}-\frac{7}{4a^4d\sin(dx+c)^4}+\frac{8}{3a^4d\sin(dx+c)^3}-\frac{4}{a^4d\sin(dx+c)^2}+\frac{8}{a^4d\sin(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^7/(a+a*sin(d*x+c))^4,x)
```

```
[Out] -1/6/a^4/d/sin(d*x+c)^6+4/5/a^4/d/sin(d*x+c)^5-7/4/a^4/d/sin(d*x+c)^4+8/3/a
^4/d/sin(d*x+c)^3-4/a^4/d/sin(d*x+c)^2+8/a^4/d/sin(d*x+c)+8*ln(sin(d*x+c))/
a^4/d-8*ln(1+sin(d*x+c))/a^4/d
```

maxima [A] time = 0.32, size = 95, normalized size = 0.70

$$-\frac{\frac{480\log(\sin(dx+c)+1)}{a^4}-\frac{480\log(\sin(dx+c))}{a^4}-\frac{480\sin(dx+c)^5-240\sin(dx+c)^4+160\sin(dx+c)^3-105\sin(dx+c)^2+48\sin(dx+c)-10}{a^4\sin(dx+c)^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/60*(480*log(sin(d*x + c) + 1)/a^4 - 480*log(sin(d*x + c))/a^4 - (480*sin
(d*x + c)^5 - 240*sin(d*x + c)^4 + 160*sin(d*x + c)^3 - 105*sin(d*x + c)^2
+ 48*sin(d*x + c) - 10)/(a^4*sin(d*x + c)^6))/d
```

mupad [B] time = 6.81, size = 235, normalized size = 1.74

$$\frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a^4\,d}-\frac{189\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{128\,a^4\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{8\,a^4\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{40\,a^4\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{384\,a^4\,d}+\frac{8\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^4\,d}-\frac{16}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^7/(a + a*sin(c + d*x))^4,x)
```

```
[Out] (11*tan(c/2 + (d*x)/2)^3)/(24*a^4*d) - (189*tan(c/2 + (d*x)/2)^2)/(128*a^4*
d) - tan(c/2 + (d*x)/2)^4/(8*a^4*d) + tan(c/2 + (d*x)/2)^5/(40*a^4*d) - tan
(c/2 + (d*x)/2)^6/(384*a^4*d) + (8*log(tan(c/2 + (d*x)/2)))/(a^4*d) - (16*l
og(tan(c/2 + (d*x)/2) + 1))/(a^4*d) + (21*tan(c/2 + (d*x)/2))/(4*a^4*d) + (
cot(c/2 + (d*x)/2)^6*((8*tan(c/2 + (d*x)/2))/5 - 8*tan(c/2 + (d*x)/2)^2 + (
88*tan(c/2 + (d*x)/2)^3)/3 - (189*tan(c/2 + (d*x)/2)^4)/2 + 336*tan(c/2 + (
d*x)/2)^5 - 1/6))/(64*a^4*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^7(c+dx)}{\sin^4(c+dx)+4\sin^3(c+dx)+6\sin^2(c+dx)+4\sin(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**7/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Integral(cot(c + d*x)**7/(sin(c + d*x)**4 + 4*sin(c + d*x)**3 + 6*sin(c + d
*x)**2 + 4*sin(c + d*x) + 1), x)/a**4
```

3.87 $\int \frac{\tan^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=127

$$\frac{8\tan^9(c+dx)}{9a^4d}+\frac{16\tan^7(c+dx)}{7a^4d}+\frac{9\tan^5(c+dx)}{5a^4d}+\frac{\tan^3(c+dx)}{3a^4d}-\frac{8\sec^9(c+dx)}{9a^4d}+\frac{12\sec^7(c+dx)}{7a^4d}-\frac{4\sec^5(c+dx)}{5a^4d}$$

```
[Out] -4/5*sec(d*x+c)^5/a^4/d+12/7*sec(d*x+c)^7/a^4/d-8/9*sec(d*x+c)^9/a^4/d+1/3*
tan(d*x+c)^3/a^4/d+9/5*tan(d*x+c)^5/a^4/d+16/7*tan(d*x+c)^7/a^4/d+8/9*tan(d
*x+c)^9/a^4/d
```

Rubi [A] time = 0.31, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2711, 2607, 270, 2606, 14}

$$\frac{8\tan^9(c+dx)}{9a^4d}+\frac{16\tan^7(c+dx)}{7a^4d}+\frac{9\tan^5(c+dx)}{5a^4d}+\frac{\tan^3(c+dx)}{3a^4d}-\frac{8\sec^9(c+dx)}{9a^4d}+\frac{12\sec^7(c+dx)}{7a^4d}-\frac{4\sec^5(c+dx)}{5a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-4*Sec[c + d*x]^5)/(5*a^4*d) + (12*Sec[c + d*x]^7)/(7*a^4*d) - (8*Sec[c +
d*x]^9)/(9*a^4*d) + Tan[c + d*x]^3/(3*a^4*d) + (9*Tan[c + d*x]^5)/(5*a^4*d)
 + (16*Tan[c + d*x]^7)/(7*a^4*d) + (8*Tan[c + d*x]^9)/(9*a^4*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2711

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Dist[a^(2*m), Int[ExpandIntegrand[(g*Tan[e + f*x])
^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; Fr
eeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx &= \frac{\int \left(a^4\sec^8(c+dx)\tan^2(c+dx)-4a^4\sec^7(c+dx)\tan^3(c+dx)+6a^4\sec^6(c+dx)\tan\right.}{a^8}\\
&= \frac{\int \sec^8(c+dx)\tan^2(c+dx)\,dx}{a^4}+\frac{\int \sec^4(c+dx)\tan^6(c+dx)\,dx}{a^4}-\frac{4\int \sec^7(c+dx)}{a^4}\\
&= \frac{\text{Subst}\left(\int x^6\left(1+x^2\right)dx,x,\tan(c+dx)\right)}{a^4d}+\frac{\text{Subst}\left(\int x^2\left(1+x^2\right)^3 dx,x,\tan(c+dx)\right)}{a^4d}\\
&= \frac{\text{Subst}\left(\int \left(x^6+x^8\right)dx,x,\tan(c+dx)\right)}{a^4d}+\frac{\text{Subst}\left(\int \left(x^2+3x^4+3x^6+x^8\right)dx,x,\tan(\right.}{a^4d}\\
&= -\frac{4\sec^5(c+dx)}{5a^4d}+\frac{12\sec^7(c+dx)}{7a^4d}-\frac{8\sec^9(c+dx)}{9a^4d}+\frac{\tan^3(c+dx)}{3a^4d}+\frac{9\tan^5(c+dx)}{5a^4d}
\end{aligned}$$

Mathematica [A] time = 0.43, size = 124, normalized size = 0.98

$$\frac{\sec(c+dx)(34944\sin(c+dx)+1776\sin(2(c+dx))-9504\sin(3(c+dx))-296\sin(4(c+dx))+352\sin(5(c+}{80640a^4d(}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]*(16128 + 1554*Cos[c + d*x] - 16896*Cos[2*(c + d*x)] - 999*Cos
[3*(c + d*x)] + 2816*Cos[4*(c + d*x)] + 37*Cos[5*(c + d*x)] + 34944*Sin[c +
 d*x] + 1776*Sin[2*(c + d*x)] - 9504*Sin[3*(c + d*x)] - 296*Sin[4*(c + d*x)
] + 352*Sin[5*(c + d*x)]))/(80640*a^4*d*(1 + Sin[c + d*x])^4)
```

fricas [A] time = 0.45, size = 129, normalized size = 1.02

$$\frac{88\cos(dx+c)^4-220\cos(dx+c)^2+\left(22\cos(dx+c)^4-165\cos(dx+c)^2+175\right)\sin(dx+c)+140}{315\left(a^4d\cos(dx+c)^5-8a^4d\cos(dx+c)^3+8a^4d\cos(dx+c)-4\left(a^4d\cos(dx+c)^3-2a^4d\cos(dx+c)\right)\sin(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/315*(88*cos(d*x + c)^4 - 220*cos(d*x + c)^2 + (22*cos(d*x + c)^4 - 165*co
s(d*x + c)^2 + 175)*sin(d*x + c) + 140)/(a^4*d*cos(d*x + c)^5 - 8*a^4*d*cos
(d*x + c)^3 + 8*a^4*d*cos(d*x + c) - 4*(a^4*d*cos(d*x + c)^3 - 2*a^4*d*cos(
d*x + c))*sin(d*x + c))
```

giac [A] time = 1.81, size = 146, normalized size = 1.15

$$-\frac{\frac{315}{a^4\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}-\frac{315\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8+3150\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+1050\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+630\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-8064\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-6006\,\mathrm{ta}}{a^4\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^9}}{5040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/5040*(315/(a^4*(tan(1/2*d*x + 1/2*c) - 1)) - (315*tan(1/2*d*x + 1/2*c)^8
 + 3150*tan(1/2*d*x + 1/2*c)^7 + 1050*tan(1/2*d*x + 1/2*c)^6 + 630*tan(1/2*
d*x + 1/2*c)^5 - 8064*tan(1/2*d*x + 1/2*c)^4 - 6006*tan(1/2*d*x + 1/2*c)^3
- 5274*tan(1/2*d*x + 1/2*c)^2 - 846*tan(1/2*d*x + 1/2*c) - 59)/(a^4*(tan(1/
2*d*x + 1/2*c) + 1)^9))/d
```

maple [A] time = 0.21, size = 158, normalized size = 1.24

$$\frac{-\frac{1}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{16}{9\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^9}+\frac{8}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^8}-\frac{116}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7}+\frac{62}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^6}-\frac{83}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{1}{2\left(\tan\left(\frac{dx}{2}\right.\right.}}{d\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+a*sin(d*x+c))^4,x)
```

```
[Out] 8/d/a^4*(-1/128/(tan(1/2*d*x+1/2*c)-1)-2/9/(tan(1/2*d*x+1/2*c)+1)^9+1/(tan(
1/2*d*x+1/2*c)+1)^8-29/14/(tan(1/2*d*x+1/2*c)+1)^7+31/12/(tan(1/2*d*x+1/2*c
)+1)^6-83/40/(tan(1/2*d*x+1/2*c)+1)^5+17/16/(tan(1/2*d*x+1/2*c)+1)^4-29/96/
(tan(1/2*d*x+1/2*c)+1)^3+1/64/(tan(1/2*d*x+1/2*c)+1)^2+1/128/(tan(1/2*d*x+1
/2*c)+1))
```

maxima [B] time = 0.38, size = 356, normalized size = 2.80

$$\frac{8\left(\frac{16\sin(dx+c)}{\cos(dx+c)+1}+\frac{54\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{201\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{294\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{378\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{210\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\cdots\right.}{315\left(a^4+\frac{8a^4\sin(dx+c)}{\cos(dx+c)+1}+\frac{27a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{48a^4\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{42a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{42a^4\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{48a^4\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 8/315*(16*sin(d*x + c)/(cos(d*x + c) + 1) + 54*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 201*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 294*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 + 378*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 210*sin(d*x +
c)^6/(cos(d*x + c) + 1)^6 + 105*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 2)/((
a^4 + 8*a^4*sin(d*x + c)/(cos(d*x + c) + 1) + 27*a^4*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 + 48*a^4*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 42*a^4*sin(d*x
 + c)^4/(cos(d*x + c) + 1)^4 - 42*a^4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 -
 48*a^4*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 27*a^4*sin(d*x + c)^8/(cos(d*
x + c) + 1)^8 - 8*a^4*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - a^4*sin(d*x + c
)^10/(cos(d*x + c) + 1)^10)*d)
```

mupad [B] time = 7.58, size = 231, normalized size = 1.82

$$\frac{\frac{16\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{315}+\frac{128\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^9\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{315}+\frac{48\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{35}+\frac{536\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^7\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{105}+\frac{112\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\cdots}{15}\cdots}{a^4\,d\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + a*sin(c + d*x))^4,x)
```

```
[Out] ((16*cos(c/2 + (d*x)/2)^10)/315 + (128*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)
/2))/315 + (8*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^7)/3 + (16*cos(c/2 +
(d*x)/2)^4*sin(c/2 + (d*x)/2)^6)/3 + (48*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*
x)/2)^5)/5 + (112*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^4)/15 + (536*cos(
c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^3)/105 + (48*cos(c/2 + (d*x)/2)^8*sin(c
/2 + (d*x)/2)^2)/35)/(a^4*d*(cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2))*(cos(
c/2 + (d*x)/2) + sin(c/2 + (d*x)/2))^9)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\tan^2(c+dx)}{\sin^4(c+dx)+4\sin^3(c+dx)+6\sin^2(c+dx)+4\sin(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Integral(tan(c + d*x)**2/(sin(c + d*x)**4 + 4*sin(c + d*x)**3 + 6*sin(c + d
*x)**2 + 4*sin(c + d*x) + 1), x)/a**4
```

3.88 $\int \frac{\cot^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=108

$$-\frac{\cot(c+dx)}{a^4d}+\frac{4\tanh^{-1}(\cos(c+dx))}{a^4d}-\frac{104\cot(c+dx)}{15a^4d(\csc(c+dx)+1)}+\frac{31\cot(c+dx)}{15a^4d(\csc(c+dx)+1)^2}-\frac{2\cot(c+dx)}{5a^4d(\csc(c+dx)+1)^3}$$

```
[Out] 4*arctanh(cos(d*x+c))/a^4/d-94/15*cot(d*x+c)/a^4/d+2/5*cot(d*x+c)/a^4/d/(1+
sin(d*x+c))^3+13/15*cot(d*x+c)/a^4/d/(1+sin(d*x+c))^2+4*cot(d*x+c)/a^4/d/(1
+sin(d*x+c))
```

Rubi [A] time = 0.32, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2709, 3770, 3767, 8, 3777, 3922, 3919, 3794}

$$-\frac{\cot(c+dx)}{a^4d}+\frac{4\tanh^{-1}(\cos(c+dx))}{a^4d}-\frac{104\cot(c+dx)}{15a^4d(\csc(c+dx)+1)}+\frac{31\cot(c+dx)}{15a^4d(\csc(c+dx)+1)^2}-\frac{2\cot(c+dx)}{5a^4d(\csc(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (4*ArcTanh[Cos[c + d*x]])/(a^4*d) - Cot[c + d*x]/(a^4*d) - (2*Cot[c + d*x])
/(5*a^4*d*(1 + Csc[c + d*x])^3) + (31*Cot[c + d*x])/(15*a^4*d*(1 + Csc[c +
d*x])^2) - (104*Cot[c + d*x])/(15*a^4*d*(1 + Csc[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3922

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*
x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x
])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ
[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\int \left(\frac{9}{a^2} - \frac{4\csc(c+dx)}{a^2} + \frac{\csc^2(c+dx)}{a^2} - \frac{2}{a^2(1+\csc(c+dx))^3} + \frac{9}{a^2(1+\csc(c+dx))^2} - \frac{16}{a^2(1+\csc(c+dx))}\right) dx}{a^2}$$

$$= \frac{9x}{a^4} + \frac{\int \csc^2(c+dx)\,dx}{a^4} - \frac{2\int \frac{1}{(1+\csc(c+dx))^3}\,dx}{a^4} - \frac{4\int \csc(c+dx)\,dx}{a^4} + \frac{9\int \frac{1}{(1+\csc(c+dx))}}{a^4}$$

$$= \frac{9x}{a^4} + \frac{4\tanh^{-1}(\cos(c+dx))}{a^4d} - \frac{2\cot(c+dx)}{5a^4d(1+\csc(c+dx))^3} + \frac{3\cot(c+dx)}{a^4d(1+\csc(c+dx))^2} - \frac{}{a^4d}$$

$$= \frac{2x}{a^4} + \frac{4\tanh^{-1}(\cos(c+dx))}{a^4d} - \frac{\cot(c+dx)}{a^4d} - \frac{2\cot(c+dx)}{5a^4d(1+\csc(c+dx))^3} + \frac{31\cot(c}{15a^4d(1+\csc}$$

$$= \frac{4\tanh^{-1}(\cos(c+dx))}{a^4d} - \frac{\cot(c+dx)}{a^4d} - \frac{2\cot(c+dx)}{5a^4d(1+\csc(c+dx))^3} + \frac{31\cot(c+dx)}{15a^4d(1+\csc(c+}$$

$$= \frac{4\tanh^{-1}(\cos(c+dx))}{a^4d} - \frac{\cot(c+dx)}{a^4d} - \frac{2\cot(c+dx)}{5a^4d(1+\csc(c+dx))^3} + \frac{31\cot(c+dx)}{15a^4d(1+\csc(c+}$$

Mathematica [B] time = 0.42, size = 315, normalized size = 2.92

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right) + \cos\left(\frac{1}{2}(c+dx)\right)\right)^3 \left(24\sin\left(\frac{1}{2}(c+dx)\right) + 316\sin\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right) + \cos\left(\frac{1}{2}(c+dx)\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(24*Sin[(c + d*x)/2] - 12*(Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2]) + 76*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2])^2 - 38*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + 316*Sin[(c
+ d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 - 15*Cot[(c + d*x)/2]*(Co
s[(c + d*x)/2] + Sin[(c + d*x)/2])^5 + 120*Log[Cos[(c + d*x)/2]]*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2])^5 - 120*Log[Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2])^5 + 15*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5*Tan[(c
+ d*x)/2]))/(30*d*(a + a*Sin[c + d*x])^4)
```

fricas [B] time = 0.44, size = 369, normalized size = 3.42

$$\frac{94\cos(dx+c)^4 + 222\cos(dx+c)^3 - 115\cos(dx+c)^2 + 30\left(\cos(dx+c)^4 - 2\cos(dx+c)^3 - 5\cos(dx+c)^2 - \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/15*(94*cos(d*x + c)^4 + 222*cos(d*x + c)^3 - 115*cos(d*x + c)^2 + 30*(cos
(d*x + c)^4 - 2*cos(d*x + c)^3 - 5*cos(d*x + c)^2 - (cos(d*x + c)^3 + 3*cos
(d*x + c)^2 - 2*cos(d*x + c) - 4)*sin(d*x + c) + 2*cos(d*x + c) + 4)*log(1/
2*cos(d*x + c) + 1/2) - 30*(cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 5*cos(d*x +
 c)^2 - (cos(d*x + c)^3 + 3*cos(d*x + c)^2 - 2*cos(d*x + c) - 4)*sin(d*x +
c) + 2*cos(d*x + c) + 4)*log(-1/2*cos(d*x + c) + 1/2) + (94*cos(d*x + c)^3
- 128*cos(d*x + c)^2 - 243*cos(d*x + c) - 6)*sin(d*x + c) - 237*cos(d*x + c
) + 6)/(a^4*d*cos(d*x + c)^4 - 2*a^4*d*cos(d*x + c)^3 - 5*a^4*d*cos(d*x + c
)^2 + 2*a^4*d*cos(d*x + c) + 4*a^4*d - (a^4*d*cos(d*x + c)^3 + 3*a^4*d*cos(
d*x + c)^2 - 2*a^4*d*cos(d*x + c) - 4*a^4*d)*sin(d*x + c))
```

giac [A] time = 0.70, size = 135, normalized size = 1.25

$$-\frac{\frac{120 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^4}-\frac{15 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^4}-\frac{15\left(8 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^4 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{4\left(135 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+435 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+605 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^4\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/30*(120*log(abs(tan(1/2*d*x + 1/2*c)))/a^4 - 15*tan(1/2*d*x + 1/2*c)/a^4
 - 15*(8*tan(1/2*d*x + 1/2*c) - 1)/(a^4*tan(1/2*d*x + 1/2*c)) + 4*(135*tan(
1/2*d*x + 1/2*c)^4 + 435*tan(1/2*d*x + 1/2*c)^3 + 605*tan(1/2*d*x + 1/2*c)^
2 + 385*tan(1/2*d*x + 1/2*c) + 104)/(a^4*(tan(1/2*d*x + 1/2*c) + 1)^5))/d
```

maple [A] time = 0.31, size = 161, normalized size = 1.49

$$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^4d}-\frac{1}{2a^4d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^4d}-\frac{16}{5a^4d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{8}{a^4d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{}{3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+a*sin(d*x+c))^4,x)
```

```
[Out] 1/2/a^4/d*tan(1/2*d*x+1/2*c)-1/2/a^4/d/tan(1/2*d*x+1/2*c)-4/a^4/d*ln(tan(1/
2*d*x+1/2*c))-16/5/a^4/d/(tan(1/2*d*x+1/2*c)+1)^5+8/a^4/d/(tan(1/2*d*x+1/2*
c)+1)^4-44/3/a^4/d/(tan(1/2*d*x+1/2*c)+1)^3+14/a^4/d/(tan(1/2*d*x+1/2*c)+1)
^2-18/a^4/d/(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.33, size = 288, normalized size = 2.67

$$-\frac{\frac{\frac{491\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{1690\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{2570\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{1815\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{555\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+15}{\frac{a^4\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\,a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10\,a^4\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{10\,a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{5\,a^4\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{a^4\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{120\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}-\frac{15\,\sin(dx+c)}{a^4(\cos(dx+c)+1)}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/30*((491*sin(d*x + c)/(cos(d*x + c) + 1) + 1690*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + 2570*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1815*sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 + 555*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15)/(a^4
*sin(d*x + c)/(cos(d*x + c) + 1) + 5*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 + 10*a^4*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 10*a^4*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 + 5*a^4*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + a^4*sin(d*x
+ c)^6/(cos(d*x + c) + 1)^6) + 120*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^4
 - 15*sin(d*x + c)/(a^4*(cos(d*x + c) + 1)))/d
```

mupad [B] time = 12.05, size = 203, normalized size = 1.88

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^4\,d}-\frac{37\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+121\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{514\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+\frac{338\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+491\,\mathrm{ta}}{d\,\left(2\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+10\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+20\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+20\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+10\,a^4\,\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + a*sin(c + d*x))^4,x)
```

```
[Out] tan(c/2 + (d*x)/2)/(2*a^4*d) - ((491*tan(c/2 + (d*x)/2))/15 + (338*tan(c/2
+ (d*x)/2)^2)/3 + (514*tan(c/2 + (d*x)/2)^3)/3 + 121*tan(c/2 + (d*x)/2)^4 +
 37*tan(c/2 + (d*x)/2)^5 + 1)/(d*(10*a^4*tan(c/2 + (d*x)/2)^2 + 20*a^4*tan(
c/2 + (d*x)/2)^3 + 20*a^4*tan(c/2 + (d*x)/2)^4 + 10*a^4*tan(c/2 + (d*x)/2)^
5 + 2*a^4*tan(c/2 + (d*x)/2)^6 + 2*a^4*tan(c/2 + (d*x)/2))) - (4*log(tan(c/
2 + (d*x)/2)))/(a^4*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^2(c+dx)}{\sin^4(c+dx)+4\sin^3(c+dx)+6\sin^2(c+dx)+4\sin(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Integral(cot(c + d*x)**2/(sin(c + d*x)**4 + 4*sin(c + d*x)**3 + 6*sin(c + d
*x)**2 + 4*sin(c + d*x) + 1), x)/a**4
```

3.89 $\int \frac{\cot^4(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=120

$$-\frac{\cot^3(c+dx)}{3a^4d}-\frac{9\cot(c+dx)}{a^4d}+\frac{14\tanh^{-1}(\cos(c+dx))}{a^4d}+\frac{2\cot(c+dx)\csc(c+dx)}{a^4d}-\frac{44\cot(c+dx)}{3a^4d(\csc(c+dx)+1)}+\frac{4}{3a^4d(c}$$

```
[Out] 14*arctanh(cos(d*x+c))/a^4/d-33*cot(d*x+c)/a^4/d-11*cot(d*x+c)^3/a^4/d+14*c
ot(d*x+c)*csc(d*x+c)/a^4/d+4/3*cot(d*x+c)*csc(d*x+c)^2/a^4/d/(1+sin(d*x+c))
^2+28/3*cot(d*x+c)*csc(d*x+c)^2/a^4/d/(1+sin(d*x+c))
```

Rubi [A] time = 0.25, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2709, 3770, 3767, 8, 3768, 3777, 3919, 3794}

$$-\frac{\cot^3(c+dx)}{3a^4d}-\frac{9\cot(c+dx)}{a^4d}+\frac{14\tanh^{-1}(\cos(c+dx))}{a^4d}+\frac{2\cot(c+dx)\csc(c+dx)}{a^4d}-\frac{44\cot(c+dx)}{3a^4d(\csc(c+dx)+1)}+\frac{4}{3a^4d(c}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (14*ArcTanh[Cos[c + d*x]])/(a^4*d) - (9*Cot[c + d*x])/(a^4*d) - Cot[c + d*x
]^3/(3*a^4*d) + (2*Cot[c + d*x]*Csc[c + d*x])/(a^4*d) + (4*Cot[c + d*x])/(3
*a^4*d*(1 + Csc[c + d*x])^2) - (44*Cot[c + d*x])/(3*a^4*d*(1 + Csc[c + d*x]
))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\int \left(16-12\csc(c+dx)+8\csc^2(c+dx)-4\csc^3(c+dx)+\csc^4(c+dx)+\frac{4}{(1+\csc(c+dx)}\right.}{a^4}$$

$$= \frac{16x}{a^4}+\frac{\int \csc^4(c+dx)\,dx}{a^4}-\frac{4\int \csc^3(c+dx)\,dx}{a^4}+\frac{4\int \frac{1}{(1+\csc(c+dx))^2}\,dx}{a^4}+\frac{8\int \csc^2(c+}{a^4}$$

$$= \frac{16x}{a^4}+\frac{12\tanh^{-1}(\cos(c+dx))}{a^4d}+\frac{2\cot(c+dx)\csc(c+dx)}{a^4d}+\frac{4\cot(c+dx)}{3a^4d(1+\csc(c+dx))^2}$$

$$= \frac{14\tanh^{-1}(\cos(c+dx))}{a^4d}-\frac{9\cot(c+dx)}{a^4d}-\frac{\cot^3(c+dx)}{3a^4d}+\frac{2\cot(c+dx)\csc(c+dx)}{a^4d}+$$

$$= \frac{14\tanh^{-1}(\cos(c+dx))}{a^4d}-\frac{9\cot(c+dx)}{a^4d}-\frac{\cot^3(c+dx)}{3a^4d}+\frac{2\cot(c+dx)\csc(c+dx)}{a^4d}+$$

Mathematica [B] time = 6.09, size = 589, normalized size = 4.91

$$\frac{80\sin\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^7}{3d(a\sin(c+dx)+a)^4}-\frac{4\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6}{3d(a\sin(c+dx)+a)^4}+\frac{8\sin\left(\frac{1}{2}(c+dx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (8*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)/(3*d*(a + a*Si
n[c + d*x])^4) - (4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)/(3*d*(a + a*Si
n[c + d*x])^4) + (80*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
^7)/(3*d*(a + a*Sin[c + d*x])^4) - (13*Cot[(c + d*x)/2]*(Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2])^8)/(3*d*(a + a*Sin[c + d*x])^4) + (Csc[(c + d*x)/2]^2*(C
os[(c + d*x)/2] + Sin[(c + d*x)/2])^8)/(2*d*(a + a*Sin[c + d*x])^4) - (Cot[
(c + d*x)/2]*Csc[(c + d*x)/2]^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8)/(2
4*d*(a + a*Sin[c + d*x])^4) + (14*Log[Cos[(c + d*x)/2]]*(Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2])^8)/(d*(a + a*Sin[c + d*x])^4) - (14*Log[Sin[(c + d*x)/2]
]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8)/(d*(a + a*Sin[c + d*x])^4) - (Se
c[(c + d*x)/2]^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8)/(2*d*(a + a*Sin[c
 + d*x])^4) + (13*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8*Tan[(c + d*x)/2])
/(3*d*(a + a*Sin[c + d*x])^4) + (Sec[(c + d*x)/2]^2*(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2])^8*Tan[(c + d*x)/2])/(24*d*(a + a*Sin[c + d*x])^4)
```

fricas [B] time = 0.45, size = 445, normalized size = 3.71

$$-\frac{66\cos(dx+c)^5-24\cos(dx+c)^4-147\cos(dx+c)^3+29\cos(dx+c)^2-21\left(\cos(dx+c)^5+2\cos(dx+c)^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(66*cos(d*x + c)^5 - 24*cos(d*x + c)^4 - 147*cos(d*x + c)^3 + 29*cos(d
*x + c)^2 - 21*(cos(d*x + c)^5 + 2*cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 4*co
s(d*x + c)^2 + (cos(d*x + c)^4 - cos(d*x + c)^3 - 3*cos(d*x + c)^2 + cos(d*
x + c) + 2)*sin(d*x + c) + cos(d*x + c) + 2)*log(1/2*cos(d*x + c) + 1/2) +
21*(cos(d*x + c)^5 + 2*cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 4*cos(d*x + c)^2
 + (cos(d*x + c)^4 - cos(d*x + c)^3 - 3*cos(d*x + c)^2 + cos(d*x + c) + 2)*
sin(d*x + c) + cos(d*x + c) + 2)*log(-1/2*cos(d*x + c) + 1/2) - (66*cos(d*x
 + c)^4 + 90*cos(d*x + c)^3 - 57*cos(d*x + c)^2 - 86*cos(d*x + c) - 4)*sin(
d*x + c) + 82*cos(d*x + c) - 4)/(a^4*d*cos(d*x + c)^5 + 2*a^4*d*cos(d*x + c
)^4 - 2*a^4*d*cos(d*x + c)^3 - 4*a^4*d*cos(d*x + c)^2 + a^4*d*cos(d*x + c)
+ 2*a^4*d + (a^4*d*cos(d*x + c)^4 - a^4*d*cos(d*x + c)^3 - 3*a^4*d*cos(d*x
+ c)^2 + a^4*d*cos(d*x + c) + 2*a^4*d)*sin(d*x + c))
```

giac [A] time = 0.42, size = 179, normalized size = 1.49

$$-\frac{\frac{336 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^4}-\frac{308 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+51 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-723 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-676 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-72 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+9 \tan\left(\frac{1}{2}\,a\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)^3 a^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/24*(336*log(abs(tan(1/2*d*x + 1/2*c)))/a^4 - (308*tan(1/2*d*x + 1/2*c)^6
 + 51*tan(1/2*d*x + 1/2*c)^5 - 723*tan(1/2*d*x + 1/2*c)^4 - 676*tan(1/2*d*x
 + 1/2*c)^3 - 72*tan(1/2*d*x + 1/2*c)^2 + 9*tan(1/2*d*x + 1/2*c) - 1)/((tan
(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x + 1/2*c))^3*a^4) - (a^8*tan(1/2*d*x + 1/2
*c)^3 - 12*a^8*tan(1/2*d*x + 1/2*c)^2 + 105*a^8*tan(1/2*d*x + 1/2*c))/a^12)
/d
```

maple [A] time = 0.32, size = 195, normalized size = 1.62

$$\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{24a^4d}-\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^4d}+\frac{35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^4d}-\frac{1}{24a^4d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{1}{2a^4d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\frac{35}{8a^4d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+a*sin(d*x+c))^4,x)
```

```
[Out] 1/24/a^4/d*tan(1/2*d*x+1/2*c)^3-1/2/a^4/d*tan(1/2*d*x+1/2*c)^2+35/8/a^4/d*t
an(1/2*d*x+1/2*c)-1/24/a^4/d/tan(1/2*d*x+1/2*c)^3+1/2/a^4/d/tan(1/2*d*x+1/2
*c)^2-35/8/a^4/d/tan(1/2*d*x+1/2*c)-14/a^4/d*ln(tan(1/2*d*x+1/2*c))-16/3/a^
4/d/(tan(1/2*d*x+1/2*c)+1)^3+8/a^4/d/(tan(1/2*d*x+1/2*c)+1)^2-32/a^4/d/(tan
(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.33, size = 285, normalized size = 2.38

$$\frac{\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{72\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{984\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{1647\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{873\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-1}{\frac{a^4\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{3a^4\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{a^4\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}-\frac{12\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^4}-\frac{336\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/24*((9*sin(d*x + c)/(cos(d*x + c) + 1) - 72*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - 984*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1647*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 - 873*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1)/(a^4*sin(d*
x + c)^3/(cos(d*x + c) + 1)^3 + 3*a^4*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 +
 3*a^4*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + a^4*sin(d*x + c)^6/(cos(d*x +
c) + 1)^6) + (105*sin(d*x + c)/(cos(d*x + c) + 1) - 12*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^4 - 336*log(sin(d*
x + c)/(cos(d*x + c) + 1))/a^4)/d
```

mupad [B] time = 7.83, size = 171, normalized size = 1.42

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a^4\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2\,a^4\,d}-\frac{14\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^4\,d}+\frac{35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a^4\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(\frac{291\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{8}+\frac{549\tan\left(\frac{c}{2}+\right.}{8}\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4/(a + a*sin(c + d*x))^4,x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(24*a^4*d) - tan(c/2 + (d*x)/2)^2/(2*a^4*d) - (14*log(
tan(c/2 + (d*x)/2)))/(a^4*d) + (35*tan(c/2 + (d*x)/2))/(8*a^4*d) - (cot(c/2
 + (d*x)/2)^3*(3*tan(c/2 + (d*x)/2)^2 - (3*tan(c/2 + (d*x)/2))/8 + 41*tan(c
/2 + (d*x)/2)^3 + (549*tan(c/2 + (d*x)/2)^4)/8 + (291*tan(c/2 + (d*x)/2)^5)
/8 + 1/24))/(a^4*d*(tan(c/2 + (d*x)/2) + 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^4(c+dx)}{\sin^4(c+dx)+4\sin^3(c+dx)+6\sin^2(c+dx)+4\sin(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Integral(cot(c + d*x)**4/(sin(c + d*x)**4 + 4*sin(c + d*x)**3 + 6*sin(c + d
*x)**2 + 4*sin(c + d*x) + 1), x)/a**4
```

3.90 $\int \frac{\cot^6(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=133

$$-\frac{\cot^5(c+dx)}{5a^4d}-\frac{3\cot^3(c+dx)}{a^4d}-\frac{16\cot(c+dx)}{a^4d}+\frac{27\tanh^{-1}(\cos(c+dx))}{2a^4d}+\frac{\cot(c+dx)\csc^3(c+dx)}{a^4d}+\frac{11\cot(c+dx)}{2a^4}$$

```
[Out] 27/2*arctanh(cos(d*x+c))/a^4/d-40*cot(d*x+c)/a^4/d-27*cot(d*x+c)^3/a^4/d-41
/5*cot(d*x+c)^5/a^4/d+27/2*cot(d*x+c)*csc(d*x+c)/a^4/d+9*cot(d*x+c)*csc(d*x
+c)^3/a^4/d+8*cot(d*x+c)*csc(d*x+c)^4/a^4/d/(1+sin(d*x+c))
```

Rubi [A] time = 0.25, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2709, 3770, 3767, 8, 3768, 3777}

$$-\frac{\cot^5(c+dx)}{5a^4d}-\frac{3\cot^3(c+dx)}{a^4d}-\frac{16\cot(c+dx)}{a^4d}+\frac{27\tanh^{-1}(\cos(c+dx))}{2a^4d}+\frac{\cot(c+dx)\csc^3(c+dx)}{a^4d}+\frac{11\cot(c+dx)}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (27*ArcTanh[Cos[c + d*x]])/(2*a^4*d) - (16*Cot[c + d*x])/(a^4*d) - (3*Cot[c
 + d*x]^3)/(a^4*d) - Cot[c + d*x]^5/(5*a^4*d) + (11*Cot[c + d*x]*Csc[c + d*
x])/(2*a^4*d) + (Cot[c + d*x]*Csc[c + d*x]^3)/(a^4*d) - (8*Cot[c + d*x])/(a
^4*d*(1 + Csc[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\int \left(8a^2 - 8a^2\csc(c+dx) + 8a^2\csc^2(c+dx) - 8a^2\csc^3(c+dx) + 7a^2\csc^4(c+dx) -\right.}{a^6}$$

$$= \frac{8x}{a^4} + \frac{\int \csc^6(c+dx)\,dx}{a^4} - \frac{4\int \csc^5(c+dx)\,dx}{a^4} + \frac{7\int \csc^4(c+dx)\,dx}{a^4} - \frac{8\int \csc(c+}{a^4}$$

$$= \frac{8x}{a^4} + \frac{8\tanh^{-1}(\cos(c+dx))}{a^4d} + \frac{4\cot(c+dx)\csc(c+dx)}{a^4d} + \frac{\cot(c+dx)\csc^3(c+dx)}{a^4d}$$

$$= \frac{12\tanh^{-1}(\cos(c+dx))}{a^4d} - \frac{16\cot(c+dx)}{a^4d} - \frac{3\cot^3(c+dx)}{a^4d} - \frac{\cot^5(c+dx)}{5a^4d} + \frac{11\cot(}{}$$

$$= \frac{27\tanh^{-1}(\cos(c+dx))}{2a^4d} - \frac{16\cot(c+dx)}{a^4d} - \frac{3\cot^3(c+dx)}{a^4d} - \frac{\cot^5(c+dx)}{5a^4d} + \frac{11\cot(}{}$$

Mathematica [B] time = 6.13, size = 733, normalized size = 5.51

$$\frac{16\sin\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^7}{d(a\sin(c+dx)+a)^4} + \frac{27\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+\right.\right.}{2d(a\sin(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (16*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^7)/(d*(a + a*Sin
[c + d*x])^4) - (33*Cot[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^
8)/(5*d*(a + a*Sin[c + d*x])^4) + (11*Csc[(c + d*x)/2]^2*(Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2])^8)/(8*d*(a + a*Sin[c + d*x])^4) - (53*Cot[(c + d*x)/2]*
Csc[(c + d*x)/2]^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8)/(160*d*(a + a*S
in[c + d*x])^4) + (Csc[(c + d*x)/2]^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
^8)/(16*d*(a + a*Sin[c + d*x])^4) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^4*(C
os[(c + d*x)/2] + Sin[(c + d*x)/2])^8)/(160*d*(a + a*Sin[c + d*x])^4) + (27
*Log[Cos[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8)/(2*d*(a + a
*Sin[c + d*x])^4) - (27*Log[Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c +
d*x)/2])^8)/(2*d*(a + a*Sin[c + d*x])^4) - (11*Sec[(c + d*x)/2]^2*(Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2])^8)/(8*d*(a + a*Sin[c + d*x])^4) - (Sec[(c + d*
x)/2]^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8)/(16*d*(a + a*Sin[c + d*x])
^4) + (33*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8*Tan[(c + d*x)/2])/(5*d*(a
 + a*Sin[c + d*x])^4) + (53*Sec[(c + d*x)/2]^2*(Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2])^8*Tan[(c + d*x)/2])/(160*d*(a + a*Sin[c + d*x])^4) + (Sec[(c + d*
x)/2]^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8*Tan[(c + d*x)/2])/(160*d*(a
 + a*Sin[c + d*x])^4)
```

fricas [B] time = 0.47, size = 439, normalized size = 3.30

$$\frac{424\,\cos\left(dx+c\right)^{6}+154\,\cos\left(dx+c\right)^{5}-1060\,\cos\left(dx+c\right)^{4}-340\,\cos\left(dx+c\right)^{3}+800\,\cos\left(dx+c\right)^{2}+135\,\Big(\cos\left(d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/20*(424*cos(d*x + c)^6 + 154*cos(d*x + c)^5 - 1060*cos(d*x + c)^4 - 340*c
os(d*x + c)^3 + 800*cos(d*x + c)^2 + 135*(cos(d*x + c)^6 - 3*cos(d*x + c)^4
 + 3*cos(d*x + c)^2 - (cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 -
 2*cos(d*x + c)^2 + cos(d*x + c) + 1)*sin(d*x + c) - 1)*log(1/2*cos(d*x + c
) + 1/2) - 135*(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - (cos
(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*
x + c) + 1)*sin(d*x + c) - 1)*log(-1/2*cos(d*x + c) + 1/2) + 2*(212*cos(d*x
 + c)^5 + 135*cos(d*x + c)^4 - 395*cos(d*x + c)^3 - 225*cos(d*x + c)^2 + 17
5*cos(d*x + c) + 80)*sin(d*x + c) + 190*cos(d*x + c) - 160)/(a^4*d*cos(d*x
+ c)^6 - 3*a^4*d*cos(d*x + c)^4 + 3*a^4*d*cos(d*x + c)^2 - a^4*d - (a^4*d*c
os(d*x + c)^5 + a^4*d*cos(d*x + c)^4 - 2*a^4*d*cos(d*x + c)^3 - 2*a^4*d*cos
(d*x + c)^2 + a^4*d*cos(d*x + c) + a^4*d)*sin(d*x + c))
```

giac [A] time = 1.29, size = 204, normalized size = 1.53

$$-\frac{\frac{2160 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^4}+\frac{2560}{a^4\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}-\frac{4932 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-1110 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+240 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-55 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+}{a^4 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}}{160}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/160*(2160*log(abs(tan(1/2*d*x + 1/2*c)))/a^4 + 2560/(a^4*(tan(1/2*d*x +
1/2*c) + 1)) - (4932*tan(1/2*d*x + 1/2*c)^5 - 1110*tan(1/2*d*x + 1/2*c)^4 +
 240*tan(1/2*d*x + 1/2*c)^3 - 55*tan(1/2*d*x + 1/2*c)^2 + 10*tan(1/2*d*x +
1/2*c) - 1)/(a^4*tan(1/2*d*x + 1/2*c)^5) - (a^16*tan(1/2*d*x + 1/2*c)^5 - 1
0*a^16*tan(1/2*d*x + 1/2*c)^4 + 55*a^16*tan(1/2*d*x + 1/2*c)^3 - 240*a^16*t
an(1/2*d*x + 1/2*c)^2 + 1110*a^16*tan(1/2*d*x + 1/2*c))/a^20)/d
```

maple [A] time = 0.34, size = 229, normalized size = 1.72

$$\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{160a^4d}-\frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{16a^4d}+\frac{11\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32a^4d}-\frac{3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^4d}+\frac{111\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16a^4d}-\frac{1}{160a^4d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6/(a+a*sin(d*x+c))^4,x)
```

```
[Out] 1/160/a^4/d*tan(1/2*d*x+1/2*c)^5-1/16/a^4/d*tan(1/2*d*x+1/2*c)^4+11/32/a^4/
d*tan(1/2*d*x+1/2*c)^3-3/2/a^4/d*tan(1/2*d*x+1/2*c)^2+111/16/a^4/d*tan(1/2*
d*x+1/2*c)-1/160/a^4/d/tan(1/2*d*x+1/2*c)^5+1/16/a^4/d/tan(1/2*d*x+1/2*c)^4
-11/32/a^4/d/tan(1/2*d*x+1/2*c)^3+3/2/a^4/d/tan(1/2*d*x+1/2*c)^2-111/16/a^4
/d/tan(1/2*d*x+1/2*c)-27/2/a^4/d*ln(tan(1/2*d*x+1/2*c))-16/a^4/d/(tan(1/2*d
*x+1/2*c)+1)
```

maxima [B] time = 0.32, size = 279, normalized size = 2.10

$$\frac{\frac{\frac{9 \sin(dx+c)}{\cos(dx+c)+1}-\frac{45 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{185 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{870 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{3670 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}-1}{\frac{a^4 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{a^4 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\frac{1110 \sin(dx+c)}{\cos(dx+c)+1}-\frac{240 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{55 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{10 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\sin(dx}{(\cos(dx+}}{a^4}}{160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/160*((9*sin(d*x + c)/(cos(d*x + c) + 1) - 45*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 185*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 870*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 - 3670*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1)/(a^4*sin(d
*x + c)^5/(cos(d*x + c) + 1)^5 + a^4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) +
 (1110*sin(d*x + c)/(cos(d*x + c) + 1) - 240*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 55*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 10*sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^4 - 2160*log(sin(d*x
+ c)/(cos(d*x + c) + 1))/a^4)/d
```

mupad [B] time = 7.67, size = 209, normalized size = 1.57

$$\frac{11\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{32\,a^4\,d}-\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{2\,a^4\,d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{16\,a^4\,d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{160\,a^4\,d}-\frac{27\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{2\,a^4\,d}+\frac{111\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{16\,a^4\,d}-\frac{\mathrm{cot}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6/(a + a*sin(c + d*x))^4,x)
```

```
[Out] (11*tan(c/2 + (d*x)/2)^3)/(32*a^4*d) - (3*tan(c/2 + (d*x)/2)^2)/(2*a^4*d) -
 tan(c/2 + (d*x)/2)^4/(16*a^4*d) + tan(c/2 + (d*x)/2)^5/(160*a^4*d) - (27*l
og(tan(c/2 + (d*x)/2)))/(2*a^4*d) + (111*tan(c/2 + (d*x)/2))/(16*a^4*d) - (
cot(c/2 + (d*x)/2)^5*((9*tan(c/2 + (d*x)/2)^2)/32 - (9*tan(c/2 + (d*x)/2))/
160 - (37*tan(c/2 + (d*x)/2)^3)/32 + (87*tan(c/2 + (d*x)/2)^4)/16 + (367*ta
n(c/2 + (d*x)/2)^5)/16 + 1/160))/(a^4*d*(tan(c/2 + (d*x)/2) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^6(c+dx)}{\sin^4(c+dx)+4\sin^3(c+dx)+6\sin^2(c+dx)+4\sin(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Integral(cot(c + d*x)**6/(sin(c + d*x)**4 + 4*sin(c + d*x)**3 + 6*sin(c + d
*x)**2 + 4*sin(c + d*x) + 1), x)/a**4
```

3.91 $\int \sqrt{a+a\sin(e+fx)}\,\tan^4(e+fx)\,dx$

Optimal. Leaf size=162

$$\frac{5\tan^3(e+fx)\sqrt{a(\sin(e+fx)+1)}}{12f}+\frac{29\tan(e+fx)\sqrt{a\sin(e+fx)+a}}{12f}-\frac{\sec^3(e+fx)\sqrt{a(\sin(e+fx)+1)}}{12f}-\frac{27\,\mathrm{s}}{}$$

```
[Out] 11/16*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))*a^(1/2
)/f*2^(1/2)-27/8*sec(f*x+e)*(a*(1+sin(f*x+e)))^(1/2)/f-1/12*sec(f*x+e)^3*(a
*(1+sin(f*x+e)))^(1/2)/f+29/12*(a+a*sin(f*x+e))^(1/2)*tan(f*x+e)/f+5/12*(a*
(1+sin(f*x+e)))^(1/2)*tan(f*x+e)^3/f
```

Rubi [A] time = 0.92, antiderivative size = 195, normalized size of antiderivative = 1.20, number of steps used = 15, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {2714, 2646, 4401, 2675, 2687, 2650, 2649, 206, 2878, 2855}

$$\frac{11a^2\cos(e+fx)}{8f(a\sin(e+fx)+a)^{3/2}}-\frac{2a\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}+\frac{4\sec^3(e+fx)(a\sin(e+fx)+a)^{3/2}}{3af}-\frac{7\sec^3(e+fx)\sqrt{a\sin(e+}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*Tan[e + f*x]^4,x]
```

```
[Out] (11*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]
])])/(8*Sqrt[2]*f) + (11*a^2*Cos[e + f*x])/(8*f*(a + a*Sin[e + f*x])^(3/2))
 - (2*a*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]) - (11*a*Sec[e + f*x])/(6
*f*Sqrt[a + a*Sin[e + f*x]]) - (7*Sec[e + f*x]^3*Sqrt[a + a*Sin[e + f*x]])/
(3*f) + (4*Sec[e + f*x]^3*(a + a*Sin[e + f*x])^(3/2))/(3*a*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2714

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] - Int[((a + b*Sin[e + f*x])^m*(
1 - 2*Sin[e + f*x]^2))/Cos[e + f*x]^4, x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2878

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(
p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b*f*g*(m + p + 2)), x] + Dist[1/(b*(m
 + p + 2)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*(p
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 -
 b^2, 0] && NeQ[m + p + 2, 0]
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\,\tan^4(e+fx)\,dx = \int \sqrt{a+a\sin(e+fx)}\,dx - \int \sec^4(e+fx)\sqrt{a+a\sin(e+fx)}\,\left(1-2\,\mathrm{s}\right.$$

$$= -\frac{2a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} - \int \left(\sec^4(e+fx)\sqrt{a(1+\sin(e+fx))} - 2\sec^2\right.$$

$$= -\frac{2a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} + 2\int \sec^2(e+fx)\sqrt{a(1+\sin(e+fx))}\,\tan^2(e$$

$$= -\frac{2a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} - \frac{\sec^3(e+fx)\sqrt{a+a\sin(e+fx)}}{3f} + \frac{4\sec^3(e+}{}$$

$$= -\frac{2a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} - \frac{5a\sec(e+fx)}{6f\sqrt{a+a\sin(e+fx)}} - \frac{7\sec^3(e+fx)\sqrt{a+}}{3f}$$

$$= \frac{5a^2\cos(e+fx)}{8f(a+a\sin(e+fx))^{3/2}} - \frac{2a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} - \frac{11a\sec(e+fx)}{6f\sqrt{a+a\sin(e+f}}$$

$$= \frac{11a^2\cos(e+fx)}{8f(a+a\sin(e+fx))^{3/2}} - \frac{2a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} - \frac{11a\sec(e+fx)}{6f\sqrt{a+a\sin(e+f}}$$

$$= \frac{5\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{8\sqrt{2}f} + \frac{11a^2\cos(e+fx)}{8f(a+a\sin(e+fx))^{3/2}} - \frac{2a\cos}{f\sqrt{a+a\,\mathrm{s}}}$$

$$= \frac{11\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{8\sqrt{2}f} + \frac{11a^2\cos(e+fx)}{8f(a+a\sin(e+fx))^{3/2}} - \frac{2a\,\mathrm{co}}{f\sqrt{a+a}}$$

Mathematica [C] time = 5.57, size = 394, normalized size = 2.43

$$\frac{\sqrt{a(\sin(e+fx)+1)}\left(-48\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\cos\left(\frac{fx}{2}\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2+48\left(\sin\left(\frac{e}{2}\right)+\cos\left(\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*Tan[e + f*x]^4,x]
```

```
[Out] (((6*Sin[(f*x)/2])/(Cos[e/2] + Sin[e/2]) - (3*(Cos[e/2] - Sin[e/2])*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2]))/(Cos[e/2] + Sin[e/2]) + (33 + 33*I)*(-1)^(3
/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec[(f*x)/4]*(Cos[(2*e + f*x)/4] - Sin[(
2*e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - 48*Cos[(f*x)/2]*(
Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 48*(Cos[e/2]
 + Sin[e/2])*Sin[(f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (4*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
3 - (36*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2]))*Sqrt[a*(1 + Sin[e + f*x])])/(24*f*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])^3)
```

fricas [A] time = 0.47, size = 200, normalized size = 1.23

$$\frac{33\sqrt{2}\sqrt{a}\cos\left(fx+e\right)^3\log\left(-\frac{a\cos\left(fx+e\right)^2+2\sqrt{a\sin\left(fx+e\right)+a}\left(\sqrt{2}\cos\left(fx+e\right)-\sqrt{2}\sin\left(fx+e\right)+\sqrt{2}\right)\sqrt{a}+3a\cos\left(fx+e\right)-\left(a\cos\left(fx+e\right)-2a\right.}{\cos\left(fx+e\right)^2-\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2}\right.}{96f\cos\left(fx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] 1/96*(33*sqrt(2)*sqrt(a)*cos(f*x + e)^3*log(-(a*cos(f*x + e)^2 + 2*sqrt(a*s
in(f*x + e) + a)*(sqrt(2)*cos(f*x + e) - sqrt(2)*sin(f*x + e) + sqrt(2))*sq
rt(a) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(
f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(81*c
os(f*x + e)^2 - 2*(24*cos(f*x + e)^2 + 5)*sin(f*x + e) + 2)*sqrt(a*sin(f*x
+ e) + a))/(f*cos(f*x + e)^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\sin\left(fx+e\right)+a}\,\tan\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*tan(f*x + e)^4, x)
```

maple [A] time = 0.70, size = 172, normalized size = 1.06

$$-\frac{96a^{\frac{5}{2}}\sin\left(fx+e\right)\left(\cos^{2}\left(fx+e\right)\right)+\left(33\left(a-a\sin\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\,\sqrt{2}}{2\sqrt{a}}\right)a+20a^{\frac{5}{2}}\right)\sin\left(f\right.}{48a^{\frac{3}{2}}\left(\sin\left(fx+e\right)-1\right)\cos\left(fx+e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)*tan(f*x+e)^4,x)
```

```
[Out] -1/48/a^(3/2)*(96*a^(5/2)*sin(f*x+e)*cos(f*x+e)^2+(33*(a-a*sin(f*x+e))^(3/2
)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a+20*a^(5/2))
*sin(f*x+e)-162*a^(5/2)*cos(f*x+e)^2+33*(a-a*sin(f*x+e))^(3/2)*2^(1/2)*arct
anh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a-4*a^(5/2))/(sin(f*x+e)-1)
/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\sin\left(fx+e\right)+a}\,\tan\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*tan(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\tan\left(e+f\,x\right)^{4}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(a + a*sin(e + f*x))^(1/2),x)
```

```
[Out] int(tan(e + f*x)^4*(a + a*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\,\tan^{4}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)*tan(f*x+e)**4,x)
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*tan(e + f*x)**4, x)
```

3.92 $\int \sqrt{a+a\sin(e+fx)}\,\tan^2(e+fx)\,dx$

Optimal. Leaf size=101

$$-\frac{2\sec(e+fx)(a\sin(e+fx)+a)^{3/2}}{af}+\frac{5\sec(e+fx)\sqrt{a\sin(e+fx)+a}}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{2}f}$$

```
[Out] -2*sec(f*x+e)*(a+a*sin(f*x+e))^(3/2)/a/f-1/2*arctanh(1/2*cos(f*x+e)*a^(1/2)
*2^(1/2)/(a+a*sin(f*x+e))^(1/2))*a^(1/2)/f*2^(1/2)+5*sec(f*x+e)*(a+a*sin(f*
x+e))^(1/2)/f
```

Rubi [A] time = 0.18, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2713, 2855, 2649, 206}

$$-\frac{2\sec(e+fx)(a\sin(e+fx)+a)^{3/2}}{af}+\frac{5\sec(e+fx)\sqrt{a\sin(e+fx)+a}}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*Tan[e + f*x]^2,x]
```

```
[Out] -((Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]]
)])/(Sqrt[2]*f)) + (5*Sec[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/f - (2*Sec[e +
 f*x]*(a + a*Sin[e + f*x])^(3/2))/(a*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2713

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> -Simp[(a + b*Sin[e + f*x])^(m + 1)/(b*f*m*Cos[e + f*x]), x] +
Dist[1/(b*m), Int[((a + b*Sin[e + f*x])^m*(b*(m + 1) + a*Sin[e + f*x]))/Cos
```

```
[e + f*x]^2, x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !
IntegerQ[m] &&  !LtQ[m, 0]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\,\tan^2(e+fx)\,dx = -\frac{2\sec(e+fx)(a+a\sin(e+fx))^{3/2}}{af}+\frac{2\int \sec^2(e+fx)\sqrt{a+a\sin(e+f}{a}$$

$$=\frac{5\sec(e+fx)\sqrt{a+a\sin(e+fx)}}{f}-\frac{2\sec(e+fx)(a+a\sin(e+fx))^{3/2}}{af}+$$

$$=\frac{5\sec(e+fx)\sqrt{a+a\sin(e+fx)}}{f}-\frac{2\sec(e+fx)(a+a\sin(e+fx))^{3/2}}{af}-$$

$$=-\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{2}\,f}+\frac{5\sec(e+fx)\sqrt{a+a\sin(e+fx)}}{f}-2$$

Mathematica [C] time = 0.34, size = 114, normalized size = 1.13

$$\frac{\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\left(-2\sin(e+fx)+(1-i)\sqrt[4]{-1}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\tanh^{-1}\left(\left(\frac{1}{2}+\right.\right.\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*Tan[e + f*x]^2,x]
```

```
[Out] (Sec[e + f*x]*(3 + (1 - I)*(-1)^(1/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec[(f
*x)/4]*(Cos[(2*e + f*x)/4] - Sin[(2*e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(
e + f*x)/2]) - 2*Sin[e + f*x])*Sqrt[a*(1 + Sin[e + f*x])])/f
```

fricas [A] time = 0.43, size = 169, normalized size = 1.67

$$\frac{\sqrt{2}\sqrt{a}\cos\left(fx+e\right)\log\left(-\frac{a\cos\left(fx+e\right)^{2}-2\sqrt{2}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{a}\left(\cos\left(fx+e\right)-\sin\left(fx+e\right)+1\right)+3a\cos\left(fx+e\right)-\left(a\cos\left(fx+e\right)-2a\right)\sin\left(fx+e\right)}{\cos\left(fx+e\right)^{2}-\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2}\right.}{4f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*sqrt(a)*cos(f*x + e)*log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a
*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x
+ e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*
x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*sqrt(a*sin(f*x + e) + a)*
(2*sin(f*x + e) - 3))/(f*cos(f*x + e))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sin\left(fx+e\right)+a}\,\tan\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*tan(f*x + e)^2, x)
```

maple [A] time = 0.73, size = 89, normalized size = 0.88

$$-\frac{\left(1+\sin\left(fx+e\right)\right)\left(\sqrt{a}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\,\sqrt{2}}{2\sqrt{a}}\right)\sqrt{a-a\sin\left(fx+e\right)}+4a\sin\left(fx+e\right)-6a\right)}{2\cos\left(fx+e\right)\sqrt{a+a\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)*tan(f*x+e)^2,x)
```

```
[Out] -1/2*(1+sin(f*x+e))*(a^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(
1/2)/a^(1/2))*(a-a*sin(f*x+e))^(1/2)+4*a*sin(f*x+e)-6*a)/cos(f*x+e)/(a+a*si
n(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sin\left(fx+e\right)+a}\,\tan\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*tan(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan\left(e+f\,x\right)^2\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(a + a*sin(e + f*x))^(1/2),x)
```

```
[Out] int(tan(e + f*x)^2*(a + a*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\,\tan^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)*tan(f*x+e)**2,x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*tan(e + f*x)**2, x)
```

3.93 $\int \cot^2(e+fx)\sqrt{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=89

$$\frac{3a\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}-\frac{\cot(e+fx)\sqrt{a\sin(e+fx)+a}}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}$$

```
[Out] -arctanh(cos(f*x+e)*a^(1/2)/(a+a*sin(f*x+e))^(1/2))*a^(1/2)/f+3*a*cos(f*x+e
)/f/(a+a*sin(f*x+e))^(1/2)-cot(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.19, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2716, 2981, 2773, 206}

$$\frac{3a\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}-\frac{\cot(e+fx)\sqrt{a\sin(e+fx)+a}}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/f) + (
3*a*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]) - (Cot[e + f*x]*Sqrt[a + a*S
in[e + f*x]])/f
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2716

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)/tan[(e_.) + (f_.)*(x_)]^2,
 x_Symbol] :> -Simp[(a + b*Sin[e + f*x])^m/(f*Tan[e + f*x]), x] + Dist[1/a,
 Int[((a + b*Sin[e + f*x])^m*(b*m - a*(m + 1)*Sin[e + f*x]))/Sin[e + f*x],
x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/
2] && !LtQ[m, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
```

```
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \cot^2(e+fx)\sqrt{a+a\sin(e+fx)}\,dx = -\frac{\cot(e+fx)\sqrt{a+a\sin(e+fx)}}{f} + \frac{\int \csc(e+fx)\left(\frac{a}{2}-\frac{3}{2}a\sin(e+fx)\right)\sqrt{}}{a}$$

$$= \frac{3a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\sqrt{a+a\sin(e+fx)}}{f} + \frac{1}{2}\int \csc(e+f$$

$$= \frac{3a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\sqrt{a+a\sin(e+fx)}}{f} - \frac{a\,\mathrm{Subst}\left(\int \frac{1}{a-x^2}\right.}{}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{f} + \frac{3a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\sqrt{a+}}{f}$$

Mathematica [B] time = 0.99, size = 206, normalized size = 2.31

$$\frac{\csc^4\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sin(e+fx)+1)}\left(4\sin\left(\frac{1}{2}(e+fx)\right)+2\sin\left(\frac{3}{2}(e+fx)\right)-4\cos\left(\frac{1}{2}(e+fx)\right)+2\cos\left(\frac{3}{2}(e+f\right.\right.}{f\left(\cot\left(\frac{1}{2}(e+fx)\right)+1\right)\left(\csc\left(\frac{1}{4}(e+fx)\right)-s\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (Csc[(e + f*x)/2]^4*Sqrt[a*(1 + Sin[e + f*x])]*(-4*Cos[(e + f*x)/2] + 2*Cos
[(3*(e + f*x))/2] + 4*Sin[(e + f*x)/2] - Log[1 + Cos[(e + f*x)/2] - Sin[(e
+ f*x)/2]]*Sin[e + f*x] + Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[
e + f*x] + 2*Sin[(3*(e + f*x))/2]))/(f*(1 + Cot[(e + f*x)/2])*(Csc[(e + f*x
)/4] - Sec[(e + f*x)/4])*(Csc[(e + f*x)/4] + Sec[(e + f*x)/4]))
```

fricas [B] time = 0.44, size = 279, normalized size = 3.13

$$\frac{\left(\cos\left(fx+e\right)^{2}-\left(\cos\left(fx+e\right)+1\right)\sin\left(fx+e\right)-1\right)\sqrt{a}\log\left(\frac{a\cos\left(fx+e\right)^{3}-7\,a\cos\left(fx+e\right)^{2}-4\left(\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)+3\right)\sin\cdots\right.}{\cos\left(fx+e\right)^{3}+\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*((cos(f*x + e)^2 - (cos(f*x + e) + 1)*sin(f*x + e) - 1)*sqrt(a)*log((a*
cos(f*x + e)^3 - 7*a*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3
)*sin(f*x + e) - 2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a
*cos(f*x + e) + (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)
/(cos(f*x + e)^3 + cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos
(f*x + e) - 1)) - 4*(2*cos(f*x + e)^2 + (2*cos(f*x + e) + 3)*sin(f*x + e) -
 cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a))/(f*cos(f*x + e)^2 - (f*cos(f*x
 + e) + f)*sin(f*x + e) - f)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.79, size = 125, normalized size = 1.40

$$\frac{\left(1+\sin\left(fx+e\right)\right)\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\,\left(\sin\left(fx+e\right)\left(2\sqrt{a-a\sin\left(fx+e\right)}\,a^{\frac{3}{2}}-\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}}{\sqrt{a}}\right)a^{2}\cdots\right.\right.}{\sin\left(fx+e\right)a^{\frac{3}{2}}\cos\left(fx+e\right)\sqrt{a+a\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] (1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(sin(f*x+e)*(2*(a-a*sin(f*x+e))^(1
/2)*a^(3/2)-arctanh((a-a*sin(f*x+e))^(1/2)/a^(1/2))*a^2)-(a-a*sin(f*x+e))^(
1/2)*a^(3/2))/sin(f*x+e)/a^(3/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sin\left(fx+e\right)+a}\,\cot\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*cot(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(e+f\,x\right)^{2}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(a + a*sin(e + f*x))^(1/2),x)
```

```
[Out] int(cot(e + f*x)^2*(a + a*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\,\cot^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*cot(e + f*x)**2, x)
```

3.94 $\int \cot^4(e+fx)\sqrt{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=163

$$-\frac{2a\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}+\frac{11a\cot(e+fx)}{8f\sqrt{a\sin(e+fx)+a}}+\frac{11\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{8f}-\frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a\sin(}}{3f}$$

```
[Out] 11/8*arctanh(cos(f*x+e)*a^(1/2)/(a+a*sin(f*x+e))^(1/2))*a^(1/2)/f-2*a*cos(f
*x+e)/f/(a+a*sin(f*x+e))^(1/2)+11/8*a*cot(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-1
/12*a*cot(f*x+e)*csc(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-1/3*cot(f*x+e)*csc(f*x
+e)^2*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.38, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2718, 2646, 3044, 2980, 2772, 2773, 206}

$$-\frac{2a\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}+\frac{11a\cot(e+fx)}{8f\sqrt{a\sin(e+fx)+a}}+\frac{11\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{8f}-\frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a\sin(}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (11*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/(8*f)
 - (2*a*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]) + (11*a*Cot[e + f*x])/(8
*f*Sqrt[a + a*Sin[e + f*x]]) - (a*Cot[e + f*x]*Csc[e + f*x])/(12*f*Sqrt[a +
 a*Sin[e + f*x]]) - (Cot[e + f*x]*Csc[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]])/
(3*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2718

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] + Int[((a + b*Sin[e + f*x])^m*(
1 - 2*Sin[e + f*x]^2))/Sin[e + f*x]^4, x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&  !LtQ[m, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \cot^4(e+fx)\sqrt{a+a\sin(e+fx)}\,dx = \int \sqrt{a+a\sin(e+fx)}\,dx + \int \csc^4(e+fx)\sqrt{a+a\sin(e+fx)}\,\left(1-2\sin\right.$$

$$= -\frac{2a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+a\sin(e+fx)}}{3f} +$$

$$= -\frac{2a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} - \frac{a\cot(e+fx)\csc(e+fx)}{12f\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\csc^2(e}{}$$

$$= -\frac{2a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} + \frac{11a\cot(e+fx)}{8f\sqrt{a+a\sin(e+fx)}} - \frac{a\cot(e+fx)\csc(e+}{12f\sqrt{a+a\sin(e+}}$$

$$= -\frac{2a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} + \frac{11a\cot(e+fx)}{8f\sqrt{a+a\sin(e+fx)}} - \frac{a\cot(e+fx)\csc(e+}{12f\sqrt{a+a\sin(e+}}$$

$$= \frac{11\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{8f} - \frac{2a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} + \frac{11a\cot(e+}{8f\sqrt{a+a\sin(e}}$$

Mathematica [A] time = 1.60, size = 309, normalized size = 1.90

$$\frac{\csc^{10}\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sin(e+fx)+1)}\left(-252\sin\left(\frac{1}{2}(e+fx)\right)-250\sin\left(\frac{3}{2}(e+fx)\right)+114\sin\left(\frac{5}{2}(e+fx)\right)+48\,\mathrm{s}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4*Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (Csc[(e + f*x)/2]^10*Sqrt[a*(1 + Sin[e + f*x])]*(252*Cos[(e + f*x)/2] - 250
*Cos[(3*(e + f*x))/2] - 114*Cos[(5*(e + f*x))/2] + 48*Cos[(7*(e + f*x))/2]
- 252*Sin[(e + f*x)/2] + 99*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Si
n[e + f*x] - 99*Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[e + f*x] -
 250*Sin[(3*(e + f*x))/2] + 114*Sin[(5*(e + f*x))/2] - 33*Log[1 + Cos[(e +
f*x)/2] - Sin[(e + f*x)/2]]*Sin[3*(e + f*x)] + 33*Log[1 - Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2]]*Sin[3*(e + f*x)] + 48*Sin[(7*(e + f*x))/2]))/(24*f*(1 +
 Cot[(e + f*x)/2])*(Csc[(e + f*x)/4]^2 - Sec[(e + f*x)/4]^2)^3)
```

fricas [B] time = 0.44, size = 380, normalized size = 2.33

$$\frac{33\left(\cos\left(fx+e\right)^{4}-2\,\cos\left(fx+e\right)^{2}-\left(\cos\left(fx+e\right)^{3}+\cos\left(fx+e\right)^{2}-\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)+1\right)\sqrt{a}\,\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/96*(33*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 - (cos(f*x + e)^3 + cos(f*x + e
)^2 - cos(f*x + e) - 1)*sin(f*x + e) + 1)*sqrt(a)*log((a*cos(f*x + e)^3 - 7
*a*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3)*sin(f*x + e) - 2
*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a*cos(f*x + e) + (a
*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)/(cos(f*x + e)^3 +
 cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos(f*x + e) - 1)) +
4*(48*cos(f*x + e)^4 - 33*cos(f*x + e)^3 - 139*cos(f*x + e)^2 + (48*cos(f*x
 + e)^3 + 81*cos(f*x + e)^2 - 58*cos(f*x + e) - 83)*sin(f*x + e) + 25*cos(f
*x + e) + 83)*sqrt(a*sin(f*x + e) + a))/(f*cos(f*x + e)^4 - 2*f*cos(f*x + e
)^2 - (f*cos(f*x + e)^3 + f*cos(f*x + e)^2 - f*cos(f*x + e) - f)*sin(f*x +
e) + f)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.93, size = 170, normalized size = 1.04

$$-\frac{\left(1+\sin\left(fx+e\right)\right)\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\,\left(48\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\,a^{\frac{7}{2}}\left(\sin^{3}\left(fx+e\right)\right)-15\sqrt{-a\left(\sin\left(fx+e\right)\right.}}{24a^{\frac{7}{2}}\sin\left(fx+e\right)^{3}\,\mathrm{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/24*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(48*(-a*(sin(f*x+e)-1))^(1/2
)*a^(7/2)*sin(f*x+e)^3-15*(-a*(sin(f*x+e)-1))^(1/2)*a^(7/2)+56*(-a*(sin(f*x
```

```
+e)-1))^(3/2)*a^(5/2)-33*(-a*(sin(f*x+e)-1))^(5/2)*a^(3/2)-33*arctanh((-a*(
sin(f*x+e)-1))^(1/2)/a^(1/2))*a^4*sin(f*x+e)^3)/a^(7/2)/sin(f*x+e)^3/cos(f*
x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \sin\left(fx+e\right)+a}\, \cot\left(fx+e\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*sin(f*x + e) + a)*cot(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(e+f\,x\right)^{4}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4*(a + a*sin(e + f*x))^(1/2),x)
[Out] int(cot(e + f*x)^4*(a + a*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\, \cot^{4}\left(e+fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+a*sin(f*x+e))**(1/2),x)
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*cot(e + f*x)**4, x)
```

3.95 $\int (a+a\sin(e+fx))^{3/2}\tan^4(e+fx)\,dx$

Optimal. Leaf size=167

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}f}+\frac{2a^3\cos^3(e+fx)}{3f(a\sin(e+fx)+a)^{3/2}}-\frac{4a^2\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}+\frac{\sec^3(e+fx)(a\sin(e+fx)+a)}{3f}$$

```
[Out] 2/3*a^3*cos(f*x+e)^3/f/(a+a*sin(f*x+e))^(3/2)+1/3*sec(f*x+e)^3*(a+a*sin(f*x
+e))^(3/2)/f-1/4*a^(3/2)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*
x+e))^(1/2))/f*2^(1/2)-4*a^2*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-7/2*a*sec(
f*x+e)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.98, antiderivative size = 195, normalized size of antiderivative = 1.17, number of steps used = 14, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {2714, 2647, 2646, 4401, 2675, 2649, 206, 2878, 2855}

$$-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}f}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}+\frac{4\sec^3(e+fx)(a\sin}{af}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*Tan[e + f*x]^4,x]
```

```
[Out] -(a^(3/2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])
])/(2*Sqrt[2]*f) - (8*a^2*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) - (2
*a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*f) + (a*Sec[e + f*x]*Sqrt[a +
a*Sin[e + f*x]])/(2*f) - (23*Sec[e + f*x]^3*(a + a*Sin[e + f*x])^(3/2))/(3*
f) + (4*Sec[e + f*x]^3*(a + a*Sin[e + f*x])^(5/2))/(a*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rule 2714

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] - Int[((a + b*Sin[e + f*x])^m*(
1 - 2*Sin[e + f*x]^2))/Cos[e + f*x]^4, x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2878

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(
p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b*f*g*(m + p + 2)), x] + Dist[1/(b*(m
 + p + 2)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*(p
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 -
 b^2, 0] && NeQ[m + p + 2, 0]
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}\tan^4(e+fx)\,dx = \int (a+a\sin(e+fx))^{3/2}\,dx - \int \sec^4(e+fx)(a+a\sin(e+fx))^{3/2}\left(1-\right.$$

$$= -\frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f} + \frac{1}{3}(4a)\int \sqrt{a+a\sin(e+fx)}\,dx$$

$$= -\frac{8a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} - \frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f} + 2\int \mathrm{se}$$

$$= -\frac{8a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} - \frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f} - \frac{\sec^3(e}{}$$

$$= -\frac{8a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} - \frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f} - \frac{a\sec(e}{}$$

$$= -\frac{8a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} - \frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f} + \frac{a\sec(e}{}$$

$$= \frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}f} - \frac{8a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} - \frac{2a\cos(e+f}{}$$

$$= -\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}f} - \frac{8a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} - \frac{2a\cos(e+}{}$$

Mathematica [C] time = 5.55, size = 141, normalized size = 0.84

$$\frac{a\sec^3(e+fx)\sqrt{a(\sin(e+fx)+1)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2\left(54\sin(e+fx)+\sin(3(e+fx))+6\cos\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*Tan[e + f*x]^4,x]
```

```
[Out] (a*Sec[e + f*x]^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*Sqrt[a*(1 + Sin[e
 + f*x])]*(-45 + 6*Cos[2*(e + f*x)] + (3 + 3*I)*(-1)^(3/4)*ArcTanh[(1/2 + I
/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2
])^3 + 54*Sin[e + f*x] + Sin[3*(e + f*x)]))/(6*f)
```

fricas [A] time = 0.44, size = 239, normalized size = 1.43

$$\frac{3\left(\sqrt{2}\,a\cos\left(fx+e\right)\sin\left(fx+e\right)-\sqrt{2}\,a\cos\left(fx+e\right)\right)\sqrt{a}\,\log\left(-\frac{a\cos\left(fx+e\right)^{2}-2\,\sqrt{a\sin\left(fx+e\right)+a}\,\left(\sqrt{2}\,\cos\left(fx+e\right)-\sqrt{2}\,\sin\left(fx+\right.\right.}{\cos\left(fx+e\right)^{2}-\left(\cos\left(fx+e\right)+2\right.}}{24\left(f\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(sqrt(2)*a*cos(f*x + e)*sin(f*x + e) - sqrt(2)*a*cos(f*x + e))*sqrt
(a)*log(-(a*cos(f*x + e)^2 - 2*sqrt(a*sin(f*x + e) + a)*(sqrt(2)*cos(f*x +
e) - sqrt(2)*sin(f*x + e) + sqrt(2))*sqrt(a) + 3*a*cos(f*x + e) - (a*cos(f*
x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(
f*x + e) - cos(f*x + e) - 2)) - 4*(12*a*cos(f*x + e)^2 + (4*a*cos(f*x + e)^
2 + 53*a)*sin(f*x + e) - 51*a)*sqrt(a*sin(f*x + e) + a))/(f*cos(f*x + e)*si
n(f*x + e) - f*cos(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.75, size = 139, normalized size = 0.83

$$\frac{\left(1+\sin\left(fx+e\right)\right)\left(3a^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\,\sqrt{2}}{2\sqrt{a}}\right)\left(a-a\sin\left(fx+e\right)\right)^{\frac{3}{2}}-8a^{3}\sin\left(fx+e\right)\left(\cos^{2}\left(fx+e\right)\right)\right.}{12a\left(\sin\left(fx+e\right)-1\right)\cos\left(fx+e\right)\sqrt{a+a\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*tan(f*x+e)^4,x)
```

```
[Out] 1/12*(1+sin(f*x+e))/a/(sin(f*x+e)-1)*(3*a^(3/2)*2^(1/2)*arctanh(1/2*(a-a*si
n(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*(a-a*sin(f*x+e))^(3/2)-8*a^3*sin(f*x+e)*co
```

```
s(f*x+e)^2-24*a^3*cos(f*x+e)^2-106*sin(f*x+e)*a^3+102*a^3)/cos(f*x+e)/(a+a*
sin(f*x+e))^(1/2)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*tan(f*x+e)^4,x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan\left(e+f\,x\right)^4\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(a + a*sin(e + f*x))^(3/2),x)
[Out] int(tan(e + f*x)^4*(a + a*sin(e + f*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*tan(f*x+e)**4,x)
[Out] Timed out
```

3.96 $\int (a+a\sin(e+fx))^{3/2}\tan^2(e+fx)\,dx$

Optimal. Leaf size=88

$$\frac{11a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2\sec(e+fx)(a\sin(e+fx)+a)^{5/2}}{3af}+\frac{7\sec(e+fx)(a\sin(e+fx)+a)^{3/2}}{3f}$$

```
[Out] 7/3*sec(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f-2/3*sec(f*x+e)*(a+a*sin(f*x+e))^(5/
2)/a/f+11/3*a^2*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2713, 2855, 2646}

$$\frac{11a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2\sec(e+fx)(a\sin(e+fx)+a)^{5/2}}{3af}+\frac{7\sec(e+fx)(a\sin(e+fx)+a)^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*Tan[e + f*x]^2,x]
```

```
[Out] (11*a^2*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) + (7*Sec[e + f*x]*(a +
 a*Sin[e + f*x])^(3/2))/(3*f) - (2*Sec[e + f*x]*(a + a*Sin[e + f*x])^(5/2))
/(3*a*f)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2713

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> -Simp[(a + b*Sin[e + f*x])^(m + 1)/(b*f*m*Cos[e + f*x]), x] +
Dist[1/(b*m), Int[((a + b*Sin[e + f*x])^m*(b*(m + 1) + a*Sin[e + f*x]))/Cos
[e + f*x]^2, x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !
IntegerQ[m] &&  !LtQ[m, 0]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
```

```
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rubi steps

$$\int (a + a\sin(e+fx))^{3/2}\tan^2(e+fx)\,dx = -\frac{2\sec(e+fx)(a+a\sin(e+fx))^{5/2}}{3af} + \frac{2\int \sec^2(e+fx)(a+a\sin(e+\cdots}{3a\cdots}$$
$$= \frac{7\sec(e+fx)(a+a\sin(e+fx))^{3/2}}{3f} - \frac{2\sec(e+fx)(a+a\sin(e+fx))^{5/\cdots}}{3af}$$
$$= \frac{11a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} + \frac{7\sec(e+fx)(a+a\sin(e+fx))^{3/2}}{3f} - \frac{2\sec(e\cdots}{}$$

Mathematica [A] time = 4.15, size = 46, normalized size = 0.52

$$\frac{a\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}(-8\sin(e+fx)+\cos(2(e+fx))+15)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*Tan[e + f*x]^2,x]
```

```
[Out] (a*Sec[e + f*x]*(15 + Cos[2*(e + f*x)] - 8*Sin[e + f*x])*Sqrt[a*(1 + Sin[e
+ f*x])])/(3*f)
```

fricas [A] time = 0.41, size = 48, normalized size = 0.55

$$\frac{2\left(a\cos\left(fx+e\right)^2 - 4a\sin\left(fx+e\right) + 7a\right)\sqrt{a\sin\left(fx+e\right)+a}}{3f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] 2/3*(a*cos(f*x + e)^2 - 4*a*sin(f*x + e) + 7*a)*sqrt(a*sin(f*x + e) + a)/(f
*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
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))-1/4*pi))+21504*a*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))^2-12
288*a*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))^3+75264*a*sign(cos
(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))^4+12288*a*sign(cos(1/2*(f*x+exp(
1))-1/4*pi))*tan(1/4*exp(1))^5+21504*a*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*t
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(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*f*x)^2-12288*a*sign(cos(1/2*(f*x+exp
(1))-1/4*pi))*tan(1/4*f*x)^3+75264*a*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan
(1/4*f*x)^4+12288*a*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*f*x)^5+21504
*a*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*f*x)^6+12288*a*sign(cos(1/2*(
f*x+exp(1))-1/4*pi))*tan(1/4*f*x)^7+8448*a*sign(cos(1/2*(f*x+exp(1))-1/4*pi
))*tan(1/4*f*x)^8-12288*a*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1)
)-12288*a*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*f*x)+430080*a*sign(cos
(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))^2*tan(1/4*f*x)^2+49152*a*sign(co
s(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))^2*tan(1/4*f*x)^3-559104*a*sign(
cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))^2*tan(1/4*f*x)^4-49152*a*sign
(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))^2*tan(1/4*f*x)^5+430080*a*si
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sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))^2*tan(1/4*f*x)^8+49152*a
*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))^2*tan(1/4*f*x)+49152*a*
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*a*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))^3*tan(1/4*f*x)^3+1228
80*a*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))^3*tan(1/4*f*x)^4+12
04224*a*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))^3*tan(1/4*f*x)^5
+49152*a*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))^3*tan(1/4*f*x)^
6-172032*a*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))^3*tan(1/4*f*x
)^7-12288*a*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))^3*tan(1/4*f*
x)^8+172032*a*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))^3*tan(1/4*
f*x)-559104*a*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))^4*tan(1/4*
f*x)^2+122880*a*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))^4*tan(1/
4*f*x)^3+2171904*a*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))^4*tan
(1/4*f*x)^4-122880*a*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))^4*t
an(1/4*f*x)^5-559104*a*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))^4
*tan(1/4*f*x)^6-122880*a*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))
^4*tan(1/4*f*x)^7+75264*a*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1)
)^4*tan(1/4*f*x)^8+122880*a*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(
1))^4*tan(1/4*f*x)-49152*a*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1
))^5*tan(1/4*f*x)^2+1204224*a*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*ex
```

```
p(1))^5*tan(1/4*f*x)^3-122880*a*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*
exp(1))^5*tan(1/4*f*x)^4-1204224*a*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1
/4*exp(1))^5*tan(1/4*f*x)^5-49152*a*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(
1/4*exp(1))^5*tan(1/4*f*x)^6+172032*a*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*ta
n(1/4*exp(1))^5*tan(1/4*f*x)^7+12288*a*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*t
an(1/4*exp(1))^5*tan(1/4*f*x)^8-172032*a*sign(cos(1/2*(f*x+exp(1))-1/4*pi))
*tan(1/4*exp(1))^5*tan(1/4*f*x)+430080*a*sign(cos(1/2*(f*x+exp(1))-1/4*pi))
*tan(1/4*exp(1))^6*tan(1/4*f*x)^2+49152*a*sign(cos(1/2*(f*x+exp(1))-1/4*pi)
)*tan(1/4*exp(1))^6*tan(1/4*f*x)^3-559104*a*sign(cos(1/2*(f*x+exp(1))-1/4*p
i))*tan(1/4*exp(1))^6*tan(1/4*f*x)^4-49152*a*sign(cos(1/2*(f*x+exp(1))-1/4*
pi))*tan(1/4*exp(1))^6*tan(1/4*f*x)^5+430080*a*sign(cos(1/2*(f*x+exp(1))-1/
4*pi))*tan(1/4*exp(1))^6*tan(1/4*f*x)^6-49152*a*sign(cos(1/2*(f*x+exp(1))-1
/4*pi))*tan(1/4*exp(1))^6*tan(1/4*f*x)^7+21504*a*sign(cos(1/2*(f*x+exp(1))-
1/4*pi))*tan(1/4*exp(1))^6*tan(1/4*f*x)^8+49152*a*sign(cos(1/2*(f*x+exp(1))
-1/4*pi))*tan(1/4*exp(1))^6*tan(1/4*f*x)-49152*a*sign(cos(1/2*(f*x+exp(1))-
1/4*pi))*tan(1/4*exp(1))^7*tan(1/4*f*x)^2-172032*a*sign(cos(1/2*(f*x+exp(1)
)-1/4*pi))*tan(1/4*exp(1))^7*tan(1/4*f*x)^3-122880*a*sign(cos(1/2*(f*x+exp(
1))-1/4*pi))*tan(1/4*exp(1))^7*tan(1/4*f*x)^4+172032*a*sign(cos(1/2*(f*x+ex
p(1))-1/4*pi))*tan(1/4*exp(1))^7*tan(1/4*f*x)^5-49152*a*sign(cos(1/2*(f*x+e
xp(1))-1/4*pi))*tan(1/4*exp(1))^7*tan(1/4*f*x)^6-24576*a*sign(cos(1/2*(f*x+
exp(1))-1/4*pi))*tan(1/4*exp(1))^7*tan(1/4*f*x)^7+12288*a*sign(cos(1/2*(f*x
+exp(1))-1/4*pi))*tan(1/4*exp(1))^7*tan(1/4*f*x)^8+24576*a*sign(cos(1/2*(f*
x+exp(1))-1/4*pi))*tan(1/4*exp(1))^7*tan(1/4*f*x)+21504*a*sign(cos(1/2*(f*x
+exp(1))-1/4*pi))*tan(1/4*exp(1))^8*tan(1/4*f*x)^2-12288*a*sign(cos(1/2*(f*
x+exp(1))-1/4*pi))*tan(1/4*exp(1))^8*tan(1/4*f*x)^3+75264*a*sign(cos(1/2*(f
*x+exp(1))-1/4*pi))*tan(1/4*exp(1))^8*tan(1/4*f*x)^4+12288*a*sign(cos(1/2*(
f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))^8*tan(1/4*f*x)^5+21504*a*sign(cos(1/2*
(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))^8*tan(1/4*f*x)^6+12288*a*sign(cos(1/2
*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))^8*tan(1/4*f*x)^7+8448*a*sign(cos(1/2
*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))^8*tan(1/4*f*x)^8-12288*a*sign(cos(1/
2*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))^8*tan(1/4*f*x)+49152*a*sign(cos(1/2
*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))*tan(1/4*f*x)^2+172032*a*sign(cos(1/2
*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))*tan(1/4*f*x)^3+122880*a*sign(cos(1/2
*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))*tan(1/4*f*x)^4-172032*a*sign(cos(1/2
*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))*tan(1/4*f*x)^5+49152*a*sign(cos(1/2*
(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))*tan(1/4*f*x)^6+24576*a*sign(cos(1/2*(
f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))*tan(1/4*f*x)^7-12288*a*sign(cos(1/2*(f
*x+exp(1))-1/4*pi))*tan(1/4*exp(1))*tan(1/4*f*x)^8-24576*a*sign(cos(1/2*(f*
x+exp(1))-1/4*pi))*tan(1/4*exp(1))*tan(1/4*f*x))/(-2304*sqrt(2)*f*tan(1/4*e
xp(1))-2304*sqrt(2)*f*tan(1/4*f*x)-4608*sqrt(2)*f*tan(1/4*exp(1))^2*tan(1/4
*f*x)+1152*sqrt(2)*f*tan(1/4*exp(1))^8*tan(1/4*f*x)^8+2304*sqrt(2)*f*tan(1/
4*exp(1))^7*tan(1/4*f*x)^8+2304*sqrt(2)*f*tan(1/4*exp(1))^8*tan(1/4*f*x)^7+
2304*sqrt(2)*f*tan(1/4*exp(1))^6*tan(1/4*f*x)^8-4608*sqrt(2)*f*tan(1/4*exp(
1))^7*tan(1/4*f*x)^7+2304*sqrt(2)*f*tan(1/4*exp(1))^8*tan(1/4*f*x)^6+6912*s
qrt(2)*f*tan(1/4*exp(1))^5*tan(1/4*f*x)^8+4608*sqrt(2)*f*tan(1/4*exp(1))^6*
```

```
tan(1/4*f*x)^7+4608*sqrt(2)*f*tan(1/4*exp(1))^7*tan(1/4*f*x)^6+6912*sqrt(2)
*f*tan(1/4*exp(1))^8*tan(1/4*f*x)^5-13824*sqrt(2)*f*tan(1/4*exp(1))^5*tan(1
/4*f*x)^7+4608*sqrt(2)*f*tan(1/4*exp(1))^6*tan(1/4*f*x)^6-13824*sqrt(2)*f*t
an(1/4*exp(1))^7*tan(1/4*f*x)^5+6912*sqrt(2)*f*tan(1/4*exp(1))^3*tan(1/4*f*
x)^8+13824*sqrt(2)*f*tan(1/4*exp(1))^5*tan(1/4*f*x)^6+13824*sqrt(2)*f*tan(1
/4*exp(1))^6*tan(1/4*f*x)^5+6912*sqrt(2)*f*tan(1/4*exp(1))^8*tan(1/4*f*x)^3
-2304*sqrt(2)*f*tan(1/4*exp(1))^2*tan(1/4*f*x)^8-13824*sqrt(2)*f*tan(1/4*ex
p(1))^3*tan(1/4*f*x)^7-41472*sqrt(2)*f*tan(1/4*exp(1))^5*tan(1/4*f*x)^5-138
24*sqrt(2)*f*tan(1/4*exp(1))^7*tan(1/4*f*x)^3-2304*sqrt(2)*f*tan(1/4*exp(1)
)^8*tan(1/4*f*x)^2-4608*sqrt(2)*f*tan(1/4*exp(1))^2*tan(1/4*f*x)^7+13824*sq
rt(2)*f*tan(1/4*exp(1))^3*tan(1/4*f*x)^6+13824*sqrt(2)*f*tan(1/4*exp(1))^6*
tan(1/4*f*x)^3-4608*sqrt(2)*f*tan(1/4*exp(1))^7*tan(1/4*f*x)^2-1152*sqrt(2)
*f*tan(1/4*exp(1))^8-1152*sqrt(2)*f*tan(1/4*f*x)^8-4608*sqrt(2)*f*tan(1/4*e
xp(1))^2*tan(1/4*f*x)^6-41472*sqrt(2)*f*tan(1/4*exp(1))^3*tan(1/4*f*x)^5-41
472*sqrt(2)*f*tan(1/4*exp(1))^5*tan(1/4*f*x)^3-4608*sqrt(2)*f*tan(1/4*exp(1
))^6*tan(1/4*f*x)^2-2304*sqrt(2)*f*tan(1/4*exp(1))^7-2304*sqrt(2)*f*tan(1/4
*f*x)^7-13824*sqrt(2)*f*tan(1/4*exp(1))^2*tan(1/4*f*x)^5-13824*sqrt(2)*f*ta
n(1/4*exp(1))^5*tan(1/4*f*x)^2-2304*sqrt(2)*f*tan(1/4*exp(1))^6-2304*sqrt(2
)*f*tan(1/4*f*x)^6-41472*sqrt(2)*f*tan(1/4*exp(1))^3*tan(1/4*f*x)^3-6912*sq
rt(2)*f*tan(1/4*exp(1))^5-6912*sqrt(2)*f*tan(1/4*f*x)^5-13824*sqrt(2)*f*tan
(1/4*exp(1))^2*tan(1/4*f*x)^3-13824*sqrt(2)*f*tan(1/4*exp(1))^3*tan(1/4*f*x
)^2+4608*sqrt(2)*f*tan(1/4*exp(1))^2*tan(1/4*f*x)^2-6912*sqrt(2)*f*tan(1/4*
exp(1))^3-6912*sqrt(2)*f*tan(1/4*f*x)^3+2304*sqrt(2)*f*tan(1/4*exp(1))^2+23
04*sqrt(2)*f*tan(1/4*f*x)^2+1152*sqrt(2)*f-13824*sqrt(2)*f*tan(1/4*exp(1))^
3*tan(1/4*f*x)-13824*sqrt(2)*f*tan(1/4*exp(1))^5*tan(1/4*f*x)+4608*sqrt(2)*
f*tan(1/4*exp(1))^6*tan(1/4*f*x)-4608*sqrt(2)*f*tan(1/4*exp(1))^7*tan(1/4*f
*x)+2304*sqrt(2)*f*tan(1/4*exp(1))^8*tan(1/4*f*x)-4608*sqrt(2)*f*tan(1/4*ex
p(1))*tan(1/4*f*x)^2-13824*sqrt(2)*f*tan(1/4*exp(1))*tan(1/4*f*x)^3-13824*s
qrt(2)*f*tan(1/4*exp(1))*tan(1/4*f*x)^5+4608*sqrt(2)*f*tan(1/4*exp(1))*tan(
1/4*f*x)^6-4608*sqrt(2)*f*tan(1/4*exp(1))*tan(1/4*f*x)^7+2304*sqrt(2)*f*tan
(1/4*exp(1))*tan(1/4*f*x)^8-4608*sqrt(2)*f*tan(1/4*exp(1))*tan(1/4*f*x))
```

maple [A] time = 0.58, size = 55, normalized size = 0.62

$$-\frac{2a^2\left(1+\sin\left(fx+e\right)\right)\left(\sin^2\left(fx+e\right)+4\sin\left(fx+e\right)-8\right)}{3\cos\left(fx+e\right)\sqrt{a+a\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*tan(f*x+e)^2,x)
```

```
[Out] -2/3*a^2*(1+sin(f*x+e))*(sin(f*x+e)^2+4*sin(f*x+e)-8)/cos(f*x+e)/(a+a*sin(f
*x+e))^(1/2)/f
```

maxima [A] time = 0.58, size = 145, normalized size = 1.65

$$-\frac{8\left(2\,a^{\frac{3}{2}}-\frac{2\,a^{\frac{3}{2}}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3\,a^{\frac{3}{2}}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{2\,a^{\frac{3}{2}}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{2\,a^{\frac{3}{2}}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}\right)}{3\,f\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)\left(\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] -8/3*(2*a^(3/2) - 2*a^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^(3/2)*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2*a^(3/2)*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 + 2*a^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)/(f*(sin(f*x + e)/(co
s(f*x + e) + 1) - 1)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan\left(e+f\,x\right)^2\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(a + a*sin(e + f*x))^(3/2),x)
```

```
[Out] int(tan(e + f*x)^2*(a + a*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\tan^{2}\left(e+fx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*tan(f*x+e)**2,x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*tan(e + f*x)**2, x)
```

3.97 $\int \cot^2(e+fx)(a+a\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=121

$$-\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}+\frac{11a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}+\frac{5a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}-\frac{\cot(e+fx)(a\sin(e+}{f}$$

```
[Out] -3*a^(3/2)*arctanh(cos(f*x+e)*a^(1/2)/(a+a*sin(f*x+e))^(1/2))/f-cot(f*x+e)*
(a+a*sin(f*x+e))^(3/2)/f+11/3*a^2*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)+5/3*a
*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.32, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2716, 2976, 2981, 2773, 206}

$$\frac{11a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}+\frac{5a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}-\frac{\cot(e+fx)(a\sin(e+f}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-3*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/f + (
11*a^2*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) + (5*a*Cos[e + f*x]*Sqr
t[a + a*Sin[e + f*x]])/(3*f) - (Cot[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/f
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2716

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)/tan[(e_.) + (f_.)*(x_)]^2,
 x_Symbol] :> -Simp[(a + b*Sin[e + f*x])^m/(f*Tan[e + f*x]), x] + Dist[1/a,
 Int[((a + b*Sin[e + f*x])^m*(b*m - a*(m + 1)*Sin[e + f*x]))/Sin[e + f*x],
x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/
2] &&  !LtQ[m, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
```

```
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \cot^2(e+fx)(a+a\sin(e+fx))^{3/2}\,dx = -\frac{\cot(e+fx)(a+a\sin(e+fx))^{3/2}}{f} + \frac{\int \csc(e+fx)\left(\frac{3a}{2}-\frac{5}{2}a\sin(e+f\right.}{a}$$

$$= \frac{5a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f} - \frac{\cot(e+fx)(a+a\sin(e+fx))^{3/2}}{f}$$

$$= \frac{11a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} + \frac{5a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f} - \frac{\cot(e+}{}$$

$$= \frac{11a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} + \frac{5a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f} - \frac{\cot(e+}{}$$

$$= -\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{f} + \frac{11a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} + \frac{5a\cos(e+}{}$$

Mathematica [A] time = 0.76, size = 233, normalized size = 1.93

$$-\frac{a\csc^4\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sin(e+fx)+1)}\left(-14\sin\left(\frac{1}{2}(e+fx)\right)-9\sin\left(\frac{3}{2}(e+fx)\right)-\sin\left(\frac{5}{2}(e+fx)\right)+14\cos\left(\frac{1}{2}(e\right.\right.}{3f\left(\cot\left(\frac{1}{2}(e+fx)\right)\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -1/3*(a*Csc[(e + f*x)/2]^4*Sqrt[a*(1 + Sin[e + f*x])]*(14*Cos[(e + f*x)/2]
- 9*Cos[(3*(e + f*x))/2] + Cos[(5*(e + f*x))/2] - 14*Sin[(e + f*x)/2] + 9*L
og[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sin[e + f*x] - 9*Log[1 - Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2]]*Sin[e + f*x] - 9*Sin[(3*(e + f*x))/2] - Sin[
(5*(e + f*x))/2]))/(f*(1 + Cot[(e + f*x)/2])*(Csc[(e + f*x)/4] - Sec[(e + f
*x)/4])*(Csc[(e + f*x)/4] + Sec[(e + f*x)/4]))
```

fricas [B] time = 0.43, size = 315, normalized size = 2.60

$$\frac{9\left(a\cos\left(fx+e\right)^2-\left(a\cos\left(fx+e\right)+a\right)\sin\left(fx+e\right)-a\right)\sqrt{a}\,\log\left(\frac{a\cos\left(fx+e\right)^3-7\,a\cos\left(fx+e\right)^2-4\left(\cos\left(fx+e\right)^2+\left(\cos\left(fx+e\right)+3\right.\right.}{\cos\left(fx+\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/12*(9*(a*cos(f*x + e)^2 - (a*cos(f*x + e) + a)*sin(f*x + e) - a)*sqrt(a)*
log((a*cos(f*x + e)^3 - 7*a*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + (cos(f*x +
 e) + 3)*sin(f*x + e) - 2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a
) - 9*a*cos(f*x + e) + (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x +
e) - a)/(cos(f*x + e)^3 + cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e
) - cos(f*x + e) - 1)) + 4*(2*a*cos(f*x + e)^3 - 8*a*cos(f*x + e)^2 + a*cos
(f*x + e) - (2*a*cos(f*x + e)^2 + 10*a*cos(f*x + e) + 11*a)*sin(f*x + e) +
11*a)*sqrt(a*sin(f*x + e) + a))/(f*cos(f*x + e)^2 - (f*cos(f*x + e) + f)*si
n(f*x + e) - f)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.90, size = 144, normalized size = 1.19

$$\frac{\left(1+\sin\left(fx+e\right)\right)\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\left(\sin\left(fx+e\right)\left(12\sqrt{a-a\sin\left(fx+e\right)}\,a^{\frac{3}{2}}-2\sqrt{a}\,\left(a-a\sin\left(fx+e\right)\right)^{\frac{3}{2}}\right.\right.}{3\sin\left(fx+e\right)\sqrt{a}\,\cos\left(fx+e\right)\sqrt{a+a\sin\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/3*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(sin(f*x+e)*(12*(a-a*sin(f*x+e
))^(1/2)*a^(3/2)-2*a^(1/2)*(a-a*sin(f*x+e))^(3/2)-9*arctanh((a-a*sin(f*x+e)
)^(1/2)/a^(1/2))*a^2)-3*(a-a*sin(f*x+e))^(1/2)*a^(3/2))/sin(f*x+e)/a^(1/2)/
cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*cot(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\cot\left(e+f\,x\right)^2\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(a + a*sin(e + f*x))^(3/2),x)
```

```
[Out] int(cot(e + f*x)^2*(a + a*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\cot^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*cot(e + f*x)**2, x)
```

3.98 $\int \cot^4(e+fx)(a+a\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=197

$$\frac{37a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{8f}-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}+\frac{29a^2\cot(e+fx)}{24f\sqrt{a\sin(e+fx)+a}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+}}{3f}$$

```
[Out] 37/8*a^(3/2)*arctanh(cos(f*x+e)*a^(1/2)/(a+a*sin(f*x+e))^(1/2))/f-1/3*cot(f
*x+e)*csc(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/f-8/3*a^2*cos(f*x+e)/f/(a+a*sin(f
*x+e))^(1/2)+29/24*a^2*cot(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-2/3*a*cos(f*x+e)
*(a+a*sin(f*x+e))^(1/2)/f-1/4*a*cot(f*x+e)*csc(f*x+e)*(a+a*sin(f*x+e))^(1/2
)/f
```

Rubi [A] time = 0.50, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2718, 2647, 2646, 3044, 2975, 2980, 2773, 206}

$$-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}+\frac{29a^2\cot(e+fx)}{24f\sqrt{a\sin(e+fx)+a}}+\frac{37a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{8f}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (37*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/(8*f)
 - (8*a^2*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) + (29*a^2*Cot[e + f*
x])/(24*f*Sqrt[a + a*Sin[e + f*x]]) - (2*a*Cos[e + f*x]*Sqrt[a + a*Sin[e +
f*x]])/(3*f) - (a*Cot[e + f*x]*Csc[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(4*f)
 - (Cot[e + f*x]*Csc[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2))/(3*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2718

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] + Int[((a + b*Sin[e + f*x])^m*(
1 - 2*Sin[e + f*x]^2))/Sin[e + f*x]^4, x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&  !LtQ[m, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
```

```
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \cot^4(e+fx)(a+a\sin(e+fx))^{3/2}\,dx = \int (a+a\sin(e+fx))^{3/2}\,dx + \int \csc^4(e+fx)(a+a\sin(e+fx))^{3/2}\,(1-$$

$$= -\frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f} - \frac{\cot(e+fx)\csc^2(e+fx)(a+a\,\mathrm{si}}{3f}$$

$$= -\frac{8a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} - \frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f} - \frac{a\cot(e}{}$$

$$= -\frac{8a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} + \frac{29a^2\cot(e+fx)}{24f\sqrt{a+a\sin(e+fx)}} - \frac{2a\cos(e+fx)\sqrt{a}}{3f}$$

$$= -\frac{8a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} + \frac{29a^2\cot(e+fx)}{24f\sqrt{a+a\sin(e+fx)}} - \frac{2a\cos(e+fx)\sqrt{a}}{3f}$$

$$= \frac{37a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{8f} - \frac{8a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} + \frac{29a^2\cot(e}{24f\sqrt{a+a\,\mathrm{si}}}$$

Mathematica [A] time = 1.63, size = 334, normalized size = 1.70

$$-\frac{a\csc^{10}\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sin(e+fx)+1)}\left(276\sin\left(\frac{1}{2}(e+fx)\right)+326\sin\left(\frac{3}{2}(e+fx)\right)-78\sin\left(\frac{5}{2}(e+fx)\right)-72\,\mathrm{si}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4*(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -1/24*(a*Csc[(e + f*x)/2]^10*Sqrt[a*(1 + Sin[e + f*x])]*(-276*Cos[(e + f*x)
/2] + 326*Cos[(3*(e + f*x))/2] + 78*Cos[(5*(e + f*x))/2] - 72*Cos[(7*(e + f
*x))/2] + 8*Cos[(9*(e + f*x))/2] + 276*Sin[(e + f*x)/2] - 333*Log[1 + Cos[(
```

```
e + f*x)/2] - Sin[(e + f*x)/2]]*Sin[e + f*x] + 333*Log[1 - Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2]]*Sin[e + f*x] + 326*Sin[(3*(e + f*x))/2] - 78*Sin[(5*(e
 + f*x))/2] + 111*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sin[3*(e + f
*x)] - 111*Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[3*(e + f*x)] -
72*Sin[(7*(e + f*x))/2] - 8*Sin[(9*(e + f*x))/2]))/(f*(1 + Cot[(e + f*x)/2]
)*(Csc[(e + f*x)/4]^2 - Sec[(e + f*x)/4]^2)^3)
```

fricas [B] time = 0.45, size = 424, normalized size = 2.15

$$111\left(a\cos\left(fx+e\right)^4-2\,a\cos\left(fx+e\right)^2-\left(a\cos\left(fx+e\right)^3+a\cos\left(fx+e\right)^2-a\cos\left(fx+e\right)-a\right)\sin\left(fx+e\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/96*(111*(a*cos(f*x + e)^4 - 2*a*cos(f*x + e)^2 - (a*cos(f*x + e)^3 + a*co
s(f*x + e)^2 - a*cos(f*x + e) - a)*sin(f*x + e) + a)*sqrt(a)*log((a*cos(f*x
 + e)^3 - 7*a*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3)*sin(f
*x + e) - 2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a*cos(f*
x + e) + (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)/(cos(f
*x + e)^3 + cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos(f*x +
e) - 1)) - 4*(16*a*cos(f*x + e)^5 - 64*a*cos(f*x + e)^4 - 17*a*cos(f*x + e)
^3 + 165*a*cos(f*x + e)^2 + 9*a*cos(f*x + e) - (16*a*cos(f*x + e)^4 + 80*a*
cos(f*x + e)^3 + 63*a*cos(f*x + e)^2 - 102*a*cos(f*x + e) - 93*a)*sin(f*x +
 e) - 93*a)*sqrt(a*sin(f*x + e) + a))/(f*cos(f*x + e)^4 - 2*f*cos(f*x + e)^
2 - (f*cos(f*x + e)^3 + f*cos(f*x + e)^2 - f*cos(f*x + e) - f)*sin(f*x + e)
 + f)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 1.00, size = 196, normalized size = 0.99

$$\left(1+\sin\left(fx+e\right)\right)\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\,\left(16\left(-a\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{3}{2}}\left(\sin^3\left(fx+e\right)\right)a^{\frac{3}{2}}-96a^{\frac{5}{2}}\sqrt{-a\left(\sin\left(fx\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/24*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)/a^(3/2)*(16*(-a*(sin(f*x+e)-1
))^(3/2)*sin(f*x+e)^3*a^(3/2)-96*a^(5/2)*(-a*(sin(f*x+e)-1))^(1/2)*sin(f*x+
e)^3+111*arctanh((-a*(sin(f*x+e)-1))^(1/2)/a^(1/2))*sin(f*x+e)^3*a^3+15*(-a
*(sin(f*x+e)-1))^(5/2)*a^(1/2)-8*(-a*(sin(f*x+e)-1))^(3/2)*a^(3/2)-15*(-a*(
sin(f*x+e)-1))^(1/2)*a^(5/2))/sin(f*x+e)^3/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2
)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \sin\left(fx+e\right)+a\right)^{\frac{3}{2}} \cot\left(fx+e\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*cot(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(e+f\,x\right)^4\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4*(a + a*sin(e + f*x))^(3/2),x)
```

```
[Out] int(cot(e + f*x)^4*(a + a*sin(e + f*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.99 $\int (a+a\sin(e+fx))^{5/2}\tan^4(e+fx)\,dx$

Optimal. Leaf size=151

$$-\frac{2a^5\cos^5(e+fx)}{5f(a\sin(e+fx)+a)^{5/2}}+\frac{8a^4\cos^3(e+fx)}{3f(a\sin(e+fx)+a)^{3/2}}-\frac{12a^3\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}-\frac{8a^2\sec(e+fx)\sqrt{a\sin(e+fx)+a}}{f}$$

```
[Out] -2/5*a^5*cos(f*x+e)^5/f/(a+a*sin(f*x+e))^(5/2)+8/3*a^4*cos(f*x+e)^3/f/(a+a*
sin(f*x+e))^(3/2)+2/3*a*sec(f*x+e)^3*(a+a*sin(f*x+e))^(3/2)/f-12*a^3*cos(f*
x+e)/f/(a+a*sin(f*x+e))^(1/2)-8*a^2*sec(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.98, antiderivative size = 208, normalized size of antiderivative = 1.38, number of steps used = 10, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2714, 2647, 2646, 4401, 2673, 2878, 2855}

$$-\frac{16a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15f}-\frac{64a^3\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}-\frac{46a^2\sec(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}-\frac{2a\cos(e}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*Tan[e + f*x]^4,x]
```

```
[Out] (-64*a^3*Cos[e + f*x])/(15*f*Sqrt[a + a*Sin[e + f*x]]) - (16*a^2*Cos[e + f*
x]*Sqrt[a + a*Sin[e + f*x]])/(15*f) - (46*a^2*Sec[e + f*x]*Sqrt[a + a*Sin[e
 + f*x]])/(3*f) - (2*a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(5*f) - (2*
a*Sec[e + f*x]^3*(a + a*Sin[e + f*x])^(3/2))/(3*f) + (26*Sec[e + f*x]^3*(a
+ a*Sin[e + f*x])^(5/2))/(3*f) - (4*Sec[e + f*x]^3*(a + a*Sin[e + f*x])^(7/
2))/(a*f)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
```

```
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2714

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] - Int[((a + b*Sin[e + f*x])^m*(
1 - 2*Sin[e + f*x]^2))/Cos[e + f*x]^4, x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2878

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(
p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b*f*g*(m + p + 2)), x] + Dist[1/(b*(m
 + p + 2)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*(p
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 -
 b^2, 0] && NeQ[m + p + 2, 0]
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\int (a + a\sin(e + fx))^{5/2}\tan^4(e + fx)\,dx = \int (a + a\sin(e + fx))^{5/2}\,dx - \int \sec^4(e + fx)(a + a\sin(e + fx))^{5/2}\left(1 \right.$$

$$= -\frac{2a\cos(e + fx)(a + a\sin(e + fx))^{3/2}}{5f} + \frac{1}{5}(8a)\int (a + a\sin(e + fx))^{3/}$$

$$= -\frac{16a^2\cos(e + fx)\sqrt{a + a\sin(e + fx)}}{15f} - \frac{2a\cos(e + fx)(a + a\sin(e +}{5f}$$

$$= -\frac{64a^3\cos(e + fx)}{15f\sqrt{a + a\sin(e + fx)}} - \frac{16a^2\cos(e + fx)\sqrt{a + a\sin(e + fx)}}{15f} - 2a$$

$$= -\frac{64a^3\cos(e + fx)}{15f\sqrt{a + a\sin(e + fx)}} - \frac{16a^2\cos(e + fx)\sqrt{a + a\sin(e + fx)}}{15f} - 2a$$

$$= -\frac{64a^3\cos(e + fx)}{15f\sqrt{a + a\sin(e + fx)}} - \frac{16a^2\cos(e + fx)\sqrt{a + a\sin(e + fx)}}{15f} - 46$$

Mathematica [A] time = 5.47, size = 112, normalized size = 0.74

$$\frac{a^2\sqrt{a(\sin(e + fx) + 1)}\,(1488\sin(e + fx) + 16\sin(3(e + fx)) + 204\cos(2(e + fx)) - 3\cos(4(e + fx)) - 1225)}{60f\left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)\right)^3\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*Tan[e + f*x]^4,x]
```

```
[Out] (a^2*Sqrt[a*(1 + Sin[e + f*x])]*(-1225 + 204*Cos[2*(e + f*x)] - 3*Cos[4*(e
+ f*x)] + 1488*Sin[e + f*x] + 16*Sin[3*(e + f*x)]))/(60*f*(Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

fricas [A] time = 0.43, size = 98, normalized size = 0.65

$$\frac{2\left(3\,a^2\cos\left(fx + e\right)^4 - 54\,a^2\cos\left(fx + e\right)^2 + 179\,a^2 - 8\left(a^2\cos\left(fx + e\right)^2 + 23\,a^2\right)\sin\left(fx + e\right)\right)\sqrt{a\sin\left(fx + e\right)}}{15\left(f\cos\left(fx + e\right)\sin\left(fx + e\right) - f\cos\left(fx + e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] 2/15*(3*a^2*cos(f*x + e)^4 - 54*a^2*cos(f*x + e)^2 + 179*a^2 - 8*(a^2*cos(f
*x + e)^2 + 23*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e)*
sin(f*x + e) - f*cos(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.69, size = 87, normalized size = 0.58

$$\frac{2a^3\left(1+\sin\left(fx+e\right)\right)\left(3\left(\sin^4\left(fx+e\right)\right)+8\left(\sin^3\left(fx+e\right)\right)+48\left(\sin^2\left(fx+e\right)\right)-192\sin\left(fx+e\right)+128\right)}{15\left(\sin\left(fx+e\right)-1\right)\cos\left(fx+e\right)\sqrt{a+a\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*tan(f*x+e)^4,x)
```

```
[Out] 2/15*a^3*(1+sin(f*x+e))/(sin(f*x+e)-1)*(3*sin(f*x+e)^4+8*sin(f*x+e)^3+48*si
n(f*x+e)^2-192*sin(f*x+e)+128)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [B] time = 1.29, size = 277, normalized size = 1.83

$$\frac{32\left(8\,a^{\frac{5}{2}}-\frac{24\,a^{\frac{5}{2}}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{44\,a^{\frac{5}{2}}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{68\,a^{\frac{5}{2}}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{75\,a^{\frac{5}{2}}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}-\frac{68\,a^{\frac{5}{2}}\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}+\frac{44\,a^{\frac{5}{2}}\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}-\cdots\right.}{15\,f\left(\frac{3\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{3\,\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}-1\right)\left(\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] 32/15*(8*a^(5/2) - 24*a^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 44*a^(5/2)*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 68*a^(5/2)*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 75*a^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 68*a^(5/2)*sin
(f*x + e)^5/(cos(f*x + e) + 1)^5 + 44*a^(5/2)*sin(f*x + e)^6/(cos(f*x + e)
+ 1)^6 - 24*a^(5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 8*a^(5/2)*sin(f*x
 + e)^8/(cos(f*x + e) + 1)^8)/(f*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 + sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)
*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(5/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan\left(e+f\,x\right)^4\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(a + a*sin(e + f*x))^(5/2),x)
```

```
[Out] int(tan(e + f*x)^4*(a + a*sin(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*tan(f*x+e)**4,x)
```

```
[Out] Timed out
```

3.100 $\int (a+a\sin(e+fx))^{5/2}\tan^2(e+fx)\,dx$

Optimal. Leaf size=118

$$\frac{124a^3\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}+\frac{31a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15f}-\frac{2\sec(e+fx)(a\sin(e+fx)+a)^{7/2}}{5af}+\frac{9\sec(e+fx)(}{}$$

```
[Out] 9/5*sec(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f-2/5*sec(f*x+e)*(a+a*sin(f*x+e))^(7/
2)/a/f+124/15*a^3*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)+31/15*a^2*cos(f*x+e)*
(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.21, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2713, 2855, 2647, 2646}

$$\frac{31a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15f}+\frac{124a^3\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}-\frac{2\sec(e+fx)(a\sin(e+fx)+a)^{7/2}}{5af}+\frac{9\sec(e+fx)(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*Tan[e + f*x]^2,x]
```

```
[Out] (124*a^3*Cos[e + f*x])/(15*f*Sqrt[a + a*Sin[e + f*x]]) + (31*a^2*Cos[e + f*
x]*Sqrt[a + a*Sin[e + f*x]])/(15*f) + (9*Sec[e + f*x]*(a + a*Sin[e + f*x])^
(5/2))/(5*f) - (2*Sec[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(5*a*f)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2713

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> -Simp[(a + b*Sin[e + f*x])^(m + 1)/(b*f*m*Cos[e + f*x]), x] +
Dist[1/(b*m), Int[((a + b*Sin[e + f*x])^m*(b*(m + 1) + a*Sin[e + f*x]))/Cos
[e + f*x]^2, x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !
```

```
IntegerQ[m] &&  !LtQ[m, 0]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}\tan^2(e+fx)\,dx = -\frac{2\sec(e+fx)(a+a\sin(e+fx))^{7/2}}{5af} + \frac{2\int \sec^2(e+fx)(a+a\sin(e+ \ldots}{5 \ldots}$$

$$= \frac{9\sec(e+fx)(a+a\sin(e+fx))^{5/2}}{5f} - \frac{2\sec(e+fx)(a+a\sin(e+fx)) \ldots}{5af}$$

$$= \frac{31a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f} + \frac{9\sec(e+fx)(a+a\sin(e+fx \ldots}{5f}$$

$$= \frac{124a^3\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} + \frac{31a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f} + \frac{9\,\mathrm{se}\ldots}{}$$

Mathematica [A] time = 5.47, size = 60, normalized size = 0.51

$$\frac{a^2\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\,(-185\sin(e+fx)+3\sin(3(e+fx))+22\cos(2(e+fx))+330)}{30f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*Tan[e + f*x]^2,x]
```

```
[Out] (a^2*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*(330 + 22*Cos[2*(e + f*x)] - 1
85*Sin[e + f*x] + 3*Sin[3*(e + f*x)]))/(30*f)
```

fricas [A] time = 0.43, size = 70, normalized size = 0.59

$$\frac{2\left(11\,a^2\cos\left(fx+e\right)^2+77\,a^2+\left(3\,a^2\cos\left(fx+e\right)^2-47\,a^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}}{15\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] 2/15*(11*a^2*cos(f*x + e)^2 + 77*a^2 + (3*a^2*cos(f*x + e)^2 - 47*a^2)*sin(
f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.49, size = 67, normalized size = 0.57

$$-\frac{2a^{3}\left(1+\sin\left(fx+e\right)\right)\left(3\left(\sin^{3}\left(fx+e\right)\right)+11\left(\sin^{2}\left(fx+e\right)\right)+44\sin\left(fx+e\right)-88\right)}{15\cos\left(fx+e\right)\sqrt{a+a\sin\left(fx+e\right)}\, f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*tan(f*x+e)^2,x)
```

```
[Out] -2/15*a^3*(1+sin(f*x+e))*(3*sin(f*x+e)^3+11*sin(f*x+e)^2+44*sin(f*x+e)-88)/
cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [A] time = 0.48, size = 191, normalized size = 1.62

$$-\frac{8\left(22\,a^{\frac{5}{2}}-\frac{22\,a^{\frac{5}{2}}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{55\,a^{\frac{5}{2}}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{50\,a^{\frac{5}{2}}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{55\,a^{\frac{5}{2}}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}-\frac{22\,a^{\frac{5}{2}}\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}+\frac{22\,a^{\frac{5}{2}}\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}\right)}{15\,f\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)\left(\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] -8/15*(22*a^(5/2) - 22*a^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 55*a^(5/2)
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 50*a^(5/2)*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 55*a^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 22*a^(5/2)*si
n(f*x + e)^5/(cos(f*x + e) + 1)^5 + 22*a^(5/2)*sin(f*x + e)^6/(cos(f*x + e)
```

```
 + 1)^6)/(f*(sin(f*x + e)/(cos(f*x + e) + 1) - 1)*(sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 1)^(5/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan\left(e+f\,x\right)^2\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(a + a*sin(e + f*x))^(5/2),x)
[Out] int(tan(e + f*x)^2*(a + a*sin(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*tan(f*x+e)**2,x)
[Out] Timed out
```

3.101 $\int \cot^2(e+fx)(a+a\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=151

$$-\frac{5a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}+\frac{49a^3\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}+\frac{31a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15f}+\frac{7a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}$$

```
[Out] -5*a^(5/2)*arctanh(cos(f*x+e)*a^(1/2)/(a+a*sin(f*x+e))^(1/2))/f+7/5*a*cos(f
*x+e)*(a+a*sin(f*x+e))^(3/2)/f-cot(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f+49/15*a^
3*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)+31/15*a^2*cos(f*x+e)*(a+a*sin(f*x+e))
^(1/2)/f
```

Rubi [A] time = 0.43, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2716, 2976, 2981, 2773, 206}

$$\frac{49a^3\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}+\frac{31a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15f}-\frac{5a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}+\frac{7a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-5*a^(5/2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/f + (
49*a^3*Cos[e + f*x])/(15*f*Sqrt[a + a*Sin[e + f*x]]) + (31*a^2*Cos[e + f*x]
*Sqrt[a + a*Sin[e + f*x]])/(15*f) + (7*a*Cos[e + f*x]*(a + a*Sin[e + f*x])^
(3/2))/(5*f) - (Cot[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/f
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2716

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)/tan[(e_.) + (f_.)*(x_)]^2,
 x_Symbol] :> -Simp[(a + b*Sin[e + f*x])^m/(f*Tan[e + f*x]), x] + Dist[1/a,
 Int[((a + b*Sin[e + f*x])^m*(b*m - a*(m + 1)*Sin[e + f*x]))/Sin[e + f*x],
x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/
2] && !LtQ[m, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \cot^2(e+fx)(a+a\sin(e+fx))^{5/2}\,dx = -\frac{\cot(e+fx)(a+a\sin(e+fx))^{5/2}}{f} + \frac{\int \csc(e+fx)\left(\frac{5a}{2}-\frac{7}{2}a\sin(e+fx\right.}{a}$$

$$= \frac{7a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5f} - \frac{\cot(e+fx)(a+a\sin(e+fx))^{5/2}}{f}$$

$$= \frac{31a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f} + \frac{7a\cos(e+fx)(a+a\sin(e+fx}{5f}$$

$$= \frac{49a^3\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} + \frac{31a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f} + \frac{7a\,\mathrm{co}}{}$$

$$= \frac{49a^3\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} + \frac{31a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f} + \frac{7a\,\mathrm{co}}{}$$

$$= -\frac{5a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{f} + \frac{49a^3\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} + \frac{31a^2\cos(e}{}$$

Mathematica [A] time = 1.26, size = 261, normalized size = 1.73

$$-\frac{a^2\csc^4\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sin(e+fx)+1)}\left(-125\sin\left(\frac{1}{2}(e+fx)\right)-93\sin\left(\frac{3}{2}(e+fx)\right)-25\sin\left(\frac{5}{2}(e+fx)\right)+3\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] -1/30*(a^2*Csc[(e + f*x)/2]^4*Sqrt[a*(1 + Sin[e + f*x])]*(125*Cos[(e + f*x)
/2] - 93*Cos[(3*(e + f*x))/2] + 25*Cos[(5*(e + f*x))/2] + 3*Cos[(7*(e + f*x
))/2] - 125*Sin[(e + f*x)/2] + 150*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)
/2]]*Sin[e + f*x] - 150*Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[e
+ f*x] - 93*Sin[(3*(e + f*x))/2] - 25*Sin[(5*(e + f*x))/2] + 3*Sin[(7*(e +
f*x))/2]))/(f*(1 + Cot[(e + f*x)/2])*(Csc[(e + f*x)/4] - Sec[(e + f*x)/4])*
(Csc[(e + f*x)/4] + Sec[(e + f*x)/4]))
```

fricas [B] time = 0.45, size = 363, normalized size = 2.40

$$\frac{75\left(a^2\cos\left(fx+e\right)^2-a^2-\left(a^2\cos\left(fx+e\right)+a^2\right)\sin\left(fx+e\right)\right)\sqrt{a}\,\log\left(\frac{a\cos\left(fx+e\right)^3-7\,a\cos\left(fx+e\right)^2-4\left(\cos\left(fx+e\right)^2+\left(\cos\left(f\right.\right.\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/60*(75*(a^2*cos(f*x + e)^2 - a^2 - (a^2*cos(f*x + e) + a^2)*sin(f*x + e))
*sqrt(a)*log((a*cos(f*x + e)^3 - 7*a*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + (
cos(f*x + e) + 3)*sin(f*x + e) - 2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) +
a)*sqrt(a) - 9*a*cos(f*x + e) + (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*s
in(f*x + e) - a)/(cos(f*x + e)^3 + cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*si
n(f*x + e) - cos(f*x + e) - 1)) + 4*(6*a^2*cos(f*x + e)^4 + 28*a^2*cos(f*x
+ e)^3 - 40*a^2*cos(f*x + e)^2 - 13*a^2*cos(f*x + e) + 49*a^2 + (6*a^2*cos(
f*x + e)^3 - 22*a^2*cos(f*x + e)^2 - 62*a^2*cos(f*x + e) - 49*a^2)*sin(f*x
+ e))*sqrt(a*sin(f*x + e) + a))/(f*cos(f*x + e)^2 - (f*cos(f*x + e) + f)*si
n(f*x + e) - f)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.73, size = 162, normalized size = 1.07

$$\frac{\left(1+\sin\left(fx+e\right)\right)\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\,\left(\sin\left(fx+e\right)\left(90\sqrt{a-a\sin\left(fx+e\right)}\,a^{\frac{5}{2}}-40a^{\frac{3}{2}}\left(a-a\sin\left(fx+e\right)\right)^{\frac{3}{2}}\right.\right.}{15\sin\left(fx+e\right)\sqrt{a}\,\cos\left(fx+e\right)\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/15*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(sin(f*x+e)*(90*(a-a*sin(f*x+
e))^(1/2)*a^(5/2)-40*a^(3/2)*(a-a*sin(f*x+e))^(3/2)+6*a^(1/2)*(a-a*sin(f*x+
e))^(5/2)-75*arctanh((a-a*sin(f*x+e))^(1/2)/a^(1/2))*a^3)-15*(a-a*sin(f*x+e
))^(1/2)*a^(5/2))/sin(f*x+e)/a^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\cot\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*cot(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(e+f\,x\right)^2\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(a + a*sin(e + f*x))^(5/2),x)
[Out] int(cot(e + f*x)^2*(a + a*sin(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+a*sin(f*x+e))**(5/2),x)
[Out] Timed out
```

3.102 $\int \cot^4(e+fx)(a+a\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=227

$$\frac{55a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{8f}-\frac{9a^3\cos(e+fx)}{40f\sqrt{a\sin(e+fx)+a}}-\frac{16a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15f}+\frac{17a^2\cot(e+fx)}{2}$$

```
[Out] 55/8*a^(5/2)*arctanh(cos(f*x+e)*a^(1/2)/(a+a*sin(f*x+e))^(1/2))/f-2/5*a*cos
(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f-5/12*a*cot(f*x+e)*csc(f*x+e)*(a+a*sin(f*x+
e))^(3/2)/f-1/3*cot(f*x+e)*csc(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/f-9/40*a^3*c
os(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-16/15*a^2*cos(f*x+e)*(a+a*sin(f*x+e))^(1
/2)/f+17/24*a^2*cot(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.63, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2718, 2647, 2646, 3044, 2975, 2981, 2773, 206}

$$-\frac{9a^3\cos(e+fx)}{40f\sqrt{a\sin(e+fx)+a}}-\frac{16a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15f}+\frac{17a^2\cot(e+fx)\sqrt{a\sin(e+fx)+a}}{24f}+\frac{55a^{5/2}\tanh^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (55*a^(5/2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/(8*f)
 - (9*a^3*Cos[e + f*x])/(40*f*Sqrt[a + a*Sin[e + f*x]]) - (16*a^2*Cos[e + f
*x]*Sqrt[a + a*Sin[e + f*x]])/(15*f) + (17*a^2*Cot[e + f*x]*Sqrt[a + a*Sin[
e + f*x]])/(24*f) - (2*a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(5*f) - (
5*a*Cot[e + f*x]*Csc[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(12*f) - (Cot[e +
 f*x]*Csc[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/(3*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2718

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] + Int[((a + b*Sin[e + f*x])^m*(
1 - 2*Sin[e + f*x]^2))/Sin[e + f*x]^4, x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&  !LtQ[m, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \cot^4(e+fx)(a+a\sin(e+fx))^{5/2}\,dx = \int (a+a\sin(e+fx))^{5/2}\,dx + \int \csc^4(e+fx)(a+a\sin(e+fx))^{5/2}\left(1-\right.$$

$$= -\frac{2a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5f} - \frac{\cot(e+fx)\csc^2(e+fx)(a+}{3f}$$

$$= -\frac{16a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f} - \frac{2a\cos(e+fx)(a+a\sin(e+}{5f}$$

$$= -\frac{64a^3\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} - \frac{16a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f} + \frac{17}{}$$

$$= -\frac{9a^3\cos(e+fx)}{40f\sqrt{a+a\sin(e+fx)}} - \frac{16a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f} + \frac{17}{}$$

$$= -\frac{9a^3\cos(e+fx)}{40f\sqrt{a+a\sin(e+fx)}} - \frac{16a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f} + \frac{17}{}$$

$$= \frac{55a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{8f} - \frac{9a^3\cos(e+fx)}{40f\sqrt{a+a\sin(e+fx)}} - \frac{16a^2\cos(e}{}$$

Mathematica [A] time = 1.74, size = 360, normalized size = 1.59

$$-\frac{a^2\csc^{10}\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sin(e+fx)+1)}\left(-108\sin\left(\frac{1}{2}(e+fx)\right)+706\sin\left(\frac{3}{2}(e+fx)\right)+450\sin\left(\frac{5}{2}(e+fx)\right)-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4*(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] -1/120*(a^2*Csc[(e + f*x)/2]^10*Sqrt[a*(1 + Sin[e + f*x])]*(108*Cos[(e + f*
x)/2] + 706*Cos[(3*(e + f*x))/2] - 450*Cos[(5*(e + f*x))/2] - 156*Cos[(7*(e
 + f*x))/2] + 100*Cos[(9*(e + f*x))/2] + 12*Cos[(11*(e + f*x))/2] - 108*Sin
[(e + f*x)/2] - 2475*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sin[e + f
*x] + 2475*Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[e + f*x] + 706*
Sin[(3*(e + f*x))/2] + 450*Sin[(5*(e + f*x))/2] + 825*Log[1 + Cos[(e + f*x)
/2] - Sin[(e + f*x)/2]]*Sin[3*(e + f*x)] - 825*Log[1 - Cos[(e + f*x)/2] + S
in[(e + f*x)/2]]*Sin[3*(e + f*x)] - 156*Sin[(7*(e + f*x))/2] - 100*Sin[(9*(
e + f*x))/2] + 12*Sin[(11*(e + f*x))/2]))/(f*(1 + Cot[(e + f*x)/2])*(Csc[(e
 + f*x)/4]^2 - Sec[(e + f*x)/4]^2)^3)
```

fricas [B] time = 0.44, size = 485, normalized size = 2.14

$$825\left(a^2\cos\left(fx+e\right)^4-2\,a^2\cos\left(fx+e\right)^2+a^2-\left(a^2\cos\left(fx+e\right)^3+a^2\cos\left(fx+e\right)^2-a^2\cos\left(fx+e\right)-a^2\right)\sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/480*(825*(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2 + a^2 - (a^2*cos(f*x
+ e)^3 + a^2*cos(f*x + e)^2 - a^2*cos(f*x + e) - a^2)*sin(f*x + e))*sqrt(a)
*log((a*cos(f*x + e)^3 - 7*a*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 + (cos(f*x
+ e) + 3)*sin(f*x + e) - 2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(
a) - 9*a*cos(f*x + e) + (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x +
 e) - a)/(cos(f*x + e)^3 + cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x +
e) - cos(f*x + e) - 1)) - 4*(48*a^2*cos(f*x + e)^6 + 224*a^2*cos(f*x + e)^5
 - 128*a^2*cos(f*x + e)^4 - 583*a^2*cos(f*x + e)^3 + 147*a^2*cos(f*x + e)^2
 + 399*a^2*cos(f*x + e) - 27*a^2 + (48*a^2*cos(f*x + e)^5 - 176*a^2*cos(f*x
 + e)^4 - 304*a^2*cos(f*x + e)^3 + 279*a^2*cos(f*x + e)^2 + 426*a^2*cos(f*x
 + e) + 27*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(f*cos(f*x + e)^4 -
 2*f*cos(f*x + e)^2 - (f*cos(f*x + e)^3 + f*cos(f*x + e)^2 - f*cos(f*x + e)
 - f)*sin(f*x + e) + f)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.91, size = 222, normalized size = 0.98

$$-\frac{\left(1+\sin\left(fx+e\right)\right)\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\,\left(48\left(-a\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{5}{2}}\left(\sin^{3}\left(fx+e\right)\right)\sqrt{a}-320\left(-a\left(\sin\left(fx\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/120*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(48*(-a*(sin(f*x+e)-1))^(5/
2)*sin(f*x+e)^3*a^(1/2)-320*(-a*(sin(f*x+e)-1))^(3/2)*sin(f*x+e)^3*a^(3/2)+
480*a^(5/2)*(-a*(sin(f*x+e)-1))^(1/2)*sin(f*x+e)^3-825*arctanh((-a*(sin(f*x
+e)-1))^(1/2)/a^(1/2))*sin(f*x+e)^3*a^3+135*(-a*(sin(f*x+e)-1))^(5/2)*a^(1/
2)-440*(-a*(sin(f*x+e)-1))^(3/2)*a^(3/2)+345*(-a*(sin(f*x+e)-1))^(1/2)*a^(5
/2))/sin(f*x+e)^3/a^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\cot\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*cot(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cot\left(e+f\,x\right)^4\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4*(a + a*sin(e + f*x))^(5/2),x)
```

```
[Out] int(cot(e + f*x)^4*(a + a*sin(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.103 $\int \frac{\tan^4(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=150

$$\frac{\tan^3(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}+\frac{a\sin(e+fx)\tan(e+fx)}{24f(a\sin(e+fx)+a)^{3/2}}-\frac{(127\sin(e+fx)+53)\sec(e+fx)}{192f\sqrt{a\sin(e+fx)+a}}-\frac{67\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+}}\right.}{64\sqrt{2}\sqrt{a}f}$$

```
[Out] -67/128*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/f*2^
(1/2)/a^(1/2)-1/192*sec(f*x+e)*(53+127*sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)
+1/24*a*sin(f*x+e)*tan(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)+1/3*tan(f*x+e)^3/f/(
a+a*sin(f*x+e))^(1/2)
```

Rubi **[A]** time = 0.93, antiderivative size = 241, normalized size of antiderivative = 1.61, number of steps used = 17, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {2714, 2649, 206, 4401, 2687, 2681, 2650, 2877, 2855}

$$\frac{61a\cos(e+fx)}{64f(a\sin(e+fx)+a)^{3/2}}+\frac{7\sec^3(e+fx)\sqrt{a\sin(e+fx)+a}}{12af}-\frac{5\sec^3(e+fx)}{6f\sqrt{a\sin(e+fx)+a}}-\frac{61\sec(e+fx)}{48f\sqrt{a\sin(e+fx)+a}}+\frac{}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (61*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(64
*Sqrt[2]*Sqrt[a]*f) - (Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt
[a + a*Sin[e + f*x]])])/(Sqrt[a]*f) + (61*a*Cos[e + f*x])/(64*f*(a + a*Sin[
e + f*x])^(3/2)) + (7*a*Sec[e + f*x])/(24*f*(a + a*Sin[e + f*x])^(3/2)) - (
61*Sec[e + f*x])/(48*f*Sqrt[a + a*Sin[e + f*x]]) - (5*Sec[e + f*x]^3)/(6*f*
Sqrt[a + a*Sin[e + f*x]]) + (7*Sec[e + f*x]^3*Sqrt[a + a*Sin[e + f*x]])/(12
*a*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2714

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] - Int[((a + b*Sin[e + f*x])^m*(
1 - 2*Sin[e + f*x]^2))/Cos[e + f*x]^4, x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2877

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^
(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1)), x] - Dist[1/(a^2*(2*
m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(
2*m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a
^2 - b^2, 0] && LeQ[m, -2^(-1)] && NeQ[2*m + p + 1, 0]
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx - \int \frac{\sec^4(e+fx)\left(1-2\sin^2(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}}\,dx$$

$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{f} - \int \left(\frac{\sec^4(e+fx)}{\sqrt{a(1+\sin(e+fx))}} - \frac{2\sec^2(e+fx)}{\sqrt{a(1+\sin}}\right.$$

$$= -\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,f} + 2\int \frac{\sec^2(e+fx)\tan^2(e+fx)}{\sqrt{a(1+\sin(e+fx))}}\,dx - \int \frac{\sec^4(}{\sqrt{a(1+\sin}}$$

$$= -\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,f} - \frac{5\sec^3(e+fx)}{6f\sqrt{a+a\sin(e+fx)}} + \frac{\int \sec^4(e+fx)\sqrt{a+a\,\mathrm{s}}}{}$$

$$= -\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,f} + \frac{7a\sec(e+fx)}{24f(a+a\sin(e+fx))^{3/2}} - \frac{5\sec^3(e+fx)}{6f\sqrt{a+a\sin(e+fx}}$$

$$= -\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,f} + \frac{7a\sec(e+fx)}{24f(a+a\sin(e+fx))^{3/2}} - \frac{61\sec(e+fx)}{48f\sqrt{a+a\sin(e+f}}$$

$$= -\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,f} + \frac{61a\cos(e+fx)}{64f(a+a\sin(e+fx))^{3/2}} + \frac{7a\sec(e+fx)}{24f(a+a\sin(e+f}$$

$$= -\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,f} + \frac{61a\cos(e+fx)}{64f(a+a\sin(e+fx))^{3/2}} + \frac{7a\sec(e+fx)}{24f(a+a\sin(e+f}$$

$$= \frac{61\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}}\right)}{64\sqrt{2}\,\sqrt{a}\,f} - \frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,f} + \frac{61a\cos(e+f}{64f(a+a\sin(e+}$$

Mathematica [C] time = 0.69, size = 118, normalized size = 0.79

$$\frac{-\sec^3(e+fx)(-41\sin(e+fx)+183\sin(3(e+fx))+122\cos(2(e+fx))+90)+(804+804i)(-1)^{3/4}\left(\sin\left(\frac{1}{2}(e\right.\right.}{768f\sqrt{a(\sin(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] ((804 + 804*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x
)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - Sec[e + f*x]^3*(90 + 122*Cos
[2*(e + f*x)] - 41*Sin[e + f*x] + 183*Sin[3*(e + f*x)]))/(768*f*Sqrt[a*(1 +
 Sin[e + f*x])])
```

fricas [A] time = 0.44, size = 229, normalized size = 1.53

$$\frac{201\sqrt{2}\left(\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)+\cos\left(fx+e\right)^{3}\right)\sqrt{a}\log\left(-\frac{a\cos\left(fx+e\right)^{2}-2\sqrt{2}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{a}\left(\cos\left(fx+e\right)-\sin\left(fx+e\right)+1\right.}{\cos\left(fx+e\right)^{2}-\left(\cos\left(fx+e\right)+2\right)\sin\left(fx\right.}\right.}{768\left(af\cos\left(fx+e\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/768*(201*sqrt(2)*(cos(f*x + e)^3*sin(f*x + e) + cos(f*x + e)^3)*sqrt(a)*l
og(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x
 + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f
*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x
+ e) - 2)) - 4*(61*cos(f*x + e)^2 + (183*cos(f*x + e)^2 - 56)*sin(f*x + e)
- 8)*sqrt(a*sin(f*x + e) + a))/(a*f*cos(f*x + e)^3*sin(f*x + e) + a*f*cos(f
*x + e)^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*2*(-1/48*(9*(-sqrt(
a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^5+39*sqrt(a)*(-sqrt
(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^4-26*a*(-sqrt(a)*t
an((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3+69*a^2*(-sqrt(a)*tan(
(f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))-78*sqrt(a)*a*(-sqrt(a)*tan
((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2-13*sqrt(a)*a^2)/(-(-sqr
t(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2-2*sqrt(a)*(-sqr
t(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))+a)^3/sign(tan((f*
x+exp(1))/2)+1)+1/128*(43*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp
(1))/2)^2+a))^7-237*sqrt(a)*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+e
xp(1))/2)^2+a))^6+161*a*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1
))/2)^2+a))^5+221*sqrt(a)*a*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+e
```

```
xp(1))/2)^2+a))^4+25*a^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(
1))/2)^2+a))^3-93*a^3*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))
/2)^2+a))-103*sqrt(a)*a^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp
(1))/2)^2+a))^2-17*sqrt(a)*a^3)/(-(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan(
(f*x+exp(1))/2)^2+a))^2+2*sqrt(a)*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan(
(f*x+exp(1))/2)^2+a))+a)^4/sign(tan((f*x+exp(1))/2)+1)+67/128*atan((-sqrt(a
)*tan((f*x+exp(1))/2)-sqrt(a)+sqrt(a*tan((f*x+exp(1))/2)^2+a))/sqrt(2)/sqrt
(-a))/sqrt(2)/sqrt(-a)/sign(tan((f*x+exp(1))/2)+1))
```

maple [A] time = 1.01, size = 231, normalized size = 1.54

$$\frac{366a^{\frac{7}{2}}\sin\left(fx+e\right)\left(\cos^{2}\left(fx+e\right)\right)+\left(402\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\ \sqrt{2}}{2\sqrt{a}}\right)\left(a-a\sin\left(fx+e\right)\right)^{\frac{3}{2}}a^{2}-112a^{\frac{7}{2}}\right)\sin}{384a^{\frac{7}{2}}\left(\sin\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/384*(366*a^(7/2)*sin(f*x+e)*cos(f*x+e)^2+(402*2^(1/2)*arctanh(1/2*(a-a*si
n(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*(a-a*sin(f*x+e))^(3/2)*a^2-112*a^(7/2))*si
n(f*x+e)+(-201*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*
(a-a*sin(f*x+e))^(3/2)*a^2+122*a^(7/2))*cos(f*x+e)^2+402*2^(1/2)*arctanh(1/
2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*(a-a*sin(f*x+e))^(3/2)*a^2-16*a^(
7/2))/a^(7/2)/(sin(f*x+e)-1)/(1+sin(f*x+e))/cos(f*x+e)/(a+a*sin(f*x+e))^(1/
2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(fx+e\right)^{4}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^4/sqrt(a*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan\left(e+f\,x\right)^{4}}{\sqrt{a+a\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4/(a + a*sin(e + f*x))^(1/2),x)
```

```
[Out] int(tan(e + f*x)^4/(a + a*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4\left(e+fx\right)}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**4/sqrt(a*(sin(e + f*x) + 1)), x)
```

3.104 $\int \frac{\tan^2(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=107

$$\frac{3\sec(e+fx)\sqrt{a\sin(e+fx)+a}}{4af}-\frac{\sec(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}+\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{4\sqrt{2}\sqrt{a}f}$$

```
[Out] 5/8*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/f*2^(1/2
)/a^(1/2)-1/2*sec(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)+3/4*sec(f*x+e)*(a+a*sin(f
*x+e))^(1/2)/a/f
```

Rubi [A] time = 0.20, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2712, 2855, 2649, 206}

$$\frac{3\sec(e+fx)\sqrt{a\sin(e+fx)+a}}{4af}-\frac{\sec(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}+\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{4\sqrt{2}\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (5*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(4*S
qrt[2]*Sqrt[a]*f) - Sec[e + f*x]/(2*f*Sqrt[a + a*Sin[e + f*x]]) + (3*Sec[e
+ f*x]*Sqrt[a + a*Sin[e + f*x]])/(4*a*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2712

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[(b*(a + b*Sin[e + f*x])^m)/(a*f*(2*m - 1)*Cos[e + f*x]),
```

```
x] - Dist[1/(a^2*(2*m - 1)), Int[((a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*
m - 1)*Sin[e + f*x]))/Cos[e + f*x]^2, x], x] /; FreeQ[{a, b, e, f}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[m] && LtQ[m, 0]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^2(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx &= -\frac{\sec(e+fx)}{2f\sqrt{a+a\sin(e+fx)}}+\frac{\int \sec^2(e+fx)\sqrt{a+a\sin(e+fx)}\left(-\frac{a}{2}+2a\sin(e+fx)\right)\,d\\ &= -\frac{\sec(e+fx)}{2f\sqrt{a+a\sin(e+fx)}}+\frac{3\sec(e+fx)\sqrt{a+a\sin(e+fx)}}{4af}-\frac{5}{8}\int\frac{1}{\sqrt{a+a\sin(e+f}}\\ &= -\frac{\sec(e+fx)}{2f\sqrt{a+a\sin(e+fx)}}+\frac{3\sec(e+fx)\sqrt{a+a\sin(e+fx)}}{4af}+\frac{5\,\mathrm{Subst}\left(\int\frac{1}{2a-x^2}\,dx,x}{4f}\\ &= \frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{4\sqrt{2}\sqrt{a}f}-\frac{\sec(e+fx)}{2f\sqrt{a+a\sin(e+fx)}}+\frac{3\sec(e+fx)\sqrt{a+a\sin(e+}}{4af}\end{aligned}$$

Mathematica [C] time = 0.26, size = 118, normalized size = 1.10

$$-\frac{\sec(e+fx)\left(-3\sin(e+fx)+(5+5i)(-1)^{3/4}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+}{4f\sqrt{a(\sin(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -1/4*(Sec[e + f*x]*(-1 + (5 + 5*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4
)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])^2 - 3*Sin[e + f*x]))/(f*Sqrt[a*(1 + Sin[e + f*x
])])
```

fricas [B] time = 0.43, size = 200, normalized size = 1.87

$$\frac{5\sqrt{2}\left(\cos\left(fx+e\right)\sin\left(fx+e\right)+\cos\left(fx+e\right)\right)\sqrt{a}\log\left(-\frac{a\cos\left(fx+e\right)^{2}+2\sqrt{2}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{a}\left(\cos\left(fx+e\right)-\sin\left(fx+e\right)+1\right)+3}{\cos\left(fx+e\right)^{2}-\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)}\right.}{16\left(af\cos\left(fx+e\right)\sin\left(fx+e\right)+af\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(5*sqrt(2)*(cos(f*x + e)*sin(f*x + e) + cos(f*x + e))*sqrt(a)*log(-(a*
cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) -
 sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e)
 + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) -
2)) + 4*sqrt(a*sin(f*x + e) + a)*(3*sin(f*x + e) + 1))/(a*f*cos(f*x + e)*si
n(f*x + e) + a*f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*2*(-1/4*(sqrt(a)*ta
n((f*x+exp(1))/2)+sqrt(a)-sqrt(a*tan((f*x+exp(1))/2)^2+a))/(-(-sqrt(a)*tan(
(f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2-2*sqrt(a)*(-sqrt(a)*tan(
(f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))+a)/sign(tan((f*x+exp(1))/2
)+1)+1/8*(-3*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))
^3+a*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))+sqrt(a)
*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2+sqrt(a)*a
)/(-(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2+2*sqrt
(a)*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))+a)^2/sig
n(tan((f*x+exp(1))/2)+1)-5/8*atan((-sqrt(a)*tan((f*x+exp(1))/2)-sqrt(a)+sqr
t(a*tan((f*x+exp(1))/2)^2+a))/sqrt(2)/sqrt(-a))/sqrt(2)/sqrt(-a)/sign(tan((
f*x+exp(1))/2)+1))
```

maple [A] time = 0.58, size = 130, normalized size = 1.21

$$\frac{\sin\left(fx+e\right)\left(5\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\ \sqrt{2}}{2\sqrt{a}}\right)\sqrt{a-a\sin\left(fx+e\right)}\ a+6a^{\frac{3}{2}}\right)+5\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\ \sqrt{2}}{2\sqrt{a}}\right.}{8a^{\frac{3}{2}}\cos\left(fx+e\right)\sqrt{a+a\sin\left(fx+e\right)}\ f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/8*(sin(f*x+e)*(5*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/
2))*(a-a*sin(f*x+e))^(1/2)*a+6*a^(3/2))+5*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+
e))^(1/2)*2^(1/2)/a^(1/2))*(a-a*sin(f*x+e))^(1/2)*a+2*a^(3/2))/a^(3/2)/cos(
f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(fx+e\right)^{2}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^2/sqrt(a*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan\left(e+f\,x\right)^{2}}{\sqrt{a+a\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(a + a*sin(e + f*x))^(1/2),x)
```

```
[Out] int(tan(e + f*x)^2/(a + a*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{2}\left(e+fx\right)}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**2/sqrt(a*(sin(e + f*x) + 1)), x)
```

3.105 $\int \frac{\cot^2(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=62

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}\,f}-\frac{\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] arctanh(cos(f*x+e)*a^(1/2)/(a+a*sin(f*x+e))^(1/2))/f/a^(1/2)-cot(f*x+e)/f/(
a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2716, 21, 2773, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}\,f}-\frac{\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]]/(Sqrt[a]*f) - Cot[
e + f*x]/(f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2716

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)/tan[(e_.) + (f_.)*(x_)]^2,
 x_Symbol] :> -Simp[(a + b*Sin[e + f*x])^m/(f*Tan[e + f*x]), x] + Dist[1/a,
 Int[((a + b*Sin[e + f*x])^m*(b*m - a*(m + 1)*Sin[e + f*x]))/Sin[e + f*x],
x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/
```

```
2] && !LtQ[m, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^2(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx &= -\frac{\cot(e+fx)}{f\sqrt{a+a\sin(e+fx)}}+\frac{\int \frac{\csc(e+fx)\left(-\frac{a}{2}-\frac{1}{2}a\sin(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}}\,dx}{a}\\
&= -\frac{\cot(e+fx)}{f\sqrt{a+a\sin(e+fx)}}-\frac{\int \csc(e+fx)\sqrt{a+a\sin(e+fx)}\,dx}{2a}\\
&= -\frac{\cot(e+fx)}{f\sqrt{a+a\sin(e+fx)}}+\frac{\text{Subst}\left(\int \frac{1}{a-x^2}\,dx,x,\frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{f}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,f}-\frac{\cot(e+fx)}{f\sqrt{a+a\sin(e+fx)}}
\end{aligned}
$$

Mathematica [B] time = 0.32, size = 138, normalized size = 2.23

$$
\frac{\left(\tan\left(\frac{1}{2}(e+fx)\right)+1\right)\csc\left(\frac{1}{4}(e+fx)\right)\sec\left(\frac{1}{4}(e+fx)\right)\left(2\sin\left(\frac{1}{2}(e+fx)\right)-2\cos\left(\frac{1}{2}(e+fx)\right)+\sin(e+fx)\left(\log\left(-\right.\right.\right.}{8f\sqrt{a(\sin(e+fx)+1)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (Csc[(e + f*x)/4]*Sec[(e + f*x)/4]*(-2*Cos[(e + f*x)/2] + 2*Sin[(e + f*x)/2
] + (Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[1 - Cos[(e + f*x)/2
] + Sin[(e + f*x)/2]])*Sin[e + f*x])*(1 + Tan[(e + f*x)/2]))/(8*f*Sqrt[a*(1
 + Sin[e + f*x])])
```

fricas [B] time = 0.43, size = 263, normalized size = 4.24

$$\frac{\left(\cos\left(fx+e\right)^{2}-\left(\cos\left(fx+e\right)+1\right)\sin\left(fx+e\right)-1\right)\sqrt{a}\,\log\left(\frac{a\cos\left(fx+e\right)^{3}-7\,a\cos\left(fx+e\right)^{2}+4\left(\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)+3\right)\sin\cdots}{\cos\left(fx+e\right)^{3}+\cdots}\right.}{4\left(af\cos\left(fx+e\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*((cos(f*x + e)^2 - (cos(f*x + e) + 1)*sin(f*x + e) - 1)*sqrt(a)*log((a*
cos(f*x + e)^3 - 7*a*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3
)*sin(f*x + e) - 2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a
*cos(f*x + e) + (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)
/(cos(f*x + e)^3 + cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos
(f*x + e) - 1)) + 4*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) +
 1))/(a*f*cos(f*x + e)^2 - a*f - (a*f*cos(f*x + e) + a*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*(1/4*sqrt(a*tan((f*x+exp
(1))/2)^2+a)/sign(tan((f*x+exp(1))/2)+1)/a+2*(1/4*sqrt(a)/((-sqrt(a)*tan((f
*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2-a)/sign(tan((f*x+exp(1))/2
)+1)-1/4*atan((-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a)
)/sqrt(-a))/sqrt(-a)/sign(tan((f*x+exp(1))/2)+1)+1/8*sqrt(a)*ln(abs(-sqrt(a
)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a)))/a/sign(tan((f*x+exp
(1))/2)+1))+(-sqrt(-a)*sqrt(2)*ln(sqrt(2)*sqrt(a)+sqrt(a))-sqrt(-a)*sqrt(2)
-sqrt(-a)*ln(sqrt(2)*sqrt(a)+sqrt(a))-3*sqrt(-a)+2*sqrt(2)*sqrt(a)*atan((sq
rt(2)*sqrt(a)+sqrt(a))/sqrt(-a))+2*sqrt(a)*atan((sqrt(2)*sqrt(a)+sqrt(a))/s
qrt(-a)))/(4*sqrt(-a)*sqrt(2)*sqrt(a)+4*sqrt(-a)*sqrt(a))*sign(tan((f*x+exp
(1))/2)+1))
```

maple [A] time = 0.69, size = 103, normalized size = 1.66

$$-\frac{\left(1+\sin \left(f x+e\right)\right) \sqrt{-a \left(\sin \left(f x+e\right)-1\right)}\, \left(-\operatorname{arctanh}\left(\frac{\sqrt{a-a \sin \left(f x+e\right)}}{\sqrt{a}}\right) a \sin \left(f x+e\right)+\sqrt{a-a \sin \left(f x+e\right)}\, \sqrt{a}\right)}{a^{\frac{3}{2}} \sin \left(f x+e\right) \cos \left(f x+e\right) \sqrt{a+a \sin \left(f x+e\right)}\, f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(-arctanh((a-a*sin(f*x+e))^(1/2)/
a^(1/2))*a*sin(f*x+e)+(a-a*sin(f*x+e))^(1/2)*a^(1/2))/a^(3/2)/sin(f*x+e)/co
s(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot \left(f x+e\right)^{2}}{\sqrt{a \sin \left(f x+e\right)+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(f*x + e)^2/sqrt(a*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cot \left(e+f\,x\right)^{2}}{\sqrt{a+a\,\sin \left(e+f\,x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2/(a + a*sin(e + f*x))^(1/2),x)
```

```
[Out] int(cot(e + f*x)^2/(a + a*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{2}\left(e+f x\right)}{\sqrt{a \left(\sin \left(e+f x\right)+1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+a*sin(f*x+e))**(1/2),x)
[Out] Integral(cot(e + f*x)**2/sqrt(a*(sin(e + f*x) + 1)), x)
```

3.106 $\int \frac{\cot^4(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=135

$$\frac{9\cot(e+fx)}{8f\sqrt{a\sin(e+fx)+a}}-\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{8\sqrt{a}f}-\frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}+\frac{\cot(e+fx)\csc(e+fx)}{12f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -7/8*arctanh(cos(f*x+e)*a^(1/2)/(a+a*sin(f*x+e))^(1/2))/f/a^(1/2)+9/8*cot(f
*x+e)/f/(a+a*sin(f*x+e))^(1/2)+1/12*cot(f*x+e)*csc(f*x+e)/f/(a+a*sin(f*x+e)
)^(1/2)-1/3*cot(f*x+e)*csc(f*x+e)^2/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.62, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2718, 2649, 206, 3044, 2984, 2985, 2773}

$$\frac{9\cot(e+fx)}{8f\sqrt{a\sin(e+fx)+a}}-\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{8\sqrt{a}f}-\frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}+\frac{\cot(e+fx)\csc(e+fx)}{12f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (-7*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/(8*Sqrt[a]*f)
 + (9*Cot[e + f*x])/(8*f*Sqrt[a + a*Sin[e + f*x]]) + (Cot[e + f*x]*Csc[e +
f*x])/(12*f*Sqrt[a + a*Sin[e + f*x]]) - (Cot[e + f*x]*Csc[e + f*x]^2)/(3*f*
Sqrt[a + a*Sin[e + f*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2718

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] + Int[((a + b*Sin[e + f*x])^m*(
```

```
1 - 2*Sin[e + f*x]^2))/Sin[e + f*x]^4, x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&  !LtQ[m, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx + \int \frac{\csc^4(e+fx)\left(1-2\sin^2(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}}\,dx$$

$$= -\frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} + \frac{\int \frac{\csc^3(e+fx)\left(-\frac{a}{2}-\frac{7}{2}a\sin(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}}\,dx}{3a} - \frac{2\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \cdots\right.}{f}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} + \frac{\cot(e+fx)\csc(e+fx)}{12f\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} + \frac{9\cot(e+fx)}{8f\sqrt{a+a\sin(e+fx)}} + \frac{\cot(e+fx)\csc(e+fx)}{12f\sqrt{a+a\sin(e+fx)}} -$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} + \frac{9\cot(e+fx)}{8f\sqrt{a+a\sin(e+fx)}} + \frac{\cot(e+fx)\csc(e+fx)}{12f\sqrt{a+a\sin(e+fx)}} -$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} + \frac{9\cot(e+fx)}{8f\sqrt{a+a\sin(e+fx)}} + \frac{\cot(e+fx)\csc(e+fx)}{12f\sqrt{a+a\sin(e+fx)}} -$$

$$= -\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{8\sqrt{a}f} + \frac{9\cot(e+fx)}{8f\sqrt{a+a\sin(e+fx)}} + \frac{\cot(e+fx)\csc(e+fx)}{12f\sqrt{a+a\sin(e+fx)}} - \frac{\cot\cdots}{3f\cdots}$$

Mathematica [B] time = 0.61, size = 292, normalized size = 2.16

$$\frac{\csc^9\left(\frac{1}{2}(e+fx)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(-36\sin\left(\frac{1}{2}(e+fx)\right)-46\sin\left(\frac{3}{2}(e+fx)\right)+54\sin\left(\frac{5}{2}(e+f\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (Csc[(e + f*x)/2]^9*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(36*Cos[(e + f*x)
/2] - 46*Cos[(3*(e + f*x))/2] - 54*Cos[(5*(e + f*x))/2] - 36*Sin[(e + f*x)/
2] - 63*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sin[e + f*x] + 63*Log[
1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[e + f*x] - 46*Sin[(3*(e + f*x)
)/2] + 54*Sin[(5*(e + f*x))/2] + 21*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x
```

```
)/2]]*Sin[3*(e + f*x)] - 21*Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Si
n[3*(e + f*x)]))/(24*f*(Csc[(e + f*x)/4]^2 - Sec[(e + f*x)/4]^2)^3*Sqrt[a*(
1 + Sin[e + f*x])])
```

fricas [B] time = 0.44, size = 369, normalized size = 2.73

$$21\left(\cos\left(fx+e\right)^4 - 2\cos\left(fx+e\right)^2 - \left(\cos\left(fx+e\right)^3 + \cos\left(fx+e\right)^2 - \cos\left(fx+e\right) - 1\right)\sin\left(fx+e\right) + 1\right)\sqrt{a}\ \mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/96*(21*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 - (cos(f*x + e)^3 + cos(f*x + e
)^2 - cos(f*x + e) - 1)*sin(f*x + e) + 1)*sqrt(a)*log((a*cos(f*x + e)^3 - 7
*a*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3)*sin(f*x + e) - 2
*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a*cos(f*x + e) + (a
*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)/(cos(f*x + e)^3 +
 cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos(f*x + e) - 1)) -
4*(27*cos(f*x + e)^3 + 25*cos(f*x + e)^2 - (27*cos(f*x + e)^2 + 2*cos(f*x +
 e) - 17)*sin(f*x + e) - 19*cos(f*x + e) - 17)*sqrt(a*sin(f*x + e) + a))/(a
*f*cos(f*x + e)^4 - 2*a*f*cos(f*x + e)^2 + a*f - (a*f*cos(f*x + e)^3 + a*f*
cos(f*x + e)^2 - a*f*cos(f*x + e) - a*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*(2*sqrt(a*t
an((f*x+exp(1))/2)^2+a)*(tan((f*x+exp(1))/2)*(1/96*tan((f*x+exp(1))/2)/a/si
gn(tan((f*x+exp(1))/2)+1)-1/64/a/sign(tan((f*x+exp(1))/2)+1))-11/96/a/sign(
tan((f*x+exp(1))/2)+1))+2*(1/96*(3*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan
((f*x+exp(1))/2)^2+a))^5-18*sqrt(a)*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*ta
n((f*x+exp(1))/2)^2+a))^4-3*a^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f
```

```
*x+exp(1))/2)^2+a))+48*sqrt(a)*a*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((
f*x+exp(1))/2)^2+a))^2-22*sqrt(a)*a^2)/((-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(
a*tan((f*x+exp(1))/2)^2+a))^2-a)^3/sign(tan((f*x+exp(1))/2)+1)+7/32*atan((-
sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))/sqrt(-a))/sqrt
(-a)/sign(tan((f*x+exp(1))/2)+1)-7/64*ln(abs(-sqrt(a)*tan((f*x+exp(1))/2)+s
qrt(a*tan((f*x+exp(1))/2)^2+a)))/sqrt(a)/sign(tan((f*x+exp(1))/2)+1))+(105*
sqrt(-a)*sqrt(2)*ln(sqrt(2)*sqrt(a)+sqrt(a))+128*sqrt(-a)*sqrt(2)+147*sqrt(
-a)*ln(sqrt(2)*sqrt(a)+sqrt(a))+186*sqrt(-a)-210*sqrt(2)*sqrt(a)*atan((sqrt
(2)*sqrt(a)+sqrt(a))/sqrt(-a))-294*sqrt(a)*atan((sqrt(2)*sqrt(a)+sqrt(a))/s
qrt(-a)))/(480*sqrt(-a)*sqrt(2)*sqrt(a)+672*sqrt(-a)*sqrt(a))*sign(tan((f*x
+exp(1))/2)+1))
```

maple [A] time = 0.82, size = 144, normalized size = 1.07

$$\frac{\left(1+\sin\left(fx+e\right)\right)\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\left(-21\,\mathrm{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}}{\sqrt{a}}\right)\left(\sin^{3}\left(fx+e\right)\right)a^{3}+27\left(-a\left(\sin\left(fx+\cdots\right.\right.\right.\right.}{24\sin\left(fx+e\right)^{3}a^{\frac{7}{2}}\cos\left(fx+e\right)\sqrt{a+a\sin\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/24*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(-21*arctanh((-a*(sin(f*x+e)-
1))^(1/2)/a^(1/2))*sin(f*x+e)^3*a^3+27*(-a*(sin(f*x+e)-1))^(5/2)*a^(1/2)-56
*(-a*(sin(f*x+e)-1))^(3/2)*a^(3/2)+21*(-a*(sin(f*x+e)-1))^(1/2)*a^(5/2))/si
n(f*x+e)^3/a^(7/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(fx+e\right)^{4}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(f*x + e)^4/sqrt(a*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot\left(e+f\,x\right)^{4}}{\sqrt{a+a\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4/(a + a*sin(e + f*x))^(1/2),x)
[Out] int(cot(e + f*x)^4/(a + a*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4\left(e+fx\right)}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+a*sin(f*x+e))**(1/2),x)
[Out] Integral(cot(e + f*x)**4/sqrt(a*(sin(e + f*x) + 1)), x)
```

3.107 $\int \frac{\tan^4(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=177

$$\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{256\sqrt{2}a^{3/2}f}+\frac{\tan^3(e+fx)}{3f(a\sin(e+fx)+a)^{3/2}}+\frac{a\sin(e+fx)\tan(e+fx)}{12f(a\sin(e+fx)+a)^{5/2}}+\frac{7\cos(e+fx)}{256f(a\sin(e+fx)+a)^{3/2}}-\frac{(8}{}$$

```
[Out] 7/256*cos(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)-1/192*sec(f*x+e)*(65+87*sin(f*x+e
))/f/(a+a*sin(f*x+e))^(3/2)+7/512*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a
+a*sin(f*x+e))^(1/2))/a^(3/2)/f*2^(1/2)+1/12*a*sin(f*x+e)*tan(f*x+e)/f/(a+a
*sin(f*x+e))^(5/2)+1/3*tan(f*x+e)^3/f/(a+a*sin(f*x+e))^(3/2)
```

Rubi [A] time = 1.20, antiderivative size = 195, normalized size of antiderivative = 1.10, number of steps used = 20, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {2714, 2650, 2649, 206, 4401, 2681, 2687, 2877, 2855}

$$\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{256\sqrt{2}a^{3/2}f}+\frac{7\cos(e+fx)}{256f(a\sin(e+fx)+a)^{3/2}}+\frac{\sec^3(e+fx)}{4af\sqrt{a\sin(e+fx)+a}}-\frac{\sec^3(e+fx)}{6f(a\sin(e+fx)+a)^{3/2}}-\frac{}{64a}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (7*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(256
*Sqrt[2]*a^(3/2)*f) + (7*Cos[e + f*x])/(256*f*(a + a*Sin[e + f*x])^(3/2)) +
 (9*Sec[e + f*x])/(32*f*(a + a*Sin[e + f*x])^(3/2)) - Sec[e + f*x]^3/(6*f*(
a + a*Sin[e + f*x])^(3/2)) - (45*Sec[e + f*x])/(64*a*f*Sqrt[a + a*Sin[e + f
*x]]) + Sec[e + f*x]^3/(4*a*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2714

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] - Int[((a + b*Sin[e + f*x])^m*(
1 - 2*Sin[e + f*x]^2))/Cos[e + f*x]^4, x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2877

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^
(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1)), x] - Dist[1/(a^2*(2*
m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(
2*m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a
```

```
^2 - b^2, 0] && LeQ[m, -2^(-1)] && NeQ[2*m + p + 1, 0]
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{1}{(a+a\sin(e+fx))^{3/2}}\,dx - \int \frac{\sec^4(e+fx)\left(1-2\sin^2(e+fx)\right)}{(a+a\sin(e+fx))^{3/2}}\,dx$$

$$= -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} + \frac{\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx}{4a} - \int \left(\frac{\sec^4(e+fx)}{(a(1+\sin(e+fx)))^{3/2}} - \frac{2}{}\right.$$

$$= -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} + 2\int \frac{\sec^2(e+fx)\tan^2(e+fx)}{(a(1+\sin(e+fx)))^{3/2}}\,dx - \frac{\text{Subst}\left(\int \frac{1}{2a-x^2}\right.}{2}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} - \frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} - \frac{\sec^3(e+fx)}{6f(a+a\sin(e+fx))^{3/2}}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} - \frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} - \frac{\sec^3(e+fx)}{6f(a+a\sin(e+fx))^{3/2}}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} - \frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} + \frac{9\sec(e+fx)}{32f(a+a\sin(e+fx))^{3/}}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} - \frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} + \frac{9\sec(e+fx)}{32f(a+a\sin(e+fx))^{3/}}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} + \frac{7\cos(e+fx)}{256f(a+a\sin(e+fx))^{3/2}} + \frac{9\sec(e+fx)}{32f(a+a\sin(e+fx}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} + \frac{7\cos(e+fx)}{256f(a+a\sin(e+fx))^{3/2}} + \frac{9\sec(e+fx)}{32f(a+a\sin(e+fx}$$

$$= \frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{256\sqrt{2}a^{3/2}f} + \frac{7\cos(e+fx)}{256f(a+a\sin(e+fx))^{3/2}} + \frac{9\sec(e+fx)}{32f(a+a\sin(e+fx}$$

Mathematica [C] time = 0.37, size = 334, normalized size = 1.89

$$-\frac{192\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3}{\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)} + \frac{32\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3}{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3} - 171\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2 + 342\sin$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (124 + (64*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - 32/(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - (248*Sin[(e + f*x)/2])/(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2]) + 342*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2]) - 171*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - (21 + 21*I)*(-
1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])^3 + (32*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3
)/(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 - (192*(Cos[(e + f*x)/2] + Sin[(e
 + f*x)/2])^3)/(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/(768*f*(a*(1 + Sin[e
+ f*x]))^(3/2))
```

fricas [A] time = 0.48, size = 270, normalized size = 1.53

$$\frac{21\sqrt{2}\left(\cos\left(fx+e\right)^{5}-2\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)-2\cos\left(fx+e\right)^{3}\right)\sqrt{a}\log\left(-\frac{a\cos\left(fx+e\right)^{2}+2\sqrt{2}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{a}\left(\cdots\right.}{\cos\left(fx+e\cdots\right.}\right.}{3072\left(a^{2}f\cos\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3072*(21*sqrt(2)*(cos(f*x + e)^5 - 2*cos(f*x + e)^3*sin(f*x + e) - 2*cos(
f*x + e)^3)*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e)
+ a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(
f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*si
n(f*x + e) - cos(f*x + e) - 2)) - 4*(21*cos(f*x + e)^4 - 324*cos(f*x + e)^2
 - 12*(45*cos(f*x + e)^2 - 16)*sin(f*x + e) + 64)*sqrt(a*sin(f*x + e) + a))
/(a^2*f*cos(f*x + e)^5 - 2*a^2*f*cos(f*x + e)^3*sin(f*x + e) - 2*a^2*f*cos(
f*x + e)^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*2*(-1/48*(3*(-sqrt(
a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^5+15*sqrt(a)*(-sqrt
(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^4-10*a*(-sqrt(a)*t
```

```
an((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3+27*a^2*(-sqrt(a)*tan(
(f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))-30*sqrt(a)*a*(-sqrt(a)*tan
((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2-5*sqrt(a)*a^2)/(-(-sqrt
(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2-2*sqrt(a)*(-sqrt
(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))+a)^3/a/sign(tan((f
*x+exp(1))/2)+1)+1/1536*(-117*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x
+exp(1))/2)^2+a))^11+1479*sqrt(a)*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan(
(f*x+exp(1))/2)^2+a))^10-6645*a*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f
*x+exp(1))/2)^2+a))^9+4875*sqrt(a)*a*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*t
an((f*x+exp(1))/2)^2+a))^8+7710*a^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*ta
n((f*x+exp(1))/2)^2+a))^7-3002*sqrt(a)*a^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sq
rt(a*tan((f*x+exp(1))/2)^2+a))^6-5562*a^3*(-sqrt(a)*tan((f*x+exp(1))/2)+sqr
t(a*tan((f*x+exp(1))/2)^2+a))^5-3690*sqrt(a)*a^3*(-sqrt(a)*tan((f*x+exp(1))
/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^4+1255*a^4*(-sqrt(a)*tan((f*x+exp(1))/
2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3+831*a^5*(-sqrt(a)*tan((f*x+exp(1))/2)
+sqrt(a*tan((f*x+exp(1))/2)^2+a))+2787*sqrt(a)*a^4*(-sqrt(a)*tan((f*x+exp(1
))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2+79*sqrt(a)*a^5)/(-(-sqrt(a)*tan((f
*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2+2*sqrt(a)*(-sqrt(a)*tan((f
*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))+a)^6/a/sign(tan((f*x+exp(1))
/2)+1)-7/512*atan((-sqrt(a)*tan((f*x+exp(1))/2)-sqrt(a)+sqrt(a*tan((f*x+exp
(1))/2)^2+a))/sqrt(2)/sqrt(-a))/sqrt(2)/sqrt(-a)/a/sign(tan((f*x+exp(1))/2)
+1))
```

maple [A] time = 0.80, size = 289, normalized size = 1.63

$$-\frac{\left(-1080a^{\frac{9}{2}}-21\left(a-a\sin\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\ \sqrt{2}}{2\sqrt{a}}\right)a^{3}\right)\sin\left(fx+e\right)\left(\cos^{2}\left(fx+e\right)\right)+\left(384a^{\frac{9}{2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/1536/a^(11/2)*((-1080*a^(9/2)-21*(a-a*sin(f*x+e))^(3/2)*2^(1/2)*arctanh(
1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3)*sin(f*x+e)*cos(f*x+e)^2+(3
84*a^(9/2)+84*(a-a*sin(f*x+e))^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(
1/2)*2^(1/2)/a^(1/2))*a^3)*sin(f*x+e)+42*a^(9/2)*cos(f*x+e)^4+(-648*a^(9/2)
-63*(a-a*sin(f*x+e))^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/
2)/a^(1/2))*a^3)*cos(f*x+e)^2+128*a^(9/2)+84*(a-a*sin(f*x+e))^(3/2)*2^(1/2)
*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3)/(sin(f*x+e)-1)/(1
+sin(f*x+e))^2/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan\left(e+f\,x\right)^4}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4/(a + a*sin(e + f*x))^(3/2),x)
[Out] int(tan(e + f*x)^4/(a + a*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{4}\left(e+fx\right)}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+a*sin(f*x+e))**(3/2),x)
[Out] Integral(tan(e + f*x)**4/(a*(sin(e + f*x) + 1))**(3/2), x)
```

3.108 $\int \frac{\tan^2(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=134

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{32\sqrt{2}\,a^{3/2}f}+\frac{\cos(e+fx)}{32f(a\sin(e+fx)+a)^{3/2}}+\frac{5\sec(e+fx)}{8af\sqrt{a\sin(e+fx)+a}}-\frac{\sec(e+fx)}{4f(a\sin(e+fx)+a)^{3/2}}$$

```
[Out] 1/32*cos(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)-1/4*sec(f*x+e)/f/(a+a*sin(f*x+e))^
(3/2)+1/64*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a
^(3/2)/f*2^(1/2)+5/8*sec(f*x+e)/a/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2712, 2855, 2650, 2649, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{32\sqrt{2}\,a^{3/2}f}+\frac{\cos(e+fx)}{32f(a\sin(e+fx)+a)^{3/2}}+\frac{5\sec(e+fx)}{8af\sqrt{a\sin(e+fx)+a}}-\frac{\sec(e+fx)}{4f(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])]/(32*Sqrt
[2]*a^(3/2)*f) + Cos[e + f*x]/(32*f*(a + a*Sin[e + f*x])^(3/2)) - Sec[e + f
*x]/(4*f*(a + a*Sin[e + f*x])^(3/2)) + (5*Sec[e + f*x])/(8*a*f*Sqrt[a + a*S
in[e + f*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
```

```
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2712

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[(b*(a + b*Sin[e + f*x])^m)/(a*f*(2*m - 1)*Cos[e + f*x]),
x] - Dist[1/(a^2*(2*m - 1)), Int[((a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*
m - 1)*Sin[e + f*x]))/Cos[e + f*x]^2, x], x] /; FreeQ[{a, b, e, f}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[m] && LtQ[m, 0]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{\sec(e+fx)}{4f(a+a\sin(e+fx))^{3/2}} + \frac{\int \frac{\sec^2(e+fx)\left(-\frac{3a}{2}+4a\sin(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}}\,dx}{4a^2}$$

$$= -\frac{\sec(e+fx)}{4f(a+a\sin(e+fx))^{3/2}} + \frac{5\sec(e+fx)}{8af\sqrt{a+a\sin(e+fx)}} - \frac{1}{16}\int \frac{1}{(a+a\sin(e+fx))^{3/2}}$$

$$= \frac{\cos(e+fx)}{32f(a+a\sin(e+fx))^{3/2}} - \frac{\sec(e+fx)}{4f(a+a\sin(e+fx))^{3/2}} + \frac{5\sec(e+fx)}{8af\sqrt{a+a\sin(e+fx)}} -$$

$$= \frac{\cos(e+fx)}{32f(a+a\sin(e+fx))^{3/2}} - \frac{\sec(e+fx)}{4f(a+a\sin(e+fx))^{3/2}} + \frac{5\sec(e+fx)}{8af\sqrt{a+a\sin(e+fx)}} +$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{32\sqrt{2}a^{3/2}f} + \frac{\cos(e+fx)}{32f(a+a\sin(e+fx))^{3/2}} - \frac{\sec(e+fx)}{4f(a+a\sin(e+fx))^{3/2}} +$$

Mathematica [C] time = 0.46, size = 128, normalized size = 0.96

$$-\frac{\sec(e+fx)\left(-40\sin(e+fx)-\cos(2(e+fx))+(2+2i)(-1)^{3/4}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e\right.\right.\right.}{64f(a(\sin(e+fx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -1/64*(Sec[e + f*x]*(-25 - Cos[2*(e + f*x)] + (2 + 2*I)*(-1)^(3/4)*ArcTanh[
(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e
+ f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - 40*Sin[e + f*x]))/(f*(
a*(1 + Sin[e + f*x]))^(3/2))
```

fricas [B] time = 0.47, size = 237, normalized size = 1.77

$$\frac{\sqrt{2}\left(\cos\left(fx+e\right)^{3}-2\cos\left(fx+e\right)\sin\left(fx+e\right)-2\cos\left(fx+e\right)\right)\sqrt{a}\log\left(-\frac{a\cos\left(fx+e\right)^{2}+2\sqrt{2}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{a}\left(\cos\left(\right.\right.}{\cos\left(fx+e\right)^{2}-\left(\right.}\right.}{128\left(a^{2}f\cos\left(fx+e\right)^{3}-2a^{2}f\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/128*(sqrt(2)*(cos(f*x + e)^3 - 2*cos(f*x + e)*sin(f*x + e) - 2*cos(f*x +
e))*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqr
t(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e)
 - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x +
e) - cos(f*x + e) - 2)) - 4*(cos(f*x + e)^2 + 20*sin(f*x + e) + 12)*sqrt(a*
sin(f*x + e) + a))/(a^2*f*cos(f*x + e)^3 - 2*a^2*f*cos(f*x + e)*sin(f*x + e
) - 2*a^2*f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*2*(-1/8*(sqrt(a)*ta
n((f*x+exp(1))/2)+sqrt(a)-sqrt(a*tan((f*x+exp(1))/2)^2+a))/a/(-(-sqrt(a)*ta
```

```
n((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2-2*sqrt(a)*(-sqrt(a)*ta
n((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))+a)/sign(tan((f*x+exp(1))
/2)+1)+1/64*(7*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a
))^7-81*sqrt(a)*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+
a))^6+53*a*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^5
+65*sqrt(a)*a*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a)
)^4+13*a^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3
-33*a^3*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))-19*s
qrt(a)*a^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2
-5*sqrt(a)*a^3)/(-(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^
2+a))^2+2*sqrt(a)*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^
2+a))+a)^4/a/sign(tan((f*x+exp(1))/2)+1)-1/64*atan((-sqrt(a)*tan((f*x+exp(1
))/2)-sqrt(a)+sqrt(a*tan((f*x+exp(1))/2)^2+a))/sqrt(2)/sqrt(-a))/sqrt(2)/sq
rt(-a)/a/sign(tan((f*x+exp(1))/2)+1))
```

maple [A] time = 0.70, size = 202, normalized size = 1.51

$$\frac{\sin\left(fx+e\right)\left(2\sqrt{a-a\sin\left(fx+e\right)}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\,\sqrt{2}}{2\sqrt{a}}\right)a^{2}+40a^{\frac{5}{2}}\right)+\left(-\sqrt{a-a\sin\left(fx+e\right)}\,\sqrt{2}\,\operatorname{arct}\right.}{64a^{\frac{7}{2}}\left(1+\sin\left(fx+e\right)\right)\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/64/a^(7/2)*(sin(f*x+e)*(2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(a-a
*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2+40*a^(5/2))+(-(a-a*sin(f*x+e))^(1/2
)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2+2*a^(5/2)
)*cos(f*x+e)^2+2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e)
)^(1/2)*2^(1/2)/a^(1/2))*a^2+24*a^(5/2))/(1+sin(f*x+e))/cos(f*x+e)/(a+a*sin
(f*x+e))^(1/2)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan\left(e+f\,x\right)^2}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(a + a*sin(e + f*x))^(3/2),x)
```

```
[Out] int(tan(e + f*x)^2/(a + a*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{2}\left(e+fx\right)}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**2/(a*(sin(e + f*x) + 1))**(3/2), x)
```

3.109 $\int \frac{\cot^2(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=113

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{a^{3/2}f}-\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{a^{3/2}f}-\frac{\cot(e+fx)}{af\sqrt{a\sin(e+fx)+a}}$$

```
[Out] 3*arctanh(cos(f*x+e)*a^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(3/2)/f-2*arctanh(1/
2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(3/2)/f*2^(1/2)-cot(
f*x+e)/a/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2715, 2985, 2649, 206, 2773}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{a^{3/2}f}-\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{a^{3/2}f}-\frac{\cot(e+fx)}{af\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (3*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/(a^(3/2)*f) -
(2*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]]
)])/(a^(3/2)*f) - Cot[e + f*x]/(a*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2715

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> -Simp[(a + b*Sin[e + f*x])^(m + 1)/(a*f*Tan[e + f*x]), x] + Di
```

```
st[1/b^2, Int[((a + b*Sin[e + f*x])^(m + 1)*(b*m - a*(m + 1)*Sin[e + f*x]))
/Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egerQ[m - 1/2] && LtQ[m, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^2(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx &= -\frac{\cot(e+fx)}{af\sqrt{a+a\sin(e+fx)}}+\frac{\int \frac{\csc(e+fx)\left(-\frac{3a}{2}+\frac{1}{2}a\sin(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}}\,dx}{a^2}\\
&= -\frac{\cot(e+fx)}{af\sqrt{a+a\sin(e+fx)}}-\frac{3\int \csc(e+fx)\sqrt{a+a\sin(e+fx)}\,dx}{2a^2}+\frac{2\int \frac{1}{\sqrt{a+a\sin(e+}}}{a}\\
&= -\frac{\cot(e+fx)}{af\sqrt{a+a\sin(e+fx)}}+\frac{3\,\mathrm{Subst}\left(\int \frac{1}{a-x^2}\,dx,x,\frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{af}-\frac{4\,\mathrm{Subst}\left(\int \frac{1}{2a-x}\right.}{}\\
&= \frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{a^{3/2}f}-\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{a^{3/2}f}-\frac{\cot(e+fx)}{af\sqrt{a+a\sin(e+f}}\end{aligned}$$

Mathematica [C] time = 2.11, size = 206, normalized size = 1.82

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3\left(-\cot\left(\frac{1}{4}(e+fx)\right)+(16+16i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(\tan\left(\frac{1}{4}(e+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*((16 + 16*I)*(-1)^(3/4)*ArcTanh[(1
/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] - Cot[(e + f*x)/4] + 2*(3*Log
[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 3*Log[1 - Cos[(e + f*x)/2] + Si
n[(e + f*x)/2]] + Sec[(e + f*x)/2] + Csc[e + f*x]*Sin[(e + f*x)/4]^2 - Csc[
e + f*x]*Sin[(e + f*x)/4]*Sin[(3*(e + f*x))/4])))/(4*f*(a*(1 + Sin[e + f*x]
))^(3/2))
```

fricas [B] time = 0.46, size = 421, normalized size = 3.73

$$\frac{3\left(\cos\left(fx+e\right)^{2}-\left(\cos\left(fx+e\right)+1\right)\sin\left(fx+e\right)-1\right)\sqrt{a}\,\log\left(\frac{a\cos\left(fx+e\right)^{3}-7\,a\cos\left(fx+e\right)^{2}+4\left(\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)+3\right)\sin\cdots}{\cos\left(fx+e\right)^{3}\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(3*(cos(f*x + e)^2 - (cos(f*x + e) + 1)*sin(f*x + e) - 1)*sqrt(a)*log((
a*cos(f*x + e)^3 - 7*a*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 + (cos(f*x + e) +
 3)*sin(f*x + e) - 2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9
*a*cos(f*x + e) + (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) -
a)/(cos(f*x + e)^3 + cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - c
os(f*x + e) - 1)) + 4*sqrt(2)*(a*cos(f*x + e)^2 - (a*cos(f*x + e) + a)*sin(
f*x + e) - a)*log(-(cos(f*x + e)^2 - (cos(f*x + e) - 2)*sin(f*x + e) - 2*sq
rt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1)/sqrt(a) +
3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos
(f*x + e) - 2))/sqrt(a) + 4*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*
x + e) + 1))/(a^2*f*cos(f*x + e)^2 - a^2*f - (a^2*f*cos(f*x + e) + a^2*f)*s
in(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
```

```
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)2/f*(1/4*sqrt(a*tan((f*x+exp(1))/2)^2+a)/a^2/sign(tan((f*x+
exp(1))/2)+1)+2*(-3/4*atan((-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+ex
p(1))/2)^2+a))/sqrt(-a))/sqrt(-a)/a/sign(tan((f*x+exp(1))/2)+1)+3/8*ln(abs(
-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a)))/sqrt(a)/a/si
gn(tan((f*x+exp(1))/2)+1)+sqrt(2)*atan((-sqrt(a)*tan((f*x+exp(1))/2)-sqrt(a
)+sqrt(a*tan((f*x+exp(1))/2)^2+a))/sqrt(2)/sqrt(-a))/sqrt(-a)/a/sign(tan((f
*x+exp(1))/2)+1)+1/4/sqrt(a)/((-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x
+exp(1))/2)^2+a))^2-a)/sign(tan((f*x+exp(1))/2)+1))+(-3*sqrt(-a)*sqrt(2)*ln
(sqrt(2)*sqrt(a)+sqrt(a))-sqrt(-a)*sqrt(2)-3*sqrt(-a)*ln(sqrt(2)*sqrt(a)+sq
rt(a))-3*sqrt(-a)+6*sqrt(2)*sqrt(a)*atan((sqrt(2)*sqrt(a)+sqrt(a))/sqrt(-a)
)-8*sqrt(2)*sqrt(a)*atan(sqrt(a)/sqrt(-a))+6*sqrt(a)*atan((sqrt(2)*sqrt(a)+
sqrt(a))/sqrt(-a))-16*sqrt(a)*atan(sqrt(a)/sqrt(-a)))/(4*a*sqrt(-a)*sqrt(2)
*sqrt(a)+4*a*sqrt(-a)*sqrt(a))*sign(tan((f*x+exp(1))/2)+1))
```

maple [A] time = 0.80, size = 134, normalized size = 1.19

$$-\frac{\left(1+\sin\left(fx+e\right)\right)\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\,\left(\sin\left(fx+e\right)a^{2}\left(2\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\ \sqrt{2}}{2\sqrt{a}}\right)-3\operatorname{arctanh}\left(\frac{\sqrt{a}}{}\right.\right.\right.}{a^{\frac{7}{2}}\sin\left(fx+e\right)\cos\left(fx+e\right)\sqrt{a+a\sin\left(fx+e\right)}\ f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/a^(7/2)*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(sin(f*x+e)*a^2*(2*2^(1
/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))-3*arctanh((a-a*sin(
f*x+e))^(1/2)/a^(1/2)))+(a-a*sin(f*x+e))^(1/2)*a^(3/2))/sin(f*x+e)/cos(f*x+
e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cot\left(fx+e\right)^{2}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(f*x + e)^2/(a*sin(f*x + e) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot \left(e+f\,x\right)^2}{{\left(a+a\,\sin \left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2/(a + a*sin(e + f*x))^(3/2),x)
```

```
[Out] int(cot(e + f*x)^2/(a + a*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{2}{\left(e + f x \right)}}{\left(a \left(\sin{\left(e + f x \right)} + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**2/(a*(sin(e + f*x) + 1))**(3/2), x)
```

3.110 $\int \frac{\cot^4(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=144

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{8a^{3/2}f}-\frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a\sin(e+fx)+a}}{3a^2f}-\frac{\cot(e+fx)}{8af\sqrt{a\sin(e+fx)+a}}+\frac{11\cot(e+fx)}{12af\sqrt{a\sin(e}$$

```
[Out] -1/8*arctanh(cos(f*x+e)*a^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(3/2)/f-1/8*cot(f
*x+e)/a/f/(a+a*sin(f*x+e))^(1/2)+11/12*cot(f*x+e)*csc(f*x+e)/a/f/(a+a*sin(f
*x+e))^(1/2)-1/3*cot(f*x+e)*csc(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/a^2/f
```

Rubi [A] time = 0.55, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2717, 2772, 2773, 206, 3044, 2980}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{8a^{3/2}f}-\frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a\sin(e+fx)+a}}{3a^2f}-\frac{\cot(e+fx)}{8af\sqrt{a\sin(e+fx)+a}}+\frac{11\cot(e+fx)}{12af\sqrt{a\sin(e}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]]/(8*a^(3/2)*f) - C
ot[e + f*x]/(8*a*f*Sqrt[a + a*Sin[e + f*x]]) + (11*Cot[e + f*x]*Csc[e + f*x
])/(12*a*f*Sqrt[a + a*Sin[e + f*x]]) - (Cot[e + f*x]*Csc[e + f*x]^2*Sqrt[a
+ a*Sin[e + f*x]])/(3*a^2*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2717

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Dist[-2/(a*b), Int[(a + b*Sin[e + f*x])^(m + 2)/Sin[e + f*x]^3
, x], x] + Dist[1/a^2, Int[((a + b*Sin[e + f*x])^(m + 2)*(1 + Sin[e + f*x]^
2))/Sin[e + f*x]^4, x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] &
& IntegerQ[m - 1/2] && LtQ[m, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\int \csc^4(e+fx)\sqrt{a+a\sin(e+fx)}\left(1+\sin^2(e+fx)\right)dx}{a^2} - \frac{2\int \csc^3(e+fx)\sqrt{a+}}{a^2}$$

$$= \frac{\cot(e+fx)\csc(e+fx)}{af\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+a\sin(e+fx)}}{3a^2f} + \frac{\int \csc^3(e}{}$$

$$= \frac{3\cot(e+fx)}{2af\sqrt{a+a\sin(e+fx)}} + \frac{11\cot(e+fx)\csc(e+fx)}{12af\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{}}{3a^2f}$$

$$= -\frac{\cot(e+fx)}{8af\sqrt{a+a\sin(e+fx)}} + \frac{11\cot(e+fx)\csc(e+fx)}{12af\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\csc^2(e+fx)}{3a^2f}$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{2a^{3/2}f} - \frac{\cot(e+fx)}{8af\sqrt{a+a\sin(e+fx)}} + \frac{11\cot(e+fx)\csc(e+fx)}{12af\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{8a^{3/2}f} - \frac{\cot(e+fx)}{8af\sqrt{a+a\sin(e+fx)}} + \frac{11\cot(e+fx)\csc(e+fx)}{12af\sqrt{a+a\sin(e+fx)}}$$

Mathematica [B] time = 0.76, size = 294, normalized size = 2.04

$$\frac{\csc^9\left(\frac{1}{2}(e+fx)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3\left(132\sin\left(\frac{1}{2}(e+fx)\right)+62\sin\left(\frac{3}{2}(e+fx)\right)-6\sin\left(\frac{5}{2}(e+f\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^4/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (Csc[(e + f*x)/2]^9*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(-132*Cos[(e +
f*x)/2] + 62*Cos[(3*(e + f*x))/2] + 6*Cos[(5*(e + f*x))/2] + 132*Sin[(e + f
*x)/2] - 9*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sin[e + f*x] + 9*Lo
g[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[e + f*x] + 62*Sin[(3*(e + f*
x))/2] - 6*Sin[(5*(e + f*x))/2] + 3*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x
)/2]]*Sin[3*(e + f*x)] - 3*Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin
[3*(e + f*x)]))/(24*f*(Csc[(e + f*x)/4]^2 - Sec[(e + f*x)/4]^2)^3*(a*(1 + S
in[e + f*x]))^(3/2))
```

fricas [B] time = 0.45, size = 383, normalized size = 2.66

$$\frac{3\left(\cos\left(fx+e\right)^{4}-2\,\cos\left(fx+e\right)^{2}-\left(\cos\left(fx+e\right)^{3}+\cos\left(fx+e\right)^{2}-\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)+1\right)\sqrt{a}\,\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/96*(3*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 - (cos(f*x + e)^3 + cos(f*x + e)
^2 - cos(f*x + e) - 1)*sin(f*x + e) + 1)*sqrt(a)*log((a*cos(f*x + e)^3 - 7*
a*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3)*sin(f*x + e) - 2*
cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a*cos(f*x + e) + (a*
cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)/(cos(f*x + e)^3 +
cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos(f*x + e) - 1)) + 4
*(3*cos(f*x + e)^3 + 17*cos(f*x + e)^2 - (3*cos(f*x + e)^2 - 14*cos(f*x + e
) - 25)*sin(f*x + e) - 11*cos(f*x + e) - 25)*sqrt(a*sin(f*x + e) + a))/(a^2
*f*cos(f*x + e)^4 - 2*a^2*f*cos(f*x + e)^2 + a^2*f - (a^2*f*cos(f*x + e)^3
+ a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - a^2*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*(2*sqrt(a*t
an((f*x+exp(1))/2)^2+a)*(tan((f*x+exp(1))/2)*(1/96*tan((f*x+exp(1))/2)/a^2/
sign(tan((f*x+exp(1))/2)+1)-3/64/a^2/sign(tan((f*x+exp(1))/2)+1))+7/96/a^2/
sign(tan((f*x+exp(1))/2)+1))+2*(1/96*(9*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(
a*tan((f*x+exp(1))/2)^2+a))^5+18*sqrt(a)*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt
(a*tan((f*x+exp(1))/2)^2+a))^4-9*a^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*t
an((f*x+exp(1))/2)^2+a))-24*sqrt(a)*a*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*
tan((f*x+exp(1))/2)^2+a))^2+14*sqrt(a)*a^2)/((-sqrt(a)*tan((f*x+exp(1))/2)+
sqrt(a*tan((f*x+exp(1))/2)^2+a))^2-a)^3/a/sign(tan((f*x+exp(1))/2)+1)+1/32*
```

```
atan((-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))/sqrt(-a
))/sqrt(-a)/a/sign(tan((f*x+exp(1))/2)+1)-1/64*ln(abs(-sqrt(a)*tan((f*x+exp
(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a)))/sqrt(a)/a/sign(tan((f*x+exp(1))/2
)+1))+(15*sqrt(-a)*sqrt(2)*ln(sqrt(2)*sqrt(a)+sqrt(a))-280*sqrt(-a)*sqrt(2)
+21*sqrt(-a)*ln(sqrt(2)*sqrt(a)+sqrt(a))-402*sqrt(-a)-30*sqrt(2)*sqrt(a)*at
an((sqrt(2)*sqrt(a)+sqrt(a))/sqrt(-a))-42*sqrt(a)*atan((sqrt(2)*sqrt(a)+sqr
t(a))/sqrt(-a)))/(480*a*sqrt(-a)*sqrt(2)*sqrt(a)+672*a*sqrt(-a)*sqrt(a))*si
gn(tan((f*x+exp(1))/2)+1))
```

maple [A] time = 0.89, size = 144, normalized size = 1.00

$$-\frac{\left(1+\sin\left(fx+e\right)\right)\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\left(3\left(-a\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{5}{2}}a^{\frac{3}{2}}+3\,\mathrm{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}}{\sqrt{a}}\right)a^{4}\left(\sin\right.\right.}{24a^{\frac{11}{2}}\sin\left(fx+e\right)^{3}\cos\left(fx+e\right)\sqrt{a+a\sin\left(\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/24/a^(11/2)*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(3*(-a*(sin(f*x+e)-
1))^(5/2)*a^(3/2)+3*arctanh((-a*(sin(f*x+e)-1))^(1/2)/a^(1/2))*a^4*sin(f*x+
e)^3+8*(-a*(sin(f*x+e)-1))^(3/2)*a^(5/2)-3*(-a*(sin(f*x+e)-1))^(1/2)*a^(7/2
))/sin(f*x+e)^3/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot\left(e+f\,x\right)^4}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4/(a + a*sin(e + f*x))^(3/2),x)
```

```
[Out] int(cot(e + f*x)^4/(a + a*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4\left(e+fx\right)}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+a*sin(f*x+e))**(3/2),x)
[Out] Integral(cot(e + f*x)**4/(a*(sin(e + f*x) + 1))**(3/2), x)
```

3.111 $\int \frac{\tan^4(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=207

$$\frac{317\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{4096\sqrt{2}\,a^{5/2}f}+\frac{\tan^3(e+fx)}{3f(a\sin(e+fx)+a)^{5/2}}+\frac{5a\sin(e+fx)\tan(e+fx)}{48f(a\sin(e+fx)+a)^{7/2}}+\frac{317\cos(e+fx)}{4096af(a\sin(e+fx)+a$$

```
[Out] 317/3072*cos(f*x+e)/f/(a+a*sin(f*x+e))^(5/2)-1/384*sec(f*x+e)*(115+129*sin(
f*x+e))/f/(a+a*sin(f*x+e))^(5/2)+317/4096*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(
3/2)+317/8192*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2)
)/a^(5/2)/f*2^(1/2)+5/48*a*sin(f*x+e)*tan(f*x+e)/f/(a+a*sin(f*x+e))^(7/2)+1
/3*tan(f*x+e)^3/f/(a+a*sin(f*x+e))^(5/2)
```

Rubi [A] time = 1.43, antiderivative size = 260, normalized size of antiderivative = 1.26, number of steps used = 23, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {2714, 2650, 2649, 206, 4401, 2681, 2687, 2877, 2859}

$$-\frac{31\sec^3(e+fx)}{192a^2f\sqrt{a\sin(e+fx)+a}}-\frac{1085\sec(e+fx)}{3072a^2f\sqrt{a\sin(e+fx)+a}}+\frac{317\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{4096\sqrt{2}\,a^{5/2}f}+\frac{317\cos(e+f}{4096af(a\sin(e+f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (317*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(4
096*Sqrt[2]*a^(5/2)*f) - Cos[e + f*x]/(4*f*(a + a*Sin[e + f*x])^(5/2)) - Se
c[e + f*x]^3/(8*f*(a + a*Sin[e + f*x])^(5/2)) + (317*Cos[e + f*x])/(4096*a*
f*(a + a*Sin[e + f*x])^(3/2)) + (217*Sec[e + f*x])/(1536*a*f*(a + a*Sin[e +
 f*x])^(3/2)) + (53*Sec[e + f*x]^3)/(96*a*f*(a + a*Sin[e + f*x])^(3/2)) - (
1085*Sec[e + f*x])/(3072*a^2*f*Sqrt[a + a*Sin[e + f*x]]) - (31*Sec[e + f*x]
^3)/(192*a^2*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2714

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] - Int[((a + b*Sin[e + f*x])^m*(
1 - 2*Sin[e + f*x]^2))/Cos[e + f*x]^4, x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2877

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^
(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1)), x] - Dist[1/(a^2*(2*
m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(
2*m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a
^2 - b^2, 0] && LeQ[m, -2^(-1)] && NeQ[2*m + p + 1, 0]
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{1}{(a+a\sin(e+fx))^{5/2}}\,dx - \int \frac{\sec^4(e+fx)\left(1-2\sin^2(e+fx)\right)}{(a+a\sin(e+fx))^{5/2}}\,dx$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} + \frac{3\int \frac{1}{(a+a\sin(e+fx))^{3/2}}\,dx}{8a} - \int\left(\frac{\sec^4(e+fx)}{(a(1+\sin(e+fx)))^{5/2}} - \right.$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{3\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} + 2\int \frac{\sec^2(e+fx)\tan^2(e}{(a(1+\sin(e+fx)}$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{\sec^3(e+fx)}{8f(a+a\sin(e+fx))^{5/2}} - \frac{3\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}}$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}\,a^{5/2}f} - \frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{\sec^3(e+fx)}{8f(a+a\sin(e+fx))^{5/2}}$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}\,a^{5/2}f} - \frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{\sec^3(e+fx)}{8f(a+a\sin(e+fx))^{5/2}}$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}\,a^{5/2}f} - \frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{\sec^3(e+fx)}{8f(a+a\sin(e+fx))^{5/2}}$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}\,a^{5/2}f} - \frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{\sec^3(e+fx)}{8f(a+a\sin(e+fx))^{5/2}}$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}\,a^{5/2}f} - \frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{\sec^3(e+fx)}{8f(a+a\sin(e+fx))^{5/2}}$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}\,a^{5/2}f} - \frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{\sec^3(e+fx)}{8f(a+a\sin(e+fx))^{5/2}}$$

$$= \frac{317\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{4096\sqrt{2}\,a^{5/2}f} - \frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{\sec^3(e+fx)}{8f(a+a\sin(e+fx))^{5/2}}$$

Mathematica [C] time = 0.54, size = 394, normalized size = 1.90

$$-\frac{1152\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5}{\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)}+\frac{256\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5}{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}-201\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4+402\,\mathrm{s}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (1312 + (768*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - 38
4/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - (2624*Sin[(e + f*x)/2])/(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2]) + 2584*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2]) - 1292*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 402*Sin[
(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - 201*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2])^4 - (951 + 951*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)
^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 + (
256*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/(Cos[(e + f*x)/2] - Sin[(e + f
*x)/2])^3 - (1152*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/(Cos[(e + f*x)/2
] - Sin[(e + f*x)/2]))/(12288*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

fricas [A] time = 0.48, size = 307, normalized size = 1.48

$$951\sqrt{2}\left(3\cos\left(fx+e\right)^5-4\cos\left(fx+e\right)^3+\left(\cos\left(fx+e\right)^5-4\cos\left(fx+e\right)^3\right)\sin\left(fx+e\right)\right)\sqrt{a}\log\left(-\frac{a\cos\left(fx+e\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/49152*(951*sqrt(2)*(3*cos(f*x + e)^5 - 4*cos(f*x + e)^3 + (cos(f*x + e)^5
 - 4*cos(f*x + e)^3)*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(
2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a
*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2
- (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(2219*cos(f*x +
e)^4 - 4960*cos(f*x + e)^2 + (951*cos(f*x + e)^4 - 6944*cos(f*x + e)^2 + 28
16)*sin(f*x + e) + 1280)*sqrt(a*sin(f*x + e) + a))/(3*a^3*f*cos(f*x + e)^5
- 4*a^3*f*cos(f*x + e)^3 + (a^3*f*cos(f*x + e)^5 - 4*a^3*f*cos(f*x + e)^3)*
sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*2*(-1/192*(3*(-sqrt
(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^5+21*sqrt(a)*(-sqr
t(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^4-14*a*(-sqrt(a)*
tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3+39*a^2*(-sqrt(a)*tan
((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))-42*sqrt(a)*a*(-sqrt(a)*ta
n((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2-7*sqrt(a)*a^2)/a^2/(-(
-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2-2*sqrt(a)*(
-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))+a)^3/sign(tan
((f*x+exp(1))/2)+1)+1/24576*(-1335*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan
((f*x+exp(1))/2)^2+a))^15+20025*sqrt(a)*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(
a*tan((f*x+exp(1))/2)^2+a))^14-111513*a*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(
a*tan((f*x+exp(1))/2)^2+a))^13+173379*sqrt(a)*a*(-sqrt(a)*tan((f*x+exp(1))/
2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^12-26091*a^2*(-sqrt(a)*tan((f*x+exp(1))
/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^11-247283*sqrt(a)*a^2*(-sqrt(a)*tan((f
*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^10+157019*a^3*(-sqrt(a)*tan(
(f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^9+345215*sqrt(a)*a^3*(-sqr
t(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^8-87061*a^4*(-sqr
t(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^7-353877*sqrt(a)*
a^4*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^6-89371*
a^5*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^5+95561*
sqrt(a)*a^5*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^
4+78327*a^6*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^
3+7705*a^7*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))+3
8735*sqrt(a)*a^6*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2
+a))^2+565*sqrt(a)*a^7)/a^2/(-(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x
+exp(1))/2)^2+a))^2+2*sqrt(a)*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x
+exp(1))/2)^2+a))+a)^8/sign(tan((f*x+exp(1))/2)+1)-317/8192*atan((-sqrt(a)*
tan((f*x+exp(1))/2)-sqrt(a)+sqrt(a*tan((f*x+exp(1))/2)^2+a))/sqrt(2)/sqrt(-
a))/sqrt(2)/a^2/sqrt(-a)/sign(tan((f*x+exp(1))/2)+1))
```

maple [A] time = 0.99, size = 353, normalized size = 1.71

$$-\frac{1902a^{\frac{11}{2}}\sin\left(fx+e\right)\left(\cos^{4}\left(fx+e\right)\right)+\left(-13888a^{\frac{11}{2}}-3804\left(a-a\sin\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\ \sqrt{2}}{2\sqrt{a}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+a*sin(f*x+e))^(5/2),x)
```

[Out] -1/24576/a^(15/2)*(1902*a^(11/2)*sin(f*x+e)*cos(f*x+e)^4+(-13888*a^(11/2)-3804*(a-a*sin(f*x+e))^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^4)*cos(f*x+e)^2*sin(f*x+e)+(5632*a^(11/2)+7608*(a-a*sin(f*x+e))^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^4)*sin(f*x+e)+(4438*a^(11/2)+951*(a-a*sin(f*x+e))^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^4)*cos(f*x+e)^4+(-9920*a^(11/2)-7608*(a-a*sin(f*x+e))^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^4)*cos(f*x+e)^2+2560*a^(11/2)+7608*(a-a*sin(f*x+e))^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^4)/(sin(f*x+e)-1)/(1+sin(f*x+e))^3/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan\left(e+f\,x\right)^4}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(e + f*x)^4/(a + a*sin(e + f*x))^(5/2),x)

[Out] int(tan(e + f*x)^4/(a + a*sin(e + f*x))^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{4}{\left(e + f x \right)}}{\left(a \left(\sin{\left(e + f x \right)} + 1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)**4/(a+a*sin(f*x+e))**(5/2),x)

[Out] Integral(tan(e + f*x)**4/(a*(sin(e + f*x) + 1))**(5/2), x)

3.112 $\int \frac{\tan^2(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=167

$$-\frac{11\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{128\sqrt{2}a^{5/2}f}+\frac{11\sec(e+fx)}{96a^2f\sqrt{a\sin(e+fx)+a}}-\frac{11\cos(e+fx)}{128af(a\sin(e+fx)+a)^{3/2}}+\frac{17\sec(e+fx)}{48af(a\sin(e+fx)+a)^{3/2}}$$

```
[Out] -1/6*sec(f*x+e)/f/(a+a*sin(f*x+e))^(5/2)-11/128*cos(f*x+e)/a/f/(a+a*sin(f*x
+e))^(3/2)+17/48*sec(f*x+e)/a/f/(a+a*sin(f*x+e))^(3/2)-11/256*arctanh(1/2*c
os(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/f*2^(1/2)+11/96*s
ec(f*x+e)/a^2/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2712, 2859, 2687, 2650, 2649, 206}

$$\frac{11\sec(e+fx)}{96a^2f\sqrt{a\sin(e+fx)+a}}-\frac{11\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{128\sqrt{2}a^{5/2}f}-\frac{11\cos(e+fx)}{128af(a\sin(e+fx)+a)^{3/2}}+\frac{17\sec(e+fx)}{48af(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-11*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(1
28*Sqrt[2]*a^(5/2)*f) - Sec[e + f*x]/(6*f*(a + a*Sin[e + f*x])^(5/2)) - (11
*Cos[e + f*x])/(128*a*f*(a + a*Sin[e + f*x])^(3/2)) + (17*Sec[e + f*x])/(48
*a*f*(a + a*Sin[e + f*x])^(3/2)) + (11*Sec[e + f*x])/(96*a^2*f*Sqrt[a + a*S
in[e + f*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2712

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[(b*(a + b*Sin[e + f*x])^m)/(a*f*(2*m - 1)*Cos[e + f*x]),
x] - Dist[1/(a^2*(2*m - 1)), Int[((a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*
m - 1)*Sin[e + f*x]))/Cos[e + f*x]^2, x], x] /; FreeQ[{a, b, e, f}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[m] && LtQ[m, 0]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{\sec(e+fx)}{6f(a+a\sin(e+fx))^{5/2}} + \frac{\int \frac{\sec^2(e+fx)\left(-\frac{5a}{2}+6a\sin(e+fx)\right)}{(a+a\sin(e+fx))^{3/2}}\,dx}{6a^2}$$

$$= -\frac{\sec(e+fx)}{6f(a+a\sin(e+fx))^{5/2}} + \frac{17\sec(e+fx)}{48af(a+a\sin(e+fx))^{3/2}} + \frac{11\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx}{96a^2}$$

$$= -\frac{\sec(e+fx)}{6f(a+a\sin(e+fx))^{5/2}} + \frac{17\sec(e+fx)}{48af(a+a\sin(e+fx))^{3/2}} + \frac{11\sec(e+fx)}{96a^2f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{\sec(e+fx)}{6f(a+a\sin(e+fx))^{5/2}} - \frac{11\cos(e+fx)}{128af(a+a\sin(e+fx))^{3/2}} + \frac{17\sec(e+fx)}{48af(a+a\sin(e+fx))}$$

$$= -\frac{\sec(e+fx)}{6f(a+a\sin(e+fx))^{5/2}} - \frac{11\cos(e+fx)}{128af(a+a\sin(e+fx))^{3/2}} + \frac{17\sec(e+fx)}{48af(a+a\sin(e+fx))}$$

$$= -\frac{11\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{128\sqrt{2}a^{5/2}f} - \frac{\sec(e+fx)}{6f(a+a\sin(e+fx))^{5/2}} - \frac{11\cos(e+fx)}{128af(a+a\sin(e+fx}$$

Mathematica [C] time = 0.40, size = 284, normalized size = 1.70

$$\frac{\frac{48\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5}{\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)} + 15\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4 - 30\sin\left(\frac{1}{2}(e+fx)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-32 + (64*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 104*Si
n[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 52*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^2 - 30*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^3 + 15*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 + (33 + 33*I)*(-1)^(
3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^5 + (48*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/(C
os[(e + f*x)/2] - Sin[(e + f*x)/2]))/(384*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

fricas [A] time = 0.45, size = 279, normalized size = 1.67

$$\frac{33\sqrt{2}\left(3\cos\left(fx+e\right)^{3}+\left(\cos\left(fx+e\right)^{3}-4\cos\left(fx+e\right)\right)\sin\left(fx+e\right)-4\cos\left(fx+e\right)\right)\sqrt{a}\log\left(-\frac{a\cos\left(fx+e\right)^{2}-2\cdots}{\cdots}\cdots\right.}{1536\left(3a^{3}f\cos\left(fx+\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/1536*(33*sqrt(2)*(3*cos(f*x + e)^3 + (cos(f*x + e)^3 - 4*cos(f*x + e))*si
n(f*x + e) - 4*cos(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqr
t(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f
*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos
(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*(77*cos(f*x + e)^2 + (
33*cos(f*x + e)^2 - 224)*sin(f*x + e) - 160)*sqrt(a*sin(f*x + e) + a))/(3*a
^3*f*cos(f*x + e)^3 - 4*a^3*f*cos(f*x + e) + (a^3*f*cos(f*x + e)^3 - 4*a^3*
f*cos(f*x + e))*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*2*(-1/16*(sqrt(a)*t
an((f*x+exp(1))/2)+sqrt(a)-sqrt(a*tan((f*x+exp(1))/2)^2+a))/a^2/(-(-sqrt(a)
*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2-2*sqrt(a)*(-sqrt(a)
*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))+a)/sign(tan((f*x+exp(
1))/2)+1)+1/768*(81*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2
)^2+a))^11-1083*sqrt(a)*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1
))/2)^2+a))^10+2001*a*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))
/2)^2+a))^9-1071*sqrt(a)*a*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+ex
p(1))/2)^2+a))^8-2502*a^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp
(1))/2)^2+a))^7+1330*sqrt(a)*a^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((
f*x+exp(1))/2)^2+a))^6+3378*a^3*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f
*x+exp(1))/2)^2+a))^5+546*sqrt(a)*a^3*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*
tan((f*x+exp(1))/2)^2+a))^4-1499*a^4*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*t
an((f*x+exp(1))/2)^2+a))^3-339*a^5*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan
((f*x+exp(1))/2)^2+a))-807*sqrt(a)*a^4*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a
*tan((f*x+exp(1))/2)^2+a))^2-35*sqrt(a)*a^5)/a^2/(-(-sqrt(a)*tan((f*x+exp(1
```

```
))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2+2*sqrt(a)*(-sqrt(a)*tan((f*x+exp(1
))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))+a)^6/sign(tan((f*x+exp(1))/2)+1)+11/
256*atan((-sqrt(a)*tan((f*x+exp(1))/2)-sqrt(a)+sqrt(a*tan((f*x+exp(1))/2)^2
+a))/sqrt(2)/sqrt(-a))/sqrt(2)/a^2/sqrt(-a)/sign(tan((f*x+exp(1))/2)+1))
```

maple [A] time = 0.96, size = 266, normalized size = 1.59

$$-\frac{\left(66a^{\frac{7}{2}}-33\sqrt{a-a\sin\left(fx+e\right)}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\,\sqrt{2}}{2\sqrt{a}}\right)a^{3}\right)\sin\left(fx+e\right)\left(\cos^{2}\left(fx+e\right)\right)+\left(-448a^{\frac{7}{2}}+13\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/768/a^(11/2)*((66*a^(7/2)-33*(a-a*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*
(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3)*sin(f*x+e)*cos(f*x+e)^2+(-448*
a^(7/2)+132*(a-a*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/
2)*2^(1/2)/a^(1/2))*a^3)*sin(f*x+e)+(154*a^(7/2)-99*(a-a*sin(f*x+e))^(1/2)*
2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3)*cos(f*x+e)
^2-320*a^(7/2)+132*(a-a*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+
e))^(1/2)*2^(1/2)/a^(1/2))*a^3)/(1+sin(f*x+e))^2/cos(f*x+e)/(a+a*sin(f*x+e)
)^(1/2)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan\left(e+f\,x\right)^2}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(a + a*sin(e + f*x))^(5/2),x)
```

```
[Out] int(tan(e + f*x)^2/(a + a*sin(e + f*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2\left(e+fx\right)}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral(tan(e + f*x)**2/(a*(sin(e + f*x) + 1))**(5/2), x)
```

3.113 $\int \frac{\cot^2(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=141

$$\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{a^{5/2}f}-\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{2}a^{5/2}f}-\frac{2\cos(e+fx)}{af(a\sin(e+fx)+a)^{3/2}}-\frac{\cot(e+fx)}{af(a\sin(e+fx)+a)^{3/2}}$$

```
[Out] 5*arctanh(cos(f*x+e)*a^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/f-2*cos(f*x+e)
/a/f/(a+a*sin(f*x+e))^(3/2)-cot(f*x+e)/a/f/(a+a*sin(f*x+e))^(3/2)-7/2*arcta
nh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/f*2^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2715, 2978, 2985, 2649, 206, 2773}

$$\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{a^{5/2}f}-\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{2}a^{5/2}f}-\frac{2\cos(e+fx)}{af(a\sin(e+fx)+a)^{3/2}}-\frac{\cot(e+fx)}{af(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (5*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/(a^(5/2)*f) -
(7*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(Sqr
t[2]*a^(5/2)*f) - (2*Cos[e + f*x])/(a*f*(a + a*Sin[e + f*x])^(3/2)) - Cot[e
 + f*x]/(a*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2715

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> -Simp[(a + b*Sin[e + f*x])^(m + 1)/(a*f*Tan[e + f*x]), x] + Di
```

```
st[1/b^2, Int[((a + b*Sin[e + f*x])^(m + 1)*(b*m - a*(m + 1)*Sin[e + f*x]))
/Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egerQ[m - 1/2] && LtQ[m, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{\cot(e+fx)}{af(a+a\sin(e+fx))^{3/2}} + \frac{\int \frac{\csc(e+fx)\left(-\frac{5a}{2}+\frac{3}{2}a\sin(e+fx)\right)}{(a+a\sin(e+fx))^{3/2}}\,dx}{a^2}$$

$$= -\frac{2\cos(e+fx)}{af(a+a\sin(e+fx))^{3/2}} - \frac{\cot(e+fx)}{af(a+a\sin(e+fx))^{3/2}} + \frac{\int \frac{\csc(e+fx)\left(-5a^2+2a^2\sin(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}}}{2a^4}$$

$$= -\frac{2\cos(e+fx)}{af(a+a\sin(e+fx))^{3/2}} - \frac{\cot(e+fx)}{af(a+a\sin(e+fx))^{3/2}} - \frac{5\int \csc(e+fx)\sqrt{a+a\sin(e}{2a^3}$$

$$= -\frac{2\cos(e+fx)}{af(a+a\sin(e+fx))^{3/2}} - \frac{\cot(e+fx)}{af(a+a\sin(e+fx))^{3/2}} + \frac{5\,\mathrm{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \frac{a\cos}{\sqrt{a+a\,s}}\right.}{a^2f}$$

$$= \frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{a^{5/2}f} - \frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{2}a^{5/2}f} - \frac{2\cos(e+fx)}{af(a+a\sin(e+fx))^{3/2}}$$

Mathematica [C] time = 0.74, size = 451, normalized size = 3.20

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3\left(8\sin\left(\frac{1}{2}(e+fx)\right)+\frac{2\sin\left(\frac{1}{4}(e+fx)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2}{\cos\left(\frac{1}{4}(e+fx)\right)-\sin\left(\frac{1}{4}(e+fx)\right)} - \frac{2\sin\left(\frac{1}{4}(e+fx)\right)\left(\sin\right.}{\sin\left(\frac{1}{4}(e+fx\right.}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(8*Sin[(e + f*x)/2] - 4*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2]) + 2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (
28 + 28*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4]
)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - Cot[(e + f*x)/4]*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2])^2 + 10*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - 10*Log[1 - Cos[(e + f*x)/2] + S
in[(e + f*x)/2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (2*Sin[(e + f*x)
/4]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/(Cos[(e + f*x)/4] - Sin[(e + f
*x)/4]) - (2*Sin[(e + f*x)/4]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/(Cos
[(e + f*x)/4] + Sin[(e + f*x)/4]) - (Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2
*Tan[(e + f*x)/4]))/(4*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

fricas [B] time = 0.47, size = 539, normalized size = 3.82

$$5\left(\cos\left(fx+e\right)^{3}+2\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)^{2}-\cos\left(fx+e\right)-2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2\right)\sqrt{a}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(5*(cos(f*x + e)^3 + 2*cos(f*x + e)^2 + (cos(f*x + e)^2 - cos(f*x + e)
- 2)*sin(f*x + e) - cos(f*x + e) - 2)*sqrt(a)*log((a*cos(f*x + e)^3 - 7*a*c
os(f*x + e)^2 + 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3)*sin(f*x + e) - 2*cos
(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a*cos(f*x + e) + (a*cos
(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)/(cos(f*x + e)^3 + cos
(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos(f*x + e) - 1)) + 7*sq
rt(2)*(a*cos(f*x + e)^3 + 2*a*cos(f*x + e)^2 - a*cos(f*x + e) + (a*cos(f*x
+ e)^2 - a*cos(f*x + e) - 2*a)*sin(f*x + e) - 2*a)*log(-(cos(f*x + e)^2 - (
cos(f*x + e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*
x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 -
(cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a) + 4*(2*cos(f*x
 + e)^2 + (2*cos(f*x + e) + 1)*sin(f*x + e) + cos(f*x + e) - 1)*sqrt(a*sin(
f*x + e) + a))/(a^3*f*cos(f*x + e)^3 + 2*a^3*f*cos(f*x + e)^2 - a^3*f*cos(f
*x + e) - 2*a^3*f + (a^3*f*cos(f*x + e)^2 - a^3*f*cos(f*x + e) - 2*a^3*f)*s
in(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)2/f*(1/4*sqrt(a*tan((f*x+exp(1))/2)^2+a)/a^3/sign(tan((f*x+
exp(1))/2)+1)+2*(1/2*(-3*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(
```

```
1))/2)^2+a))^3+a*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2
+a))+sqrt(a)*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))
^2+sqrt(a)*a)/a^2/(-(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2
)^2+a))^2+2*sqrt(a)*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2
)^2+a))+a)^2/sign(tan((f*x+exp(1))/2)+1)-5/4*atan((-sqrt(a)*tan((f*x+exp(1)
)/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))/sqrt(-a))/a^2/sqrt(-a)/sign(tan((f*x+
exp(1))/2)+1)+7/2*atan((-sqrt(a)*tan((f*x+exp(1))/2)-sqrt(a)+sqrt(a*tan((f*
x+exp(1))/2)^2+a))/sqrt(2)/sqrt(-a))/sqrt(2)/a^2/sqrt(-a)/sign(tan((f*x+exp
(1))/2)+1)+5/8/sqrt(a)/a^2*ln(abs(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((
f*x+exp(1))/2)^2+a)))/sign(tan((f*x+exp(1))/2)+1)-1/4/sqrt(a)/a/(-(-sqrt(a)
*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2+a)/sign(tan((f*x+ex
p(1))/2)+1)))
```

maple [A] time = 0.69, size = 219, normalized size = 1.55

$$-\left(7\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\ \sqrt{2}}{2\sqrt{a}}\right)\left(\sin^{2}\left(fx+e\right)\right)a-10\operatorname{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}}{\sqrt{a}}\right)\left(\sin^{2}\left(fx+e\right)\right)a+7\sqrt{2}\ a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/2/a^(7/2)*(7*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/
2))*sin(f*x+e)^2*a-10*arctanh((-a*(sin(f*x+e)-1))^(1/2)/a^(1/2))*sin(f*x+e)
^2*a+7*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a*sin
(f*x+e)+4*(-a*(sin(f*x+e)-1))^(1/2)*a^(1/2)*sin(f*x+e)-10*arctanh((-a*(sin(
f*x+e)-1))^(1/2)/a^(1/2))*a*sin(f*x+e)+2*(-a*(sin(f*x+e)-1))^(1/2)*a^(1/2))
*(-a*(sin(f*x+e)-1))^(1/2)/sin(f*x+e)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cot\left(e+f\,x\right)}^2}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2/(a + a*sin(e + f*x))^(5/2),x)
```

```
[Out] int(cot(e + f*x)^2/(a + a*sin(e + f*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2\left(e+fx\right)}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral(cot(e + f*x)**2/(a*(sin(e + f*x) + 1))**(5/2), x)
```

3.114 $\int \frac{\cot^4(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=191

$$\frac{45\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{8a^{5/2}f}-\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{a^{5/2}f}-\frac{19\cot(e+fx)}{8a^2f\sqrt{a\sin(e+fx)+a}}-\frac{\cot(e+fx)\csc^2(e+fx)}{3a^2f\sqrt{a\sin(e+fx)+a}}+$$

```
[Out] 45/8*arctanh(cos(f*x+e)*a^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/f-4*arctanh
(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/f*2^(1/2)-1
9/8*cot(f*x+e)/a^2/f/(a+a*sin(f*x+e))^(1/2)+13/12*cot(f*x+e)*csc(f*x+e)/a^2
/f/(a+a*sin(f*x+e))^(1/2)-1/3*cot(f*x+e)*csc(f*x+e)^2/a^2/f/(a+a*sin(f*x+e)
)^(1/2)
```

Rubi [A] time = 0.96, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2717, 2779, 2984, 2985, 2649, 206, 2773, 3044}

$$-\frac{19\cot(e+fx)}{8a^2f\sqrt{a\sin(e+fx)+a}}+\frac{45\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{8a^{5/2}f}-\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{a^{5/2}f}-\frac{\cot(e+fx)\csc^2(e+fx)}{3a^2f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (45*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/(8*a^(5/2)*f)
 - (4*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*
x]])])/(a^(5/2)*f) - (19*Cot[e + f*x])/(8*a^2*f*Sqrt[a + a*Sin[e + f*x]]) +
 (13*Cot[e + f*x]*Csc[e + f*x])/(12*a^2*f*Sqrt[a + a*Sin[e + f*x]]) - (Cot[
e + f*x]*Csc[e + f*x]^2)/(3*a^2*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2717

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Dist[-2/(a*b), Int[(a + b*Sin[e + f*x])^(m + 2)/Sin[e + f*x]^3
, x], x] + Dist[1/a^2, Int[((a + b*Sin[e + f*x])^(m + 2)*(1 + Sin[e + f*x]^
2))/Sin[e + f*x]^4, x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] &
& IntegerQ[m - 1/2] && LtQ[m, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\int \frac{\csc^4(e+fx)\left(1+\sin^2(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}}\,dx}{a^2} - \frac{2\int \frac{\csc^3(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx}{a^2}$$

$$= \frac{\cot(e+fx)\csc(e+fx)}{a^2 f\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\csc^2(e+fx)}{3a^2 f\sqrt{a+a\sin(e+fx)}} + \frac{\int \frac{\csc^3(e+fx)\left(-\frac{a}{2}+\frac{11}{2}a\sin(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}}\,d}{3a^3}$$

$$= -\frac{\cot(e+fx)}{2a^2 f\sqrt{a+a\sin(e+fx)}} + \frac{13\cot(e+fx)\csc(e+fx)}{12a^2 f\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\csc^2(e+fx)}{3a^2 f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{19\cot(e+fx)}{8a^2 f\sqrt{a+a\sin(e+fx)}} + \frac{13\cot(e+fx)\csc(e+fx)}{12a^2 f\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\csc^2(e+fx)}{3a^2 f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{19\cot(e+fx)}{8a^2 f\sqrt{a+a\sin(e+fx)}} + \frac{13\cot(e+fx)\csc(e+fx)}{12a^2 f\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\csc^2(e+fx)}{3a^2 f\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{2a^{5/2}f} - \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{a^{5/2}f} - \frac{19\cot(e+fx)}{8a^2 f\sqrt{a+a\sin(e+f}}$$

$$= \frac{45\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{8a^{5/2}f} - \frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{a^{5/2}f} - \frac{19\cot(e+fx)}{8a^2 f\sqrt{a+a\sin(e+}}$$

Mathematica [C] time = 2.41, size = 332, normalized size = 1.74

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5\left(-\frac{8\csc^9\left(\frac{1}{2}(e+fx)\right)\left(-396\sin\left(\frac{1}{2}(e+fx)\right)-218\sin\left(\frac{3}{2}(e+fx)\right)+114\sin\left(\frac{5}{2}(e+fx)\right)+396\cos\left(\frac{1}{2}(e+fx)\right)\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^4/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*((1536 + 1536*I)*(-1)^(3/4)*ArcTan
h[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] - (8*Csc[(e + f*x)/2]^9*(
396*Cos[(e + f*x)/2] - 218*Cos[(3*(e + f*x))/2] - 114*Cos[(5*(e + f*x))/2]
- 396*Sin[(e + f*x)/2] - 405*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*S
in[e + f*x] + 405*Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[e + f*x]
 - 218*Sin[(3*(e + f*x))/2] + 114*Sin[(5*(e + f*x))/2] + 135*Log[1 + Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2]]*Sin[3*(e + f*x)] - 135*Log[1 - Cos[(e + f*x)
/2] + Sin[(e + f*x)/2]]*Sin[3*(e + f*x)]))/(Csc[(e + f*x)/4]^2 - Sec[(e + f
*x)/4]^2)^3))/(192*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

fricas [B] time = 0.46, size = 564, normalized size = 2.95

$$\frac{135\left(\cos\left(fx+e\right)^4-2\cos\left(fx+e\right)^2-\left(\cos\left(fx+e\right)^3+\cos\left(fx+e\right)^2-\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)+1\right)\sqrt{a}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/96*(135*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 - (cos(f*x + e)^3 + cos(f*x +
e)^2 - cos(f*x + e) - 1)*sin(f*x + e) + 1)*sqrt(a)*log((a*cos(f*x + e)^3 -
7*a*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3)*sin(f*x + e) -
2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a*cos(f*x + e) + (
a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)/(cos(f*x + e)^3
+ cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos(f*x + e) - 1)) +
 192*sqrt(2)*(a*cos(f*x + e)^4 - 2*a*cos(f*x + e)^2 - (a*cos(f*x + e)^3 + a
*cos(f*x + e)^2 - a*cos(f*x + e) - a)*sin(f*x + e) + a)*log(-(cos(f*x + e)^
2 - (cos(f*x + e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(c
os(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)
^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a) + 4*(57*c
```

```
os(f*x + e)^3 + 83*cos(f*x + e)^2 - (57*cos(f*x + e)^2 - 26*cos(f*x + e) -
91)*sin(f*x + e) - 65*cos(f*x + e) - 91)*sqrt(a*sin(f*x + e) + a))/(a^3*f*c
os(f*x + e)^4 - 2*a^3*f*cos(f*x + e)^2 + a^3*f - (a^3*f*cos(f*x + e)^3 + a^
3*f*cos(f*x + e)^2 - a^3*f*cos(f*x + e) - a^3*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)2/f*(2*sqrt(a*tan((f*x+exp(1))/2)^2+a)*(tan((f
*x+exp(1))/2)*(1/96*tan((f*x+exp(1))/2)/a^3/sign(tan((f*x+exp(1))/2)+1)-5/6
4/a^3/sign(tan((f*x+exp(1))/2)+1))+37/96/a^3/sign(tan((f*x+exp(1))/2)+1))+2
*(1/96*(15*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^5
+78*sqrt(a)*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^
4-15*a^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))-144
*sqrt(a)*a*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2
+74*sqrt(a)*a^2)/a^2/((-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))
/2)^2+a))^2-a)^3/sign(tan((f*x+exp(1))/2)+1)-45/32*atan((-sqrt(a)*tan((f*x+
exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))/sqrt(-a))/a^2/sqrt(-a)/sign(tan
((f*x+exp(1))/2)+1)+4*atan((-sqrt(a)*tan((f*x+exp(1))/2)-sqrt(a)+sqrt(a*tan
((f*x+exp(1))/2)^2+a))/sqrt(2)/sqrt(-a))/sqrt(2)/a^2/sqrt(-a)/sign(tan((f*x
+exp(1))/2)+1)+45/64/sqrt(a)/a^2*ln(abs(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a
*tan((f*x+exp(1))/2)^2+a)))/sign(tan((f*x+exp(1))/2)+1))+(-945*sqrt(-a)*sqr
t(2)*ln(sqrt(2)*sqrt(a)+sqrt(a))-1302*sqrt(-a)*sqrt(2)-1350*sqrt(-a)*ln(sqr
t(2)*sqrt(a)+sqrt(a))-1808*sqrt(-a)+1890*sqrt(2)*sqrt(a)*atan((sqrt(2)*sqrt
(a)+sqrt(a))/sqrt(-a))-3840*sqrt(2)*sqrt(a)*atan(sqrt(a)/sqrt(-a))+2700*sqr
t(a)*atan((sqrt(2)*sqrt(a)+sqrt(a))/sqrt(-a))-5376*sqrt(a)*atan(sqrt(a)/sqr
t(-a)))/(672*a^2*sqrt(-a)*sqrt(2)*sqrt(a)+960*a^2*sqrt(-a)*sqrt(a))*sign(ta
n((f*x+exp(1))/2)+1))
```

maple [A] time = 1.01, size = 182, normalized size = 0.95

$$\frac{\left(1+\sin\left(fx+e\right)\right)\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\,\left(-96\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\ \sqrt{2}}{2\sqrt{a}}\right)a^{5}\left(\sin^{3}\left(fx+e\right)\right)-57\left(-a\left(\sin\right.\right.\right.}{24a^{\frac{15}{2}}\sin\left(fx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/24*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(-96*2^(1/2)*arctanh(1/2*(-a*
(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^5*sin(f*x+e)^3-57*(-a*(sin(f*x+e)-
1))^(5/2)*a^(5/2)+88*(-a*(sin(f*x+e)-1))^(3/2)*a^(7/2)-39*(-a*(sin(f*x+e)-1
))^(1/2)*a^(9/2)+135*a^5*arctanh((-a*(sin(f*x+e)-1))^(1/2)/a^(1/2))*sin(f*x
+e)^3)/a^(15/2)/sin(f*x+e)^3/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot\left(e+f\,x\right)^4}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4/(a + a*sin(e + f*x))^(5/2),x)
```

```
[Out] int(cot(e + f*x)^4/(a + a*sin(e + f*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4\left(e+fx\right)}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral(cot(e + f*x)**4/(a*(sin(e + f*x) + 1))**(5/2), x)
```

3.115 $\int \sqrt[3]{a+a\sin(e+fx)}\,\tan^4(e+fx)\,dx$

Optimal. Leaf size=982

$$-\frac{3\sin^2(e+fx)\tan(e+fx)a^2}{f(a-a\sin(e+fx))(\sin(e+fx)a+a)^{2/3}}+\frac{3\sin(e+fx)\tan(e+fx)a^2}{2f(a-a\sin(e+fx))(\sin(e+fx)a+a)^{2/3}}-\frac{\sec(e+fx)\left(65a^2-14\right.}{42f(a-a\sin(e+fx))(\mathrm{si}}$$

```
[Out] -361/126*sec(f*x+e)*(a+a*sin(f*x+e))^(1/3)/f+361/63*sec(f*x+e)*(1-sin(f*x+e
))*(a+a*sin(f*x+e))^(1/3)/f-1/42*sec(f*x+e)*(65*a^2-142*a^2*sin(f*x+e))/f/(
a-a*sin(f*x+e))/(a+a*sin(f*x+e))^(2/3)+361/63*sec(f*x+e)*(1-sin(f*x+e))*(a+
a*sin(f*x+e))^(2/3)*(1+3^(1/2))/f/(2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3)*(
1+3^(1/2)))-361/63*2^(1/3)*((2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3)*(1-3^(1
/2)))^2/(2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3)*(1+3^(1/2)))^2)^(1/2)/(2^(1
/3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3)*(1-3^(1/2)))*(2^(1/3)*a^(1/3)-(a+a*sin(f
*x+e))^(1/3)*(1+3^(1/2)))*EllipticE((1-(2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1
/3)*(1-3^(1/2)))^2/(2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3)*(1+3^(1/2)))^2)^
(1/2),1/4*6^(1/2)+1/4*2^(1/2))*sec(f*x+e)*(a+a*sin(f*x+e))^(2/3)*(2^(1/3)*a
^(1/3)-(a+a*sin(f*x+e))^(1/3))*((2^(2/3)*a^(2/3)+2^(1/3)*a^(1/3)*(a+a*sin(f
*x+e))^(1/3)+(a+a*sin(f*x+e))^(2/3))/(2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3
)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(2/3)/f/(-(a+a*sin(f*x+e))^(1/3)*(2^(1/3)
*a^(1/3)-(a+a*sin(f*x+e))^(1/3))/(2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3)*(1
+3^(1/2)))^2)^(1/2)-361/378*((2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3)*(1-3^(
1/2)))^2/(2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3)*(1+3^(1/2)))^2)^(1/2)/(2^(
1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3)*(1-3^(1/2)))*(2^(1/3)*a^(1/3)-(a+a*sin(
f*x+e))^(1/3)*(1+3^(1/2)))*EllipticF((1-(2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(
1/3)*(1-3^(1/2)))^2/(2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3)*(1+3^(1/2)))^2)
^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*sec(f*x+e)*(a+a*sin(f*x+e))^(2/3)*(2^(1/3)*
a^(1/3)-(a+a*sin(f*x+e))^(1/3))*(1-3^(1/2))*((2^(2/3)*a^(2/3)+2^(1/3)*a^(1/
3)*(a+a*sin(f*x+e))^(1/3)+(a+a*sin(f*x+e))^(2/3))/(2^(1/3)*a^(1/3)-(a+a*sin
(f*x+e))^(1/3)*(1+3^(1/2)))^2)^(1/2)*2^(1/3)*3^(3/4)/a^(2/3)/f/(-(a+a*sin(f
*x+e))^(1/3)*(2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3))/(2^(1/3)*a^(1/3)-(a+a
*sin(f*x+e))^(1/3)*(1+3^(1/2)))^2)^(1/2)+3/2*a^2*sin(f*x+e)*tan(f*x+e)/f/(a
-a*sin(f*x+e))/(a+a*sin(f*x+e))^(2/3)-3*a^2*sin(f*x+e)^2*tan(f*x+e)/f/(a-a*
sin(f*x+e))/(a+a*sin(f*x+e))^(2/3)
```

Rubi [A] time = 1.27, antiderivative size = 982, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23,

$\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {2719, 100, 153, 144, 51, 63, 308, 225, 1881}

$$-\frac{3\sin^2(e+fx)\tan(e+fx)a^2}{f(a-a\sin(e+fx))(\sin(e+fx)a+a)^{2/3}}+\frac{3\sin(e+fx)\tan(e+fx)a^2}{2f(a-a\sin(e+fx))(\sin(e+fx)a+a)^{2/3}}-\frac{\sec(e+fx)\left(65a^2-1\right.}{42f(a-a\sin(e+fx))(}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(1/3)*Tan[e + f*x]^4,x]
```

```
[Out] (-361*Sec[e + f*x]*(a + a*Sin[e + f*x])^(1/3))/(126*f) + (361*Sec[e + f*x]*
(1 - Sin[e + f*x])*(a + a*Sin[e + f*x])^(1/3))/(63*f) - (Sec[e + f*x]*(65*a
^2 - 142*a^2*Sin[e + f*x]))/(42*f*(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])
^(2/3)) + (361*(1 + Sqrt[3])*Sec[e + f*x]*(1 - Sin[e + f*x])*(a + a*Sin[e +
 f*x])^(2/3))/(63*f*(2^(1/3)*a^(1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*x])^(
1/3))) - (361*2^(1/3)*EllipticE[ArcCos[(2^(1/3)*a^(1/3) - (1 - Sqrt[3])*(a
+ a*Sin[e + f*x])^(1/3))/(2^(1/3)*a^(1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*
x])^(1/3))], (2 + Sqrt[3])/4]*Sec[e + f*x]*(a + a*Sin[e + f*x])^(2/3)*(2^(1
/3)*a^(1/3) - (a + a*Sin[e + f*x])^(1/3))*Sqrt[(2^(2/3)*a^(2/3) + 2^(1/3)*a
^(1/3)*(a + a*Sin[e + f*x])^(1/3) + (a + a*Sin[e + f*x])^(2/3))/(2^(1/3)*a^
(1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*x])^(1/3))^2])/(21*3^(3/4)*a^(2/3)*f
*Sqrt[-(((a + a*Sin[e + f*x])^(1/3)*(2^(1/3)*a^(1/3) - (a + a*Sin[e + f*x])
^(1/3)))/(2^(1/3)*a^(1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*x])^(1/3))^2)])
- (361*(1 - Sqrt[3])*EllipticF[ArcCos[(2^(1/3)*a^(1/3) - (1 - Sqrt[3])*(a +
 a*Sin[e + f*x])^(1/3))/(2^(1/3)*a^(1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*x
])^(1/3))], (2 + Sqrt[3])/4]*Sec[e + f*x]*(a + a*Sin[e + f*x])^(2/3)*(2^(1/
3)*a^(1/3) - (a + a*Sin[e + f*x])^(1/3))*Sqrt[(2^(2/3)*a^(2/3) + 2^(1/3)*a^
(1/3)*(a + a*Sin[e + f*x])^(1/3) + (a + a*Sin[e + f*x])^(2/3))/(2^(1/3)*a^(
1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*x])^(1/3))^2])/(63*2^(2/3)*3^(1/4)*a^
(2/3)*f*Sqrt[-(((a + a*Sin[e + f*x])^(1/3)*(2^(1/3)*a^(1/3) - (a + a*Sin[e
+ f*x])^(1/3)))/(2^(1/3)*a^(1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*x])^(1/3)
)^2)]) + (3*a^2*Sin[e + f*x]*Tan[e + f*x])/(2*f*(a - a*Sin[e + f*x])*(a + a
*Sin[e + f*x])^(2/3)) - (3*a^2*Sin[e + f*x]^2*Tan[e + f*x])/(f*(a - a*Sin[e
 + f*x])*(a + a*Sin[e + f*x])^(2/3))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 144

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))
*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*c*d*e*g*(n + 1) + a^2*c*d*f*
h*(n + 1) + a*b*(d^2*e*g*(m + 1) + c^2*f*h*(m + 1) - c*d*(f*g + e*h)*(m + n
 + 2)) + (a^2*d^2*f*h*(n + 1) - a*b*d^2*(f*g + e*h)*(n + 1) + b^2*(c^2*f*h*
(m + 1) - c*d*(f*g + e*h)*(m + 1) + d^2*e*g*(m + n + 2)))*x)*(a + b*x)^(m +
 1)*(c + d*x)^(n + 1))/(b*d*(b*c - a*d)^2*(m + 1)*(n + 1)), x] - Dist[(a^2*
d^2*f*h*(2 + 3*n + n^2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
 + n + 3)) + b^2*(c^2*f*h*(2 + 3*m + m^2) - c*d*(f*g + e*h)*(m + 1)*(m + n
+ 3) + d^2*e*g*(6 + m^2 + 5*n + n^2 + m*(2*n + 5))))/(b*d*(b*c - a*d)^2*(m
+ 1)*(n + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, h}, x] && LtQ[m, -1] && LtQ[n, -1]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rule 2719

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[(Sqrt[a + b*Sin[e + f*x]]*Sqrt[a - b*Sin[e + f*x]])/(b
*f*Cos[e + f*x]), Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] && IntegerQ[p/2]
```

Rubi steps

$$\int \sqrt[3]{a+a\sin(e+fx)}\,\tan^4(e+fx)\,dx = \frac{\left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}\right)\text{Subst}\left(\int \frac{x^4}{(a-x)^{5/2}(a+x}\right.}{af}$$

$$= -\frac{3a^2\sin^2(e+fx)\tan(e+fx)}{f(a-a\sin(e+fx))(a+a\sin(e+fx))^{2/3}} - \frac{\left(3\sec(e+fx)\sqrt{a-a\sin(e}\right.}{}$$

$$= \frac{3a^2\sin(e+fx)\tan(e+fx)}{2f(a-a\sin(e+fx))(a+a\sin(e+fx))^{2/3}} - \frac{3a^2\sin^2(e+fx)\tan}{f(a-a\sin(e+fx))(a+a}$$

$$= -\frac{\sec(e+fx)\left(65a^2-142a^2\sin(e+fx)\right)}{42f(a-a\sin(e+fx))(a+a\sin(e+fx))^{2/3}} + \frac{3a^2\sin(e+fx)}{2f(a-a\sin(e+fx))(a}$$

$$= -\frac{361\sec(e+fx)\sqrt[3]{a+a\sin(e+fx)}}{126f} - \frac{\sec(e+fx)\left(65a^2-142a^2\sin\right.}{42f(a-a\sin(e+fx))(a+a\sin}$$

$$= -\frac{361\sec(e+fx)\sqrt[3]{a+a\sin(e+fx)}}{126f} + \frac{361\sec(e+fx)(1-\sin(e+fx))}{63f}$$

$$= -\frac{361\sec(e+fx)\sqrt[3]{a+a\sin(e+fx)}}{126f} + \frac{361\sec(e+fx)(1-\sin(e+fx))}{63f}$$

$$= -\frac{361\sec(e+fx)\sqrt[3]{a+a\sin(e+fx)}}{126f} + \frac{361\sec(e+fx)(1-\sin(e+fx))}{63f}$$

$$= -\frac{361\sec(e+fx)\sqrt[3]{a+a\sin(e+fx)}}{126f} + \frac{361\sec(e+fx)(1-\sin(e+fx))}{63f}$$

<u>**Mathematica**</u> [C] time = 3.24, size = 318, normalized size = 0.32

$$\frac{\sqrt[3]{a(\sin(e+fx)+1)}\left(3\left(-172\tan(e+fx)-3\sec^3(e+fx)+86\sec(e+fx)+24\tan(e+fx)\sec^2(e+fx)+361\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^(1/3)*Tan[e + f*x]^4,x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(1/3)*(((1083/10 + (1083*I)/10)*(-1)^(3/4)*(20*E^(I
*(e + f*x))*Sqrt[Cos[(2*e + Pi + 2*f*x)/4]^2]*Hypergeometric2F1[-1/3, 1/3,
2/3, (-I)/E^(I*(e + f*x))] - 2*(1 + I/E^(I*(e + f*x)))^(2/3)*(1 + E^((2*I)*
(e + f*x)))*Hypergeometric2F1[1/2, 5/6, 11/6, Sin[(2*e + Pi + 2*f*x)/4]^2]
+ (5*I)*Hypergeometric2F1[1/3, 2/3, 5/3, (-I)/E^(I*(e + f*x))]*Sqrt[2 - 2*S
in[e + f*x]]))/(Sqrt[2]*E^(I*(e + f*x))*(1 + I/E^(I*(e + f*x)))^(2/3)*Sqrt[
(I*(-I + E^(I*(e + f*x)))^2)/E^(I*(e + f*x))]) + 3*(361 + 86*Sec[e + f*x] -
 3*Sec[e + f*x]^3 - 172*Tan[e + f*x] + 24*Sec[e + f*x]^2*Tan[e + f*x])))/(1
89*f)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{\frac{1}{3}}\tan\left(fx+e\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^(1/3)*tan(f*x + e)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{1}{3}}\tan\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(1/3)*tan(f*x + e)^4, x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int\left(a+a\sin\left(fx+e\right)\right)^{\frac{1}{3}}\left(\tan^{4}\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^4,x)
```

```
[Out] int((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{1}{3}}\tan\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^4,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^(1/3)*tan(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \tan\left(e+f\,x\right)^4\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(a + a*sin(e + f*x))^(1/3),x)
[Out] int(tan(e + f*x)^4*(a + a*sin(e + f*x))^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{a\left(\sin\left(e+fx\right)+1\right)}\,\tan^4\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/3)*tan(f*x+e)**4,x)
[Out] Integral((a*(sin(e + f*x) + 1))**(1/3)*tan(e + f*x)**4, x)
```

3.116 $\int \sqrt[3]{a+a\sin(e+fx)}\,\tan^2(e+fx)\,dx$

Optimal. Leaf size=123

$$-\frac{5a\sqrt[6]{\sin(e+fx)+1}\,\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{3\sqrt[6]{2}\,f(a\sin(e+fx)+a)^{2/3}}-\frac{3\sec(e+fx)(a\sin(e+fx)+a)^{4/3}}{af}+\frac{7\sec(e}{}$$

[Out] -5/6*a*cos(f*x+e)*hypergeom([1/2, 7/6],[3/2],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(1/6)*2^(5/6)/f/(a+a*sin(f*x+e))^(2/3)+7*sec(f*x+e)*(a+a*sin(f*x+e))^(1/3)/f-3*sec(f*x+e)*(a+a*sin(f*x+e))^(4/3)/a/f

Rubi [A] time = 0.19, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2713, 2855, 2652, 2651}

$$-\frac{5a\sqrt[6]{\sin(e+fx)+1}\,\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{3\sqrt[6]{2}\,f(a\sin(e+fx)+a)^{2/3}}-\frac{3\sec(e+fx)(a\sin(e+fx)+a)^{4/3}}{af}+\frac{7\sec(e}{}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^(1/3)*Tan[e + f*x]^2,x]

[Out] (-5*a*Cos[e + f*x]*Hypergeometric2F1[1/2, 7/6, 3/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(1/6))/(3*2^(1/6)*f*(a + a*Sin[e + f*x])^(2/3)) + (7*Sec[e + f*x]*(a + a*Sin[e + f*x])^(1/3))/f - (3*Sec[e + f*x]*(a + a*Sin[e + f*x])^(4/3))/(a*f)

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n + 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]
```

Rule 2713

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> -Simp[(a + b*Sin[e + f*x])^(m + 1)/(b*f*m*Cos[e + f*x]), x] +
Dist[1/(b*m), Int[((a + b*Sin[e + f*x])^m*(b*(m + 1) + a*Sin[e + f*x]))/Cos
[e + f*x]^2, x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !
IntegerQ[m] &&  !LtQ[m, 0]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rubi steps

$$\int \sqrt[3]{a+a\sin(e+fx)}\,\tan^2(e+fx)\,dx = -\frac{3\sec(e+fx)(a+a\sin(e+fx))^{4/3}}{af} + \frac{3\int \sec^2(e+fx)\sqrt[3]{a+a\sin(e+f}{a}$$
$$= \frac{7\sec(e+fx)\sqrt[3]{a+a\sin(e+fx)}}{f} - \frac{3\sec(e+fx)(a+a\sin(e+fx))^{4/3}}{af} +$$
$$= \frac{7\sec(e+fx)\sqrt[3]{a+a\sin(e+fx)}}{f} - \frac{3\sec(e+fx)(a+a\sin(e+fx))^{4/3}}{af} +$$
$$= -\frac{5a\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)\sqrt[6]{1+\sin(e+fx)}}{3\sqrt[6]{2}\,f(a+a\sin(e+fx))^{2/3}} + \frac{7\,\mathrm{s}}{}$$

Mathematica [C] time = 2.71, size = 290, normalized size = 2.36

$$\frac{\sqrt[3]{a(\sin(e+fx)+1)}\left(-3(-2\tan(e+fx)+\sec(e+fx)+5)+\frac{\left(\frac{3}{2}+\frac{3i}{2}\right)(-1)^{3/4}e^{-i(e+fx)}\left(2\left(1+ie^{-i(e+fx)}\right)^{2/3}\left(1+e^{2i(e+fx)}\right){}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6}\right.\right.}{}\right.}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(1/3)*Tan[e + f*x]^2,x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(1/3)*(((3/2 + (3*I)/2)*(-1)^(3/4)*(-20*E^(I*(e + f
*x))*Sqrt[Cos[(2*e + Pi + 2*f*x)/4]^2]*Hypergeometric2F1[-1/3, 1/3, 2/3, (-
```

```
I)/E^(I*(e + f*x))] + 2*(1 + I/E^(I*(e + f*x)))^(2/3)*(1 + E^((2*I)*(e + f*
x)))*Hypergeometric2F1[1/2, 5/6, 11/6, Sin[(2*e + Pi + 2*f*x)/4]^2] - (5*I)
*Hypergeometric2F1[1/3, 2/3, 5/3, (-I)/E^(I*(e + f*x))]*Sqrt[2 - 2*Sin[e +
f*x]]))/(Sqrt[2]*E^(I*(e + f*x))*(1 + I/E^(I*(e + f*x)))^(2/3)*Sqrt[(I*(-I
+ E^(I*(e + f*x)))^2)/E^(I*(e + f*x))]) - 3*(5 + Sec[e + f*x] - 2*Tan[e + f
*x])))/(3*f)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{\frac{1}{3}}\tan\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^2,x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)^(1/3)*tan(f*x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{1}{3}}\tan\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^2,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^(1/3)*tan(f*x + e)^2, x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int\left(a+a\sin\left(fx+e\right)\right)^{\frac{1}{3}}\left(\tan^{2}\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^2,x)
[Out] int((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{1}{3}}\tan\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(1/3)*tan(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan\left(e+f\,x\right)^2\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(a + a*sin(e + f*x))^(1/3),x)
[Out] int(tan(e + f*x)^2*(a + a*sin(e + f*x))^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{a\left(\sin\left(e+fx\right)+1\right)}\,\tan^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/3)*tan(f*x+e)**2,x)
[Out] Integral((a*(sin(e + f*x) + 1))**(1/3)*tan(e + f*x)**2, x)
```

3.117 $\int \cot^2(e+fx)\sqrt[3]{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=80

$$\frac{6\sqrt{2}\sqrt{1-\sin(e+fx)}\sec(e+fx)(a\sin(e+fx)+a)^{7/3}F_1\left(\frac{11}{6};-\frac{1}{2},2;\frac{17}{6};\frac{1}{2}(\sin(e+fx)+1),\sin(e+fx)+1\right)}{11a^2f}$$

```
[Out] 6/11*AppellF1(11/6,2,-1/2,17/6,1+sin(f*x+e),1/2+1/2*sin(f*x+e))*sec(f*x+e)*
(a+a*sin(f*x+e))^(7/3)*2^(1/2)*(1-sin(f*x+e))^(1/2)/a^2/f
```

Rubi [A] time = 0.10, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2719, 137, 136}

$$\frac{6\sqrt{2}\sqrt{1-\sin(e+fx)}\sec(e+fx)(a\sin(e+fx)+a)^{7/3}F_1\left(\frac{11}{6};-\frac{1}{2},2;\frac{17}{6};\frac{1}{2}(\sin(e+fx)+1),\sin(e+fx)+1\right)}{11a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(a + a*Sin[e + f*x])^(1/3),x]
```

```
[Out] (6*Sqrt[2]*AppellF1[11/6, -1/2, 2, 17/6, (1 + Sin[e + f*x])/2, 1 + Sin[e +
f*x]]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(7/3))/(11*a
^2*f)
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !GtQ[b/(b*c
- a*d), 0] && !SimplerQ[c + d*x, a + b*x]
```

Rule 2719

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[(Sqrt[a + b*Sin[e + f*x]]*Sqrt[a - b*Sin[e + f*x]])/(b
*f*Cos[e + f*x]), Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] && IntegerQ[p/2]
```

Rubi steps

$$\int \cot^2(e+fx)\sqrt[3]{a+a\sin(e+fx)}\,dx = \frac{\left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{a-x}\,(a+x)^{5/}}{x^2}\right.}{af}$$

$$= \frac{\left(\sqrt{2}\,\sec(e+fx)(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}\right)\text{Subst}\left(\int \frac{(a+x)^{5/6}}{x^2}\right.}{af\sqrt{\frac{a-a\sin(e+fx)}{a}}}$$

$$= \frac{6\sqrt{2}\,F_1\left(\frac{11}{6};-\frac{1}{2},2;\frac{17}{6};\frac{1}{2}(1+\sin(e+fx)),1+\sin(e+fx)\right)\sec(e+fx)\sqrt{1}}{11a^2f}$$

Mathematica [C] time = 23.71, size = 2692, normalized size = 33.65

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^2*(a + a*Sin[e + f*x])^(1/3),x]
```

```
[Out] ((15/2 + (15*I)/2)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(e + f
*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])]*(a*(1 + Sin[e + f*x]))^(1/3))/
(f*((5 + 5*I)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2
]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])] + (AppellF1[5/3, 1/3, 4/3, 8/3, (1/
2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])] + I*Ap
pellF1[5/3, 4/3, 1/3, 8/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*
(1 + Cot[(e + f*x)/2])])*(1 + Cot[(e + f*x)/2]))) + ((-4 - Cot[e + f*x])*(a
*(1 + Sin[e + f*x]))^(1/3))/f + ((5/2 + (5*I)/2)*AppellF1[2/3, 1/3, 1/3, 5/
3, (1/2 + I/2)*(1 + Tan[(e + f*x)/2]), (1/2 - I/2)*(1 + Tan[(e + f*x)/2])]*
(a*(1 + Sin[e + f*x]))^(1/3))/(f*((5 + 5*I)*AppellF1[2/3, 1/3, 1/3, 5/3, (1
/2 + I/2)*(1 + Tan[(e + f*x)/2]), (1/2 - I/2)*(1 + Tan[(e + f*x)/2])] + (Ap
pellF1[5/3, 1/3, 4/3, 8/3, (1/2 + I/2)*(1 + Tan[(e + f*x)/2]), (1/2 - I/2)*
(1 + Tan[(e + f*x)/2])] + I*AppellF1[5/3, 4/3, 1/3, 8/3, (1/2 + I/2)*(1 + T
an[(e + f*x)/2]), (1/2 - I/2)*(1 + Tan[(e + f*x)/2])])*(1 + Tan[(e + f*x)/2
]))) + (Cos[(3*(e + f*x))/2]*Csc[(e + f*x)/2]*Sec[(e + f*x)/2]*(a*(1 + Sin[
```

```
e + f*x]))^(1/3)*((1 + Tan[(e + f*x)/2])/Sqrt[Sec[(e + f*x)/2]^2])^(2/3)*(8
 + (1 + I)*2^(2/3)*(((1 - I)*(I + Cot[(e + f*x)/2]))/(1 + Cot[(e + f*x)/2])
)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(e + f*x)/2
])/(2 + 2*Tan[(e + f*x)/2])]*(I + Tan[(e + f*x)/2]) - AppellF1[2/3, 1/3, 1/
3, 5/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/
2])]*((2 + 2*I) - (2 - 2*I)*Cot[(e + f*x)/2])^(1/3)*((-1 - I)*(I + Cot[(e +
 f*x)/2]))^(1/3)*(1 + Tan[(e + f*x)/2])))/(f*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])*(1 + Tan[(e + f*x)/2])*((-3*Sec[(e + f*x)/2]^2*((1 + Tan[(e + f*x)/
2])/Sqrt[Sec[(e + f*x)/2]^2])^(2/3)*(8 + (1 + I)*2^(2/3)*(((1 - I)*(I + Cot
[(e + f*x)/2]))/(1 + Cot[(e + f*x)/2]))^(1/3)*Hypergeometric2F1[1/3, 2/3, 5
/3, ((1 + I) + (1 - I)*Tan[(e + f*x)/2])/(2 + 2*Tan[(e + f*x)/2])]*(I + Tan
[(e + f*x)/2]) - AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(e + f*x
)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])]*((2 + 2*I) - (2 - 2*I)*Cot[(e +
f*x)/2])^(1/3)*((-1 - I)*(I + Cot[(e + f*x)/2]))^(1/3)*(1 + Tan[(e + f*x)/2
])))/(4*(1 + Tan[(e + f*x)/2])^2) + ((8 + (1 + I)*2^(2/3)*(((1 - I)*(I + Co
t[(e + f*x)/2]))/(1 + Cot[(e + f*x)/2]))^(1/3)*Hypergeometric2F1[1/3, 2/3,
5/3, ((1 + I) + (1 - I)*Tan[(e + f*x)/2])/(2 + 2*Tan[(e + f*x)/2])]*(I + Ta
n[(e + f*x)/2]) - AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(e + f*
x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])]*((2 + 2*I) - (2 - 2*I)*Cot[(e +
 f*x)/2])^(1/3)*((-1 - I)*(I + Cot[(e + f*x)/2]))^(1/3)*(1 + Tan[(e + f*x)/
2]))*(Sqrt[Sec[(e + f*x)/2]^2]/2 - (Tan[(e + f*x)/2]*(1 + Tan[(e + f*x)/2])
)/(2*Sqrt[Sec[(e + f*x)/2]^2])))/((1 + Tan[(e + f*x)/2])*((1 + Tan[(e + f*x
)/2])/Sqrt[Sec[(e + f*x)/2]^2])^(1/3)) + (3*((1 + Tan[(e + f*x)/2])/Sqrt[Se
c[(e + f*x)/2]^2])^(2/3)*(-1/2*(AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1
 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])]*((2 + 2*I) - (2 -
 2*I)*Cot[(e + f*x)/2])^(1/3)*((-1 - I)*(I + Cot[(e + f*x)/2]))^(1/3)*Sec[(
e + f*x)/2]^2) + ((1 + I)*(((1 - I)*(I + Cot[(e + f*x)/2]))/(1 + Cot[(e + f
*x)/2]))^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(e +
 f*x)/2])/(2 + 2*Tan[(e + f*x)/2])]*Sec[(e + f*x)/2]^2)/2^(1/3) + ((1/3 + I
/3)*2^(2/3)*(((1/2 - I/2)*(I + Cot[(e + f*x)/2])*Csc[(e + f*x)/2]^2)/(1 + C
ot[(e + f*x)/2])^2 - ((1/2 - I/2)*Csc[(e + f*x)/2]^2)/(1 + Cot[(e + f*x)/2]
))*Hypergeometric2F1[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(e + f*x)/2])/(2
 + 2*Tan[(e + f*x)/2])]*(I + Tan[(e + f*x)/2]))/(((1 - I)*(I + Cot[(e + f*x
)/2]))/(1 + Cot[(e + f*x)/2]))^(2/3) - ((1/6 + I/6)*AppellF1[2/3, 1/3, 1/3,
 5/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2]
)]*((2 + 2*I) - (2 - 2*I)*Cot[(e + f*x)/2])^(1/3)*Csc[(e + f*x)/2]^2*(1 + T
an[(e + f*x)/2]))/((-1 - I)*(I + Cot[(e + f*x)/2]))^(2/3) - ((1/3 - I/3)*Ap
pellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*
(1 + Cot[(e + f*x)/2])]*((-1 - I)*(I + Cot[(e + f*x)/2]))^(1/3)*Csc[(e + f*
x)/2]^2*(1 + Tan[(e + f*x)/2]))/((2 + 2*I) - (2 - 2*I)*Cot[(e + f*x)/2])^(2
/3) - ((2 + 2*I) - (2 - 2*I)*Cot[(e + f*x)/2])^(1/3)*((-1 - I)*(I + Cot[(e
+ f*x)/2]))^(1/3)*((-1/30 + I/30)*AppellF1[5/3, 1/3, 4/3, 8/3, (1/2 + I/2)*
(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])]*Csc[(e + f*x)/2
]^2 - (1/30 + I/30)*AppellF1[5/3, 4/3, 1/3, 8/3, (1/2 + I/2)*(1 + Cot[(e +
f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])]*Csc[(e + f*x)/2]^2)*(1 + Tan[
```

```
(e + f*x)/2]) + ((2/3 + (2*I)/3)*2^(2/3)*(((1 - I)*(I + Cot[(e + f*x)/2]))/
(1 + Cot[(e + f*x)/2]))^(1/3)*(I + Tan[(e + f*x)/2])*(2 + 2*Tan[(e + f*x)/2
])*(-((Sec[(e + f*x)/2]^2*((1 + I) + (1 - I)*Tan[(e + f*x)/2]))/(2 + 2*Tan[
(e + f*x)/2])^2) + ((1/2 - I/2)*Sec[(e + f*x)/2]^2)/(2 + 2*Tan[(e + f*x)/2]
))*(-Hypergeometric2F1[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(e + f*x)/2])/
(2 + 2*Tan[(e + f*x)/2])] + (1 - ((1 + I) + (1 - I)*Tan[(e + f*x)/2])/(2 +
2*Tan[(e + f*x)/2]))^(-1/3)))/((1 + I) + (1 - I)*Tan[(e + f*x)/2])))/(2*(1
+ Tan[(e + f*x)/2]))))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \sin\left(fx+e\right)+a\right)^{\frac{1}{3}} \cot\left(fx+e\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(1/3)*cot(f*x + e)^2, x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \left(\cot^{2}\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+a*sin(f*x+e))^(1/3),x)
```

```
[Out] int(cot(f*x+e)^2*(a+a*sin(f*x+e))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \sin\left(fx+e\right)+a\right)^{\frac{1}{3}} \cot\left(fx+e\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(1/3),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^(1/3)*cot(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(e+f\,x\right)^2\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(a + a*sin(e + f*x))^(1/3),x)
[Out] int(cot(e + f*x)^2*(a + a*sin(e + f*x))^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{a\left(\sin\left(e+fx\right)+1\right)}\,\cot^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+a*sin(f*x+e))**(1/3),x)
[Out] Integral((a*(sin(e + f*x) + 1))**(1/3)*cot(e + f*x)**2, x)
```

3.118 $\int \cot^4(e+fx)\sqrt[3]{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=80

$$\frac{12\sqrt{2}\sqrt{1-\sin(e+fx)}\sec(e+fx)(a\sin(e+fx)+a)^{10/3}F_1\left(\frac{17}{6};-\frac{3}{2},4;\frac{23}{6};\frac{1}{2}(\sin(e+fx)+1),\sin(e+fx)+1\right)}{17a^3f}$$

```
[Out] 12/17*AppellF1(17/6,4,-3/2,23/6,1+sin(f*x+e),1/2+1/2*sin(f*x+e))*sec(f*x+e)
*(a+a*sin(f*x+e))^(10/3)*2^(1/2)*(1-sin(f*x+e))^(1/2)/a^3/f
```

Rubi [A] time = 0.09, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2719, 137, 136}

$$\frac{12\sqrt{2}\sqrt{1-\sin(e+fx)}\sec(e+fx)(a\sin(e+fx)+a)^{10/3}F_1\left(\frac{17}{6};-\frac{3}{2},4;\frac{23}{6};\frac{1}{2}(\sin(e+fx)+1),\sin(e+fx)+1\right)}{17a^3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*(a + a*Sin[e + f*x])^(1/3),x]
```

```
[Out] (12*Sqrt[2]*AppellF1[17/6, -3/2, 4, 23/6, (1 + Sin[e + f*x])/2, 1 + Sin[e +
 f*x]]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(10/3))/(17
*a^3*f)
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !GtQ[b/(b*c
- a*d), 0] && !SimplerQ[c + d*x, a + b*x]
```

Rule 2719

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[(Sqrt[a + b*Sin[e + f*x]]*Sqrt[a - b*Sin[e + f*x]])/(b
*f*Cos[e + f*x]), Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] && IntegerQ[p/2]
```

Rubi steps

$$\int \cot^4(e+fx)\sqrt[3]{a+a\sin(e+fx)}\,dx = \frac{\left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}\right)\text{Subst}\left(\int \frac{(a-x)^{3/2}(a+}{x^4}\right.}{af}$$

$$= \frac{\left(2\sqrt{2}\,\sec(e+fx)(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}\right)\text{Subst}\left(\int \frac{(a+x)}{}\right.}{f\sqrt{\frac{a-a\sin(e+fx)}{a}}}$$

$$= \frac{12\sqrt{2}\,F_1\left(\frac{17}{6};-\frac{3}{2},4;\frac{23}{6};\frac{1}{2}(1+\sin(e+fx)),1+\sin(e+fx)\right)\sec(e+fx)}{17a^3f}$$

Mathematica [C] time = 22.33, size = 2796, normalized size = 34.95

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^4*(a + a*Sin[e + f*x])^(1/3),x]
```

```
[Out] ((239/54 + (77*Cot[e + f*x])/54 - (Cot[e + f*x]*Csc[e + f*x])/18 - (Cot[e +
 f*x]*Csc[e + f*x]^2)/3)*(a*(1 + Sin[e + f*x]))^(1/3))/f - ((70/9 + (70*I)/
9)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 -
I/2)*(1 + Cot[(e + f*x)/2])]*(a*(1 + Sin[e + f*x]))^(1/3)*(1 + Tan[(e + f*x
)/2]))/(f*((5 + 5*I)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(e +
 f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])]*Sec[(e + f*x)/2] + AppellF1[
5/3, 1/3, 4/3, 8/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1 + Co
t[(e + f*x)/2])]*(Csc[(e + f*x)/2] + Sec[(e + f*x)/2]) + I*AppellF1[5/3, 4/
3, 1/3, 8/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e +
f*x)/2])]*(Csc[(e + f*x)/2] + Sec[(e + f*x)/2]))*(Cos[(e + f*x)/2] + Sin[(e
 + f*x)/2])) - ((355/108 + (355*I)/108)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 +
 I/2)*(1 + Tan[(e + f*x)/2]), (1/2 - I/2)*(1 + Tan[(e + f*x)/2])]*(a*(1 + S
in[e + f*x]))^(1/3))/(f*((5 + 5*I)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)
*(1 + Tan[(e + f*x)/2]), (1/2 - I/2)*(1 + Tan[(e + f*x)/2])] + (AppellF1[5/
```

```
3, 1/3, 4/3, 8/3, (1/2 + I/2)*(1 + Tan[(e + f*x)/2]), (1/2 - I/2)*(1 + Tan[
(e + f*x)/2])] + I*AppellF1[5/3, 4/3, 1/3, 8/3, (1/2 + I/2)*(1 + Tan[(e + f
*x)/2]), (1/2 - I/2)*(1 + Tan[(e + f*x)/2])])*(1 + Tan[(e + f*x)/2]))) - (2
39*Cos[(3*(e + f*x))/2]*Csc[e + f*x]*(a*(1 + Sin[e + f*x]))^(1/3)*((1 + Tan
[(e + f*x)/2])/Sqrt[Sec[(e + f*x)/2]^2])^(2/3)*(8 + (1 + I)*2^(2/3)*(((1 -
I)*(I + Cot[(e + f*x)/2]))/(1 + Cot[(e + f*x)/2]))^(1/3)*Hypergeometric2F1[
1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(e + f*x)/2])/(2 + 2*Tan[(e + f*x)/2]
)]*(I + Tan[(e + f*x)/2]) - AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + C
ot[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])]*((2 + 2*I) - (2 - 2*I
)*Cot[(e + f*x)/2])^(1/3)*((-1 - I)*(I + Cot[(e + f*x)/2]))^(1/3)*(1 + Tan[
(e + f*x)/2])))/(216*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(1 + Tan[(e +
f*x)/2])*((-3*Sec[(e + f*x)/2]^2*((1 + Tan[(e + f*x)/2])/Sqrt[Sec[(e + f*x)
/2]^2])^(2/3)*(8 + (1 + I)*2^(2/3)*(((1 - I)*(I + Cot[(e + f*x)/2]))/(1 + C
ot[(e + f*x)/2]))^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, ((1 + I) + (1 - I)
*Tan[(e + f*x)/2])/(2 + 2*Tan[(e + f*x)/2])]*(I + Tan[(e + f*x)/2]) - Appel
lF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1
+ Cot[(e + f*x)/2])]*((2 + 2*I) - (2 - 2*I)*Cot[(e + f*x)/2])^(1/3)*((-1 -
I)*(I + Cot[(e + f*x)/2]))^(1/3)*(1 + Tan[(e + f*x)/2])))/(8*(1 + Tan[(e +
f*x)/2])^2) + ((8 + (1 + I)*2^(2/3)*(((1 - I)*(I + Cot[(e + f*x)/2]))/(1 +
Cot[(e + f*x)/2]))^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, ((1 + I) + (1 - I
)*Tan[(e + f*x)/2])/(2 + 2*Tan[(e + f*x)/2])]*(I + Tan[(e + f*x)/2]) - Appe
llF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1
 + Cot[(e + f*x)/2])]*((2 + 2*I) - (2 - 2*I)*Cot[(e + f*x)/2])^(1/3)*((-1 -
 I)*(I + Cot[(e + f*x)/2]))^(1/3)*(1 + Tan[(e + f*x)/2]))*(Sqrt[Sec[(e + f*
x)/2]^2]/2 - (Tan[(e + f*x)/2]*(1 + Tan[(e + f*x)/2]))/(2*Sqrt[Sec[(e + f*x
)/2]^2])))/(2*(1 + Tan[(e + f*x)/2])*((1 + Tan[(e + f*x)/2])/Sqrt[Sec[(e +
f*x)/2]^2])^(1/3)) + (3*((1 + Tan[(e + f*x)/2])/Sqrt[Sec[(e + f*x)/2]^2])^(
2/3)*(-1/2*(AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2])
, (1/2 - I/2)*(1 + Cot[(e + f*x)/2])]*((2 + 2*I) - (2 - 2*I)*Cot[(e + f*x)/
2])^(1/3)*((-1 - I)*(I + Cot[(e + f*x)/2]))^(1/3)*Sec[(e + f*x)/2]^2) + ((1
 + I)*(((1 - I)*(I + Cot[(e + f*x)/2]))/(1 + Cot[(e + f*x)/2]))^(1/3)*Hyper
geometric2F1[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(e + f*x)/2])/(2 + 2*Tan
[(e + f*x)/2])]*Sec[(e + f*x)/2]^2)/2^(1/3) + ((1/3 + I/3)*2^(2/3)*(((1/2 -
 I/2)*(I + Cot[(e + f*x)/2])*Csc[(e + f*x)/2]^2)/(1 + Cot[(e + f*x)/2])^2 -
 ((1/2 - I/2)*Csc[(e + f*x)/2]^2)/(1 + Cot[(e + f*x)/2]))*Hypergeometric2F1
[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(e + f*x)/2])/(2 + 2*Tan[(e + f*x)/2
])]*(I + Tan[(e + f*x)/2]))/(((1 - I)*(I + Cot[(e + f*x)/2]))/(1 + Cot[(e +
 f*x)/2]))^(2/3) - ((1/6 + I/6)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1
 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])]*((2 + 2*I) - (2 -
 2*I)*Cot[(e + f*x)/2])^(1/3)*Csc[(e + f*x)/2]^2*(1 + Tan[(e + f*x)/2]))/((
-1 - I)*(I + Cot[(e + f*x)/2]))^(2/3) - ((1/3 - I/3)*AppellF1[2/3, 1/3, 1/3
, 5/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2
])]*((-1 - I)*(I + Cot[(e + f*x)/2]))^(1/3)*Csc[(e + f*x)/2]^2*(1 + Tan[(e
+ f*x)/2]))/((2 + 2*I) - (2 - 2*I)*Cot[(e + f*x)/2])^(2/3) - ((2 + 2*I) - (
2 - 2*I)*Cot[(e + f*x)/2])^(1/3)*((-1 - I)*(I + Cot[(e + f*x)/2]))^(1/3)*((
```

```
-1/30 + I/30)*AppellF1[5/3, 1/3, 4/3, 8/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2
]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])]*Csc[(e + f*x)/2]^2 - (1/30 + I/30)*
AppellF1[5/3, 4/3, 1/3, 8/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2
)*(1 + Cot[(e + f*x)/2])]*Csc[(e + f*x)/2]^2)*(1 + Tan[(e + f*x)/2]) + ((2/
3 + (2*I)/3)*2^(2/3)*(((1 - I)*(I + Cot[(e + f*x)/2]))/(1 + Cot[(e + f*x)/2
]))^(1/3)*(I + Tan[(e + f*x)/2])*(2 + 2*Tan[(e + f*x)/2])*(-((Sec[(e + f*x)
/2]^2*((1 + I) + (1 - I)*Tan[(e + f*x)/2]))/(2 + 2*Tan[(e + f*x)/2])^2) + (
(1/2 - I/2)*Sec[(e + f*x)/2]^2)/(2 + 2*Tan[(e + f*x)/2]))*(-Hypergeometric2
F1[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(e + f*x)/2])/(2 + 2*Tan[(e + f*x)
/2])] + (1 - ((1 + I) + (1 - I)*Tan[(e + f*x)/2])/(2 + 2*Tan[(e + f*x)/2]))
^(-1/3)))/((1 + I) + (1 - I)*Tan[(e + f*x)/2])))/(4*(1 + Tan[(e + f*x)/2]))
))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(1/3),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \sin\left(fx+e\right)+a\right)^{\frac{1}{3}} \cot\left(fx+e\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(1/3),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^(1/3)*cot(f*x + e)^4, x)
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \left(\cot^{4}\left(fx+e\right)\right) \left(a+a \sin\left(fx+e\right)\right)^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+a*sin(f*x+e))^(1/3),x)
[Out] int(cot(f*x+e)^4*(a+a*sin(f*x+e))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \sin\left(fx+e\right)+a\right)^{\frac{1}{3}} \cot\left(fx+e\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(1/3)*cot(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(e+f\,x\right)^4\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4*(a + a*sin(e + f*x))^(1/3),x)
```

```
[Out] int(cot(e + f*x)^4*(a + a*sin(e + f*x))^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{a\left(\sin\left(e+fx\right)+1\right)}\,\cot^4\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+a*sin(f*x+e))**(1/3),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(1/3)*cot(e + f*x)**4, x)
```

3.119 $\int \frac{\tan^4(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=551

$$\frac{973\sec(e+fx)(a\sin(e+fx)+a)^{2/3}\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a\sin(e+fx)+a}\right)\sqrt{\frac{2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{a\sin(e+fx)+a}+(a\sin(e+fx)+a)^{2/3}}{\left(\sqrt[3]{2}\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a\sin(e+fx)+a}\right)^2}}}{495\sqrt[3]{2}\sqrt[4]{3}a^{4/3}f\sqrt{-\frac{\sqrt[3]{a\sin(e+fx)+a}\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a\sin(e+fx)+a}\right)}{\left(\sqrt[3]{2}\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a\sin(e+fx)+a}\right)^2}}}$$

```
[Out] 973/396*sec(f*x+e)/f/(a+a*sin(f*x+e))^(1/3)-973/495*sec(f*x+e)*(1-sin(f*x+e
))/f/(a+a*sin(f*x+e))^(1/3)-1/132*sec(f*x+e)*(95*a+356*a*sin(f*x+e))/f/(1-s
in(f*x+e))/(a+a*sin(f*x+e))^(4/3)+973/2970*((2^(1/3)*a^(1/3)-(a+a*sin(f*x+e
))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3)*(1+3^(1/2))
)^2)^(1/2)/(2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3)*(1-3^(1/2)))*(2^(1/3)*a^
(1/3)-(a+a*sin(f*x+e))^(1/3)*(1+3^(1/2)))*EllipticF((1-(2^(1/3)*a^(1/3)-(a+
a*sin(f*x+e))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3)*
(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*sec(f*x+e)*(a+a*sin(f*x+e))^
(2/3)*(2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3))*((2^(2/3)*a^(2/3)+2^(1/3)*a^
(1/3)*(a+a*sin(f*x+e))^(1/3)+(a+a*sin(f*x+e))^(2/3))/(2^(1/3)*a^(1/3)-(a+a*
sin(f*x+e))^(1/3)*(1+3^(1/2)))^2)^(1/2)*2^(2/3)*3^(3/4)/a^(4/3)/f/(-(a+a*si
n(f*x+e))^(1/3)*(2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3))/(2^(1/3)*a^(1/3)-(
a+a*sin(f*x+e))^(1/3)*(1+3^(1/2)))^2)^(1/2)+3/4*a^2*sin(f*x+e)*tan(f*x+e)/f
/(a-a*sin(f*x+e))/(a+a*sin(f*x+e))^(4/3)+3*a^2*sin(f*x+e)^2*tan(f*x+e)/f/(a
-a*sin(f*x+e))/(a+a*sin(f*x+e))^(4/3)
```

Rubi [A] time = 0.49, antiderivative size = 551, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2719, 100, 153, 144, 51, 63, 225}

$$\frac{3a^2\sin^2(e+fx)\tan(e+fx)}{f(a-a\sin(e+fx))(a\sin(e+fx)+a)^{4/3}}+\frac{3a^2\sin(e+fx)\tan(e+fx)}{4f(a-a\sin(e+fx))(a\sin(e+fx)+a)^{4/3}}+\frac{973\sec(e+fx)(a\sin(e}{}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(a + a*Sin[e + f*x])^(1/3),x]
```

```
[Out] (973*Sec[e + f*x])/(396*f*(a + a*Sin[e + f*x])^(1/3)) - (973*Sec[e + f*x]*(
1 - Sin[e + f*x]))/(495*f*(a + a*Sin[e + f*x])^(1/3)) - (Sec[e + f*x]*(95*a
```

```
 + 356*a*Sin[e + f*x]))/(132*f*(1 - Sin[e + f*x])*(a + a*Sin[e + f*x])^(4/3
)) + (973*EllipticF[ArcCos[(2^(1/3)*a^(1/3) - (1 - Sqrt[3])*(a + a*Sin[e +
f*x])^(1/3))/(2^(1/3)*a^(1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*x])^(1/3))],
 (2 + Sqrt[3])/4]*Sec[e + f*x]*(a + a*Sin[e + f*x])^(2/3)*(2^(1/3)*a^(1/3)
- (a + a*Sin[e + f*x])^(1/3))*Sqrt[(2^(2/3)*a^(2/3) + 2^(1/3)*a^(1/3)*(a +
a*Sin[e + f*x])^(1/3) + (a + a*Sin[e + f*x])^(2/3))/(2^(1/3)*a^(1/3) - (1 +
 Sqrt[3])*(a + a*Sin[e + f*x])^(1/3))^2])/(495*2^(1/3)*3^(1/4)*a^(4/3)*f*Sq
rt[-(((a + a*Sin[e + f*x])^(1/3)*(2^(1/3)*a^(1/3) - (a + a*Sin[e + f*x])^(1
/3)))/(2^(1/3)*a^(1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*x])^(1/3))^2)]) + (
3*a^2*Sin[e + f*x]*Tan[e + f*x])/(4*f*(a - a*Sin[e + f*x])*(a + a*Sin[e + f
*x])^(4/3)) + (3*a^2*Sin[e + f*x]^2*Tan[e + f*x])/(f*(a - a*Sin[e + f*x])*(
a + a*Sin[e + f*x])^(4/3))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 144

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))
*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*c*d*e*g*(n + 1) + a^2*c*d*f*
h*(n + 1) + a*b*(d^2*e*g*(m + 1) + c^2*f*h*(m + 1) - c*d*(f*g + e*h)*(m + n
 + 2)) + (a^2*d^2*f*h*(n + 1) - a*b*d^2*(f*g + e*h)*(n + 1) + b^2*(c^2*f*h*
```

```
(m + 1) - c*d*(f*g + e*h)*(m + 1) + d^2*e*g*(m + n + 2)))*x)*(a + b*x)^(m +
 1)*(c + d*x)^(n + 1))/(b*d*(b*c - a*d)^2*(m + 1)*(n + 1)), x] - Dist[(a^2*
d^2*f*h*(2 + 3*n + n^2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
 + n + 3)) + b^2*(c^2*f*h*(2 + 3*m + m^2) - c*d*(f*g + e*h)*(m + 1)*(m + n
+ 3) + d^2*e*g*(6 + m^2 + 5*n + n^2 + m*(2*n + 5))))/(b*d*(b*c - a*d)^2*(m
+ 1)*(n + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, h}, x] && LtQ[m, -1] && LtQ[n, -1]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 2719

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[(Sqrt[a + b*Sin[e + f*x]]*Sqrt[a - b*Sin[e + f*x]])/(b
*f*Cos[e + f*x]), Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx = \frac{\left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}\right)\text{Subst}\left(\int \frac{x^4}{(a-x)^{5/2}(a+x)^{17/6}}\,dx,x,a\,\text{s}\right.}{af}$$

$$= \frac{3a^2\sin^2(e+fx)\tan(e+fx)}{f(a-a\sin(e+fx))(a+a\sin(e+fx))^{4/3}} + \frac{\left(3\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\right.}{}$$

$$= \frac{3a^2\sin(e+fx)\tan(e+fx)}{4f(a-a\sin(e+fx))(a+a\sin(e+fx))^{4/3}} + \frac{3a^2\sin^2(e+fx)\tan(e+fx)}{f(a-a\sin(e+fx))(a+a\sin(e+fx))}$$

$$= -\frac{\sec(e+fx)(95a+356a\sin(e+fx))}{132f(1-\sin(e+fx))(a+a\sin(e+fx))^{4/3}} + \frac{3a^2\sin(e+fx)\tan(e+fx)}{4f(a-a\sin(e+fx))(a+a\sin(e+f}$$

$$= \frac{973\sec(e+fx)}{396f\sqrt[3]{a+a\sin(e+fx)}} - \frac{\sec(e+fx)(95a+356a\sin(e+fx))}{132f(1-\sin(e+fx))(a+a\sin(e+fx))^{4/3}} + \frac{3a^2}{4f(a-a\,\text{si}}$$

$$= \frac{973\sec(e+fx)}{396f\sqrt[3]{a+a\sin(e+fx)}} - \frac{973\sec(e+fx)(1-\sin(e+fx))}{495f\sqrt[3]{a+a\sin(e+fx)}} - \frac{\sec(e+fx)(95a+}{132f(1-\sin(e+fx)}$$

$$= \frac{973\sec(e+fx)}{396f\sqrt[3]{a+a\sin(e+fx)}} - \frac{973\sec(e+fx)(1-\sin(e+fx))}{495f\sqrt[3]{a+a\sin(e+fx)}} - \frac{\sec(e+fx)(95a+}{132f(1-\sin(e+fx)}$$

$$= \frac{973\sec(e+fx)}{396f\sqrt[3]{a+a\sin(e+fx)}} - \frac{973\sec(e+fx)(1-\sin(e+fx))}{495f\sqrt[3]{a+a\sin(e+fx)}} - \frac{\sec(e+fx)(95a+}{132f(1-\sin(e+fx)}$$

Mathematica [C] time = 0.79, size = 128, normalized size = 0.23

$$\frac{973\sqrt{2}\cos(e+fx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)\right)+\sqrt{1-\sin(e+fx)}\,\sec^3(e+fx)(22\sin(e+fx)-128\,\text{si}}{495f\sqrt{1-\sin(e+fx)}\,\sqrt[3]{a(\sin(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/(a + a*Sin[e + f*x])^(1/3),x]
```

```
[Out] (973*Sqrt[2]*Cos[e + f*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[(2*e + Pi +
2*f*x)/4]^2] + Sec[e + f*x]^3*Sqrt[1 - Sin[e + f*x]]*(-49 - 64*Cos[2*(e + f
*x)] + 22*Sin[e + f*x] - 128*Sin[3*(e + f*x)]))/(495*f*Sqrt[1 - Sin[e + f*x
]]*(a*(1 + Sin[e + f*x]))^(1/3))
```

fricas [F] time = 1.31, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\tan\left(fx+e\right)^{4}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{1}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(1/3),x, algorithm="fricas")
[Out] integral(tan(f*x + e)^4/(a*sin(f*x + e) + a)^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(fx+e\right)^{4}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(1/3),x, algorithm="giac")
[Out] integrate(tan(f*x + e)^4/(a*sin(f*x + e) + a)^(1/3), x)
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{\tan^{4}\left(fx+e\right)}{\left(a+a\sin\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+a*sin(f*x+e))^(1/3),x)
[Out] int(tan(f*x+e)^4/(a+a*sin(f*x+e))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(fx+e\right)^{4}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^4/(a*sin(f*x + e) + a)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan\left(e+f\,x\right)^4}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4/(a + a*sin(e + f*x))^(1/3),x)
```

```
[Out] int(tan(e + f*x)^4/(a + a*sin(e + f*x))^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4\left(e+fx\right)}{\sqrt[3]{a\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+a*sin(f*x+e))**(1/3),x)
```

```
[Out] Integral(tan(e + f*x)**4/(a*(sin(e + f*x) + 1))**(1/3), x)
```

3.120 $\int \frac{\tan^2(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=126

$$\frac{11\sqrt[6]{2}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{15f\sqrt[6]{\sin(e+fx)+1}\sqrt[3]{a\sin(e+fx)+a}}+\frac{4\sec(e+fx)(a\sin(e+fx)+a)^{2/3}}{5af}-\frac{3\sec(e+fx)}{5f\sqrt[3]{a\sin(e+fx)+a}}$$

```
[Out] -3/5*sec(f*x+e)/f/(a+a*sin(f*x+e))^(1/3)+11/15*2^(1/6)*cos(f*x+e)*hypergeom
([1/2, 5/6],[3/2],1/2-1/2*sin(f*x+e))/f/(1+sin(f*x+e))^(1/6)/(a+a*sin(f*x+e
))^(1/3)+4/5*sec(f*x+e)*(a+a*sin(f*x+e))^(2/3)/a/f
```

Rubi [A] time = 0.22, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2712, 2855, 2652, 2651}

$$\frac{11\sqrt[6]{2}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{15f\sqrt[6]{\sin(e+fx)+1}\sqrt[3]{a\sin(e+fx)+a}}+\frac{4\sec(e+fx)(a\sin(e+fx)+a)^{2/3}}{5af}-\frac{3\sec(e+fx)}{5f\sqrt[3]{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + a*Sin[e + f*x])^(1/3),x]
```

```
[Out] (-3*Sec[e + f*x])/(5*f*(a + a*Sin[e + f*x])^(1/3)) + (11*2^(1/6)*Cos[e + f*
x]*Hypergeometric2F1[1/2, 5/6, 3/2, (1 - Sin[e + f*x])/2])/(15*f*(1 + Sin[e
 + f*x])^(1/6)*(a + a*Sin[e + f*x])^(1/3)) + (4*Sec[e + f*x]*(a + a*Sin[e +
 f*x])^(2/3))/(5*a*f)
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]
```

Rule 2712

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[(b*(a + b*Sin[e + f*x])^m)/(a*f*(2*m - 1)*Cos[e + f*x]),
x] - Dist[1/(a^2*(2*m - 1)), Int[((a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*
m - 1)*Sin[e + f*x]))/Cos[e + f*x]^2, x], x] /; FreeQ[{a, b, e, f}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[m] && LtQ[m, 0]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx = -\frac{3\sec(e+fx)}{5f\sqrt[3]{a+a\sin(e+fx)}} + \frac{3\int \sec^2(e+fx)(a+a\sin(e+fx))^{2/3}\left(-\frac{a}{3}+\frac{5}{3}a\sin(e+fx\right.}{5a^2}$$

$$= -\frac{3\sec(e+fx)}{5f\sqrt[3]{a+a\sin(e+fx)}} + \frac{4\sec(e+fx)(a+a\sin(e+fx))^{2/3}}{5af} - \frac{11}{15}\int \frac{1}{\sqrt[3]{a+a\sin(e-}}$$

$$= -\frac{3\sec(e+fx)}{5f\sqrt[3]{a+a\sin(e+fx)}} + \frac{4\sec(e+fx)(a+a\sin(e+fx))^{2/3}}{5af} - \frac{\left(11\sqrt[3]{1+\sin(e+fx)}\right.}{15\sqrt[3]{a+a\,\mathrm{s}}}$$

$$= -\frac{3\sec(e+fx)}{5f\sqrt[3]{a+a\sin(e+fx)}} + \frac{11\sqrt[6]{2}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{15f\sqrt[6]{1+\sin(e+fx)}\sqrt[3]{a+a\sin(e+fx)}} + \frac{4\sec}{}$$

Mathematica [A] time = 0.50, size = 100, normalized size = 0.79

$$\frac{\sqrt{2-2\sin(e+fx)}\,(4\tan(e+fx)+\sec(e+fx))-22\cos(e+fx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)\right)}{5f\sqrt{2-2\sin(e+fx)}\sqrt[3]{a(\sin(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/(a + a*Sin[e + f*x])^(1/3),x]
```

```
[Out] (-22*Cos[e + f*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[(2*e + Pi + 2*f*x)/4
]^2] + Sqrt[2 - 2*Sin[e + f*x]]*(Sec[e + f*x] + 4*Tan[e + f*x]))/(5*f*Sqrt[
2 - 2*Sin[e + f*x]]*(a*(1 + Sin[e + f*x]))^(1/3))
```

fricas [F] time = 1.69, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\tan\left(fx+e\right)^{2}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{1}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(1/3),x, algorithm="fricas")
[Out] integral(tan(f*x + e)^2/(a*sin(f*x + e) + a)^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(fx+e\right)^{2}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(1/3),x, algorithm="giac")
[Out] integrate(tan(f*x + e)^2/(a*sin(f*x + e) + a)^(1/3), x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{\tan^{2}\left(fx+e\right)}{\left(a+a\sin\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+a*sin(f*x+e))^(1/3),x)
[Out] int(tan(f*x+e)^2/(a+a*sin(f*x+e))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(fx+e\right)^{2}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^2/(a*sin(f*x + e) + a)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan\left(e+f\,x\right)^2}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(a + a*sin(e + f*x))^(1/3),x)
[Out] int(tan(e + f*x)^2/(a + a*sin(e + f*x))^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{2}{\left(e + f x \right)}}{\sqrt[3]{a \left(\sin{\left(e + f x \right)} + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+a*sin(f*x+e))**(1/3),x)
[Out] Integral(tan(e + f*x)**2/(a*(sin(e + f*x) + 1))**(1/3), x)
```

3.121 $\int \frac{\cot^2(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=80

$$\frac{6\sqrt{2}\sqrt{1-\sin(e+fx)}\sec(e+fx)(a\sin(e+fx)+a)^{5/3}F_1\left(\frac{7}{6};-\frac{1}{2},2;\frac{13}{6};\frac{1}{2}(\sin(e+fx)+1),\sin(e+fx)+1\right)}{7a^2f}$$

```
[Out] 6/7*AppellF1(7/6,2,-1/2,13/6,1+sin(f*x+e),1/2+1/2*sin(f*x+e))*sec(f*x+e)*(a
+a*sin(f*x+e))^(5/3)*2^(1/2)*(1-sin(f*x+e))^(1/2)/a^2/f
```

Rubi [A] time = 0.09, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2719, 137, 136}

$$\frac{6\sqrt{2}\sqrt{1-\sin(e+fx)}\sec(e+fx)(a\sin(e+fx)+a)^{5/3}F_1\left(\frac{7}{6};-\frac{1}{2},2;\frac{13}{6};\frac{1}{2}(\sin(e+fx)+1),\sin(e+fx)+1\right)}{7a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(a + a*Sin[e + f*x])^(1/3),x]
```

```
[Out] (6*Sqrt[2]*AppellF1[7/6, -1/2, 2, 13/6, (1 + Sin[e + f*x])/2, 1 + Sin[e + f
*x]]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(5/3))/(7*a^2
*f)
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 2719

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[(Sqrt[a + b*Sin[e + f*x]]*Sqrt[a - b*Sin[e + f*x]])/(b
*f*Cos[e + f*x]), Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx = \frac{\left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{a-x}\,\sqrt[6]{a+x}}{x^2}\,dx,x,a\sin(\right.}{af}$$

$$= \frac{\left(\sqrt{2}\,\sec(e+fx)(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt[6]{a+x}\,\sqrt{\frac{1}{2}-\frac{x}{2a}}}{x^2}\,dx,x,a\right.}{af\sqrt{\frac{a-a\sin(e+fx)}{a}}}$$

$$= \frac{6\sqrt{2}\,F_1\left(\frac{7}{6};-\frac{1}{2},2;\frac{13}{6};\frac{1}{2}(1+\sin(e+fx)),1+\sin(e+fx)\right)\sec(e+fx)\sqrt{1-\sin(e+fx}}{7a^2f}$$

Mathematica [F] time = 12.57, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[e + f*x]^2/(a + a*Sin[e + f*x])^(1/3),x]
[Out] Integrate[Cot[e + f*x]^2/(a + a*Sin[e + f*x])^(1/3), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(1/3),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(fx+e\right)^{2}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(1/3),x, algorithm="giac")
[Out] integrate(cot(f*x + e)^2/(a*sin(f*x + e) + a)^(1/3), x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{\cot^{2}\left(fx+e\right)}{\left(a+a\sin\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+a*sin(f*x+e))^(1/3),x)
[Out] int(cot(f*x+e)^2/(a+a*sin(f*x+e))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(fx+e\right)^{2}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(1/3),x, algorithm="maxima")
[Out] integrate(cot(f*x + e)^2/(a*sin(f*x + e) + a)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cot\left(e+f\,x\right)}^2}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2/(a + a*sin(e + f*x))^(1/3),x)
```

```
[Out] int(cot(e + f*x)^2/(a + a*sin(e + f*x))^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2\left(e+fx\right)}{\sqrt[3]{a\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+a*sin(f*x+e))**(1/3),x)
```

```
[Out] Integral(cot(e + f*x)**2/(a*(sin(e + f*x) + 1))**(1/3), x)
```

3.122 $\int \frac{\cot^4(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=80

$$\frac{12\sqrt{2}\sqrt{1-\sin(e+fx)}\sec(e+fx)(a\sin(e+fx)+a)^{8/3}F_1\left(\frac{13}{6};-\frac{3}{2},4;\frac{19}{6};\frac{1}{2}(\sin(e+fx)+1),\sin(e+fx)+1\right)}{13a^3f}$$

```
[Out] 12/13*AppellF1(13/6,4,-3/2,19/6,1+sin(f*x+e),1/2+1/2*sin(f*x+e))*sec(f*x+e)
*(a+a*sin(f*x+e))^(8/3)*2^(1/2)*(1-sin(f*x+e))^(1/2)/a^3/f
```

Rubi [A] time = 0.09, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2719, 137, 136}

$$\frac{12\sqrt{2}\sqrt{1-\sin(e+fx)}\sec(e+fx)(a\sin(e+fx)+a)^{8/3}F_1\left(\frac{13}{6};-\frac{3}{2},4;\frac{19}{6};\frac{1}{2}(\sin(e+fx)+1),\sin(e+fx)+1\right)}{13a^3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(a + a*Sin[e + f*x])^(1/3),x]
```

```
[Out] (12*Sqrt[2]*AppellF1[13/6, -3/2, 4, 19/6, (1 + Sin[e + f*x])/2, 1 + Sin[e +
 f*x]]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(8/3))/(13*
a^3*f)
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 2719

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[(Sqrt[a + b*Sin[e + f*x]]*Sqrt[a - b*Sin[e + f*x]])/(b
*f*Cos[e + f*x]), Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx = \frac{\left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}\right)\text{Subst}\left(\int \frac{(a-x)^{3/2}(a+x)^{7/6}}{x^4}\,dx, x, a\sin\right.}{af}$$

$$= \frac{\left(2\sqrt{2}\,\sec(e+fx)(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}\right)\text{Subst}\left(\int \frac{(a+x)^{7/6}\left(\frac{1}{2}-\frac{x}{2a}\right)^{3/2}}{x^4}\,dx\right.}{f\sqrt{\frac{a-a\sin(e+fx)}{a}}}$$

$$= \frac{12\sqrt{2}\,F_1\left(\frac{13}{6};-\frac{3}{2},4;\frac{19}{6};\frac{1}{2}(1+\sin(e+fx)),1+\sin(e+fx)\right)\sec(e+fx)\sqrt{1-\sin(e+}}{13a^3f}$$

Mathematica [F] time = 8.94, size = 0, normalized size = 0.00

$$\int \frac{\cot^4(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[e + f*x]^4/(a + a*Sin[e + f*x])^(1/3),x]
[Out] Integrate[Cot[e + f*x]^4/(a + a*Sin[e + f*x])^(1/3), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(1/3),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(fx+e\right)^{4}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(1/3),x, algorithm="giac")
[Out] integrate(cot(f*x + e)^4/(a*sin(f*x + e) + a)^(1/3), x)
```

maple [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{\cot^{4}\left(fx+e\right)}{\left(a+a\sin\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+a*sin(f*x+e))^(1/3),x)
[Out] int(cot(f*x+e)^4/(a+a*sin(f*x+e))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(fx+e\right)^{4}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(1/3),x, algorithm="maxima")
[Out] integrate(cot(f*x + e)^4/(a*sin(f*x + e) + a)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cot\left(e+f\,x\right)}^4}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4/(a + a*sin(e + f*x))^(1/3),x)
```

```
[Out] int(cot(e + f*x)^4/(a + a*sin(e + f*x))^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4\left(e+fx\right)}{\sqrt[3]{a\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+a*sin(f*x+e))**(1/3),x)
```

```
[Out] Integral(cot(e + f*x)**4/(a*(sin(e + f*x) + 1))**(1/3), x)
```

3.123 $\int (a+a\sin(e+fx))^3 (g\tan(e+fx))^p\,dx$

Optimal. Leaf size=269

$$\frac{3a^3(g\tan(e+fx))^{p+3}\,{}_2F_1\left(2,\frac{p+3}{2};\frac{p+5}{2};-\tan^2(e+fx)\right)}{fg^3(p+3)}+\frac{a^3(g\tan(e+fx))^{p+1}\,{}_2F_1\left(1,\frac{p+1}{2};\frac{p+3}{2};-\tan^2(e+fx)\right)}{fg(p+1)}+$$

```
[Out] a^3*hypergeom([1, 1/2+1/2*p],[3/2+1/2*p],-tan(f*x+e)^2)*(g*tan(f*x+e))^(1+p
)/f/g/(1+p)+3*a^3*(cos(f*x+e)^2)^(1/2+1/2*p)*hypergeom([1+1/2*p, 1/2+1/2*p]
,[2+1/2*p],sin(f*x+e)^2)*sin(f*x+e)*(g*tan(f*x+e))^(1+p)/f/g/(2+p)+a^3*(cos
(f*x+e)^2)^(1/2+1/2*p)*hypergeom([2+1/2*p, 1/2+1/2*p],[3+1/2*p],sin(f*x+e)^
2)*sin(f*x+e)^3*(g*tan(f*x+e))^(1+p)/f/g/(4+p)+3*a^3*hypergeom([2, 3/2+1/2*
p],[5/2+1/2*p],-tan(f*x+e)^2)*(g*tan(f*x+e))^(3+p)/f/g^3/(3+p)
```

Rubi [A] time = 0.35, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2710, 3476, 364, 2602, 2577, 2591}

$$\frac{3a^3(g\tan(e+fx))^{p+3}\,{}_2F_1\left(2,\frac{p+3}{2};\frac{p+5}{2};-\tan^2(e+fx)\right)}{fg^3(p+3)}+\frac{a^3(g\tan(e+fx))^{p+1}\,{}_2F_1\left(1,\frac{p+1}{2};\frac{p+3}{2};-\tan^2(e+fx)\right)}{fg(p+1)}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(g*Tan[e + f*x])^p,x]
```

```
[Out] (a^3*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e +
 f*x])^(1 + p))/(f*g*(1 + p)) + (3*a^3*(Cos[e + f*x]^2)^((1 + p)/2)*Hyperge
ometric2F1[(1 + p)/2, (2 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(g
*Tan[e + f*x])^(1 + p))/(f*g*(2 + p)) + (a^3*(Cos[e + f*x]^2)^((1 + p)/2)*H
ypergeometric2F1[(1 + p)/2, (4 + p)/2, (6 + p)/2, Sin[e + f*x]^2]*Sin[e + f
*x]^3*(g*Tan[e + f*x])^(1 + p))/(f*g*(4 + p)) + (3*a^3*Hypergeometric2F1[2,
 (3 + p)/2, (5 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e + f*x])^(3 + p))/(f*g^3*(3
 + p))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
```

```
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2710

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int (a + a\sin(e + fx))^3(g\tan(e + fx))^p\,dx = \int \left(a^3(g\tan(e + fx))^p + 3a^3\sin(e + fx)(g\tan(e + fx))^p + 3a^3\sin^2(\right.$$
$$= a^3\int (g\tan(e + fx))^p\,dx + a^3\int \sin^3(e + fx)(g\tan(e + fx))^p\,dx + \Big($$
$$= \frac{\left(a^3g\right)\text{Subst}\left(\int \frac{x^p}{g^2+x^2}\,dx, x, g\tan(e + fx)\right)}{f} + \frac{\left(3a^3g\right)\text{Subst}\Big(\int \frac{x^{2+p}}{\left(g^2+x^2\right.}}{f}$$
$$= \frac{a^3\,{}_2F_1\left(1, \frac{1+p}{2}; \frac{3+p}{2}; -\tan^2(e + fx)\right)(g\tan(e + fx))^{1+p}}{fg(1 + p)} + \frac{3a^3\cos^2(e}{}$$

Mathematica [C] time = 58.50, size = 5199, normalized size = 19.33

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(g*Tan[e + f*x])^p,x]
[Out] Result too large to show
```

fricas [F] time = 1.99, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(3\,a^3\cos\left(fx + e\right)^2 - 4\,a^3 + \left(a^3\cos\left(fx + e\right)^2 - 4\,a^3\right)\sin\left(fx + e\right)\right)\left(g\tan\left(fx + e\right)\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(g*tan(f*x+e))^p,x, algorithm="fricas")
[Out] integral(-(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(
f*x + e))*(g*tan(f*x + e))^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx + e\right) + a\right)^3\left(g\tan\left(fx + e\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(g*tan(f*x+e))^p,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^3*(g*tan(f*x + e))^p, x)
```

maple [F] time = 2.68, size = 0, normalized size = 0.00

$$\int \left(a + a\sin\left(fx + e\right)\right)^{3} \left(g\tan\left(fx + e\right)\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(g*tan(f*x+e))^p,x)
```

```
[Out] int((a+a*sin(f*x+e))^3*(g*tan(f*x+e))^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx + e\right) + a\right)^{3} \left(g\tan\left(fx + e\right)\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(g*tan(f*x+e))^p,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*(g*tan(f*x + e))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(g\tan\left(e+f\,x\right)\right)}^p\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*tan(e + f*x))^p*(a + a*sin(e + f*x))^3,x)
```

```
[Out] int((g*tan(e + f*x))^p*(a + a*sin(e + f*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^{3} \left(\int \left(g \tan{\left(e + f x \right)}\right)^{p}\, dx + \int 3 \left(g \tan{\left(e + f x \right)}\right)^{p} \sin{\left(e + f x \right)}\, dx + \int 3 \left(g \tan{\left(e + f x \right)}\right)^{p} \sin^{2}{\left(e + f x \right)}\, dx + \int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(g*tan(f*x+e))**p,x)
```

```
[Out] a**3*(Integral((g*tan(e + f*x))**p, x) + Integral(3*(g*tan(e + f*x))**p*sin
(e + f*x), x) + Integral(3*(g*tan(e + f*x))**p*sin(e + f*x)**2, x) + Integr
al((g*tan(e + f*x))**p*sin(e + f*x)**3, x))
```

3.124 $\int (a+a\sin(e+fx))^2(g\tan(e+fx))^p\,dx$

Optimal. Leaf size=187

$$\frac{a^2(g\tan(e+fx))^{p+3}\,{}_2F_1\left(2,\frac{p+3}{2};\frac{p+5}{2};-\tan^2(e+fx)\right)}{fg^3(p+3)}+\frac{a^2(g\tan(e+fx))^{p+1}\,{}_2F_1\left(1,\frac{p+1}{2};\frac{p+3}{2};-\tan^2(e+fx)\right)}{fg(p+1)}+\frac{2a}{}$$

```
[Out] a^2*hypergeom([1, 1/2+1/2*p],[3/2+1/2*p],-tan(f*x+e)^2)*(g*tan(f*x+e))^(1+p
)/f/g/(1+p)+2*a^2*(cos(f*x+e)^2)^(1/2+1/2*p)*hypergeom([1+1/2*p, 1/2+1/2*p]
,[2+1/2*p],sin(f*x+e)^2)*sin(f*x+e)*(g*tan(f*x+e))^(1+p)/f/g/(2+p)+a^2*hype
rgeom([2, 3/2+1/2*p],[5/2+1/2*p],-tan(f*x+e)^2)*(g*tan(f*x+e))^(3+p)/f/g^3/
(3+p)
```

Rubi [A] time = 0.23, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2710, 3476, 364, 2602, 2577, 2591}

$$\frac{a^2(g\tan(e+fx))^{p+3}\,{}_2F_1\left(2,\frac{p+3}{2};\frac{p+5}{2};-\tan^2(e+fx)\right)}{fg^3(p+3)}+\frac{a^2(g\tan(e+fx))^{p+1}\,{}_2F_1\left(1,\frac{p+1}{2};\frac{p+3}{2};-\tan^2(e+fx)\right)}{fg(p+1)}+\frac{2a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(g*Tan[e + f*x])^p,x]
```

```
[Out] (a^2*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e +
 f*x])^(1 + p))/(f*g*(1 + p)) + (2*a^2*(Cos[e + f*x]^2)^((1 + p)/2)*Hyperge
ometric2F1[(1 + p)/2, (2 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(g
*Tan[e + f*x])^(1 + p))/(f*g*(2 + p)) + (a^2*Hypergeometric2F1[2, (3 + p)/2
, (5 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e + f*x])^(3 + p))/(f*g^3*(3 + p))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
```

```
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2710

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(g\tan(e+fx))^p\,dx = \int \left(a^2(g\tan(e+fx))^p + 2a^2\sin(e+fx)(g\tan(e+fx))^p + a^2\sin^2(e+\right.$$

$$= a^2\int (g\tan(e+fx))^p\,dx + a^2\int \sin^2(e+fx)(g\tan(e+fx))^p\,dx + \left(2a\right.$$

$$= \frac{\left(a^2g\right)\text{Subst}\left(\int \frac{x^{2+p}}{\left(g^2+x^2\right)^2}\,dx, x, g\tan(e+fx)\right)}{f} + \frac{\left(a^2g\right)\text{Subst}\left(\int \frac{x^p}{g^2+x^2}\,d\right.}{f}$$

$$= \frac{a^2\,{}_2F_1\left(1,\frac{1+p}{2};\frac{3+p}{2};-\tan^2(e+fx)\right)(g\tan(e+fx))^{1+p}}{fg(1+p)} + \frac{2a^2\cos^2(e+}{}$$

Mathematica [C] time = 18.14, size = 2054, normalized size = 10.98

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(g*Tan[e + f*x])^p,x]
```

```
[Out] (2*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^p*(a + a*Sin[e + f*x])^2*Tan[(e + f*x)
/2]*((2 + p)*AppellF1[(1 + p)/2, p, 1, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2] + 4*(2 + p)*AppellF1[(1 + p)/2, p, 2, (3 + p)/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2] - 4*(2 + p)*AppellF1[(1 + p)/2, p, 3, (3 + p
)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*(1 + p)*AppellF1[1 + p/2,
 p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2])*
(g*Tan[e + f*x])^p*(Cos[(e + f*x)/2]^4*Tan[e + f*x]^p + 4*Cos[(e + f*x)/2]^
3*Sin[(e + f*x)/2]*Tan[e + f*x]^p + 6*Cos[(e + f*x)/2]^2*Sin[(e + f*x)/2]^2
*Tan[e + f*x]^p + 4*Cos[(e + f*x)/2]*Sin[(e + f*x)/2]^3*Tan[e + f*x]^p + Si
n[(e + f*x)/2]^4*Tan[e + f*x]^p))/(f*(1 + p)*(2 + p)*(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2])^4*((2*p*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^p*Sec[e + f*x]^2*
Tan[(e + f*x)/2]*((2 + p)*AppellF1[(1 + p)/2, p, 1, (3 + p)/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2] + 4*(2 + p)*AppellF1[(1 + p)/2, p, 2, (3 + p)/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 4*(2 + p)*AppellF1[(1 + p)/2,
 p, 3, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*(1 + p)*Appe
llF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(
e + f*x)/2])*Tan[e + f*x]^(-1 + p))/((1 + p)*(2 + p)) + (Sec[(e + f*x)/2]^2
*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^p*((2 + p)*AppellF1[(1 + p)/2, p, 1, (3
+ p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*(2 + p)*AppellF1[(1 +
p)/2, p, 2, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 4*(2 + p)
*AppellF1[(1 + p)/2, p, 3, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2] + 4*(1 + p)*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2]*Tan[(e + f*x)/2])*Tan[e + f*x]^p)/((1 + p)*(2 + p)) + (2*p*(
Cos[e + f*x]*Sec[(e + f*x)/2]^2)^(-1 + p)*Tan[(e + f*x)/2]*((2 + p)*AppellF
1[(1 + p)/2, p, 1, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*
(2 + p)*AppellF1[(1 + p)/2, p, 2, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2] - 4*(2 + p)*AppellF1[(1 + p)/2, p, 3, (3 + p)/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2] + 4*(1 + p)*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2])*(-(Sec[(e + f*x)/2]^
2*Sin[e + f*x]) + Cos[e + f*x]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])*Tan[e +
 f*x]^p)/((1 + p)*(2 + p)) + (2*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^p*Tan[(e
+ f*x)/2]*(2*(1 + p)*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2 + 4*(1 + p)*Tan[(e + f*x)/2]*((-2*(1
 + p/2)*AppellF1[2 + p/2, p, 3, 3 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(2 + p/2) + ((1 + p/2)*p*Appell
F1[2 + p/2, 1 + p, 2, 3 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec
[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(2 + p/2)) + (2 + p)*(-(((1 + p)*AppellF1
```

```
[1 + (1 + p)/2, p, 2, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p)) + (p*(1 + p)*AppellF1[1 +
(1 + p)/2, 1 + p, 1, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p)) + 4*(2 + p)*((-2*(1 + p)*Ap
pellF1[1 + (1 + p)/2, p, 3, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p) + (p*(1 + p)*AppellF1
[1 + (1 + p)/2, 1 + p, 2, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p)) - 4*(2 + p)*((-3*(1 +
p)*AppellF1[1 + (1 + p)/2, p, 4, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p) + (p*(1 + p)*App
ellF1[1 + (1 + p)/2, 1 + p, 3, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p)))*Tan[e + f*x]^p)/
((1 + p)*(2 + p))))
```

fricas [F] time = 1.06, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\left(a^2\cos\left(fx+e\right)^2-2\,a^2\sin\left(fx+e\right)-2\,a^2\right)\left(g\tan\left(fx+e\right)\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(g*tan(f*x+e))^p,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2)*(g*tan(f*x + e)
)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^2\left(g\tan\left(fx+e\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(g*tan(f*x+e))^p,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(g*tan(f*x + e))^p, x)
```

maple [F] time = 2.24, size = 0, normalized size = 0.00

$$\int\left(a+a\sin\left(fx+e\right)\right)^2\left(g\tan\left(fx+e\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(g*tan(f*x+e))^p,x)
```

```
[Out] int((a+a*sin(f*x+e))^2*(g*tan(f*x+e))^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{2}\left(g\tan\left(fx+e\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(g*tan(f*x+e))^p,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^2*(g*tan(f*x + e))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(g\,\tan\left(e+f\,x\right)\right)}^p\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*tan(e + f*x))^p*(a + a*sin(e + f*x))^2,x)
[Out] int((g*tan(e + f*x))^p*(a + a*sin(e + f*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int \left(g\tan\left(e+fx\right)\right)^{p}\,dx+\int 2\left(g\tan\left(e+fx\right)\right)^{p}\sin\left(e+fx\right)dx+\int \left(g\tan\left(e+fx\right)\right)^{p}\sin^{2}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(g*tan(f*x+e))**p,x)
[Out] a**2*(Integral((g*tan(e + f*x))**p, x) + Integral(2*(g*tan(e + f*x))**p*sin
(e + f*x), x) + Integral((g*tan(e + f*x))**p*sin(e + f*x)**2, x))
```

3.125 $\int (a+a\sin(e+fx))(g\tan(e+fx))^p\,dx$

Optimal. Leaf size=129

$$\frac{a(g\tan(e+fx))^{p+1}\,{}_2F_1\left(1,\frac{p+1}{2};\frac{p+3}{2};-\tan^2(e+fx)\right)}{fg(p+1)}+\frac{a\sin(e+fx)\cos^2(e+fx)^{\frac{p+1}{2}}(g\tan(e+fx))^{p+1}\,{}_2F_1\left(\frac{p+1}{2},\frac{p+}{2}\right.}{fg(p+2)}$$

```
[Out] a*hypergeom([1, 1/2+1/2*p],[3/2+1/2*p],-tan(f*x+e)^2)*(g*tan(f*x+e))^(1+p)/
f/g/(1+p)+a*(cos(f*x+e)^2)^(1/2+1/2*p)*hypergeom([1+1/2*p, 1/2+1/2*p],[2+1/
2*p],sin(f*x+e)^2)*sin(f*x+e)*(g*tan(f*x+e))^(1+p)/f/g/(2+p)
```

Rubi [A] time = 0.14, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2710, 3476, 364, 2602, 2577}

$$\frac{a(g\tan(e+fx))^{p+1}\,{}_2F_1\left(1,\frac{p+1}{2};\frac{p+3}{2};-\tan^2(e+fx)\right)}{fg(p+1)}+\frac{a\sin(e+fx)\cos^2(e+fx)^{\frac{p+1}{2}}(g\tan(e+fx))^{p+1}\,{}_2F_1\left(\frac{p+1}{2},\frac{p+}{2}\right.}{fg(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(g*Tan[e + f*x])^p,x]
```

```
[Out] (a*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e + f
*x])^(1 + p))/(f*g*(1 + p)) + (a*(Cos[e + f*x]^2)^((1 + p)/2)*Hypergeometri
c2F1[(1 + p)/2, (2 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(g*Tan[e
 + f*x])^(1 + p))/(f*g*(2 + p))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

Rule 2710

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int (a+a\sin(e+fx))(g\tan(e+fx))^p\,dx &= \int \left(a(g\tan(e+fx))^p + a\sin(e+fx)(g\tan(e+fx))^p\right)dx \\
&= a\int (g\tan(e+fx))^p\,dx + a\int \sin(e+fx)(g\tan(e+fx))^p\,dx \\
&= \frac{(ag)\,\mathrm{Subst}\left(\int \frac{x^p}{g^2+x^2}\,dx, x, g\tan(e+fx)\right)}{f} + \frac{\left(a\cos^{1+p}(e+fx)\sin^{-1-p}\right.}{} \\
&= \frac{a\,{}_2F_1\left(1,\frac{1+p}{2};\frac{3+p}{2};-\tan^2(e+fx)\right)(g\tan(e+fx))^{1+p}}{fg(1+p)} + \frac{a\cos^2(e+fx}{}
\end{aligned}$$

Mathematica [F] time = 2.06, size = 0, normalized size = 0.00

$$\int (a+a\sin(e+fx))(g\tan(e+fx))^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + a*Sin[e + f*x])*(g*Tan[e + f*x])^p,x]
[Out] Integrate[(a + a*Sin[e + f*x])*(g*Tan[e + f*x])^p, x]
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(a\sin\left(fx+e\right)+a\right)\left(g\tan\left(fx+e\right)\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(g*tan(f*x+e))^p,x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)*(g*tan(f*x + e))^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)\left(g\tan\left(fx+e\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(g*tan(f*x+e))^p,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)*(g*tan(f*x + e))^p, x)
```

maple [F] time = 1.52, size = 0, normalized size = 0.00

$$\int\left(a+a\sin\left(fx+e\right)\right)\left(g\tan\left(fx+e\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(g*tan(f*x+e))^p,x)
[Out] int((a+a*sin(f*x+e))*(g*tan(f*x+e))^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)\left(g\tan\left(fx+e\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(g*tan(f*x+e))^p,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)*(g*tan(f*x + e))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int{\left(g\tan\left(e+f\,x\right)\right)}^p\,\left(a+a\,\sin\left(e+f\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*tan(e + f*x))^p*(a + a*sin(e + f*x)),x)
```

```
[Out] int((g*tan(e + f*x))^p*(a + a*sin(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \left(g\tan\left(e+fx\right)\right)^{p}\,dx+\int \left(g\tan\left(e+fx\right)\right)^{p}\sin\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(g*tan(f*x+e))**p,x)
```

```
[Out] a*(Integral((g*tan(e + f*x))**p, x) + Integral((g*tan(e + f*x))**p*sin(e +
f*x), x))
```

3.126 $\int \frac{(g\tan(e+fx))^p}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=108

$$\frac{(g\tan(e+fx))^{p+1}}{afg(p+1)}-\frac{\sec(e+fx)\cos^2(e+fx)^{\frac{p+3}{2}}(g\tan(e+fx))^{p+2}\,{}_2F_1\left(\frac{p+2}{2},\frac{p+3}{2};\frac{p+4}{2};\sin^2(e+fx)\right)}{afg^2(p+2)}$$

```
[Out] (g*tan(f*x+e))^(1+p)/a/f/g/(1+p)-(cos(f*x+e)^2)^(3/2+1/2*p)*hypergeom([1+1/
2*p, 3/2+1/2*p],[2+1/2*p],sin(f*x+e)^2)*sec(f*x+e)*(g*tan(f*x+e))^(2+p)/a/f
/g^2/(2+p)
```

Rubi [A] time = 0.13, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2706, 2607, 32, 2617}

$$\frac{(g\tan(e+fx))^{p+1}}{afg(p+1)}-\frac{\sec(e+fx)\cos^2(e+fx)^{\frac{p+3}{2}}(g\tan(e+fx))^{p+2}\,{}_2F_1\left(\frac{p+2}{2},\frac{p+3}{2};\frac{p+4}{2};\sin^2(e+fx)\right)}{afg^2(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Tan[e + f*x])^p/(a + a*Sin[e + f*x]),x]
```

```
[Out] (g*Tan[e + f*x])^(1 + p)/(a*f*g*(1 + p)) - ((Cos[e + f*x]^2)^((3 + p)/2)*Hy
pergeometric2F1[(2 + p)/2, (3 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sec[e + f*
x]*(g*Tan[e + f*x])^(2 + p))/(a*f*g^2*(2 + p))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
```

```
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{(g\tan(e+fx))^p}{a+a\sin(e+fx)}\,dx = \frac{\int \sec^2(e+fx)(g\tan(e+fx))^p\,dx}{a} - \frac{\int \sec(e+fx)(g\tan(e+fx))^{1+p}\,dx}{ag}$$

$$= -\frac{\cos^2(e+fx)^{\frac{3+p}{2}}\,{}_2F_1\left(\frac{2+p}{2},\frac{3+p}{2};\frac{4+p}{2};\sin^2(e+fx)\right)\sec(e+fx)(g\tan(e+fx))^{2+p}}{afg^2(2+p)} + \frac{\text{Su}}{}$$

$$= \frac{(g\tan(e+fx))^{1+p}}{afg(1+p)} - \frac{\cos^2(e+fx)^{\frac{3+p}{2}}\,{}_2F_1\left(\frac{2+p}{2},\frac{3+p}{2};\frac{4+p}{2};\sin^2(e+fx)\right)\sec(e+fx)(g\,\text{ta}}{afg^2(2+p)}$$

Mathematica [B] time = 3.92, size = 232, normalized size = 2.15

$$\frac{2\tan\left(\frac{1}{2}(e+fx)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2(g\tan(e+fx))^p\left(\left(p^2+5p+6\right){}_2F_1\left(\frac{p+1}{2},p+2;\frac{p+3}{2};\tan^2\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Tan[e + f*x])^p/(a + a*Sin[e + f*x]),x]
```

```
[Out] (2*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^p*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^2*Tan[(e + f*x)/2]*((6 + 5*p + p^2)*Hypergeometric2F1[(1 + p)/2, 2 + p, (
3 + p)/2, Tan[(e + f*x)/2]^2] - (1 + p)*Tan[(e + f*x)/2]*(2*(3 + p)*Hyperge
ometric2F1[(2 + p)/2, 2 + p, (4 + p)/2, Tan[(e + f*x)/2]^2] - (2 + p)*Hyper
geometric2F1[2 + p, (3 + p)/2, (5 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)
/2]))*(g*Tan[e + f*x])^p)/(f*(1 + p)*(2 + p)*(3 + p)*(a + a*Sin[e + f*x]))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(g\tan\left(fx+e\right)\right)^p}{a\sin\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+a*sin(f*x+e)),x, algorithm="fricas")
[Out] integral((g*tan(f*x + e))^p/(a*sin(f*x + e) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\tan\left(fx+e\right)\right)^{p}}{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+a*sin(f*x+e)),x, algorithm="giac")
[Out] integrate((g*tan(f*x + e))^p/(a*sin(f*x + e) + a), x)
```

maple [F] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{\left(g\tan\left(fx+e\right)\right)^{p}}{a+a\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*tan(f*x+e))^p/(a+a*sin(f*x+e)),x)
[Out] int((g*tan(f*x+e))^p/(a+a*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\tan\left(fx+e\right)\right)^{p}}{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+a*sin(f*x+e)),x, algorithm="maxima")
[Out] integrate((g*tan(f*x + e))^p/(a*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(g\tan\left(e+f\,x\right)\right)^{p}}{a+a\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*tan(e + f*x))^p/(a + a*sin(e + f*x)),x)
```

```
[Out] int((g*tan(e + f*x))^p/(a + a*sin(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\left(g\tan\left(e+fx\right)\right)^p}{\sin\left(e+fx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))**p/(a+a*sin(f*x+e)),x)
```

```
[Out] Integral((g*tan(e + f*x))**p/(sin(e + f*x) + 1), x)/a
```

3.127 $\int \frac{(g\tan(e+fx))^p}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=138

$$\frac{2(g\tan(e+fx))^{p+3}}{a^2fg^3(p+3)}-\frac{2\sec^3(e+fx)\cos^2(e+fx)^{\frac{p+5}{2}}(g\tan(e+fx))^{p+2}\,{}_2F_1\left(\frac{p+2}{2},\frac{p+5}{2};\frac{p+4}{2};\sin^2(e+fx)\right)}{a^2fg^2(p+2)}+\frac{(g\tan(e}{a^2fg}$$

```
[Out] (g*tan(f*x+e))^(1+p)/a^2/f/g/(1+p)-2*(cos(f*x+e)^2)^(5/2+1/2*p)*hypergeom([
1+1/2*p, 5/2+1/2*p],[2+1/2*p],sin(f*x+e)^2)*sec(f*x+e)^3*(g*tan(f*x+e))^(2+
p)/a^2/f/g^2/(2+p)+2*(g*tan(f*x+e))^(3+p)/a^2/f/g^3/(3+p)
```

Rubi [A] time = 0.27, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2711, 2607, 14, 16, 2617, 32}

$$-\frac{2\sec^3(e+fx)\cos^2(e+fx)^{\frac{p+5}{2}}(g\tan(e+fx))^{p+2}\,{}_2F_1\left(\frac{p+2}{2},\frac{p+5}{2};\frac{p+4}{2};\sin^2(e+fx)\right)}{a^2fg^2(p+2)}+\frac{2(g\tan(e+fx))^{p+3}}{a^2fg^3(p+3)}+\frac{(g\tan(}{a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Tan[e + f*x])^p/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (g*Tan[e + f*x])^(1 + p)/(a^2*f*g*(1 + p)) - (2*(Cos[e + f*x]^2)^((5 + p)/2
)*Hypergeometric2F1[(2 + p)/2, (5 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sec[e
+ f*x]^3*(g*Tan[e + f*x])^(2 + p))/(a^2*f*g^2*(2 + p)) + (2*(g*Tan[e + f*x]
)^(3 + p))/(a^2*f*g^3*(3 + p))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 2711

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Dist[a^(2*m), Int[ExpandIntegrand[(g*Tan[e + f*x])
^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; Fr
eeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{(g\tan(e+fx))^p}{(a+a\sin(e+fx))^2}\,dx = \frac{\int \left(a^2\sec^4(e+fx)(g\tan(e+fx))^p - 2a^2\sec^3(e+fx)\tan(e+fx)(g\tan(e+fx))^p \cdots\right.}{a^4}$$

$$= \frac{\int \sec^4(e+fx)(g\tan(e+fx))^p\,dx}{a^2} + \frac{\int \sec^2(e+fx)\tan^2(e+fx)(g\tan(e+fx))^p\,d\cdots}{a^2}$$

$$= \frac{\text{Subst}\left(\int (gx)^p\left(1+x^2\right)\,dx, x, \tan(e+fx)\right)}{a^2 f} + \frac{\int \sec^2(e+fx)(g\tan(e+fx))^{2+p}\,dx}{a^2g^2} \cdots$$

$$= -\frac{2\cos^2(e+fx)^{\frac{5+p}{2}}\,{}_2F_1\left(\frac{2+p}{2},\frac{5+p}{2};\frac{4+p}{2};\sin^2(e+fx)\right)\sec^3(e+fx)(g\tan(e+fx))^{2+p}\cdots}{a^2fg^2(2+p)}$$

$$= \frac{(g\tan(e+fx))^{1+p}}{a^2fg(1+p)} - \frac{2\cos^2(e+fx)^{\frac{5+p}{2}}\,{}_2F_1\left(\frac{2+p}{2},\frac{5+p}{2};\frac{4+p}{2};\sin^2(e+fx)\right)\sec^3(e+f\cdots}{a^2fg^2(2+p)}$$

Mathematica [B] time = 13.96, size = 667, normalized size = 4.83

$$2^{p+1}\tan\left(\frac{1}{2}(e+fx)\right)\left(1-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)^p\left(-\frac{\tan\left(\frac{1}{2}(e+fx)\right)}{\tan^2\left(\frac{1}{2}(e+fx)\right)-1}\right)^p\tan^{-p}(e+fx)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Tan[e + f*x])^p/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (2^(1 + p)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*Tan[(e + f*x)/2]*(1 - Ta
n[(e + f*x)/2]^2)^p*(-(Tan[(e + f*x)/2]/(-1 + Tan[(e + f*x)/2]^2)))^p*(Hype
rgeometric2F1[(1 + p)/2, 2 + p, (3 + p)/2, Tan[(e + f*x)/2]^2]/(1 + p) - (2
*Hypergeometric2F1[(1 + p)/2, 3 + p, (3 + p)/2, Tan[(e + f*x)/2]^2])/(1 + p
) + (2*Hypergeometric2F1[(1 + p)/2, 4 + p, (3 + p)/2, Tan[(e + f*x)/2]^2])/
(1 + p) - (2*Hypergeometric2F1[(2 + p)/2, 2 + p, (4 + p)/2, Tan[(e + f*x)/2
]^2]*Tan[(e + f*x)/2])/(2 + p) + (6*Hypergeometric2F1[(2 + p)/2, 3 + p, (4
+ p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2])/(2 + p) - (8*Hypergeometric2F
1[(2 + p)/2, 4 + p, (4 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2])/(2 + p
) + (Hypergeometric2F1[2 + p, (3 + p)/2, (5 + p)/2, Tan[(e + f*x)/2]^2]*Tan
[(e + f*x)/2]^2)/(3 + p) - (6*Hypergeometric2F1[(3 + p)/2, 3 + p, (5 + p)/2
, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2)/(3 + p) + (12*Hypergeometric2F1[(
3 + p)/2, 4 + p, (5 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2)/(3 + p)
 + (2*Hypergeometric2F1[3 + p, (4 + p)/2, (6 + p)/2, Tan[(e + f*x)/2]^2]*Ta
n[(e + f*x)/2]^3)/(4 + p) - (8*Hypergeometric2F1[(4 + p)/2, 4 + p, (6 + p)/
2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^3)/(4 + p) + (2*Hypergeometric2F1[4
 + p, (5 + p)/2, (7 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^4)/(5 + p)
)*(g*Tan[e + f*x])^p)/(f*(a + a*Sin[e + f*x])^2*Tan[e + f*x]^p)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(g\tan\left(fx+e\right)\right)^p}{a^2\cos\left(fx+e\right)^2-2\,a^2\sin\left(fx+e\right)-2\,a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-(g*tan(f*x + e))^p/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a
^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(g\tan\left(fx+e\right)\right)^p}{\left(a\sin\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^2,x, algorithm="giac")
[Out] integrate((g*tan(f*x + e))^p/(a*sin(f*x + e) + a)^2, x)
```

maple [F] time = 1.02, size = 0, normalized size = 0.00

$$\int \frac{\left(g \tan \left(f x+e\right)\right)^{p}}{\left(a+a \sin \left(f x+e\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^2,x)
[Out] int((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g \tan \left(f x+e\right)\right)^{p}}{\left(a \sin \left(f x+e\right)+a\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((g*tan(f*x + e))^p/(a*sin(f*x + e) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(g \tan \left(e+f\,x\right)\right)^{p}}{\left(a+a\,\sin \left(e+f\,x\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*tan(e + f*x))^p/(a + a*sin(e + f*x))^2,x)
[Out] int((g*tan(e + f*x))^p/(a + a*sin(e + f*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\left(g \tan \left(e+f x\right)\right)^{p}}{\sin^{2}\left(e+f x\right)+2 \sin \left(e+f x\right)+1}\, dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))**p/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Integral((g*tan(e + f*x))**p/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1), x)/a**
2
```

3.128 $\int \frac{(g\tan(e+fx))^p}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=248

$$\frac{4(g\tan(e+fx))^{p+5}}{a^3fg^5(p+5)}-\frac{\sec^3(e+fx)\cos^2(e+fx)^{\frac{p+7}{2}}(g\tan(e+fx))^{p+4}\,{}_2F_1\left(\frac{p+4}{2},\frac{p+7}{2};\frac{p+6}{2};\sin^2(e+fx)\right)}{a^3fg^4(p+4)}+\frac{5(g\tan(e}{a^3fg^3}$$

```
[Out] (g*tan(f*x+e))^(1+p)/a^3/f/g/(1+p)-3*(cos(f*x+e)^2)^(7/2+1/2*p)*hypergeom([
1+1/2*p, 7/2+1/2*p],[2+1/2*p],sin(f*x+e)^2)*sec(f*x+e)^5*(g*tan(f*x+e))^(2+
p)/a^3/f/g^2/(2+p)+5*(g*tan(f*x+e))^(3+p)/a^3/f/g^3/(3+p)-(cos(f*x+e)^2)^(7
/2+1/2*p)*hypergeom([2+1/2*p, 7/2+1/2*p],[3+1/2*p],sin(f*x+e)^2)*sec(f*x+e)
^3*(g*tan(f*x+e))^(4+p)/a^3/f/g^4/(4+p)+4*(g*tan(f*x+e))^(5+p)/a^3/f/g^5/(5
+p)
```

Rubi [A] time = 0.42, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2711, 2607, 270, 16, 2617, 14}

$$-\frac{3\sec^5(e+fx)\cos^2(e+fx)^{\frac{p+7}{2}}(g\tan(e+fx))^{p+2}\,{}_2F_1\left(\frac{p+2}{2},\frac{p+7}{2};\frac{p+4}{2};\sin^2(e+fx)\right)}{a^3fg^2(p+2)}-\frac{\sec^3(e+fx)\cos^2(e+fx)^{\frac{p}{}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Tan[e + f*x])^p/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (g*Tan[e + f*x])^(1 + p)/(a^3*f*g*(1 + p)) - (3*(Cos[e + f*x]^2)^((7 + p)/2
)*Hypergeometric2F1[(2 + p)/2, (7 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sec[e
+ f*x]^5*(g*Tan[e + f*x])^(2 + p))/(a^3*f*g^2*(2 + p)) + (5*(g*Tan[e + f*x]
)^(3 + p))/(a^3*f*g^3*(3 + p)) - ((Cos[e + f*x]^2)^((7 + p)/2)*Hypergeometr
ic2F1[(4 + p)/2, (7 + p)/2, (6 + p)/2, Sin[e + f*x]^2]*Sec[e + f*x]^3*(g*Ta
n[e + f*x])^(4 + p))/(a^3*f*g^4*(4 + p)) + (4*(g*Tan[e + f*x])^(5 + p))/(a^
3*f*g^5*(5 + p))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 2711

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Dist[a^(2*m), Int[ExpandIntegrand[(g*Tan[e + f*x])
^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; Fr
eeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{(g\tan(e+fx))^p}{(a+a\sin(e+fx))^3}\,dx = \frac{\int \left(a^3\sec^6(e+fx)(g\tan(e+fx))^p - 3a^3\sec^5(e+fx)\tan(e+fx)(g\tan(e+fx))^p\right.}{}$$

$$= \frac{\int \sec^6(e+fx)(g\tan(e+fx))^p\,dx}{a^3} - \frac{\int \sec^3(e+fx)\tan^3(e+fx)(g\tan(e+fx))^p\,d}{a^3}$$

$$= \frac{\text{Subst}\left(\int (gx)^p\left(1+x^2\right)^2\,dx, x, \tan(e+fx)\right)}{a^3 f} - \frac{\int \sec^3(e+fx)(g\tan(e+fx))^{3+p}\,dx}{a^3 g^3}$$

$$= -\frac{3\cos^2(e+fx)^{\frac{7+p}{2}}\,{}_2F_1\left(\frac{2+p}{2},\frac{7+p}{2};\frac{4+p}{2};\sin^2(e+fx)\right)\sec^5(e+fx)(g\tan(e+fx))^{2+p}}{a^3 f g^2(2+p)}$$

$$= \frac{(g\tan(e+fx))^{1+p}}{a^3 f g(1+p)} - \frac{3\cos^2(e+fx)^{\frac{7+p}{2}}\,{}_2F_1\left(\frac{2+p}{2},\frac{7+p}{2};\frac{4+p}{2};\sin^2(e+fx)\right)\sec^5(e+f}{a^3 f g^2(2+p)}$$

$$= \frac{(g\tan(e+fx))^{1+p}}{a^3 f g(1+p)} - \frac{3\cos^2(e+fx)^{\frac{7+p}{2}}\,{}_2F_1\left(\frac{2+p}{2},\frac{7+p}{2};\frac{4+p}{2};\sin^2(e+fx)\right)\sec^5(e+f}{a^3 f g^2(2+p)}$$

Mathematica [B] time = 27.82, size = 1276, normalized size = 5.15

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Tan[e + f*x])^p/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (2^(1 + p)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*Tan[(e + f*x)/2]*(1 - Ta
n[(e + f*x)/2]^2)^p*(-(Tan[(e + f*x)/2]/(-1 + Tan[(e + f*x)/2]^2)))^p*(Hype
rgeometric2F1[(1 + p)/2, 2 + p, (3 + p)/2, Tan[(e + f*x)/2]^2]/(1 + p) - (4
*Hypergeometric2F1[(1 + p)/2, 3 + p, (3 + p)/2, Tan[(e + f*x)/2]^2])/(1 + p
) + (8*Hypergeometric2F1[(1 + p)/2, 4 + p, (3 + p)/2, Tan[(e + f*x)/2]^2])/
(1 + p) - (8*Hypergeometric2F1[(1 + p)/2, 5 + p, (3 + p)/2, Tan[(e + f*x)/2
]^2])/(1 + p) + (4*Hypergeometric2F1[(1 + p)/2, 6 + p, (3 + p)/2, Tan[(e +
f*x)/2]^2])/(1 + p) - (2*Hypergeometric2F1[(2 + p)/2, 2 + p, (4 + p)/2, Tan
[(e + f*x)/2]^2]*Tan[(e + f*x)/2])/(2 + p) + (12*Hypergeometric2F1[(2 + p)/
2, 3 + p, (4 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2])/(2 + p) - (32*Hy
pergeometric2F1[(2 + p)/2, 4 + p, (4 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f
*x)/2])/(2 + p) + (40*Hypergeometric2F1[(2 + p)/2, 5 + p, (4 + p)/2, Tan[(e
 + f*x)/2]^2]*Tan[(e + f*x)/2])/(2 + p) - (24*Hypergeometric2F1[(2 + p)/2,
6 + p, (4 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2])/(2 + p) + (Hypergeo
metric2F1[2 + p, (3 + p)/2, (5 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]
^2)/(3 + p) - (12*Hypergeometric2F1[(3 + p)/2, 3 + p, (5 + p)/2, Tan[(e + f
```

```
*x)/2]^2]*Tan[(e + f*x)/2]^2)/(3 + p) + (48*Hypergeometric2F1[(3 + p)/2, 4
+ p, (5 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2)/(3 + p) - (80*Hyper
geometric2F1[(3 + p)/2, 5 + p, (5 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)
/2]^2)/(3 + p) + (60*Hypergeometric2F1[(3 + p)/2, 6 + p, (5 + p)/2, Tan[(e
+ f*x)/2]^2]*Tan[(e + f*x)/2]^2)/(3 + p) + (4*Hypergeometric2F1[3 + p, (4 +
 p)/2, (6 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^3)/(4 + p) - (32*Hyp
ergeometric2F1[(4 + p)/2, 4 + p, (6 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*
x)/2]^3)/(4 + p) + (80*Hypergeometric2F1[(4 + p)/2, 5 + p, (6 + p)/2, Tan[(
e + f*x)/2]^2]*Tan[(e + f*x)/2]^3)/(4 + p) - (80*Hypergeometric2F1[(4 + p)/
2, 6 + p, (6 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^3)/(4 + p) + (8*H
ypergeometric2F1[4 + p, (5 + p)/2, (7 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e +
f*x)/2]^4)/(5 + p) - (40*Hypergeometric2F1[(5 + p)/2, 5 + p, (7 + p)/2, Tan
[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^4)/(5 + p) + (60*Hypergeometric2F1[(5 + p
)/2, 6 + p, (7 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^4)/(5 + p) + (8
*Hypergeometric2F1[3 + p/2, 5 + p, 4 + p/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*
x)/2]^5)/(6 + p) - (24*Hypergeometric2F1[(6 + p)/2, 6 + p, (8 + p)/2, Tan[(
e + f*x)/2]^2]*Tan[(e + f*x)/2]^5)/(6 + p) + (4*Hypergeometric2F1[6 + p, (7
 + p)/2, (9 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^6)/(7 + p))*(g*Tan
[e + f*x])^p)/(f*(a + a*Sin[e + f*x])^3*Tan[e + f*x]^p)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(g\tan\left(fx+e\right)\right)^{p}}{3\,a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}+\left(a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral(-(g*tan(f*x + e))^p/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x +
 e)^2 - 4*a^3)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(g\tan\left(fx+e\right)\right)^{p}}{\left(a\sin\left(fx+e\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((g*tan(f*x + e))^p/(a*sin(f*x + e) + a)^3, x)
```

maple [F] time = 1.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\tan\left(fx+e\right)\right)^{p}}{\left(a+a\sin\left(fx+e\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^3,x)
[Out] int((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\tan\left(fx+e\right)\right)^{p}}{\left(a\sin\left(fx+e\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
[Out] integrate((g*tan(f*x + e))^p/(a*sin(f*x + e) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g\tan\left(e+f\,x\right)\right)^{p}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*tan(e + f*x))^p/(a + a*sin(e + f*x))^3,x)
[Out] int((g*tan(e + f*x))^p/(a + a*sin(e + f*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\left(g\tan\left(e+fx\right)\right)^{p}}{\sin^{3}\left(e+fx\right)+3\sin^{2}\left(e+fx\right)+3\sin\left(e+fx\right)+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))**p/(a+a*sin(f*x+e))**3,x)
[Out] Integral((g*tan(e + f*x))**p/(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*sin(e
 + f*x) + 1), x)/a**3
```

3.129 $\int (a+a\sin(e+fx))^m(g\tan(e+fx))^p\,dx$

Optimal. Leaf size=111

$$\frac{(1-\sin(e+fx))^{\frac{p+1}{2}}(a\sin(e+fx)+a)^m(g\tan(e+fx))^{p+1}(\sin(e+fx)+1)^{\frac{1}{2}(-2m+p+1)}F_1\left(p+1;\frac{p+1}{2},\frac{1}{2}(-2m+p+\right.}{fg(p+1)}$$

```
[Out] AppellF1(1+p,1/2-m+1/2*p,1/2+1/2*p,2+p,-sin(f*x+e),sin(f*x+e))*(1-sin(f*x+e
))^(1/2+1/2*p)*(1+sin(f*x+e))^(1/2-m+1/2*p)*(a+a*sin(f*x+e))^m*(g*tan(f*x+e
))^(1+p)/f/g/(1+p)
```

Rubi [A] time = 0.12, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2720, 135, 133}

$$\frac{(1-\sin(e+fx))^{\frac{p+1}{2}}(a\sin(e+fx)+a)^m(g\tan(e+fx))^{p+1}(\sin(e+fx)+1)^{\frac{1}{2}(-2m+p+1)}F_1\left(p+1;\frac{p+1}{2},\frac{1}{2}(-2m+p+\right.}{fg(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(g*Tan[e + f*x])^p,x]
```

```
[Out] (AppellF1[1 + p, (1 + p)/2, (1 - 2*m + p)/2, 2 + p, Sin[e + f*x], -Sin[e +
f*x]]*(1 - Sin[e + f*x])^((1 + p)/2)*(1 + Sin[e + f*x])^((1 - 2*m + p)/2)*(
a + a*Sin[e + f*x])^m*(g*Tan[e + f*x])^(1 + p))/(f*g*(1 + p))
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 2720

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_), x_Symbol] :> Dist[((g*Tan[e + f*x])^(p + 1)*(a - b*Sin[e + f*x])
```

```
^((p + 1)/2)*(a + b*Sin[e + f*x])^((p + 1)/2))/(f*g*(b*Sin[e + f*x])^(p + 1
)), Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^((p + 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&
  !IntegerQ[m] &&  !IntegerQ[p]
```

Rubi steps

$$\int (a + a\sin(e + fx))^m (g\tan(e + fx))^p\,dx = \frac{\left((a\sin(e + fx))^{-1-p}(a - a\sin(e + fx))^{\frac{1+p}{2}}(a + a\sin(e + fx))^{\frac{1+p}{2}}(g\,\mathrm{ta}\right.}{}$$

$$= \frac{\left((1 - \sin(e + fx))^{\frac{1}{2}+\frac{p}{2}}(a\sin(e + fx))^{-1-p}(a - a\sin(e + fx))^{-\frac{1}{2}-\frac{p}{2}+\frac{1+}{2}}\right.}{}$$

$$= \frac{\left((1 - \sin(e + fx))^{\frac{1}{2}+\frac{p}{2}}(a\sin(e + fx))^{-1-p}(1 + \sin(e + fx))^{\frac{1}{2}-m+\frac{p}{2}}(a\right.}{}$$

$$= \frac{F_1\left(1 + p; \frac{1+p}{2}, \frac{1}{2}(1 - 2m + p); 2 + p; \sin(e + fx), -\sin(e + fx)\right)(1 -}{}$$

Mathematica [B] time = 2.18, size = 367, normalized size = 3.31

$$-\frac{2(p - 3)\sin\left(\frac{1}{4}(2e + 2fx - \pi)\right)\cos^3\left(\frac{1}{4}\right.}{f(p - 1)\left((p - 3)\cos^2\left(\frac{1}{4}(2e + 2fx - \pi)\right)F_1\left(\frac{1-p}{2}; -p, m + 1; \frac{3-p}{2}; \cot^2\left(\frac{1}{4}(2e + 2fx + \pi)\right), -\tan^2\left(\frac{1}{4}(2e + 2fx -\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(g*Tan[e + f*x])^p,x]
```

```
[Out] (-2*(-3 + p)*AppellF1[(1 - p)/2, -p, 1 + m, (3 - p)/2, Cot[(2*e + Pi + 2*f*
x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*Cos[(2*e - Pi + 2*f*x)/4]^3*(a*(1 +
Sin[e + f*x]))^m*Sin[(2*e - Pi + 2*f*x)/4]*(g*Tan[e + f*x])^p)/(f*(-1 + p)*
((-3 + p)*AppellF1[(1 - p)/2, -p, 1 + m, (3 - p)/2, Cot[(2*e + Pi + 2*f*x)/
4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*Cos[(2*e - Pi + 2*f*x)/4]^2 + 2*(p*Appe
llF1[(3 - p)/2, 1 - p, 1 + m, (5 - p)/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[
(2*e - Pi + 2*f*x)/4]^2] + (1 + m)*AppellF1[(3 - p)/2, -p, 2 + m, (5 - p)/2
, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2])*Sin[(2*e - Pi
 + 2*f*x)/4]^2))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{m}\left(g\tan\left(fx+e\right)\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(g*tan(f*x+e))^p,x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)^m*(g*tan(f*x + e))^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{m}\left(g\tan\left(fx+e\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(g*tan(f*x+e))^p,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*(g*tan(f*x + e))^p, x)
```

maple [F] time = 1.32, size = 0, normalized size = 0.00

$$\int\left(a+a\sin\left(fx+e\right)\right)^{m}\left(g\tan\left(fx+e\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(g*tan(f*x+e))^p,x)
[Out] int((a+a*sin(f*x+e))^m*(g*tan(f*x+e))^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{m}\left(g\tan\left(fx+e\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(g*tan(f*x+e))^p,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^m*(g*tan(f*x + e))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int{\left(g\,\tan\left(e+f\,x\right)\right)}^p\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*tan(e + f*x))^p*(a + a*sin(e + f*x))^m,x)
[Out] int((g*tan(e + f*x))^p*(a + a*sin(e + f*x))^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}\left(g\tan\left(e+fx\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(g*tan(f*x+e))**p,x)
[Out] Integral((a*(sin(e + f*x) + 1))**m*(g*tan(e + f*x))**p, x)
```

3.130 $\int (a+a\sin(e+fx))^m \tan^3(e+fx)\,dx$

Optimal. Leaf size=163

$$-\frac{a^2\sin^2(e+fx)(a\sin(e+fx)+a)^{m-1}}{fm(a-a\sin(e+fx))}+\frac{a(m+4)(a\sin(e+fx)+a)^{m-1}\,{}_2F_1\left(1,m-1;m;\frac{1}{2}(\sin(e+fx)+1)\right)}{4f(1-m)}+\frac{(2$$

```
[Out] 1/4*a*(4+m)*hypergeom([1, -1+m],[m],1/2+1/2*sin(f*x+e))*(a+a*sin(f*x+e))^(-
1+m)/f/(1-m)-a^2*sin(f*x+e)^2*(a+a*sin(f*x+e))^(-1+m)/f/m/(a-a*sin(f*x+e))+
1/2*(a+a*sin(f*x+e))^(-1+m)*(a*(-m^2-3*m+2)+2*a*m*sin(f*x+e))/f/(1-m)/m/(1-
sin(f*x+e))
```

Rubi [A] time = 0.15, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2707, 100, 146, 68}

$$-\frac{a^2\sin^2(e+fx)(a\sin(e+fx)+a)^{m-1}}{fm(a-a\sin(e+fx))}+\frac{a(m+4)(a\sin(e+fx)+a)^{m-1}\,{}_2F_1\left(1,m-1;m;\frac{1}{2}(\sin(e+fx)+1)\right)}{4f(1-m)}+\frac{(2$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*Tan[e + f*x]^3,x]
```

```
[Out] (a*(4 + m)*Hypergeometric2F1[1, -1 + m, m, (1 + Sin[e + f*x])/2]*(a + a*Sin
[e + f*x])^(-1 + m))/(4*f*(1 - m)) - (a^2*Sin[e + f*x]^2*(a + a*Sin[e + f*x
])^(-1 + m))/(f*m*(a - a*Sin[e + f*x])) + ((a + a*Sin[e + f*x])^(-1 + m)*(a
*(2 - 3*m - m^2) + 2*a*m*Sin[e + f*x]))/(2*f*(1 - m)*m*(1 - Sin[e + f*x]))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 146

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m
 + 1)*(m + n + 3)), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m\tan^3(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^3(a+x)^{-2+m}}{(a-x)^2}\,dx, x, a\sin(e+fx)\right)}{f}$$

$$= -\frac{a^2\sin^2(e+fx)(a+a\sin(e+fx))^{-1+m}}{fm(a-a\sin(e+fx))} - \frac{\text{Subst}\left(\int \frac{x(a+x)^{-2+m}\left(-2a^2-am\right.}{(a-x)^2}\right.}{fm}$$

$$= -\frac{a^2\sin^2(e+fx)(a+a\sin(e+fx))^{-1+m}}{fm(a-a\sin(e+fx))} + \frac{(a+a\sin(e+fx))^{-1+m}\left(a\right.}{2f(1-m)}$$

$$= \frac{a(4+m)\,{}_2F_1\left(1,-1+m;m;\frac{1}{2}(1+\sin(e+fx))\right)(a+a\sin(e+fx))^{-1+}}{4f(1-m)}$$

Mathematica [A] time = 0.26, size = 105, normalized size = 0.64

$$\frac{a(a(\sin(e+fx)+1))^{m-1}\left(-m(m+4)(\sin(e+fx)-1)\,{}_2F_1\left(1,m-1;m;\frac{1}{2}(\sin(e+fx)+1)\right)+4(m-1)\sin^2(e+\right.}{4f(m-1)m(\sin(e+fx)-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*Tan[e + f*x]^3,x]
```

```
[Out] (a*(a*(1 + Sin[e + f*x]))^(-1 + m)*(-2*(-2 + 3*m + m^2) - m*(4 + m)*Hyperge
ometric2F1[1, -1 + m, m, (1 + Sin[e + f*x])/2]*(-1 + Sin[e + f*x]) + 4*m*Si
n[e + f*x] + 4*(-1 + m)*Sin[e + f*x]^2))/(4*f*(-1 + m)*m*(-1 + Sin[e + f*x]
))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{m}\tan\left(fx+e\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*tan(f*x + e)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{m}\tan\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e)^3,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*tan(f*x + e)^3, x)
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int\left(a+a\sin\left(fx+e\right)\right)^{m}\left(\tan^{3}\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*tan(f*x+e)^3,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*tan(f*x+e)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{m}\tan\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*tan(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan\left(e+f\,x\right)^3\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3*(a + a*sin(e + f*x))^m,x)
[Out] int(tan(e + f*x)^3*(a + a*sin(e + f*x))^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^m \tan^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*tan(f*x+e)**3,x)
[Out] Integral((a*(sin(e + f*x) + 1))**m*tan(e + f*x)**3, x)
```

3.131 $\int (a+a\sin(e+fx))^m \tan(e+fx)\,dx$

Optimal. Leaf size=72

$$\frac{(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{1}{2}(\sin(e+fx)+1)\right)}{4af(m+1)}-\frac{(a\sin(e+fx)+a)^m}{2fm}$$

```
[Out] -1/2*(a+a*sin(f*x+e))^m/f/m+1/4*hypergeom([1, 1+m],[2+m],1/2+1/2*sin(f*x+e)
)*(a+a*sin(f*x+e))^(1+m)/a/f/(1+m)
```

Rubi [A] time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2707, 79, 68}

$$\frac{(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{1}{2}(\sin(e+fx)+1)\right)}{4af(m+1)}-\frac{(a\sin(e+fx)+a)^m}{2fm}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*Tan[e + f*x],x]
```

```
[Out] -(a + a*Sin[e + f*x])^m/(2*f*m) + (Hypergeometric2F1[1, 1 + m, 2 + m, (1 +
Sin[e + f*x])/2]*(a + a*Sin[e + f*x])^(1 + m))/(4*a*f*(1 + m))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
```

```
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m\tan(e+fx)\,dx=\frac{\text{Subst}\left(\int\frac{x(a+x)^{-1+m}}{a-x}\,dx,x,a\sin(e+fx)\right)}{f}$$
$$=-\frac{(a+a\sin(e+fx))^m}{2fm}+\frac{\text{Subst}\left(\int\frac{(a+x)^m}{a-x}\,dx,x,a\sin(e+fx)\right)}{2f}$$
$$=-\frac{(a+a\sin(e+fx))^m}{2fm}+\frac{{}_2F_1\left(1,1+m;2+m;\frac{1}{2}(1+\sin(e+fx))\right)(a+}{4af(1+m)}$$

Mathematica [A] time = 0.07, size = 63, normalized size = 0.88

$$\frac{(a(\sin(e+fx)+1))^m\left(m(\sin(e+fx)+1)\,{}_2F_1\left(1,m+1;m+2;\frac{1}{2}(\sin(e+fx)+1)\right)-2(m+1)\right)}{4fm(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*Tan[e + f*x],x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^m*(-2*(1 + m) + m*Hypergeometric2F1[1, 1 + m, 2 + m
, (1 + Sin[e + f*x])/2]*(1 + Sin[e + f*x])))/(4*f*m*(1 + m))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^m\tan\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e),x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*tan(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^m\tan\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*tan(f*x + e), x)
```

maple [F] time = 1.37, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\tan\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*tan(f*x+e),x)
[Out] int((a+a*sin(f*x+e))^m*tan(f*x+e),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\tan\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^m*tan(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan\left(e+f\,x\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)*(a + a*sin(e + f*x))^m,x)
[Out] int(tan(e + f*x)*(a + a*sin(e + f*x))^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}\tan\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*tan(f*x+e),x)
[Out] Integral((a*(sin(e + f*x) + 1))**m*tan(e + f*x), x)
```

3.132 $\int \cot(e+fx)(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=43

$$-\frac{(a\sin(e+fx)+a)^{m+1}\,{}_2F_1(1,m+1;m+2;\sin(e+fx)+1)}{af(m+1)}$$

```
[Out] -hypergeom([1, 1+m],[2+m],1+sin(f*x+e))*(a+a*sin(f*x+e))^(1+m)/a/f/(1+m)
```

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2707, 65}

$$-\frac{(a\sin(e+fx)+a)^{m+1}\,{}_2F_1(1,m+1;m+2;\sin(e+fx)+1)}{af(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((Hypergeometric2F1[1, 1 + m, 2 + m, 1 + Sin[e + f*x]]*(a + a*Sin[e + f*x]
)^(1 + m))/(a*f*(1 + m)))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cot(e+fx)(a+a\sin(e+fx))^m\,dx &= \frac{\text{Subst}\left(\int \frac{(a+x)^m}{x}\,dx,x,a\sin(e+fx)\right)}{f}\\ &= -\frac{{}_2F_1(1,1+m;2+m;1+\sin(e+fx))(a+a\sin(e+fx))^{1+m}}{af(1+m)}\end{aligned}$$

Mathematica [A] time = 0.05, size = 43, normalized size = 1.00

$$-\frac{(a\sin(e+fx)+a)^{m+1}\,{}_2F_1(1,m+1;m+2;\sin(e+fx)+1)}{af(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((Hypergeometric2F1[1, 1 + m, 2 + m, 1 + Sin[e + f*x]]*(a + a*Sin[e + f*x]
)^(1 + m))/(a*f*(1 + m)))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{m}\cot\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*cot(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{m}\cot\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cot(f*x + e), x)
```

maple [F] time = 1.42, size = 0, normalized size = 0.00

$$\int\cot\left(fx+e\right)\left(a+a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int(cot(f*x+e)*(a+a*sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{m}\cot\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cot(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \cot\left(e+f\,x\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)*(a + a*sin(e + f*x))^m,x)
```

```
[Out] int(cot(e + f*x)*(a + a*sin(e + f*x))^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}\cot\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*cot(e + f*x), x)
```

3.133 $\int \cot^3(e+fx)(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=83

$$-\frac{(2-m)(a\sin(e+fx)+a)^{m+2}\,{}_2F_1(2,m+2;m+3;\sin(e+fx)+1)}{2a^2f(m+2)}-\frac{\csc^2(e+fx)(a\sin(e+fx)+a)^{m+2}}{2a^2f}$$

```
[Out] -1/2*csc(f*x+e)^2*(a+a*sin(f*x+e))^(2+m)/a^2/f-1/2*(2-m)*hypergeom([2, 2+m]
,[3+m],1+sin(f*x+e))*(a+a*sin(f*x+e))^(2+m)/a^2/f/(2+m)
```

Rubi [A] time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2707, 78, 65}

$$-\frac{(2-m)(a\sin(e+fx)+a)^{m+2}\,{}_2F_1(2,m+2;m+3;\sin(e+fx)+1)}{2a^2f(m+2)}-\frac{\csc^2(e+fx)(a\sin(e+fx)+a)^{m+2}}{2a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -(Csc[e + f*x]^2*(a + a*Sin[e + f*x])^(2 + m))/(2*a^2*f) - ((2 - m)*Hyperge
ometric2F1[2, 2 + m, 3 + m, 1 + Sin[e + f*x]]*(a + a*Sin[e + f*x])^(2 + m))
/(2*a^2*f*(2 + m))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
```

```
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \cot^3(e+fx)(a+a\sin(e+fx))^m\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)(a+x)^{1+m}}{x^3}\,dx, x, a\sin(e+fx)\right)}{f}$$

$$= -\frac{\csc^2(e+fx)(a+a\sin(e+fx))^{2+m}}{2a^2 f} - \frac{(2-m)\,\text{Subst}\left(\int \frac{(a+x)^{1+m}}{x^2}\,dx, x\right.}{2f}$$

$$= -\frac{\csc^2(e+fx)(a+a\sin(e+fx))^{2+m}}{2a^2 f} - \frac{(2-m)\,{}_2F_1(2,2+m;3+m;1}{2a^2}$$

Mathematica [A] time = 0.19, size = 68, normalized size = 0.82

$$-\frac{(\sin(e+fx)+1)^2(a(\sin(e+fx)+1))^m\left((m+2)\csc^2(e+fx)-(m-2)\,{}_2F_1(2,m+2;m+3;\sin(e+fx)+1)\right)}{2f(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*(a + a*Sin[e + f*x])^m,x]
[Out] -1/2*(((2 + m)*Csc[e + f*x]^2 - (-2 + m)*Hypergeometric2F1[2, 2 + m, 3 + m,
 1 + Sin[e + f*x]])*(1 + Sin[e + f*x])^2*(a*(1 + Sin[e + f*x]))^m)/(f*(2 +
m))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^m\cot\left(fx+e\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)^m*cot(f*x + e)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^m\cot\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+a*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*cot(f*x + e)^3, x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \left(\cot^{3}\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a+a*sin(f*x+e))^m,x)
[Out] int(cot(f*x+e)^3*(a+a*sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\cot\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^m*cot(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(e+f\,x\right)^{3}\left(a+a\,\sin\left(e+f\,x\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3*(a + a*sin(e + f*x))^m,x)
[Out] int(cot(e + f*x)^3*(a + a*sin(e + f*x))^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}\cot^{3}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a+a*sin(f*x+e))**m,x)
[Out] Integral((a*(sin(e + f*x) + 1))**m*cot(e + f*x)**3, x)
```

3.134 $\int \cot^5(e+fx)(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=123

$$-\frac{\left(m^2-9m+12\right)(a\sin(e+fx)+a)^{m+3}\,{}_2F_1(3,m+3;m+4;\sin(e+fx)+1)}{12a^3f(m+3)}-\frac{\csc^4(e+fx)(a\sin(e+fx)+a)^{m}}{4a^3f}$$

```
[Out] 1/12*(9-m)*csc(f*x+e)^3*(a+a*sin(f*x+e))^(3+m)/a^3/f-1/4*csc(f*x+e)^4*(a+a*
sin(f*x+e))^(3+m)/a^3/f-1/12*(m^2-9*m+12)*hypergeom([3, 3+m],[4+m],1+sin(f*
x+e))*(a+a*sin(f*x+e))^(3+m)/a^3/f/(3+m)
```

Rubi [A] time = 0.10, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2707, 89, 78, 65}

$$-\frac{\left(m^2-9m+12\right)(a\sin(e+fx)+a)^{m+3}\,{}_2F_1(3,m+3;m+4;\sin(e+fx)+1)}{12a^3f(m+3)}-\frac{\csc^4(e+fx)(a\sin(e+fx)+a)^{m}}{4a^3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] ((9 - m)*Csc[e + f*x]^3*(a + a*Sin[e + f*x])^(3 + m))/(12*a^3*f) - (Csc[e +
 f*x]^4*(a + a*Sin[e + f*x])^(3 + m))/(4*a^3*f) - ((12 - 9*m + m^2)*Hyperge
ometric2F1[3, 3 + m, 4 + m, 1 + Sin[e + f*x]]*(a + a*Sin[e + f*x])^(3 + m))
/(12*a^3*f*(3 + m))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \cot^5(e+fx)(a+a\sin(e+fx))^m\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^{2+m}}{x^5}\,dx, x, a\sin(e+fx)\right)}{f}$$

$$= -\frac{\csc^4(e+fx)(a+a\sin(e+fx))^{3+m}}{4a^3f} + \frac{\text{Subst}\left(\int \frac{(a+x)^{2+m}\left(-a^2(9-m)+4ax\right)}{x^4}\,d\right.}{4af}$$

$$= \frac{(9-m)\csc^3(e+fx)(a+a\sin(e+fx))^{3+m}}{12a^3f} - \frac{\csc^4(e+fx)(a+a\sin(e+}{4a^3f}$$

$$= \frac{(9-m)\csc^3(e+fx)(a+a\sin(e+fx))^{3+m}}{12a^3f} - \frac{\csc^4(e+fx)(a+a\sin(e+}{4a^3f}$$

Mathematica [A] time = 0.26, size = 83, normalized size = 0.67

$$-\frac{(\sin(e+fx)+1)^3(a(\sin(e+fx)+1))^m\left(\left(m^2-9m+12\right){}_2F_1(3,m+3;m+4;\sin(e+fx)+1)+(m+3)\csc^3(e}{12f(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -1/12*(((3 + m)*Csc[e + f*x]^3*(-9 + m + 3*Csc[e + f*x]) + (12 - 9*m + m^2)
*Hypergeometric2F1[3, 3 + m, 4 + m, 1 + Sin[e + f*x]])*(1 + Sin[e + f*x])^3
*(a*(1 + Sin[e + f*x]))^m)/(f*(3 + m))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^m\cot\left(fx+e\right)^5,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)^m*cot(f*x + e)^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^m\cot\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+a*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*cot(f*x + e)^5, x)
```

maple [F] time = 0.53, size = 0, normalized size = 0.00

$$\int\left(\cot^5\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5*(a+a*sin(f*x+e))^m,x)
[Out] int(cot(f*x+e)^5*(a+a*sin(f*x+e))^m,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\cot\left(e+f\,x\right)^5\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5*(a + a*sin(e + f*x))^m,x)
[Out] int(cot(e + f*x)^5*(a + a*sin(e + f*x))^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5*(a+a*sin(f*x+e))**m,x)
[Out] Timed out
```

3.135 $\int (a+a\sin(e+fx))^m \tan^4(e+fx)\,dx$

Optimal. Leaf size=311

$$-\frac{a^2\sin^2(e+fx)\tan(e+fx)(a\sin(e+fx)+a)^{m-1}}{fm(a-a\sin(e+fx))}+\frac{a^2\sin(e+fx)\tan(e+fx)(a\sin(e+fx)+a)^{m-1}}{f(1-m)(a-a\sin(e+fx))}+\frac{2^{m-\frac{3}{2}}\left(m\right.}{}$$

```
[Out] 1/3*2^(-3/2+m)*(m^4+6*m^3-7*m^2-12*m+9)*hypergeom([1/2, 5/2-m],[3/2],1/2-1/
2*sin(f*x+e))*sec(f*x+e)*(1-sin(f*x+e))*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x
+e))^m/f/(1-m)/m-1/3*sec(f*x+e)*(a+a*sin(f*x+e))^(-1+m)*(a*(-m^3-7*m^2-m+6)
-a*(-m^3-8*m^2-6*m+9)*sin(f*x+e))/f/(1-m)/m/(1-sin(f*x+e))+a^2*sin(f*x+e)*(
a+a*sin(f*x+e))^(-1+m)*tan(f*x+e)/f/(1-m)/(a-a*sin(f*x+e))-a^2*sin(f*x+e)^2
*(a+a*sin(f*x+e))^(-1+m)*tan(f*x+e)/f/m/(a-a*sin(f*x+e))
```

Rubi [A] time = 0.36, antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2719, 100, 153, 145, 70, 69}

$$-\frac{a^2\sin^2(e+fx)\tan(e+fx)(a\sin(e+fx)+a)^{m-1}}{fm(a-a\sin(e+fx))}+\frac{a^2\sin(e+fx)\tan(e+fx)(a\sin(e+fx)+a)^{m-1}}{f(1-m)(a-a\sin(e+fx))}+\frac{2^{m-\frac{3}{2}}\left(m\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*Tan[e + f*x]^4,x]
```

```
[Out] (2^(-3/2 + m)*(9 - 12*m - 7*m^2 + 6*m^3 + m^4)*Hypergeometric2F1[1/2, 5/2 -
 m, 3/2, (1 - Sin[e + f*x])/2]*Sec[e + f*x]*(1 - Sin[e + f*x])*(1 + Sin[e +
 f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^m)/(3*f*(1 - m)*m) - (Sec[e + f*x]*(a
 + a*Sin[e + f*x])^(-1 + m)*(a*(6 - m - 7*m^2 - m^3) - a*(9 - 6*m - 8*m^2 -
 m^3)*Sin[e + f*x]))/(3*f*(1 - m)*m*(1 - Sin[e + f*x])) + (a^2*Sin[e + f*x]
*(a + a*Sin[e + f*x])^(-1 + m)*Tan[e + f*x])/(f*(1 - m)*(a - a*Sin[e + f*x]
)) - (a^2*Sin[e + f*x]^2*(a + a*Sin[e + f*x])^(-1 + m)*Tan[e + f*x])/(f*m*(
a - a*Sin[e + f*x]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 145

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(
n + 2) - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h)
 + d*e*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(
f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)*(
a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), x]
 + Dist[(f*h)/b^2 - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1
) - d*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)
)))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), Int[(a + b*x)^(m + 2)*(c + d*x)^n,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[
m + n + 3, 0] &&  !LtQ[n, -2]))
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 2719

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[(Sqrt[a + b*Sin[e + f*x]]*Sqrt[a - b*Sin[e + f*x]])/(b
```

```
*f*Cos[e + f*x]), Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] && IntegerQ[p/2]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m\tan^4(e+fx)\,dx=\frac{\left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}\right)\text{Subst}\left(\int\frac{x^4(a+x)^{-\frac{5}{2}}}{(a-x)^{5/}}\right.}{af}$$

$$=-\frac{a^2\sin^2(e+fx)(a+a\sin(e+fx))^{-1+m}\tan(e+fx)}{fm(a-a\sin(e+fx))}-\frac{\left(\sec(e+fx)\sqrt{\right.}{}$$

$$=\frac{a^2\sin(e+fx)(a+a\sin(e+fx))^{-1+m}\tan(e+fx)}{f(1-m)(a-a\sin(e+fx))}-\frac{a^2\sin^2(e+fx)(a}{fm}$$

$$=-\frac{\sec(e+fx)(a+a\sin(e+fx))^{-1+m}\left(a\left(6-m-7m^2-m^3\right)-a\left(9-6\right.\right.}{3f(1-m)m(1-\sin(e+fx))}$$

$$=-\frac{\sec(e+fx)(a+a\sin(e+fx))^{-1+m}\left(a\left(6-m-7m^2-m^3\right)-a\left(9-6\right.\right.}{3f(1-m)m(1-\sin(e+fx))}$$

$$=\frac{2^{-\frac{3}{2}+m}\left(9-12m-7m^2+6m^3+m^4\right){}_2F_1\left(\frac{1}{2},\frac{5}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+f\right.}{3f(1}$$

Mathematica [F] time = 1.12, size = 0, normalized size = 0.00

$$\int (a+a\sin(e+fx))^m\tan^4(e+fx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*Tan[e + f*x]^4,x]

[Out] Integrate[(a + a*Sin[e + f*x])^m*Tan[e + f*x]^4, x]
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{m}\tan\left(fx+e\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e)^4,x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)^m*tan(f*x + e)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{m}\tan\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e)^4,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*tan(f*x + e)^4, x)
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int\left(a+a\sin\left(fx+e\right)\right)^{m}\left(\tan^{4}\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*tan(f*x+e)^4,x)
[Out] int((a+a*sin(f*x+e))^m*tan(f*x+e)^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{m}\tan\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e)^4,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^m*tan(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\tan\left(e+fx\right)^{4}\left(a+a\,\sin\left(e+fx\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(a + a*sin(e + f*x))^m,x)
```

```
[Out] int(tan(e + f*x)^4*(a + a*sin(e + f*x))^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*tan(f*x+e)**4,x)
```

```
[Out] Timed out
```

3.136 $\int (a+a\sin(e+fx))^m \tan^2(e+fx)\,dx$

Optimal. Leaf size=157

$$\frac{2^{m-\frac{1}{2}}\left(-m^2-m+1\right)\sec(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(-\frac{1}{2},\frac{3}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f(1-m)m}+$$

```
[Out] sec(f*x+e)*(a+a*sin(f*x+e))^m/f/(1-m)/m+2^(-1/2+m)*(-m^2-m+1)*hypergeom([-1
/2, 3/2-m],[1/2],1/2-1/2*sin(f*x+e))*sec(f*x+e)*(1+sin(f*x+e))^(1/2-m)*(a+a
*sin(f*x+e))^m/f/(1-m)/m-sec(f*x+e)*(a+a*sin(f*x+e))^(1+m)/a/f/m
```

Rubi [A] time = 0.25, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2713, 2860, 2689, 70, 69}

$$\frac{2^{m-\frac{1}{2}}\left(-m^2-m+1\right)\sec(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(-\frac{1}{2},\frac{3}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f(1-m)m}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*Tan[e + f*x]^2,x]
```

```
[Out] (Sec[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 - m)*m) + (2^(-1/2 + m)*(1 - m
- m^2)*Hypergeometric2F1[-1/2, 3/2 - m, 1/2, (1 - Sin[e + f*x])/2]*Sec[e +
f*x]*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 - m)*m) - (
Sec[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*m)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2713

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> -Simp[(a + b*Sin[e + f*x])^(m + 1)/(b*f*m*Cos[e + f*x]), x] +
Dist[1/(b*m), Int[((a + b*Sin[e + f*x])^m*(b*(m + 1) + a*Sin[e + f*x]))/Cos
[e + f*x]^2, x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !
IntegerQ[m] &&  !LtQ[m, 0]
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m \tan^2(e+fx)\,dx = -\frac{\sec(e+fx)(a+a\sin(e+fx))^{1+m}}{afm} + \frac{\int \sec^2(e+fx)(a+a\sin(e+fx)}{a}$$

$$= \frac{\sec(e+fx)(a+a\sin(e+fx))^m}{f(1-m)m} - \frac{\sec(e+fx)(a+a\sin(e+fx))^{1+m}}{afm}$$

$$= \frac{\sec(e+fx)(a+a\sin(e+fx))^m}{f(1-m)m} - \frac{\sec(e+fx)(a+a\sin(e+fx))^{1+m}}{afm}$$

$$= \frac{\sec(e+fx)(a+a\sin(e+fx))^m}{f(1-m)m} - \frac{\sec(e+fx)(a+a\sin(e+fx))^{1+m}}{afm}$$

$$= \frac{\sec(e+fx)(a+a\sin(e+fx))^m}{f(1-m)m} + \frac{2^{-\frac{1}{2}+m}\left(1-m-m^2\right)\,{}_2F_1\left(-\frac{1}{2},\frac{3}{2}-m\right.}{}$$

Mathematica [C] time = 6.32, size = 4043, normalized size = 25.75

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*Tan[e + f*x]^2,x]
```

```
[Out] (-2*Cos[(-e + Pi/2 - f*x)/2]*Hypergeometric2F1[1/2, (1 + 2*m)/2, (3 + 2*m)/
2, Cos[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]*(a + a*Sin[e + f*x]
)^m)/(f*(1 + 2*m)*Sqrt[Sin[(-e + Pi/2 - f*x)/2]^2]) - ((Cos[(-e + Pi/2 - f*
x)/4]^2)^(2*m)*Cot[(-e + Pi/2 - f*x)/4]*(a + a*Sin[e + f*x])^m*(-(AppellF1[
-1/2, -2*m, 2*m, 1/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]
^2]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)) + (3*AppellF1[1/2, -2*m, 2*m, 3/2,
Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Tan[(-e + Pi/2 - f
*x)/4]^2*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(3*AppellF1[1/2, -2*m, 2*m
, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 4*m*(Appe
llF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 -
f*x)/4]^2] + AppellF1[3/2, -2*m, 1 + 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2,
-Tan[(-e + Pi/2 - f*x)/4]^2])*Tan[(-e + Pi/2 - f*x)/4]^2)))/(4*f*(Cos[Pi/4
+ (e - Pi/2 + f*x)/2] - Sin[Pi/4 + (e - Pi/2 + f*x)/2])^2*(-1/2*(m*(Cos[(-e
 + Pi/2 - f*x)/4]^2)^(2*m)*(-(AppellF1[-1/2, -2*m, 2*m, 1/2, Tan[(-e + Pi/2
 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*
m)) + (3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e
 + Pi/2 - f*x)/4]^2]*Tan[(-e + Pi/2 - f*x)/4]^2*(1 - Tan[(-e + Pi/2 - f*x)/
4]^2)^(2*m))/(3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -
Tan[(-e + Pi/2 - f*x)/4]^2] - 4*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-e
 + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[3/2, -2*m, 1 +
 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Tan[(-
e + Pi/2 - f*x)/4]^2))) - ((Cos[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Csc[(-e + Pi/
2 - f*x)/4]^2*(-(AppellF1[-1/2, -2*m, 2*m, 1/2, Tan[(-e + Pi/2 - f*x)/4]^2,
 -Tan[(-e + Pi/2 - f*x)/4]^2]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)) + (3*Appe
llF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x
)/4]^2]*Tan[(-e + Pi/2 - f*x)/4]^2*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))/
(3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/
2 - f*x)/4]^2] - 4*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-e + Pi/2 - f*x
)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[3/2, -2*m, 1 + 2*m, 5/2, Ta
n[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Tan[(-e + Pi/2 - f*
x)/4]^2)))/8 + ((Cos[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Cot[(-e + Pi/2 - f*x)/4]
*(-(m*AppellF1[-1/2, -2*m, 2*m, 1/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e +
 Pi/2 - f*x)/4]^2]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Tan[(-e + Pi/2 - f*x)
/4]) - (Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(m*AppellF1[1/2, 1 - 2*m, 2*m, 3/
2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2
- f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4] + m*AppellF1[1/2, -2*m, 1 + 2*m, 3/2,
Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f
```

*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]) + (3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(
-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4
]^2*Tan[(-e + Pi/2 - f*x)/4]*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(2*(3*
AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 -
f*x)/4]^2] - 4*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4
]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[3/2, -2*m, 1 + 2*m, 5/2, Tan[(
-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Tan[(-e + Pi/2 - f*x)/
4]^2)) + (3*Tan[(-e + Pi/2 - f*x)/4]^2*(-1/3*(m*AppellF1[3/2, 1 - 2*m, 2*m,
5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi
/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]) - (m*AppellF1[3/2, -2*m, 1 + 2*m,
5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/
2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/3)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)
^(2*m))/(3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(
-e + Pi/2 - f*x)/4]^2] - 4*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-e + Pi
/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[3/2, -2*m, 1 + 2*m,
5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Tan[(-e + P
i/2 - f*x)/4]^2) - (3*m*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)
/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi
/2 - f*x)/4]^3*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(-1 + 2*m))/(3*AppellF1[1/2
, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]
- 4*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-
e + Pi/2 - f*x)/4]^2] + AppellF1[3/2, -2*m, 1 + 2*m, 5/2, Tan[(-e + Pi/2 -
f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Tan[(-e + Pi/2 - f*x)/4]^2) - (3*A
ppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 -
f*x)/4]^2]*Tan[(-e + Pi/2 - f*x)/4]^2*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m
)*(-2*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[
(-e + Pi/2 - f*x)/4]^2] + AppellF1[3/2, -2*m, 1 + 2*m, 5/2, Tan[(-e + Pi/2
- f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(
-e + Pi/2 - f*x)/4] + 3*(-1/3*(m*AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-e +
Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*
Tan[(-e + Pi/2 - f*x)/4]) - (m*AppellF1[3/2, -2*m, 1 + 2*m, 5/2, Tan[(-e +
Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*T
an[(-e + Pi/2 - f*x)/4])/3) - 4*m*Tan[(-e + Pi/2 - f*x)/4]^2*((-6*m*AppellF
1[5/2, 1 - 2*m, 1 + 2*m, 7/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 -
f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/5 + (3*(1
- 2*m)*AppellF1[5/2, 2 - 2*m, 2*m, 7/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-
e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/
10 - (3*(1 + 2*m)*AppellF1[5/2, -2*m, 2 + 2*m, 7/2, Tan[(-e + Pi/2 - f*x)/4
]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2
- f*x)/4])/10)))/(3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]
^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 4*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Ta
n[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[3/2, -2*m
, 1 + 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*T
an[(-e + Pi/2 - f*x)/4]^2)^2))/2)) + (Hypergeometric2F1[1/2, (-1 + 2*m)/2,
(1 + 2*m)/2, Cos[(-e + Pi/2 - f*x)/2]^2]*(a + a*Sin[e + f*x])^m*Tan[(-e + P

```
i/2 - f*x)/2])/(2*f*(-1 + 2*m)*Sqrt[Sin[(-e + Pi/2 - f*x)/2]^2])
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{m}\tan\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e)^2,x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)^m*tan(f*x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{m}\tan\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e)^2,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*tan(f*x + e)^2, x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int\left(a+a\sin\left(fx+e\right)\right)^{m}\left(\tan^{2}\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*tan(f*x+e)^2,x)
[Out] int((a+a*sin(f*x+e))^m*tan(f*x+e)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{m}\tan\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e)^2,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^m*tan(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\tan\left(e+f\,x\right)^{2}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(a + a*sin(e + f*x))^m,x)
```

```
[Out] int(tan(e + f*x)^2*(a + a*sin(e + f*x))^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}\tan^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*tan(f*x+e)**2,x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*tan(e + f*x)**2, x)
```

3.137 $\int (a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=74

$$-\frac{2^{m+\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f}$$

```
[Out] -2^(1/2+m)*cos(f*x+e)*hypergeom([1/2, 1/2-m],[3/2],1/2-1/2*sin(f*x+e))*(1+s
in(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m/f
```

Rubi [A] time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2652, 2651}

$$-\frac{2^{m+\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((2^(1/2 + m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e
 + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/f)
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m\,dx = \left((1+\sin(e+fx))^{-m}(a+a\sin(e+fx))^m\right)\int (1+\sin(e+fx))^m\,dx$$
$$= -\frac{2^{\frac{1}{2}+m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{-\frac{1}{2}-m}(a+a}{f}$$

Mathematica [A] time = 0.15, size = 90, normalized size = 1.22

$$\frac{\sqrt{2}\cos(e+fx)(a(\sin(e+fx)+1))^m\,{}_2F_1\left(\frac{1}{2},m+\frac{1}{2};m+\frac{3}{2};\frac{1}{4}\cos^2(e+fx)\csc^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\right)}{(2fm+f)\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (Sqrt[2]*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 + m, 3/2 + m, (Cos[e + f*x
]^2*Csc[(2*e - Pi + 2*f*x)/4]^2)/4]*(a*(1 + Sin[e + f*x]))^m)/((f + 2*f*m)*
Sqrt[1 - Sin[e + f*x]])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m, x)
```

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int\left(a+a\sin\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m,x)
```

```
[Out] int((a+a*sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+a\,\sin\left(e+f\,x\right)\right)}^m \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m,x)
```

```
[Out] int((a + a*sin(e + f*x))^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(e+fx\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m,x)
```

```
[Out] Integral((a*sin(e + f*x) + a)**m, x)
```

3.138 $\int \cot^2(e+fx)(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=89

$$\frac{2\sqrt{2}\sqrt{1-\sin(e+fx)}\,\sec(e+fx)(a\sin(e+fx)+a)^{m+2}F_1\left(m+\frac{3}{2};-\frac{1}{2},2;m+\frac{5}{2};\frac{1}{2}(\sin(e+fx)+1),\sin(e+fx)\right.}{a^2f(2m+3)}$$

[Out] 2*AppellF1(3/2+m,2,-1/2,5/2+m,1+sin(f*x+e),1/2+1/2*sin(f*x+e))*sec(f*x+e)*(a+a*sin(f*x+e))^(2+m)*2^(1/2)*(1-sin(f*x+e))^(1/2)/a^2/f/(3+2*m)

Rubi [A] time = 0.10, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2719, 137, 136}

$$\frac{2\sqrt{2}\sqrt{1-\sin(e+fx)}\,\sec(e+fx)(a\sin(e+fx)+a)^{m+2}F_1\left(m+\frac{3}{2};-\frac{1}{2},2;m+\frac{5}{2};\frac{1}{2}(\sin(e+fx)+1),\sin(e+fx)\right.}{a^2f(2m+3)}$$

Antiderivative was successfully verified.

[In] Int[Cot[e + f*x]^2*(a + a*Sin[e + f*x])^m,x]

[Out] (2*Sqrt[2]*AppellF1[3/2 + m, -1/2, 2, 5/2 + m, (1 + Sin[e + f*x])/2, 1 + Sin[e + f*x]]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(2 + m))/(a^2*f*(3 + 2*m))

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 2719

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[(Sqrt[a + b*Sin[e + f*x]]*Sqrt[a - b*Sin[e + f*x]])/(b
*f*Cos[e + f*x]), Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] && IntegerQ[p/2]
```

Rubi steps

$$\int \cot^2(e+fx)(a+a\sin(e+fx))^m\,dx = \frac{\left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{a-x}\,(a+x)}{x^2}\right.}{af}$$

$$= \frac{\left(\sqrt{2}\,\sec(e+fx)(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}\right)\text{Subst}\left(\int \frac{(a+x)^{\frac{1}{2}}}{}\right.}{af\sqrt{\frac{a-a\sin(e+fx)}{a}}}$$

$$= \frac{2\sqrt{2}F_1\left(\frac{3}{2}+m;-\frac{1}{2},2;\frac{5}{2}+m;\frac{1}{2}(1+\sin(e+fx)),1+\sin(e+fx)\right)\sec(e}{a^2f(3+2m)}$$

Mathematica [C] time = 26.45, size = 5048, normalized size = 56.72

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^2*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] Result too large to show
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^m\cot\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*cot(f*x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^m\cot\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cot(f*x + e)^2, x)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \left(\cot^{2}\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int(cot(f*x+e)^2*(a+a*sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\cot\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cot(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\mathrm{cot}\left(e+f\,x\right)}^2\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(a + a*sin(e + f*x))^m,x)
```

```
[Out] int(cot(e + f*x)^2*(a + a*sin(e + f*x))^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin{\left(e+fx \right)}+1\right)\right)^{m}\cot^{2}{\left(e+fx \right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*cot(e + f*x)**2, x)
```

3.139 $\int \cot^4(e+fx)(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=89

$$\frac{4\sqrt{2}\sqrt{1-\sin(e+fx)}\,\sec(e+fx)(a\sin(e+fx)+a)^{m+3}F_1\left(m+\frac{5}{2};-\frac{3}{2},4;m+\frac{7}{2};\frac{1}{2}(\sin(e+fx)+1),\sin(e+fx)+\right.}{a^3f(2m+5)}$$

```
[Out] 4*AppellF1(5/2+m,4,-3/2,7/2+m,1+sin(f*x+e),1/2+1/2*sin(f*x+e))*sec(f*x+e)*(
a+a*sin(f*x+e))^(3+m)*2^(1/2)*(1-sin(f*x+e))^(1/2)/a^3/f/(5+2*m)
```

Rubi [A] time = 0.10, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2719, 137, 136}

$$\frac{4\sqrt{2}\sqrt{1-\sin(e+fx)}\,\sec(e+fx)(a\sin(e+fx)+a)^{m+3}F_1\left(m+\frac{5}{2};-\frac{3}{2},4;m+\frac{7}{2};\frac{1}{2}(\sin(e+fx)+1),\sin(e+fx)+\right.}{a^3f(2m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (4*Sqrt[2]*AppellF1[5/2 + m, -3/2, 4, 7/2 + m, (1 + Sin[e + f*x])/2, 1 + Si
n[e + f*x]]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(3 + m
))/(a^3*f*(5 + 2*m))
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !GtQ[b/(b*c
- a*d), 0] && !SimplerQ[c + d*x, a + b*x]
```

Rule 2719

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[(Sqrt[a + b*Sin[e + f*x]]*Sqrt[a - b*Sin[e + f*x]])/(b
*f*Cos[e + f*x]), Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] && IntegerQ[p/2]
```

Rubi steps

$$\int \cot^4(e+fx)(a+a\sin(e+fx))^m\,dx = \frac{\left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}\right)\text{Subst}\left(\int \frac{(a-x)^{3/2}(a}{x^4}\right.}{af}$$

$$= \frac{\left(2\sqrt{2}\sec(e+fx)(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}\right)\text{Subst}\left(\int \frac{(a+x}{}\right.}{f\sqrt{\frac{a-a\sin(e+fx)}{a}}}$$

$$= \frac{4\sqrt{2}F_1\left(\frac{5}{2}+m;-\frac{3}{2},4;\frac{7}{2}+m;\frac{1}{2}(1+\sin(e+fx)),1+\sin(e+fx)\right)\sec(}{a^3f(5+2m)}$$

Mathematica [F] time = 0.78, size = 0, normalized size = 0.00

$$\int \cot^4(e+fx)(a+a\sin(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[e + f*x]^4*(a + a*Sin[e + f*x])^m,x]
[Out] Integrate[Cot[e + f*x]^4*(a + a*Sin[e + f*x])^m, x]
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^m\cot\left(fx+e\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)^m*cot(f*x + e)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\cot\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*cot(f*x + e)^4, x)
```

maple [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \left(\cot^{4}\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+a*sin(f*x+e))^m,x)
[Out] int(cot(f*x+e)^4*(a+a*sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\cot\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^m*cot(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(e+f\,x\right)^{4}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4*(a + a*sin(e + f*x))^m,x)
[Out] int(cot(e + f*x)^4*(a + a*sin(e + f*x))^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}\cot^{4}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+a*sin(f*x+e))**m,x)
[Out] Integral((a*(sin(e + f*x) + 1))**m*cot(e + f*x)**4, x)
```

3.140 $\int (a + b\sin(c + dx))\tan^3(c + dx)\,dx$

Optimal. Leaf size=88

$$\frac{(2a+3b)\log(1-\sin(c+dx))}{4d}+\frac{(2a-3b)\log(\sin(c+dx)+1)}{4d}+\frac{\tan^2(c+dx)(a+b\sin(c+dx))}{2d}+\frac{3b\sin(c+dx)}{2d}$$

```
[Out] 1/4*(2*a+3*b)*ln(1-sin(d*x+c))/d+1/4*(2*a-3*b)*ln(1+sin(d*x+c))/d+3/2*b*sin
(d*x+c)/d+1/2*(a+b*sin(d*x+c))*tan(d*x+c)^2/d
```

Rubi [A] time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2721, 819, 774, 633, 31}

$$\frac{(2a+3b)\log(1-\sin(c+dx))}{4d}+\frac{(2a-3b)\log(\sin(c+dx)+1)}{4d}+\frac{\tan^2(c+dx)(a+b\sin(c+dx))}{2d}+\frac{3b\sin(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])*Tan[c + d*x]^3,x]
```

```
[Out] ((2*a + 3*b)*Log[1 - Sin[c + d*x]])/(4*d) + ((2*a - 3*b)*Log[1 + Sin[c + d*
x]])/(4*d) + (3*b*Sin[c + d*x])/(2*d) + ((a + b*Sin[c + d*x])*Tan[c + d*x]^
2)/(2*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
```

```
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int (a+b\sin(c+dx))\tan^3(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^3(a+x)}{\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{(a+b\sin(c+dx))\tan^2(c+dx)}{2d} - \frac{\text{Subst}\left(\int \frac{x\left(2ab^2+3b^2x\right)}{b^2-x^2}\,dx, x, b\sin(c+dx\right.}{2b^2d}$$

$$= \frac{3b\sin(c+dx)}{2d} + \frac{(a+b\sin(c+dx))\tan^2(c+dx)}{2d} + \frac{\text{Subst}\left(\int \frac{-3b^4-2ab^2x}{b^2-x^2}\,d\right.}{2b^2}$$

$$= \frac{3b\sin(c+dx)}{2d} + \frac{(a+b\sin(c+dx))\tan^2(c+dx)}{2d} - \frac{(2a-3b)\,\text{Subst}\left(\int \frac{}{-b}\right.}{4}$$

$$= \frac{(2a+3b)\log(1-\sin(c+dx))}{4d} + \frac{(2a-3b)\log(1+\sin(c+dx))}{4d} + \frac{3b\sin(}{2}$$

Mathematica [A] time = 0.13, size = 77, normalized size = 0.88

$$\frac{a\left(\tan^2(c+dx)+2\log(\cos(c+dx))\right)}{2d} - \frac{b\sin(c+dx)\tan^2(c+dx)}{d} - \frac{3b\left(\tanh^{-1}(\sin(c+dx)) - \tan(c+dx)\sec(c+\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])*Tan[c + d*x]^3,x]
```

```
[Out] -((b*Sin[c + d*x]*Tan[c + d*x]^2)/d) - (3*b*(ArcTanh[Sin[c + d*x]] - Sec[c
+ d*x]*Tan[c + d*x]))/(2*d) + (a*(2*Log[Cos[c + d*x]] + Tan[c + d*x]^2))/(2
*d)
```

fricas [A] time = 0.50, size = 90, normalized size = 1.02

$$\frac{(2a-3b)\cos(dx+c)^2\log(\sin(dx+c)+1)+(2a+3b)\cos(dx+c)^2\log(-\sin(dx+c)+1)+2\left(2b\cos(dx\right.}{4d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*((2*a - 3*b)*cos(d*x + c)^2*log(sin(d*x + c) + 1) + (2*a + 3*b)*cos(d*x
 + c)^2*log(-sin(d*x + c) + 1) + 2*(2*b*cos(d*x + c)^2 + b)*sin(d*x + c) +
2*a)/(d*cos(d*x + c)^2)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.10, size = 96, normalized size = 1.09

$$\frac{a\left(\tan^2(dx+c)\right)}{2d}+\frac{a\ln(\cos(dx+c))}{d}+\frac{b\left(\sin^5(dx+c)\right)}{2d\cos(dx+c)^2}+\frac{b\left(\sin^3(dx+c)\right)}{2d}+\frac{3b\sin(dx+c)}{2d}-\frac{3b\ln(\sec(dx+c)+}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))*tan(d*x+c)^3,x)
```

```
[Out] 1/2/d*a*tan(d*x+c)^2+1/d*a*ln(cos(d*x+c))+1/2/d*b*sin(d*x+c)^5/cos(d*x+c)^2
+1/2/d*b*sin(d*x+c)^3+3/2*b*sin(d*x+c)/d-3/2/d*b*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 1.62, size = 73, normalized size = 0.83

$$\frac{(2a-3b)\log(\sin(dx+c)+1)+(2a+3b)\log(\sin(dx+c)-1)+4b\sin(dx+c)-\frac{2(b\sin(dx+c)+a)}{\sin(dx+c)^2-1}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*((2*a - 3*b)*log(sin(d*x + c) + 1) + (2*a + 3*b)*log(sin(d*x + c) - 1)
+ 4*b*sin(d*x + c) - 2*(b*sin(d*x + c) + a)/(sin(d*x + c)^2 - 1))/d
```

mupad [B] time = 6.73, size = 176, normalized size = 2.00

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(a+\frac{3\,b}{2}\right)}{d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,\left(a-\frac{3\,b}{2}\right)}{d}-\frac{a\,\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{d}-\frac{3\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+\dots}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + b*sin(c + d*x)),x)
```

```
[Out] (log(tan(c/2 + (d*x)/2) - 1)*(a + (3*b)/2))/d + (log(tan(c/2 + (d*x)/2) + 1
)*(a - (3*b)/2))/d - (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (3*b*tan(c/2 + (
d*x)/2) + 2*a*tan(c/2 + (d*x)/2)^2 + 2*a*tan(c/2 + (d*x)/2)^4 - 2*b*tan(c/2
 + (d*x)/2)^3 + 3*b*tan(c/2 + (d*x)/2)^5)/(d*(tan(c/2 + (d*x)/2)^2 + tan(c/
2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)^6 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin(c + dx))\tan^3(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)**3,x)
```

```
[Out] Integral((a + b*sin(c + d*x))*tan(c + d*x)**3, x)
```

3.141 $\int (a + b\sin(c + dx))\tan(c + dx)\,dx$

Optimal. Leaf size=55

$$-\frac{(a+b)\log(1-\sin(c+dx))}{2d}-\frac{(a-b)\log(\sin(c+dx)+1)}{2d}-\frac{b\sin(c+dx)}{d}$$

```
[Out] -1/2*(a+b)*ln(1-sin(d*x+c))/d-1/2*(a-b)*ln(1+sin(d*x+c))/d-b*sin(d*x+c)/d
```

Rubi [A] time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2721, 774, 633, 31}

$$-\frac{(a+b)\log(1-\sin(c+dx))}{2d}-\frac{(a-b)\log(\sin(c+dx)+1)}{2d}-\frac{b\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])*Tan[c + d*x],x]
```

```
[Out] -((a + b)*Log[1 - Sin[c + d*x]])/(2*d) - ((a - b)*Log[1 + Sin[c + d*x]])/(2
*d) - (b*Sin[c + d*x])/d
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
```

```
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int (a+b\sin(c+dx))\tan(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x(a+x)}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= -\frac{b\sin(c+dx)}{d} - \frac{\text{Subst}\left(\int \frac{-b^2-ax}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= -\frac{b\sin(c+dx)}{d} + \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{-b-x}\,dx, x, b\sin(c+dx)\right)}{2d} + \frac{(a+b)\,\text{Subs}}{}$$
$$= -\frac{(a+b)\log(1-\sin(c+dx))}{2d} - \frac{(a-b)\log(1+\sin(c+dx))}{2d} - \frac{b\sin(c+dx}{d}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 0.69

$$-\frac{a\log(\cos(c+dx))}{d} - \frac{b\sin(c+dx)}{d} + \frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])*Tan[c + d*x],x]
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Log[Cos[c + d*x]])/d - (b*Sin[c + d*x])/d
```

fricas [A] time = 0.43, size = 45, normalized size = 0.82

$$-\frac{(a-b)\log(\sin(dx+c)+1) + (a+b)\log(-\sin(dx+c)+1) + 2\,b\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c),x, algorithm="fricas")
[Out] -1/2*((a - b)*log(sin(d*x + c) + 1) + (a + b)*log(-sin(d*x + c) + 1) + 2*b*
sin(d*x + c))/d
```

giac [B] time = 4.25, size = 1456, normalized size = 26.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c),x, algorithm="giac")
```

```
[Out] -1/2*(b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) +
2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c
)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + t
an(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/
2*d*x)^2*tan(1/2*c)^2 - b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*
x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/
2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*
tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2
*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + a*log(4*(tan(d*x)^4*tan(c)^2 - 2*
tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) +
1)/(tan(c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + b*log(2*(tan(1/2*d*x)^4*ta
n(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 +
tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1
/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2
*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2 - b*log(2*(tan(1/2*d*x)
^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)
^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*
tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x
) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2 + a*log(4*(tan(d*x
)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*t
an(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(1/2*d*x)^2 - 4*b*tan(1/2*d*x)^2*tan
(1/2*c) + b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c
) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1
/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2
 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*ta
n(1/2*c)^2 - b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/
2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*ta
n(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x
)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))
*tan(1/2*c)^2 + a*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*
x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(1/2
*c)^2 - 4*b*tan(1/2*d*x)*tan(1/2*c)^2 + b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^
2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d
*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*t
an(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*
c) + 1)/(tan(1/2*c)^2 + 1)) - b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(
1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*
tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^
2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(t
an(1/2*c)^2 + 1)) + a*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + ta
n(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) + 4
*b*tan(1/2*d*x) + 4*b*tan(1/2*c))/(d*tan(1/2*d*x)^2*tan(1/2*c)^2 + d*tan(1/
2*d*x)^2 + d*tan(1/2*c)^2 + d)
```

maple [A] time = 0.08, size = 46, normalized size = 0.84

$$-\frac{b\sin(dx+c)}{d}+\frac{b\ln(\sec(dx+c)+\tan(dx+c))}{d}-\frac{a\ln(\cos(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))*tan(d*x+c),x)
```

```
[Out] -b*sin(d*x+c)/d+1/d*b*ln(sec(d*x+c)+tan(d*x+c))-1/d*a*ln(cos(d*x+c))
```

maxima [A] time = 1.15, size = 43, normalized size = 0.78

$$-\frac{(a-b)\log(\sin(dx+c)+1)+(a+b)\log(\sin(dx+c)-1)+2\,b\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*((a - b)*log(sin(d*x + c) + 1) + (a + b)*log(sin(d*x + c) - 1) + 2*b*s
in(d*x + c))/d
```

mupad [B] time = 6.64, size = 74, normalized size = 1.35

$$\frac{a\,\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,(a-b)}{d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,(a+b)}{d}-\frac{b\,\sin(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + b*sin(c + d*x)),x)
```

```
[Out] (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (log(tan(c/2 + (d*x)/2) + 1)*(a - b))
/d - (log(tan(c/2 + (d*x)/2) - 1)*(a + b))/d - (b*sin(c + d*x))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin(c+dx))\tan(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*tan(c + d*x), x)
```

3.142 $\int \cot(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=24

$$\frac{a\log(\sin(c+dx))}{d}+\frac{b\sin(c+dx)}{d}$$

```
[Out] a*ln(sin(d*x+c))/d+b*sin(d*x+c)/d
```

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2721, 43}

$$\frac{a\log(\sin(c+dx))}{d}+\frac{b\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*Log[Sin[c + d*x]])/d + (b*Sin[c + d*x])/d
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)(a+b\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int \frac{a+x}{x}\,dx, x, b\sin(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(1+\frac{a}{x}\right)dx, x, b\sin(c+dx)\right)}{d}\\ &= \frac{a\log(\sin(c+dx))}{d}+\frac{b\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.04, size = 43, normalized size = 1.79

$$\frac{a(\log(\tan(c+dx))+\log(\cos(c+dx)))}{d}+\frac{b\sin(c)\cos(dx)}{d}+\frac{b\cos(c)\sin(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*(Log[Cos[c + d*x]] + Log[Tan[c + d*x]]))/d + (b*Cos[d*x]*Sin[c])/d + (b*
Cos[c]*Sin[d*x])/d
```

fricas [A] time = 0.45, size = 24, normalized size = 1.00

$$\frac{a\log\left(\frac{1}{2}\sin(dx+c)\right)+b\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] (a*log(1/2*sin(d*x + c)) + b*sin(d*x + c))/d
```

giac [A] time = 1.27, size = 23, normalized size = 0.96

$$\frac{a\log(|\sin(dx+c)|)+b\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*log(abs(sin(d*x + c))) + b*sin(d*x + c))/d
```

maple [A] time = 0.08, size = 25, normalized size = 1.04

$$\frac{a\ln(\sin(dx+c))}{d}+\frac{b\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+b*sin(d*x+c)),x)
```

```
[Out] a*ln(sin(d*x+c))/d+b*sin(d*x+c)/d
```

maxima [A] time = 0.61, size = 22, normalized size = 0.92

$$\frac{a\log(\sin(dx+c))+b\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (a*log(sin(d*x + c)) + b*sin(d*x + c))/d
```

mupad [B] time = 6.56, size = 47, normalized size = 1.96

$$\frac{a\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}+\frac{b\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + b*sin(c + d*x)),x)
```

```
[Out] (a*log(tan(c/2 + (d*x)/2)))/d - (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d + (b*si
n(c + d*x))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(a+b\sin(c+dx))\cot(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*cot(c + d*x), x)
```

3.143 $\int \cot^3(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=54

$$-\frac{a\csc^2(c+dx)}{2d}-\frac{a\log(\sin(c+dx))}{d}-\frac{b\sin(c+dx)}{d}-\frac{b\csc(c+dx)}{d}$$

```
[Out] -b*csc(d*x+c)/d-1/2*a*csc(d*x+c)^2/d-a*ln(sin(d*x+c))/d-b*sin(d*x+c)/d
```

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2721, 766}

$$-\frac{a\csc^2(c+dx)}{2d}-\frac{a\log(\sin(c+dx))}{d}-\frac{b\sin(c+dx)}{d}-\frac{b\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] -((b*Csc[c + d*x])/d) - (a*Csc[c + d*x]^2)/(2*d) - (a*Log[Sin[c + d*x]])/d
- (b*Sin[c + d*x])/d
```

Rule 766

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \cot^3(c+dx)(a+b\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)\left(b^2-x^2\right)}{x^3}\,dx,x,b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-1+\frac{ab^2}{x^3}+\frac{b^2}{x^2}-\frac{a}{x}\right)dx,x,b\sin(c+dx)\right)}{d}$$
$$= -\frac{b\csc(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{a\log(\sin(c+dx))}{d}-\frac{b\sin(c+dx)}{d}$$

Mathematica [A] time = 0.21, size = 60, normalized size = 1.11

$$-\frac{a\left(\cot^2(c+dx)+2\log(\tan(c+dx))+2\log(\cos(c+dx))\right)}{2d}-\frac{b\sin(c+dx)}{d}-\frac{b\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] -((b*Csc[c + d*x])/d) - (a*(Cot[c + d*x]^2 + 2*Log[Cos[c + d*x]] + 2*Log[Ta
n[c + d*x]]))/(2*d) - (b*Sin[c + d*x])/d
```

fricas [A] time = 0.46, size = 69, normalized size = 1.28

$$-\frac{2\left(a\cos\left(dx+c\right)^2-a\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)+2\left(b\cos\left(dx+c\right)^2-2b\right)\sin\left(dx+c\right)-a}{2\left(d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a*cos(d*x + c)^2 - a)*log(1/2*sin(d*x + c)) + 2*(b*cos(d*x + c)^2
- 2*b)*sin(d*x + c) - a)/(d*cos(d*x + c)^2 - d)
```

giac [A] time = 1.79, size = 60, normalized size = 1.11

$$-\frac{2a\log\left(\left|\sin\left(dx+c\right)\right|\right)+2b\sin\left(dx+c\right)-\frac{3a\sin(dx+c)^2-2b\sin(dx+c)-a}{\sin(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*a*log(abs(sin(d*x + c))) + 2*b*sin(d*x + c) - (3*a*sin(d*x + c)^2 -
 2*b*sin(d*x + c) - a)/sin(d*x + c)^2)/d
```

maple [A] time = 0.19, size = 83, normalized size = 1.54

$$-\frac{a\left(\cot^2(dx+c)\right)}{2d}-\frac{a\ln(\sin(dx+c))}{d}-\frac{b\left(\cos^4(dx+c)\right)}{d\sin(dx+c)}-\frac{\left(\cos^2(dx+c)\right)\sin(dx+c)\,b}{d}-\frac{2b\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+b*sin(d*x+c)),x)
```

```
[Out] -1/2/d*a*cot(d*x+c)^2-a*ln(sin(d*x+c))/d-1/d*b/sin(d*x+c)*cos(d*x+c)^4-1/d*
cos(d*x+c)^2*sin(d*x+c)*b-2*b*sin(d*x+c)/d
```

maxima [A] time = 0.96, size = 45, normalized size = 0.83

$$-\frac{2\,a\log(\sin(dx+c))+2\,b\sin(dx+c)+\frac{2\,b\sin(dx+c)+a}{\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*log(sin(d*x + c)) + 2*b*sin(d*x + c) + (2*b*sin(d*x + c) + a)/sin
(d*x + c)^2)/d
```

mupad [B] time = 6.63, size = 146, normalized size = 2.70

$$\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{10\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a}{2}}{d\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}-\frac{a\tan\left(\frac{c}{2}+\frac{d}{}\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + b*sin(c + d*x)),x)
```

```
[Out] (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (b*tan(c/2 + (d*x)/2))/(2*d) - (a/2 +
 2*b*tan(c/2 + (d*x)/2) + (a*tan(c/2 + (d*x)/2)^2)/2 + 10*b*tan(c/2 + (d*x)
/2)^3)/(d*(4*tan(c/2 + (d*x)/2)^2 + 4*tan(c/2 + (d*x)/2)^4)) - (a*tan(c/2 +
 (d*x)/2)^2)/(8*d) - (a*log(tan(c/2 + (d*x)/2)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(a+b\sin(c+dx))\cot^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*cot(c + d*x)**3, x)
```

3.144 $\int \cot^5(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$-\frac{a\csc^4(c+dx)}{4d}+\frac{a\csc^2(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}+\frac{b\sin(c+dx)}{d}-\frac{b\csc^3(c+dx)}{3d}+\frac{2b\csc(c+dx)}{d}$$

```
[Out] 2*b*csc(d*x+c)/d+a*csc(d*x+c)^2/d-1/3*b*csc(d*x+c)^3/d-1/4*a*csc(d*x+c)^4/d
+a*ln(sin(d*x+c))/d+b*sin(d*x+c)/d
```

Rubi [A] time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2721, 766}

$$-\frac{a\csc^4(c+dx)}{4d}+\frac{a\csc^2(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}+\frac{b\sin(c+dx)}{d}-\frac{b\csc^3(c+dx)}{3d}+\frac{2b\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b*Csc[c + d*x])/d + (a*Csc[c + d*x]^2)/d - (b*Csc[c + d*x]^3)/(3*d) - (a
*Csc[c + d*x]^4)/(4*d) + (a*Log[Sin[c + d*x]])/d + (b*Sin[c + d*x])/d
```

Rule 766

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \cot^5(c+dx)(a+b\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)\left(b^2-x^2\right)^2}{x^5}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{ab^4}{x^5}+\frac{b^4}{x^4}-\frac{2ab^2}{x^3}-\frac{2b^2}{x^2}+\frac{a}{x}\right)dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{2b\csc(c+dx)}{d}+\frac{a\csc^2(c+dx)}{d}-\frac{b\csc^3(c+dx)}{3d}-\frac{a\csc^4(c+dx)}{4d}+\frac{a\,\text{lc}}{}$$

Mathematica [A] time = 0.24, size = 87, normalized size = 1.07

$$\frac{a\left(-\cot^4(c+dx)+2\cot^2(c+dx)+4\log(\tan(c+dx))+4\log(\cos(c+dx))\right)}{4d}+\frac{b\sin(c+dx)}{d}-\frac{b\csc^3(c+dx)}{3d}+\frac{2b}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b*Csc[c + d*x])/d - (b*Csc[c + d*x]^3)/(3*d) + (a*(2*Cot[c + d*x]^2 - Co
t[c + d*x]^4 + 4*Log[Cos[c + d*x]] + 4*Log[Tan[c + d*x]]))/(4*d) + (b*Sin[c
 + d*x])/d
```

fricas [A] time = 0.47, size = 110, normalized size = 1.36

$$-\frac{12\,a\cos(dx+c)^2-12\left(a\cos(dx+c)^4-2\,a\cos(dx+c)^2+a\right)\log\left(\frac{1}{2}\sin(dx+c)\right)-4\left(3\,b\cos(dx+c)^4-12\,b\right.}{12\left(d\cos(dx+c)^4-2\,d\cos(dx+c)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(12*a*cos(d*x + c)^2 - 12*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)
*log(1/2*sin(d*x + c)) - 4*(3*b*cos(d*x + c)^4 - 12*b*cos(d*x + c)^2 + 8*b)
*sin(d*x + c) - 9*a)/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

giac [A] time = 1.84, size = 82, normalized size = 1.01

$$\frac{12\,a\log\left(\left|\sin(dx+c)\right|\right)+12\,b\sin(dx+c)-\frac{25\,a\sin(dx+c)^4-24\,b\sin(dx+c)^3-12\,a\sin(dx+c)^2+4\,b\sin(dx+c)+3\,a}{\sin(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(12*a*log(abs(sin(d*x + c))) + 12*b*sin(d*x + c) - (25*a*sin(d*x + c)^
4 - 24*b*sin(d*x + c)^3 - 12*a*sin(d*x + c)^2 + 4*b*sin(d*x + c) + 3*a)/sin
(d*x + c)^4)/d
```

maple [A] time = 0.18, size = 136, normalized size = 1.68

$$-\frac{a\left(\cot^{4}\left(dx+c\right)\right)}{4d}+\frac{a\left(\cot^{2}\left(dx+c\right)\right)}{2d}+\frac{a\ln\left(\sin\left(dx+c\right)\right)}{d}-\frac{b\left(\cos^{6}\left(dx+c\right)\right)}{3d\sin\left(dx+c\right)^{3}}+\frac{b\left(\cos^{6}\left(dx+c\right)\right)}{d\sin\left(dx+c\right)}+\frac{8b\sin\left(dx+c\right)}{3d}+\frac{(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+b*sin(d*x+c)),x)
```

```
[Out] -1/4/d*a*cot(d*x+c)^4+1/2/d*a*cot(d*x+c)^2+a*ln(sin(d*x+c))/d-1/3/d*b/sin(d
*x+c)^3*cos(d*x+c)^6+1/d*b/sin(d*x+c)*cos(d*x+c)^6+8/3*b*sin(d*x+c)/d+1/d*c
os(d*x+c)^4*sin(d*x+c)*b+4/3/d*cos(d*x+c)^2*sin(d*x+c)*b
```

maxima [A] time = 1.40, size = 69, normalized size = 0.85

$$\frac{12\,a\log\left(\sin\left(dx+c\right)\right)+12\,b\sin\left(dx+c\right)+\frac{24\,b\sin\left(dx+c\right)^{3}+12\,a\sin\left(dx+c\right)^{2}-4\,b\sin\left(dx+c\right)-3\,a}{\sin\left(dx+c\right)^{4}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(12*a*log(sin(d*x + c)) + 12*b*sin(d*x + c) + (24*b*sin(d*x + c)^3 + 1
2*a*sin(d*x + c)^2 - 4*b*sin(d*x + c) - 3*a)/sin(d*x + c)^4)/d
```

mupad [B] time = 6.67, size = 207, normalized size = 2.56

$$\frac{7\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}+\frac{46\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}+3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+\frac{40\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{3}+\frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{4}-\frac{2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}-\frac{a}{4}}{d\left(16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}\right)}-\frac{a\,\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5*(a + b*sin(c + d*x)),x)
```

```
[Out] (7*b*tan(c/2 + (d*x)/2))/(8*d) + ((11*a*tan(c/2 + (d*x)/2)^2)/4 - (2*b*tan(
c/2 + (d*x)/2))/3 - a/4 + 3*a*tan(c/2 + (d*x)/2)^4 + (40*b*tan(c/2 + (d*x)/
2)^3)/3 + 46*b*tan(c/2 + (d*x)/2)^5)/(d*(16*tan(c/2 + (d*x)/2)^4 + 16*tan(c
/2 + (d*x)/2)^6)) - (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d + (3*a*tan(c/2 + (d
```

```
*x)/2)^2)/(16*d) - (a*tan(c/2 + (d*x)/2)^4)/(64*d) - (b*tan(c/2 + (d*x)/2)^
3)/(24*d) + (a*log(tan(c/2 + (d*x)/2)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin(c + dx))\cot^5(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*cot(c + d*x)**5, x)
```

3.145 $\int (a+b\sin(c+dx))\tan^4(c+dx)\,dx$

Optimal. Leaf size=72

$$\frac{a\tan^3(c+dx)}{3d}-\frac{a\tan(c+dx)}{d}+ax-\frac{b\cos(c+dx)}{d}+\frac{b\sec^3(c+dx)}{3d}-\frac{2b\sec(c+dx)}{d}$$

```
[Out] a*x-b*cos(d*x+c)/d-2*b*sec(d*x+c)/d+1/3*b*sec(d*x+c)^3/d-a*tan(d*x+c)/d+1/3
*a*tan(d*x+c)^3/d
```

Rubi [A] time = 0.08, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2722, 3473, 8, 2590, 270}

$$\frac{a\tan^3(c+dx)}{3d}-\frac{a\tan(c+dx)}{d}+ax-\frac{b\cos(c+dx)}{d}+\frac{b\sec^3(c+dx)}{3d}-\frac{2b\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] a*x - (b*Cos[c + d*x])/d - (2*b*Sec[c + d*x])/d + (b*Sec[c + d*x]^3)/(3*d)
- (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
```

```
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int (a+b\sin(c+dx))\tan^4(c+dx)\,dx &= \int \left(a\tan^4(c+dx)+b\sin(c+dx)\tan^4(c+dx)\right)dx \\
&= a\int \tan^4(c+dx)\,dx+b\int \sin(c+dx)\tan^4(c+dx)\,dx \\
&= \frac{a\tan^3(c+dx)}{3d}-a\int \tan^2(c+dx)\,dx-\frac{b\,\text{Subst}\left(\int \frac{(1-x^2)^2}{x^4}\,dx,x,\cos(c+
}{d} \\
&= -\frac{a\tan(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}+a\int 1\,dx-\frac{b\,\text{Subst}\left(\int \left(1+\frac{1}{x^4}-\frac{2}{x^2}\right)
}{d} \\
&= ax-\frac{b\cos(c+dx)}{d}-\frac{2b\sec(c+dx)}{d}+\frac{b\sec^3(c+dx)}{3d}-\frac{a\tan(c+dx)}{d}+
\end{aligned}$$

Mathematica [A] time = 0.04, size = 81, normalized size = 1.12

$$\frac{a\tan^{-1}(\tan(c+dx))}{d}+\frac{a\tan^3(c+dx)}{3d}-\frac{a\tan(c+dx)}{d}-\frac{b\cos(c+dx)}{d}+\frac{b\sec^3(c+dx)}{3d}-\frac{2b\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] (a*ArcTan[Tan[c + d*x]])/d - (b*Cos[c + d*x])/d - (2*b*Sec[c + d*x])/d + (b
*Sec[c + d*x]^3)/(3*d) - (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

fricas [A] time = 0.47, size = 73, normalized size = 1.01

$$\frac{3\,adx\cos(dx+c)^3-3\,b\cos(dx+c)^4-6\,b\cos(dx+c)^2-\left(4\,a\cos(dx+c)^2-a\right)\sin(dx+c)+b}{3\,d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a*d*x*cos(d*x + c)^3 - 3*b*cos(d*x + c)^4 - 6*b*cos(d*x + c)^2 - (4*
a*cos(d*x + c)^2 - a)*sin(d*x + c) + b)/(d*cos(d*x + c)^3)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.16, size = 98, normalized size = 1.36

$$\frac{a\left(\frac{\left(\tan^{3}(dx+c)\right)}{3}-\tan(dx+c)+dx+c\right)+b\left(\frac{\sin^{6}(dx+c)}{3\cos(dx+c)^{3}}-\frac{\sin^{6}(dx+c)}{\cos(dx+c)}-\left(\frac{8}{3}+\sin^{4}(dx+c)+\frac{4\left(\sin^{2}(dx+c)\right)}{3}\right)\cos(dx+c)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))*tan(d*x+c)^4,x)
```

```
[Out] 1/d*(a*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c)+b*(1/3*sin(d*x+c)^6/cos(d*x+c)^3
-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)))
```

maxima [A] time = 1.70, size = 65, normalized size = 0.90

$$\frac{\left(\tan(dx+c)^{3}+3\,dx+3\,c-3\,\tan(dx+c)\right)a-b\left(\frac{6\,\cos(dx+c)^{2}-1}{\cos(dx+c)^{3}}+3\,\cos(dx+c)\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/3*((tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a - b*((6*cos(d*x + c)
^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x + c)))/d
```

mupad [B] time = 10.18, size = 110, normalized size = 1.53

$$a\,x+\frac{2\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}-\frac{14\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{3}-\frac{14\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{3}-\frac{32\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{3}+2\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{16\,b}{3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}-1\right)^{3}\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + b*sin(c + d*x)),x)
```

```
[Out] a*x + ((16*b)/3 + 2*a*tan(c/2 + (d*x)/2) - (14*a*tan(c/2 + (d*x)/2)^3)/3 -
(14*a*tan(c/2 + (d*x)/2)^5)/3 + 2*a*tan(c/2 + (d*x)/2)^7 - (32*b*tan(c/2 +
(d*x)/2)^2)/3)/(d*(tan(c/2 + (d*x)/2)^2 - 1)^3*(tan(c/2 + (d*x)/2)^2 + 1))
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin(c + dx))\tan^4(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)**4,x)
```

```
[Out] Integral((a + b*sin(c + d*x))*tan(c + d*x)**4, x)
```

3.146 $\int (a+b\sin(c+dx))\tan^2(c+dx)\,dx$

Optimal. Leaf size=38

$$\frac{a\tan(c+dx)}{d}-ax+\frac{b\cos(c+dx)}{d}+\frac{b\sec(c+dx)}{d}$$

```
[Out] -a*x+b*cos(d*x+c)/d+b*sec(d*x+c)/d+a*tan(d*x+c)/d
```

Rubi [A] time = 0.06, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2722, 3473, 8, 2590, 14}

$$\frac{a\tan(c+dx)}{d}-ax+\frac{b\cos(c+dx)}{d}+\frac{b\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -(a*x) + (b*Cos[c + d*x])/d + (b*Sec[c + d*x])/d + (a*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int (a+b\sin(c+dx))\tan^2(c+dx)\,dx &= \int \left(a\tan^2(c+dx)+b\sin(c+dx)\tan^2(c+dx)\right)\,dx \\
&= a\int \tan^2(c+dx)\,dx+b\int \sin(c+dx)\tan^2(c+dx)\,dx \\
&= \frac{a\tan(c+dx)}{d}-a\int 1\,dx-\frac{b\,\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx,x,\cos(c+dx)\right)}{d} \\
&= -ax+\frac{a\tan(c+dx)}{d}-\frac{b\,\text{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)\,dx,x,\cos(c+dx)\right)}{d} \\
&= -ax+\frac{b\cos(c+dx)}{d}+\frac{b\sec(c+dx)}{d}+\frac{a\tan(c+dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 47, normalized size = 1.24

$$-\frac{a\tan^{-1}(\tan(c+dx))}{d}+\frac{a\tan(c+dx)}{d}+\frac{b\cos(c+dx)}{d}+\frac{b\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -((a*ArcTan[Tan[c + d*x]])/d) + (b*Cos[c + d*x])/d + (b*Sec[c + d*x])/d + (
a*Tan[c + d*x])/d
```

fricas [A] time = 0.45, size = 47, normalized size = 1.24

$$-\frac{adx\cos(dx+c)-b\cos(dx+c)^2-a\sin(dx+c)-b}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -(a*d*x*cos(d*x + c) - b*cos(d*x + c)^2 - a*sin(d*x + c) - b)/(d*cos(d*x +
c))
```

giac [B] time = 35.58, size = 1008, normalized size = 26.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -(a*d*x*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) - a*d*x*tan(1/2*d*x)^4*
tan(1/2*c)^4 - 4*a*d*x*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) - 2*b*ta
n(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) + a*tan(d*x)*tan(1/2*d*x)^4*tan(1
/2*c)^4 + a*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) + 4*a*d*x*tan(1/2*d*x)^3*tan
(1/2*c)^3 + 2*b*tan(1/2*d*x)^4*tan(1/2*c)^4 - a*d*x*tan(d*x)*tan(1/2*d*x)^4
*tan(c) - 4*a*d*x*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 4*a*d*x*tan(d
*x)*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) + 8*b*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*
c)^3*tan(c) - a*d*x*tan(d*x)*tan(1/2*c)^4*tan(c) - 4*a*tan(d*x)*tan(1/2*d*x
)^3*tan(1/2*c)^3 - 4*a*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) + a*d*x*tan(1/2*d
*x)^4 + 4*a*d*x*tan(1/2*d*x)^3*tan(1/2*c) + 4*a*d*x*tan(1/2*d*x)*tan(1/2*c)
^3 - 8*b*tan(1/2*d*x)^3*tan(1/2*c)^3 + a*d*x*tan(1/2*c)^4 - 2*b*tan(d*x)*ta
n(1/2*d*x)^4*tan(c) - 4*a*d*x*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*tan(c) - 8*b
*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 24*b*tan(d*x)*tan(1/2*d*x)^2*t
an(1/2*c)^2*tan(c) - 8*b*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - 2*b*ta
n(d*x)*tan(1/2*c)^4*tan(c) - a*tan(d*x)*tan(1/2*d*x)^4 - 4*a*tan(d*x)*tan(1
/2*d*x)^3*tan(1/2*c) - 4*a*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3 - a*tan(d*x)*
tan(1/2*c)^4 - a*tan(1/2*d*x)^4*tan(c) - 4*a*tan(1/2*d*x)^3*tan(1/2*c)*tan(
c) - 4*a*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - a*tan(1/2*c)^4*tan(c) + 2*b*tan
(1/2*d*x)^4 + 4*a*d*x*tan(1/2*d*x)*tan(1/2*c) + 8*b*tan(1/2*d*x)^3*tan(1/2*
c) + 24*b*tan(1/2*d*x)^2*tan(1/2*c)^2 + 8*b*tan(1/2*d*x)*tan(1/2*c)^3 + 2*b
*tan(1/2*c)^4 + a*d*x*tan(d*x)*tan(c) + 8*b*tan(d*x)*tan(1/2*d*x)*tan(1/2*c
)*tan(c) - 4*a*tan(d*x)*tan(1/2*d*x)*tan(1/2*c) - 4*a*tan(1/2*d*x)*tan(1/2*
c)*tan(c) - a*d*x - 8*b*tan(1/2*d*x)*tan(1/2*c) - 2*b*tan(d*x)*tan(c) + a*t
an(d*x) + a*tan(c) + 2*b)/(d*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) -
d*tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*d*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*ta
n(c) + 4*d*tan(1/2*d*x)^3*tan(1/2*c)^3 - d*tan(d*x)*tan(1/2*d*x)^4*tan(c) -
 4*d*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 4*d*tan(d*x)*tan(1/2*d*x)*
tan(1/2*c)^3*tan(c) - d*tan(d*x)*tan(1/2*c)^4*tan(c) + d*tan(1/2*d*x)^4 + 4
*d*tan(1/2*d*x)^3*tan(1/2*c) + 4*d*tan(1/2*d*x)*tan(1/2*c)^3 + d*tan(1/2*c)
^4 - 4*d*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*tan(c) + 4*d*tan(1/2*d*x)*tan(1/2
*c) + d*tan(d*x)*tan(c) - d)
```

maple [A] time = 0.14, size = 59, normalized size = 1.55

$$\frac{a\left(\tan\left(dx+c\right)-dx-c\right)+b\left(\frac{\sin^{4}(dx+c)}{\cos(dx+c)}+\left(2+\sin^{2}\left(dx+c\right)\right)\cos\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))*tan(d*x+c)^2,x)
```

```
[Out] 1/d*(a*(tan(d*x+c)-d*x-c)+b*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d
*x+c)))
```

maxima [A] time = 1.37, size = 39, normalized size = 1.03

$$-\frac{(dx+c-\tan(dx+c))a-b\left(\frac{1}{\cos(dx+c)}+\cos(dx+c)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -((d*x + c - tan(d*x + c))*a - b*(1/cos(d*x + c) + cos(d*x + c)))/d
```

mupad [B] time = 6.60, size = 55, normalized size = 1.45

$$-a\,x-\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+4\,b}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + b*sin(c + d*x)),x)
```

```
[Out] - a*x - (4*b + 2*a*tan(c/2 + (d*x)/2) + 2*a*tan(c/2 + (d*x)/2)^3)/(d*(tan(c
/2 + (d*x)/2)^4 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(a+b\sin(c+dx))\tan^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x))*tan(c + d*x)**2, x)
```

3.147 $\int \cot^2(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=41

$$-\frac{a\cot(c+dx)}{d}-ax+\frac{b\cos(c+dx)}{d}-\frac{b\tanh^{-1}(\cos(c+dx))}{d}$$

```
[Out] -a*x-b*arctanh(cos(d*x+c))/d+b*cos(d*x+c)/d-a*cot(d*x+c)/d
```

Rubi [A] time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2722, 2592, 321, 206, 3473, 8}

$$-\frac{a\cot(c+dx)}{d}-ax+\frac{b\cos(c+dx)}{d}-\frac{b\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(a*x) - (b*ArcTanh[Cos[c + d*x]])/d + (b*Cos[c + d*x])/d - (a*Cot[c + d*x]
)/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
```

```
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \cot^2(c+dx)(a+b\sin(c+dx))\,dx &= \int \left(b\cos(c+dx)\cot(c+dx)+a\cot^2(c+dx)\right)\,dx\\ &= a\int \cot^2(c+dx)\,dx+b\int \cos(c+dx)\cot(c+dx)\,dx\\ &= -\frac{a\cot(c+dx)}{d}-a\int 1\,dx-\frac{b\,\mathrm{Subst}\left(\int \frac{x^2}{1-x^2}\,dx,x,\cos(c+dx)\right)}{d}\\ &= -ax+\frac{b\cos(c+dx)}{d}-\frac{a\cot(c+dx)}{d}-\frac{b\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\cos(c+dx)\right)}{d}\\ &= -ax-\frac{b\tanh^{-1}(\cos(c+dx))}{d}+\frac{b\cos(c+dx)}{d}-\frac{a\cot(c+dx)}{d}\end{aligned}$$

Mathematica [C] time = 0.04, size = 75, normalized size = 1.83

$$-\frac{a\cot(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(c+dx)\right)}{d}+\frac{b\cos(c+dx)}{d}+\frac{b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}-\frac{b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Cos[c + d*x])/d - (a*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c
 + d*x]^2])/d - (b*Log[Cos[(c + d*x)/2]])/d + (b*Log[Sin[(c + d*x)/2]])/d
```

fricas [B] time = 0.46, size = 84, normalized size = 2.05

$$-\frac{b\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-b\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+2\,a\cos\left(dx+c\right)+2\left(adx-b\right.}{2\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(b*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - b*log(-1/2*cos(d*x + c)
+ 1/2)*sin(d*x + c) + 2*a*cos(d*x + c) + 2*(a*d*x - b*cos(d*x + c))*sin(d*x
 + c))/(d*sin(d*x + c))
```

giac [B] time = 0.28, size = 108, normalized size = 2.63

$$-\frac{6\left(dx+c\right)a-6\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-10\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(6*(d*x + c)*a - 6*b*log(abs(tan(1/2*d*x + 1/2*c))) - 3*a*tan(1/2*d*x
+ 1/2*c) + (2*b*tan(1/2*d*x + 1/2*c)^3 + 3*a*tan(1/2*d*x + 1/2*c)^2 - 10*b*
tan(1/2*d*x + 1/2*c) + 3*a)/(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))
)/d
```

maple [A] time = 0.10, size = 57, normalized size = 1.39

$$-ax+\frac{b\cos\left(dx+c\right)}{d}-\frac{a\cot\left(dx+c\right)}{d}+\frac{b\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)}{d}-\frac{ca}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+b*sin(d*x+c)),x)
```

```
[Out] -a*x+b*cos(d*x+c)/d-a*cot(d*x+c)/d+1/d*b*ln(csc(d*x+c)-cot(d*x+c))-1/d*c*a
```

maxima [A] time = 1.38, size = 54, normalized size = 1.32

$$-\frac{2\left(dx+c+\frac{1}{\tan\left(dx+c\right)}\right)a-b\left(2\,\cos\left(dx+c\right)-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*(d*x + c + 1/tan(d*x + c))*a - b*(2*cos(d*x + c) - log(cos(d*x + c)
 + 1) + log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 6.60, size = 158, normalized size = 3.85

$$\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2d}+\frac{b\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-4b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a}{d\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}+\frac{2a\,\mathrm{atan}\left(\frac{4a^2}{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)a^2+4ba}-\frac{4}{4\tan}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + b*sin(c + d*x)),x)
```

```
[Out] (a*tan(c/2 + (d*x)/2))/(2*d) + (b*log(tan(c/2 + (d*x)/2)))/d - (a - 4*b*tan
(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)/(d*(2*tan(c/2 + (d*x)/2) + 2*tan(
c/2 + (d*x)/2)^3)) + (2*a*atan((4*a^2)/(4*a*b + 4*a^2*tan(c/2 + (d*x)/2)) -
 (4*a*b*tan(c/2 + (d*x)/2))/(4*a*b + 4*a^2*tan(c/2 + (d*x)/2))))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin(c+dx))\cot^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*cot(c + d*x)**2, x)
```

3.148 $\int \cot^4(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=82

$$-\frac{a\cot^3(c+dx)}{3d}+\frac{a\cot(c+dx)}{d}+ax-\frac{3b\cos(c+dx)}{2d}-\frac{b\cos(c+dx)\cot^2(c+dx)}{2d}+\frac{3b\tanh^{-1}(\cos(c+dx))}{2d}$$

```
[Out] a*x+3/2*b*arctanh(cos(d*x+c))/d-3/2*b*cos(d*x+c)/d+a*cot(d*x+c)/d-1/2*b*cos
(d*x+c)*cot(d*x+c)^2/d-1/3*a*cot(d*x+c)^3/d
```

Rubi [A] time = 0.08, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {2722, 2592, 288, 321, 206, 3473, 8}

$$-\frac{a\cot^3(c+dx)}{3d}+\frac{a\cot(c+dx)}{d}+ax-\frac{3b\cos(c+dx)}{2d}-\frac{b\cos(c+dx)\cot^2(c+dx)}{2d}+\frac{3b\tanh^{-1}(\cos(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] a*x + (3*b*ArcTanh[Cos[c + d*x]])/(2*d) - (3*b*Cos[c + d*x])/(2*d) + (a*Cot
[c + d*x])/d - (b*Cos[c + d*x]*Cot[c + d*x]^2)/(2*d) - (a*Cot[c + d*x]^3)/(
3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \cot^4(c+dx)(a+b\sin(c+dx))\,dx = \int \left(b\cos(c+dx)\cot^3(c+dx)+a\cot^4(c+dx)\right)\,dx$$
$$= a\int \cot^4(c+dx)\,dx + b\int \cos(c+dx)\cot^3(c+dx)\,dx$$
$$= -\frac{a\cot^3(c+dx)}{3d} - a\int \cot^2(c+dx)\,dx - \frac{b\,\mathrm{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx, x, \cos(c+\right.}{d}$$
$$= \frac{a\cot(c+dx)}{d} - \frac{b\cos(c+dx)\cot^2(c+dx)}{2d} - \frac{a\cot^3(c+dx)}{3d} + a\int 1\,dx +$$
$$= ax - \frac{3b\cos(c+dx)}{2d} + \frac{a\cot(c+dx)}{d} - \frac{b\cos(c+dx)\cot^2(c+dx)}{2d} - \frac{a\cot^3}{}$$
$$= ax + \frac{3b\tanh^{-1}(\cos(c+dx))}{2d} - \frac{3b\cos(c+dx)}{2d} + \frac{a\cot(c+dx)}{d} - \frac{b\cos(c-}{}$$

Mathematica [**C**] time = 0.05, size = 125, normalized size = 1.52

$$-\frac{a\cot^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2(c+dx)\right)}{3d} - \frac{b\cos(c+dx)}{d} - \frac{b\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{b\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{3b\log\left(\mathrm{si}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] -((b*Cos[c + d*x])/d) - (b*Csc[(c + d*x)/2]^2)/(8*d) - (a*Cot[c + d*x]^3*Hy
pergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])/(3*d) + (3*b*Log[Cos[(c +
d*x)/2]])/(2*d) - (3*b*Log[Sin[(c + d*x)/2]])/(2*d) + (b*Sec[(c + d*x)/2]^2
)/(8*d)
```

fricas [**B**] time = 0.47, size = 160, normalized size = 1.95

$$\frac{16\,a\cos(dx+c)^3 + 9\left(b\cos(dx+c)^2-b\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c) - 9\left(b\cos(dx+c)^2-b\right)\log\left(-\frac{1}{2}\right.}{12\left(d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(16*a*cos(d*x + c)^3 + 9*(b*cos(d*x + c)^2 - b)*log(1/2*cos(d*x + c) +
 1/2)*sin(d*x + c) - 9*(b*cos(d*x + c)^2 - b)*log(-1/2*cos(d*x + c) + 1/2)*
```

```
sin(d*x + c) - 12*a*cos(d*x + c) + 6*(2*a*d*x*cos(d*x + c)^2 - 2*b*cos(d*x
+ c)^3 - 2*a*d*x + 3*b*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*
sin(d*x + c))
```

giac [A] time = 0.58, size = 141, normalized size = 1.72

$$\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+24\,(dx+c)a-36\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\cdots}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(a*tan(1/2*d*x + 1/2*c)^3 + 3*b*tan(1/2*d*x + 1/2*c)^2 + 24*(d*x + c)*
a - 36*b*log(abs(tan(1/2*d*x + 1/2*c))) - 15*a*tan(1/2*d*x + 1/2*c) - 48*b/
(tan(1/2*d*x + 1/2*c)^2 + 1) + (66*b*tan(1/2*d*x + 1/2*c)^3 + 15*a*tan(1/2*
d*x + 1/2*c)^2 - 3*b*tan(1/2*d*x + 1/2*c) - a)/tan(1/2*d*x + 1/2*c)^3)/d
```

maple [A] time = 0.12, size = 106, normalized size = 1.29

$$-\frac{a\left(\cot^{3}(dx+c)\right)}{3d}+\frac{a\cot(dx+c)}{d}+ax+\frac{ca}{d}-\frac{b\left(\cos^{5}(dx+c)\right)}{2d\sin(dx+c)^{2}}-\frac{b\left(\cos^{3}(dx+c)\right)}{2d}-\frac{3b\cos(dx+c)}{2d}-\frac{3b\ln(\csc(dx\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+b*sin(d*x+c)),x)
```

```
[Out] -1/3*a*cot(d*x+c)^3/d+a*cot(d*x+c)/d+a*x+1/d*c*a-1/2/d*b/sin(d*x+c)^2*cos(d
*x+c)^5-1/2*b*cos(d*x+c)^3/d-3/2*b*cos(d*x+c)/d-3/2/d*b*ln(csc(d*x+c)-cot(d
*x+c))
```

maxima [A] time = 1.82, size = 92, normalized size = 1.12

$$\frac{4\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^{2}-1}{\tan(dx+c)^{3}}\right)a+3\,b\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^{2}-1}-4\,\cos(dx+c)+3\,\log(\cos(dx+c)+1)-3\,\log(\cos(dx+c\cdots\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(4*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a + 3*b*(2*co
s(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x + c) + 1)
- 3*log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 6.29, size = 225, normalized size = 2.74

$$\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{-5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+17\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\frac{14\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a}{3}}{d\left(8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}-\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + b*sin(c + d*x)),x)
```

```
[Out] (a*tan(c/2 + (d*x)/2)^3)/(24*d) - (a/3 + b*tan(c/2 + (d*x)/2) - (14*a*tan(c
/2 + (d*x)/2)^2)/3 - 5*a*tan(c/2 + (d*x)/2)^4 + 17*b*tan(c/2 + (d*x)/2)^3)/
(d*(8*tan(c/2 + (d*x)/2)^3 + 8*tan(c/2 + (d*x)/2)^5)) - (5*a*tan(c/2 + (d*x
)/2))/(8*d) + (b*tan(c/2 + (d*x)/2)^2)/(8*d) - (3*b*log(tan(c/2 + (d*x)/2))
)/(2*d) - (2*a*atan((4*a^2)/(6*a*b + 4*a^2*tan(c/2 + (d*x)/2)) - (6*a*b*tan
(c/2 + (d*x)/2))/(6*a*b + 4*a^2*tan(c/2 + (d*x)/2))))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin(c + dx))\cot^4(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*cot(c + d*x)**4, x)
```

3.149 $\int \cot^6(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=122

$$-\frac{a\cot^5(c+dx)}{5d}+\frac{a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-ax+\frac{15b\cos(c+dx)}{8d}-\frac{b\cos(c+dx)\cot^4(c+dx)}{4d}+\frac{5b\cos(c+dx}{8}$$

[Out] -a*x-15/8*b*arctanh(cos(d*x+c))/d+15/8*b*cos(d*x+c)/d-a*cot(d*x+c)/d+5/8*b*cos(d*x+c)*cot(d*x+c)^2/d+1/3*a*cot(d*x+c)^3/d-1/4*b*cos(d*x+c)*cot(d*x+c)^4/d-1/5*a*cot(d*x+c)^5/d

Rubi [A] time = 0.10, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {2722, 2592, 288, 321, 206, 3473, 8}

$$-\frac{a\cot^5(c+dx)}{5d}+\frac{a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-ax+\frac{15b\cos(c+dx)}{8d}-\frac{b\cos(c+dx)\cot^4(c+dx)}{4d}+\frac{5b\cos(c+dx}{8}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^6*(a + b*Sin[c + d*x]),x]

[Out] -(a*x) - (15*b*ArcTanh[Cos[c + d*x]])/(8*d) + (15*b*Cos[c + d*x])/(8*d) - (a*Cot[c + d*x])/d + (5*b*Cos[c + d*x]*Cot[c + d*x]^2)/(8*d) + (a*Cot[c + d*x]^3)/(3*d) - (b*Cos[c + d*x]*Cot[c + d*x]^4)/(4*d) - (a*Cot[c + d*x]^5)/(5*d)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \cot^6(c+dx)(a+b\sin(c+dx))\,dx = \int \left(b\cos(c+dx)\cot^5(c+dx)+a\cot^6(c+dx)\right)\,dx$$
$$= a\int \cot^6(c+dx)\,dx + b\int \cos(c+dx)\cot^5(c+dx)\,dx$$
$$= -\frac{a\cot^5(c+dx)}{5d} - a\int \cot^4(c+dx)\,dx - \frac{b\,\text{Subst}\left(\int \frac{x^6}{(1-x^2)^3}\,dx, x, \cos(c+\right.}{d}$$
$$= \frac{a\cot^3(c+dx)}{3d} - \frac{b\cos(c+dx)\cot^4(c+dx)}{4d} - \frac{a\cot^5(c+dx)}{5d} + a\int \cot^2($$
$$= -\frac{a\cot(c+dx)}{d} + \frac{5b\cos(c+dx)\cot^2(c+dx)}{8d} + \frac{a\cot^3(c+dx)}{3d} - \frac{b\cos(c}{}$$
$$= -ax + \frac{15b\cos(c+dx)}{8d} - \frac{a\cot(c+dx)}{d} + \frac{5b\cos(c+dx)\cot^2(c+dx)}{8d} + $$
$$= -ax - \frac{15b\tanh^{-1}(\cos(c+dx))}{8d} + \frac{15b\cos(c+dx)}{8d} - \frac{a\cot(c+dx)}{d} + \frac{5b}{}$$

Mathematica [C] time = 0.05, size = 164, normalized size = 1.34

$$-\frac{a\cot^5(c+dx)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-\tan^2(c+dx)\right)}{5d}+\frac{b\cos(c+dx)}{d}-\frac{b\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{9b\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{b\sec^4}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Cos[c + d*x])/d + (9*b*Csc[(c + d*x)/2]^2)/(32*d) - (b*Csc[(c + d*x)/2]^
4)/(64*d) - (a*Cot[c + d*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[c + d*x
]^2])/(5*d) - (15*b*Log[Cos[(c + d*x)/2]])/(8*d) + (15*b*Log[Sin[(c + d*x)/
2]])/(8*d) - (9*b*Sec[(c + d*x)/2]^2)/(32*d) + (b*Sec[(c + d*x)/2]^4)/(64*d
)
```

fricas [B] time = 0.50, size = 222, normalized size = 1.82

$$-\frac{368\,a\cos(dx+c)^5 - 560\,a\cos(dx+c)^3 + 225\left(b\cos(dx+c)^4 - 2\,b\cos(dx+c)^2 + b\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/240*(368*a*cos(d*x + c)^5 - 560*a*cos(d*x + c)^3 + 225*(b*cos(d*x + c)^4
 - 2*b*cos(d*x + c)^2 + b)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 225*(
b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + b)*log(-1/2*cos(d*x + c) + 1/2)*sin
(d*x + c) + 240*a*cos(d*x + c) + 30*(8*a*d*x*cos(d*x + c)^4 - 8*b*cos(d*x +
 c)^5 - 16*a*d*x*cos(d*x + c)^2 + 25*b*cos(d*x + c)^3 + 8*a*d*x - 15*b*cos(
d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*
x + c))
```

giac [A] time = 0.68, size = 199, normalized size = 1.63

$$\frac{6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+15\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-70\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-240\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-960\,(dx+c)a+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/960*(6*a*tan(1/2*d*x + 1/2*c)^5 + 15*b*tan(1/2*d*x + 1/2*c)^4 - 70*a*tan(
1/2*d*x + 1/2*c)^3 - 240*b*tan(1/2*d*x + 1/2*c)^2 - 960*(d*x + c)*a + 1800*
b*log(abs(tan(1/2*d*x + 1/2*c))) + 660*a*tan(1/2*d*x + 1/2*c) + 1920*b/(tan
(1/2*d*x + 1/2*c)^2 + 1) - (4110*b*tan(1/2*d*x + 1/2*c)^5 + 660*a*tan(1/2*d
*x + 1/2*c)^4 - 240*b*tan(1/2*d*x + 1/2*c)^3 - 70*a*tan(1/2*d*x + 1/2*c)^2
+ 15*b*tan(1/2*d*x + 1/2*c) + 6*a)/tan(1/2*d*x + 1/2*c)^5)/d
```

maple [A] time = 0.13, size = 159, normalized size = 1.30

$$-\frac{a\left(\cot^{5}(dx+c)\right)}{5d}+\frac{a\left(\cot^{3}(dx+c)\right)}{3d}-\frac{a\cot(dx+c)}{d}-ax-\frac{ca}{d}-\frac{b\left(\cos^{7}(dx+c)\right)}{4d\sin(dx+c)^{4}}+\frac{3b\left(\cos^{7}(dx+c)\right)}{8d\sin(dx+c)^{2}}+\frac{3b\left(\cos^{5}(dx\right.}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+b*sin(d*x+c)),x)
```

```
[Out] -1/5*a*cot(d*x+c)^5/d+1/3*a*cot(d*x+c)^3/d-a*cot(d*x+c)/d-a*x-1/d*c*a-1/4/d
*b/sin(d*x+c)^4*cos(d*x+c)^7+3/8/d*b/sin(d*x+c)^2*cos(d*x+c)^7+3/8*b*cos(d*
x+c)^5/d+5/8*b*cos(d*x+c)^3/d+15/8*b*cos(d*x+c)/d+15/8/d*b*ln(csc(d*x+c)-co
t(d*x+c))
```

maxima [A] time = 1.89, size = 125, normalized size = 1.02

$$-\frac{16\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^{4}-5\,\tan(dx+c)^{2}+3}{\tan(dx+c)^{5}}\right)a+15\,b\left(\frac{2\left(9\,\cos(dx+c)^{3}-7\,\cos(dx+c)\right)}{\cos(dx+c)^{4}-2\,\cos(dx+c)^{2}+1}-16\,\cos(dx+c)+15\,\log(\cos(d\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/240*(16*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(
d*x + c)^5)*a + 15*b*(2*(9*cos(d*x + c)^3 - 7*cos(d*x + c))/(cos(d*x + c)^4
 - 2*cos(d*x + c)^2 + 1) - 16*cos(d*x + c) + 15*log(cos(d*x + c) + 1) - 15*
log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 6.30, size = 288, normalized size = 2.36

$$\frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d}-\frac{22\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-72\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\frac{59\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}-\frac{15\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}-\frac{32\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{15}+\frac{b}{}\;-}{d\left(32\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+32\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6*(a + b*sin(c + d*x)),x)
```

```
[Out] (11*a*tan(c/2 + (d*x)/2))/(16*d) - (a/5 + (b*tan(c/2 + (d*x)/2))/2 - (32*a*
tan(c/2 + (d*x)/2)^2)/15 + (59*a*tan(c/2 + (d*x)/2)^4)/3 + 22*a*tan(c/2 + (
d*x)/2)^6 - (15*b*tan(c/2 + (d*x)/2)^3)/2 - 72*b*tan(c/2 + (d*x)/2)^5)/(d*(
32*tan(c/2 + (d*x)/2)^5 + 32*tan(c/2 + (d*x)/2)^7)) - (7*a*tan(c/2 + (d*x)/
2)^3)/(96*d) + (a*tan(c/2 + (d*x)/2)^5)/(160*d) - (b*tan(c/2 + (d*x)/2)^2)/
(4*d) + (b*tan(c/2 + (d*x)/2)^4)/(64*d) + (15*b*log(tan(c/2 + (d*x)/2)))/(8
*d) + (2*a*atan((4*a^2)/((15*a*b)/2 + 4*a^2*tan(c/2 + (d*x)/2)) - (15*a*b*t
an(c/2 + (d*x)/2))/(2*((15*a*b)/2 + 4*a^2*tan(c/2 + (d*x)/2)))))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin(c+dx))\cot^6(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*cot(c + d*x)**6, x)
```

3.150 $\int (a+b\sin(c+dx))^2\tan^3(c+dx)\,dx$

Optimal. Leaf size=111

$$\frac{2ab\sin(c+dx)}{d}+\frac{(a+b)(a+2b)\log(1-\sin(c+dx))}{2d}+\frac{(a-2b)(a-b)\log(\sin(c+dx)+1)}{2d}+\frac{\sec^2(c+dx)(a+b\sin}{2d}$$

```
[Out] 1/2*(a+b)*(a+2*b)*ln(1-sin(d*x+c))/d+1/2*(a-2*b)*(a-b)*ln(1+sin(d*x+c))/d+2
*a*b*sin(d*x+c)/d+1/2*b^2*sin(d*x+c)^2/d+1/2*sec(d*x+c)^2*(a+b*sin(d*x+c))^
2/d
```

Rubi [A] time = 0.17, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2721, 1645, 1629, 633, 31}

$$\frac{2ab\sin(c+dx)}{d}+\frac{(a+b)(a+2b)\log(1-\sin(c+dx))}{2d}+\frac{(a-2b)(a-b)\log(\sin(c+dx)+1)}{2d}+\frac{\sec^2(c+dx)(a+b\sin}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] ((a + b)*(a + 2*b)*Log[1 - Sin[c + d*x]])/(2*d) + ((a - 2*b)*(a - b)*Log[1
+ Sin[c + d*x]])/(2*d) + (2*a*b*Sin[c + d*x])/d + (b^2*Sin[c + d*x]^2)/(2*d
) + (Sec[c + d*x]^2*(a + b*Sin[c + d*x])^2)/(2*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int (a+b\sin(c+dx))^2\tan^3(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^3(a+x)^2}{\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^2(c+dx)(a+b\sin(c+dx))^2}{2d} + \frac{\text{Subst}\left(\int \frac{(a+x)\left(-2b^4-2ab^2x-2b^2x^2\right)}{b^2-x^2}\,dx, x\right.}{2b^2d}$$

$$= \frac{\sec^2(c+dx)(a+b\sin(c+dx))^2}{2d} + \frac{\text{Subst}\left(\int \left(4ab^2+2b^2x-\frac{2\left(3ab^4+b^2(a\right.}{b^2-x}\right.\right.}{2b^2d}$$

$$= \frac{2ab\sin(c+dx)}{d} + \frac{b^2\sin^2(c+dx)}{2d} + \frac{\sec^2(c+dx)(a+b\sin(c+dx))^2}{2d} -$$

$$= \frac{2ab\sin(c+dx)}{d} + \frac{b^2\sin^2(c+dx)}{2d} + \frac{\sec^2(c+dx)(a+b\sin(c+dx))^2}{2d} -$$

$$= \frac{(a+b)(a+2b)\log(1-\sin(c+dx))}{2d} + \frac{(a-2b)(a-b)\log(1+\sin(c+d}{2d}$$

Mathematica [A] time = 0.44, size = 108, normalized size = 0.97

$$\frac{\frac{(a-b)^2}{\sin(c+dx)+1} + 8ab\sin(c+dx) - \frac{(a+b)^2}{\sin(c+dx)-1} + 2(a-2b)(a-b)\log(\sin(c+dx)+1) + 2(a+b)(a+2b)\log(1-\sin(}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] (2*(a + b)*(a + 2*b)*Log[1 - Sin[c + d*x]] + 2*(a - 2*b)*(a - b)*Log[1 + Si
n[c + d*x]] - (a + b)^2/(-1 + Sin[c + d*x]) + 8*a*b*Sin[c + d*x] + 2*b^2*Si
n[c + d*x]^2 + (a - b)^2/(1 + Sin[c + d*x]))/(4*d)
```

fricas [A] time = 0.45, size = 140, normalized size = 1.26

$$-\frac{2\,b^2\cos\left(dx+c\right)^4 - b^2\cos\left(dx+c\right)^2 - 2\left(a^2-3\,ab+2\,b^2\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right) - 2\left(a^2+3\,ab+2\,b\right.}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*b^2*cos(d*x + c)^4 - b^2*cos(d*x + c)^2 - 2*(a^2 - 3*a*b + 2*b^2)*c
os(d*x + c)^2*log(sin(d*x + c) + 1) - 2*(a^2 + 3*a*b + 2*b^2)*cos(d*x + c)^
2*log(-sin(d*x + c) + 1) - 2*a^2 - 2*b^2 - 4*(2*a*b*cos(d*x + c)^2 + a*b)*s
in(d*x + c))/(d*cos(d*x + c)^2)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.15, size = 172, normalized size = 1.55

$$\frac{a^2\left(\tan^2\left(dx+c\right)\right)}{2d}+\frac{a^2\ln\left(\cos\left(dx+c\right)\right)}{d}+\frac{ab\left(\sin^5\left(dx+c\right)\right)}{d\cos\left(dx+c\right)^2}+\frac{ab\left(\sin^3\left(dx+c\right)\right)}{d}+\frac{3ab\sin\left(dx+c\right)}{d}-\frac{3ab\ln\left(\sec\left(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^2*tan(d*x+c)^3,x)
```

```
[Out] 1/2/d*a^2*tan(d*x+c)^2+1/d*a^2*ln(cos(d*x+c))+1/d*a*b*sin(d*x+c)^5/cos(d*x+
c)^2+1/d*a*b*sin(d*x+c)^3+3*a*b*sin(d*x+c)/d-3/d*a*b*ln(sec(d*x+c)+tan(d*x+
c))+1/2/d*b^2*sin(d*x+c)^6/cos(d*x+c)^2+1/2/d*b^2*sin(d*x+c)^4+b^2*sin(d*x+
c)^2/d+2/d*b^2*ln(cos(d*x+c))
```

maxima [A] time = 0.46, size = 105, normalized size = 0.95

$$\frac{b^2\sin(dx+c)^2+4\,ab\sin(dx+c)+\left(a^2-3\,ab+2\,b^2\right)\log(\sin(dx+c)+1)+\left(a^2+3\,ab+2\,b^2\right)\log(\sin(dx+c}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*sin(d*x + c)^2 + 4*a*b*sin(d*x + c) + (a^2 - 3*a*b + 2*b^2)*log(si
n(d*x + c) + 1) + (a^2 + 3*a*b + 2*b^2)*log(sin(d*x + c) - 1) - (2*a*b*sin(
d*x + c) + a^2 + b^2)/(sin(d*x + c)^2 - 1))/d
```

mupad [B] time = 6.72, size = 232, normalized size = 2.09

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(2\,a^2+4\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(2\,a^2+4\,b^2\right)+4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+2\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\,a\,b\tan}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^2*(2*a^2 + 4*b^2) + tan(c/2 + (d*x)/2)^6*(2*a^2 + 4*b^2
) + 4*a^2*tan(c/2 + (d*x)/2)^4 + 2*a*b*tan(c/2 + (d*x)/2)^3 + 2*a*b*tan(c/2
 + (d*x)/2)^5 + 6*a*b*tan(c/2 + (d*x)/2)^7 + 6*a*b*tan(c/2 + (d*x)/2))/(d*(
tan(c/2 + (d*x)/2)^8 - 2*tan(c/2 + (d*x)/2)^4 + 1)) - (log(tan(c/2 + (d*x)/
2)^2 + 1)*(a^2 + 2*b^2))/d + (log(tan(c/2 + (d*x)/2) - 1)*(a + b)*(a + 2*b)
)/d + (log(tan(c/2 + (d*x)/2) + 1)*(a - b)*(a - 2*b))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin(c+dx))^2\tan^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**2*tan(d*x+c)**3,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**2*tan(c + d*x)**3, x)
```

3.151 $\int (a+b\sin(c+dx))^2\tan(c+dx)\,dx$

Optimal. Leaf size=78

$$-\frac{2ab\sin(c+dx)}{d}-\frac{(a-b)^2\log(\sin(c+dx)+1)}{2d}-\frac{(a+b)^2\log(1-\sin(c+dx))}{2d}-\frac{b^2\sin^2(c+dx)}{2d}$$

```
[Out] -1/2*(a+b)^2*ln(1-sin(d*x+c))/d-1/2*(a-b)^2*ln(1+sin(d*x+c))/d-2*a*b*sin(d*
x+c)/d-1/2*b^2*sin(d*x+c)^2/d
```

Rubi [A] time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2721, 801, 633, 31}

$$-\frac{2ab\sin(c+dx)}{d}-\frac{(a-b)^2\log(\sin(c+dx)+1)}{2d}-\frac{(a+b)^2\log(1-\sin(c+dx))}{2d}-\frac{b^2\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] -((a + b)^2*Log[1 - Sin[c + d*x]])/(2*d) - ((a - b)^2*Log[1 + Sin[c + d*x]]
)/(2*d) - (2*a*b*Sin[c + d*x])/d - (b^2*Sin[c + d*x]^2)/(2*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
```

```
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int (a+b\sin(c+dx))^2\tan(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x(a+x)^2}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-2a-x+\frac{2ab^2+(a^2+b^2)x}{b^2-x^2}\right)dx, x, b\sin(c+dx)\right)}{d}$$
$$= -\frac{2ab\sin(c+dx)}{d}-\frac{b^2\sin^2(c+dx)}{2d}+\frac{\text{Subst}\left(\int \frac{2ab^2+(a^2+b^2)x}{b^2-x^2}\,dx, x, b\sin(\right.}{d}$$
$$= -\frac{2ab\sin(c+dx)}{d}-\frac{b^2\sin^2(c+dx)}{2d}+\frac{(a-b)^2\,\text{Subst}\left(\int \frac{1}{-b-x}\,dx, x, b\sin\right.}{2d}$$
$$= -\frac{(a+b)^2\log(1-\sin(c+dx))}{2d}-\frac{(a-b)^2\log(1+\sin(c+dx))}{2d}-\frac{2ab\sin}{}$$

Mathematica [A] time = 0.13, size = 64, normalized size = 0.82

$$-\frac{4ab\sin(c+dx)+(a-b)^2\log(\sin(c+dx)+1)+(a+b)^2\log(1-\sin(c+dx))+b^2\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] -1/2*((a + b)^2*Log[1 - Sin[c + d*x]] + (a - b)^2*Log[1 + Sin[c + d*x]] + 4
*a*b*Sin[c + d*x] + b^2*Sin[c + d*x]^2)/d
```

fricas [A] time = 0.47, size = 74, normalized size = 0.95

$$\frac{b^2\cos(dx+c)^2-4\,ab\sin(dx+c)-\left(a^2-2\,ab+b^2\right)\log(\sin(dx+c)+1)-\left(a^2+2\,ab+b^2\right)\log(-\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*cos(d*x + c)^2 - 4*a*b*sin(d*x + c) - (a^2 - 2*a*b + b^2)*log(sin(
d*x + c) + 1) - (a^2 + 2*a*b + b^2)*log(-sin(d*x + c) + 1))/d
```

giac [B] time = 12.16, size = 7855, normalized size = 100.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c),x, algorithm="giac")
```

```
[Out] -1/4*(4*a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c
) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1
/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2
 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*ta
n(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 - 4*a*b*log(2*(tan(1/2*d*x)^4
*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2
 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*ta
n(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x)
+ 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c
)^2*tan(c)^2 + 2*a^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan
(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(
d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + 2*b^2*log(4*(tan(d*x)^4*tan(c
)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*t
an(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2
- b^2*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + 4*a*b*log(2*(tan(1/
2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(
1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^
3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1
/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^2*t
an(1/2*c)^2 - 4*a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*t
an(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)
^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/
2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2
+ 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a^2*log(4*(tan(d*x)^4*tan(
c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*
tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*b^2*
log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + ta
n(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^2
*tan(1/2*c)^2 + 4*a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4
*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*
x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(
1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^
2 + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(c)^2 - 4*a*b*log(2*(tan(1/2*d*x)^4*ta
n(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 +
tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1
/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2
*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(c)^2 + 2
```

```
*a^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2
 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(1/2*d
*x)^2*tan(c)^2 + 2*b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + t
an(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*ta
n(d*x)^2*tan(1/2*d*x)^2*tan(c)^2 - 16*a*b*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2
*c)*tan(c)^2 + 4*a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*
tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x
)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1
/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2
 + 1))*tan(d*x)^2*tan(1/2*c)^2*tan(c)^2 - 4*a*b*log(2*(tan(1/2*d*x)^4*tan(1
/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan
(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*
d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*ta
n(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*c)^2*tan(c)^2 + 2*a^2*
log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + ta
n(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(1/2*c)^2*t
an(c)^2 + 2*b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)
^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^
2*tan(1/2*c)^2*tan(c)^2 - 16*a*b*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^2*tan(c
)^2 + 4*a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c
) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1
/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2
 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*ta
n(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 - 4*a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)
^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*
d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*
tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2
*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + 2*a^2*l
og(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan
(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^
2*tan(c)^2 + 2*b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d
*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(1/
2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + b^2*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2
 + 4*b^2*tan(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c) - b^2*tan(d*x)^2*tan(1
/2*d*x)^2*tan(c)^2 - b^2*tan(d*x)^2*tan(1/2*c)^2*tan(c)^2 + b^2*tan(1/2*d*x
)^2*tan(1/2*c)^2*tan(c)^2 + 4*a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*ta
n(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 +
2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c
)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/
(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^2 - 4*a*b*log(2*(tan(1/2*d*x)^4
*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2
 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*ta
n(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x)
+ 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^2 + 2*a^2*l
og(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan
```

```
(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^2
+ 2*b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c
)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(1/
2*d*x)^2 - 16*a*b*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c) + 4*a*b*log(2*(tan(1
/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan
(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)
^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(
1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*c)^2 -
4*a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*
tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^
2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan
(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)
^2*tan(1/2*c)^2 + 2*a^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) +
tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*t
an(d*x)^2*tan(1/2*c)^2 + 2*b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*ta
n(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2
+ 1))*tan(d*x)^2*tan(1/2*c)^2 - 16*a*b*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^2
 + 4*a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) +
 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*
c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 +
tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1
/2*d*x)^2*tan(1/2*c)^2 - 4*a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1
/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*t
an(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2
 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(ta
n(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a^2*log(4*(tan(d*x)^4*tan(
c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*
tan(c) + 1)/(tan(c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*b^2*log(4*(tan(
d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 -
2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*a*b*
log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/
2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*
tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c
)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan
(c)^2 - 4*a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2
*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan
(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)
^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*
tan(d*x)^2*tan(c)^2 + 2*a^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c
) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1
))*tan(d*x)^2*tan(c)^2 + 2*b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*ta
n(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2
+ 1))*tan(d*x)^2*tan(c)^2 + 16*a*b*tan(d*x)^2*tan(1/2*d*x)*tan(c)^2 + 4*a*b
*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1
/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2
```

*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(c)^2 - 4*a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(c)^2 + 2*a^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(1/2*d*x)^2*tan(c)^2 + 2*b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(1/2*d*x)^2*tan(c)^2 + 16*a*b*tan(d*x)^2*tan(1/2*c)*tan(c)^2 - 16*a*b*tan(1/2*d*x)^2*tan(1/2*c)*tan(c)^2 + 4*a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*c)^2*tan(c)^2 - 4*a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*c)^2*tan(c)^2 + 2*a^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(1/2*c)^2*tan(c)^2 + 2*b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(1/2*c)^2*tan(c)^2 - 16*a*b*tan(1/2*d*x)*tan(1/2*c)^2*tan(c)^2 + b^2*tan(d*x)^2*tan(1/2*d*x)^2 + b^2*tan(d*x)^2*tan(1/2*c)^2 - b^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*b^2*tan(d*x)*tan(1/2*d*x)^2*tan(c) + 4*b^2*tan(d*x)*tan(1/2*c)^2*tan(c) - b^2*tan(d*x)^2*tan(c)^2 + b^2*tan(1/2*d*x)^2*tan(c)^2 + b^2*tan(1/2*c)^2*tan(c)^2 + 4*a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2 - 4*a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2 + 2*a^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2 + 2*b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2 + 16*a*b*tan(d*x)^2*tan(1/2*d*x) + 4*a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2 - 4*a*b*log(2*(t

an(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3
*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*
d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*
tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2 + 2*a^2
log(4(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + t
an(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(1/2*d*x)^2 + 2*b^2*l
og(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan
(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(1/2*d*x)^2 + 16*a*b*ta
n(d*x)^2*tan(1/2*c) - 16*a*b*tan(1/2*d*x)^2*tan(1/2*c) + 4*a*b*log(2*(tan(1
/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan
(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)
^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(
1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*c)^2 - 4*a*b*log(2
*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x
)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1
/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 +
 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*c)^2 + 2*a^
2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 +
tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(1/2*c)^2 + 2*b^2*lo
g(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(
d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(1/2*c)^2 - 16*a*b*tan(1
/2*d*x)*tan(1/2*c)^2 + 4*a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2
*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan
(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 +
 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(
1/2*c)^2 + 1))*tan(c)^2 - 4*a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(
1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*
tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^
2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(t
an(1/2*c)^2 + 1))*tan(c)^2 + 2*a^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^
3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c
)^2 + 1))*tan(c)^2 + 2*b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c)
 + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)
)*tan(c)^2 + 16*a*b*tan(1/2*d*x)*tan(c)^2 + 16*a*b*tan(1/2*c)*tan(c)^2 + b^
2*tan(d*x)^2 - b^2*tan(1/2*d*x)^2 - b^2*tan(1/2*c)^2 + 4*b^2*tan(d*x)*tan(c
) + b^2*tan(c)^2 + 4*a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x
)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2
*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*t
an(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*
c)^2 + 1)) - 4*a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*ta
n(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^
2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2
*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 +
 1)) + 2*a^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*
tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) + 2*b^2*log(

```
4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*
x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) + 16*a*b*tan(1/2*d*x) + 16*a*
b*tan(1/2*c) - b^2)/(d*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + d*
tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + d*tan(d*x)^2*tan(1/2*d*x)^2*tan(c)
^2 + d*tan(d*x)^2*tan(1/2*c)^2*tan(c)^2 + d*tan(1/2*d*x)^2*tan(1/2*c)^2*tan
(c)^2 + d*tan(d*x)^2*tan(1/2*d*x)^2 + d*tan(d*x)^2*tan(1/2*c)^2 + d*tan(1/2
*d*x)^2*tan(1/2*c)^2 + d*tan(d*x)^2*tan(c)^2 + d*tan(1/2*d*x)^2*tan(c)^2 +
d*tan(1/2*c)^2*tan(c)^2 + d*tan(d*x)^2 + d*tan(1/2*d*x)^2 + d*tan(1/2*c)^2
+ d*tan(c)^2 + d)
```

maple [A] time = 0.15, size = 82, normalized size = 1.05

$$-\frac{a^2\ln(\cos(dx+c))}{d}-\frac{2ab\sin(dx+c)}{d}+\frac{2ab\ln(\sec(dx+c)+\tan(dx+c))}{d}-\frac{b^2\left(\sin^2(dx+c)\right)}{2d}-\frac{b^2\ln(\cos(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^2*tan(d*x+c),x)
```

```
[Out] -1/d*a^2*ln(cos(d*x+c))-2*a*b*sin(d*x+c)/d+2/d*a*b*ln(sec(d*x+c)+tan(d*x+c)
)-1/2*b^2*sin(d*x+c)^2/d-1/d*b^2*ln(cos(d*x+c))
```

maxima [A] time = 0.37, size = 70, normalized size = 0.90

$$-\frac{b^2\sin(dx+c)^2+4\,ab\sin(dx+c)+\left(a^2-2\,ab+b^2\right)\log(\sin(dx+c)+1)+\left(a^2+2\,ab+b^2\right)\log(\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*(b^2*sin(d*x + c)^2 + 4*a*b*sin(d*x + c) + (a^2 - 2*a*b + b^2)*log(sin
(d*x + c) + 1) + (a^2 + 2*a*b + b^2)*log(sin(d*x + c) - 1))/d
```

mupad [B] time = 6.37, size = 150, normalized size = 1.92

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)\,\left(a^2+b^2\right)}{d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,{\left(a+b\right)}^2}{d}-\frac{2\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+4\,a}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (log(tan(c/2 + (d*x)/2)^2 + 1)*(a^2 + b^2))/d - (log(tan(c/2 + (d*x)/2) - 1
)*(a + b)^2)/d - (2*b^2*tan(c/2 + (d*x)/2)^2 + 4*a*b*tan(c/2 + (d*x)/2)^3 +
```

```
 4*a*b*tan(c/2 + (d*x)/2))/(d*(2*tan(c/2 + (d*x)/2)^2 + tan(c/2 + (d*x)/2)^
4 + 1)) - (log(tan(c/2 + (d*x)/2) + 1)*(a - b)^2)/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin(c + dx))^2 \tan(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**2*tan(d*x+c),x)
```

```
[Out] Integral((a + b*sin(c + d*x))**2*tan(c + d*x), x)
```

3.152 $\int \cot(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=46

$$\frac{a^2\log(\sin(c+dx))}{d}+\frac{2ab\sin(c+dx)}{d}+\frac{b^2\sin^2(c+dx)}{2d}$$

```
[Out] a^2*ln(sin(d*x+c))/d+2*a*b*sin(d*x+c)/d+1/2*b^2*sin(d*x+c)^2/d
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2721, 43}

$$\frac{a^2\log(\sin(c+dx))}{d}+\frac{2ab\sin(c+dx)}{d}+\frac{b^2\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Log[Sin[c + d*x]])/d + (2*a*b*Sin[c + d*x])/d + (b^2*Sin[c + d*x]^2)/(
2*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \cot(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^2}{x}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(2a+\frac{a^2}{x}+x\right)\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{a^2\log(\sin(c+dx))}{d} + \frac{2ab\sin(c+dx)}{d} + \frac{b^2\sin^2(c+dx)}{2d}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 1.00

$$\frac{a^2\log(\sin(c+dx))}{d} + \frac{2ab\sin(c+dx)}{d} + \frac{b^2\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Log[Sin[c + d*x]])/d + (2*a*b*Sin[c + d*x])/d + (b^2*Sin[c + d*x]^2)/(
2*d)
```

fricas [A] time = 0.46, size = 42, normalized size = 0.91

$$-\frac{b^2\cos(dx+c)^2 - 2\,a^2\log\left(\frac{1}{2}\,\sin(dx+c)\right) - 4\,ab\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*cos(d*x + c)^2 - 2*a^2*log(1/2*sin(d*x + c)) - 4*a*b*sin(d*x + c)
)/d
```

giac [A] time = 0.42, size = 41, normalized size = 0.89

$$\frac{b^2\sin(dx+c)^2 + 2\,a^2\log(|\sin(dx+c)|) + 4\,ab\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*sin(d*x + c)^2 + 2*a^2*log(abs(sin(d*x + c))) + 4*a*b*sin(d*x + c)
)/d
```

maple [A] time = 0.10, size = 45, normalized size = 0.98

$$\frac{a^2 \ln(\sin(dx+c))}{d} + \frac{2ab\sin(dx+c)}{d} + \frac{b^2\left(\sin^2(dx+c)\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+b*sin(d*x+c))^2,x)
[Out] a^2*ln(sin(d*x+c))/d+2*a*b*sin(d*x+c)/d+1/2*b^2*sin(d*x+c)^2/d
```

maxima [A] time = 0.48, size = 40, normalized size = 0.87

$$\frac{b^2\sin(dx+c)^2 + 2\,a^2\log(\sin(dx+c)) + 4\,ab\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
[Out] 1/2*(b^2*sin(d*x + c)^2 + 2*a^2*log(sin(d*x + c)) + 4*a*b*sin(d*x + c))/d
```

mupad [B] time = 6.44, size = 117, normalized size = 2.54

$$\frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{2\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}-\frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + b*sin(c + d*x))^2,x)
[Out] (a^2*log(tan(c/2 + (d*x)/2)))/d + (2*b^2*tan(c/2 + (d*x)/2)^2 + 4*a*b*tan(c
/2 + (d*x)/2)^3 + 4*a*b*tan(c/2 + (d*x)/2))/(d*(2*tan(c/2 + (d*x)/2)^2 + ta
n(c/2 + (d*x)/2)^4 + 1)) - (a^2*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin(c+dx))^2\cot(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c))**2,x)
[Out] Integral((a + b*sin(c + d*x))**2*cot(c + d*x), x)
```

3.153 $\int \cot^3(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=84

$$-\frac{(a^2-b^2)\log(\sin(c+dx))}{d}-\frac{a^2\csc^2(c+dx)}{2d}-\frac{2ab\sin(c+dx)}{d}-\frac{2ab\csc(c+dx)}{d}-\frac{b^2\sin^2(c+dx)}{2d}$$

```
[Out] -2*a*b*csc(d*x+c)/d-1/2*a^2*csc(d*x+c)^2/d-(a^2-b^2)*ln(sin(d*x+c))/d-2*a*b
*sin(d*x+c)/d-1/2*b^2*sin(d*x+c)^2/d
```

Rubi [A] time = 0.07, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2721, 894}

$$-\frac{(a^2-b^2)\log(\sin(c+dx))}{d}-\frac{a^2\csc^2(c+dx)}{2d}-\frac{2ab\sin(c+dx)}{d}-\frac{2ab\csc(c+dx)}{d}-\frac{b^2\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a*b*Csc[c + d*x])/d - (a^2*Csc[c + d*x]^2)/(2*d) - ((a^2 - b^2)*Log[Sin
[c + d*x]])/d - (2*a*b*Sin[c + d*x])/d - (b^2*Sin[c + d*x]^2)/(2*d)
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \cot^3(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^2(b^2-x^2)}{x^3}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-2a+\frac{a^2b^2}{x^3}+\frac{2ab^2}{x^2}+\frac{-a^2+b^2}{x}-x\right)dx, x, b\sin(c+dx)\right)}{d}$$
$$= -\frac{2ab\csc(c+dx)}{d} - \frac{a^2\csc^2(c+dx)}{2d} - \frac{(a^2-b^2)\log(\sin(c+dx))}{d} - \frac{2ab}{}$$

Mathematica [A] time = 0.26, size = 70, normalized size = 0.83

$$-\frac{2(a^2-b^2)\log(\sin(c+dx))+a^2\csc^2(c+dx)+4ab\sin(c+dx)+4ab\csc(c+dx)+b^2\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/2*(4*a*b*Csc[c + d*x] + a^2*Csc[c + d*x]^2 + 2*(a^2 - b^2)*Log[Sin[c + d
*x]] + 4*a*b*Sin[c + d*x] + b^2*Sin[c + d*x]^2)/d
```

fricas [A] time = 0.49, size = 115, normalized size = 1.37

$$\frac{2\,b^2\cos(dx+c)^4-3\,b^2\cos(dx+c)^2+2\,a^2+b^2-4\left((a^2-b^2)\cos(dx+c)^2-a^2+b^2\right)\log\left(\frac{1}{2}\sin(dx+c)\right)-8}{4\left(d\cos(dx+c)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*cos(d*x + c)^4 - 3*b^2*cos(d*x + c)^2 + 2*a^2 + b^2 - 4*((a^2 -
b^2)*cos(d*x + c)^2 - a^2 + b^2)*log(1/2*sin(d*x + c)) - 8*(a*b*cos(d*x + c
)^2 - 2*a*b)*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

giac [A] time = 0.75, size = 99, normalized size = 1.18

$$-\frac{b^2\sin(dx+c)^2+4\,ab\sin(dx+c)+2\left(a^2-b^2\right)\log(|\sin(dx+c)|)-\frac{3\,a^2\sin(dx+c)^2-3\,b^2\sin(dx+c)^2-4\,ab\sin(dx+c)-a^2}{\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="giac")

[Out] -1/2*(b^2*sin(d*x + c)^2 + 4*a*b*sin(d*x + c) + 2*(a^2 - b^2)*log(abs(sin(d*x + c))) - (3*a^2*sin(d*x + c)^2 - 3*b^2*sin(d*x + c)^2 - 4*a*b*sin(d*x + c) - a^2)/sin(d*x + c)^2)/d

maple [A] time = 0.24, size = 120, normalized size = 1.43

$$-\frac{a^{2}\left(\cot^{2}\left(dx+c\right)\right)}{2d}-\frac{a^{2}\ln\left(\sin\left(dx+c\right)\right)}{d}-\frac{2ab\left(\cos^{4}\left(dx+c\right)\right)}{d\sin\left(dx+c\right)}-\frac{2ab\left(\cos^{2}\left(dx+c\right)\right)\sin\left(dx+c\right)}{d}-\frac{4ab\sin\left(dx+c\right)}{d}+$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^3*(a+b*sin(d*x+c))^2,x)

[Out] -1/2/d*a^2*cot(d*x+c)^2-a^2*ln(sin(d*x+c))/d-2/d*a*b/sin(d*x+c)*cos(d*x+c)^4-2/d*a*b*cos(d*x+c)^2*sin(d*x+c)-4*a*b*sin(d*x+c)/d+1/2/d*b^2*cos(d*x+c)^2+1/d*b^2*ln(sin(d*x+c))

maxima [A] time = 0.77, size = 69, normalized size = 0.82

$$-\frac{b^{2}\sin\left(dx+c\right)^{2}+4\,ab\sin\left(dx+c\right)+2\left(a^{2}-b^{2}\right)\log\left(\sin\left(dx+c\right)\right)+\frac{4\,ab\sin\left(dx+c\right)+a^{2}}{\sin\left(dx+c\right)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="maxima")

[Out] -1/2*(b^2*sin(d*x + c)^2 + 4*a*b*sin(d*x + c) + 2*(a^2 - b^2)*log(sin(d*x + c)) + (4*a*b*sin(d*x + c) + a^2)/sin(d*x + c)^2)/d

mupad [B] time = 6.39, size = 221, normalized size = 2.63

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)\,\left(a^{2}-b^{2}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}\,\left(\frac{a^{2}}{2}+8\,b^{2}\right)+a^{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+\frac{a^{2}}{2}+24\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}+20}{d\,\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+4\tan\left(\frac{c}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d*x)^3*(a + b*sin(c + d*x))^2,x)

[Out] (log(tan(c/2 + (d*x)/2)^2 + 1)*(a^2 - b^2))/d - (tan(c/2 + (d*x)/2)^4*(a^2/2 + 8*b^2) + a^2*tan(c/2 + (d*x)/2)^2 + a^2/2 + 24*a*b*tan(c/2 + (d*x)/2)^3 + 20*a*b*tan(c/2 + (d*x)/2)^5 + 4*a*b*tan(c/2 + (d*x)/2))/(d*(4*tan(c/2 + (d*x)/2)^2 + 8*tan(c/2 + (d*x)/2)^4 + 4*tan(c/2 + (d*x)/2)^6)) - (a^2*tan(c

```
/2 + (d*x)/2)^2)/(8*d) - (log(tan(c/2 + (d*x)/2))*(a^2 - b^2))/d - (a*b*tan
(c/2 + (d*x)/2))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin (c+dx))^2 \cot ^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**2*cot(c + d*x)**3, x)
```

3.154 $\int \cot^5(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=126

$$\frac{(2a^2-b^2)\csc^2(c+dx)}{2d}+\frac{(a^2-2b^2)\log(\sin(c+dx))}{d}-\frac{a^2\csc^4(c+dx)}{4d}+\frac{2ab\sin(c+dx)}{d}-\frac{2ab\csc^3(c+dx)}{3d}+\frac{4ab\,\mathrm{cs}}{}$$

```
[Out] 4*a*b*csc(d*x+c)/d+1/2*(2*a^2-b^2)*csc(d*x+c)^2/d-2/3*a*b*csc(d*x+c)^3/d-1/
4*a^2*csc(d*x+c)^4/d+(a^2-2*b^2)*ln(sin(d*x+c))/d+2*a*b*sin(d*x+c)/d+1/2*b^
2*sin(d*x+c)^2/d
```

Rubi [A] time = 0.10, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2721, 948}

$$\frac{(2a^2-b^2)\csc^2(c+dx)}{2d}+\frac{(a^2-2b^2)\log(\sin(c+dx))}{d}-\frac{a^2\csc^4(c+dx)}{4d}+\frac{2ab\sin(c+dx)}{d}-\frac{2ab\csc^3(c+dx)}{3d}+\frac{4ab\,\mathrm{cs}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (4*a*b*Csc[c + d*x])/d + ((2*a^2 - b^2)*Csc[c + d*x]^2)/(2*d) - (2*a*b*Csc[
c + d*x]^3)/(3*d) - (a^2*Csc[c + d*x]^4)/(4*d) + ((a^2 - 2*b^2)*Log[Sin[c +
 d*x]])/d + (2*a*b*Sin[c + d*x])/d + (b^2*Sin[c + d*x]^2)/(2*d)
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \cot^5(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^2\left(b^2-x^2\right)^2}{x^5}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(2a+\frac{a^2b^4}{x^5}+\frac{2ab^4}{x^4}+\frac{-2a^2b^2+b^4}{x^3}-\frac{4ab^2}{x^2}+\frac{a^2-2b^2}{x}+x\right)dx, x, b\sin(\right.}{d}$$
$$= \frac{4ab\csc(c+dx)}{d}+\frac{\left(2a^2-b^2\right)\csc^2(c+dx)}{2d}-\frac{2ab\csc^3(c+dx)}{3d}-\frac{a^2\csc^4}{4}$$

Mathematica [A] time = 0.73, size = 107, normalized size = 0.85

$$\frac{6\left(2a^2-b^2\right)\csc^2(c+dx)+6\left(2\left(a^2-2b^2\right)\log(\sin(c+dx))+4ab\sin(c+dx)+b^2\sin^2(c+dx)\right)-3a^2\csc^4(c+a}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (48*a*b*Csc[c + d*x] + 6*(2*a^2 - b^2)*Csc[c + d*x]^2 - 8*a*b*Csc[c + d*x]^
3 - 3*a^2*Csc[c + d*x]^4 + 6*(2*(a^2 - 2*b^2)*Log[Sin[c + d*x]] + 4*a*b*Sin
[c + d*x] + b^2*Sin[c + d*x]^2))/(12*d)
```

fricas [A] time = 0.50, size = 177, normalized size = 1.40

$$-\frac{6\,b^2\cos(dx+c)^6-15\,b^2\cos(dx+c)^4+6\left(2\,a^2+b^2\right)\cos(dx+c)^2-9\,a^2+3\,b^2-12\left(\left(a^2-2\,b^2\right)\cos(dx+c)^4\right.}{12\left(d\cos(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/12*(6*b^2*cos(d*x + c)^6 - 15*b^2*cos(d*x + c)^4 + 6*(2*a^2 + b^2)*cos(d
*x + c)^2 - 9*a^2 + 3*b^2 - 12*((a^2 - 2*b^2)*cos(d*x + c)^4 - 2*(a^2 - 2*b
^2)*cos(d*x + c)^2 + a^2 - 2*b^2)*log(1/2*sin(d*x + c)) - 8*(3*a*b*cos(d*x
+ c)^4 - 12*a*b*cos(d*x + c)^2 + 8*a*b)*sin(d*x + c))/(d*cos(d*x + c)^4 - 2
*d*cos(d*x + c)^2 + d)
```

giac [A] time = 0.87, size = 138, normalized size = 1.10

$$\frac{6\,b^2\sin(dx+c)^2+24\,ab\sin(dx+c)+12\left(a^2-2\,b^2\right)\log(|\sin(dx+c)|)-\frac{25\,a^2\sin(dx+c)^4-50\,b^2\sin(dx+c)^4-48\,ab\sin(dx+}{}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/12*(6*b^2*sin(d*x + c)^2 + 24*a*b*sin(d*x + c) + 12*(a^2 - 2*b^2)*log(abs
(sin(d*x + c))) - (25*a^2*sin(d*x + c)^4 - 50*b^2*sin(d*x + c)^4 - 48*a*b*s
in(d*x + c)^3 - 12*a^2*sin(d*x + c)^2 + 6*b^2*sin(d*x + c)^2 + 8*a*b*sin(d*
x + c) + 3*a^2)/sin(d*x + c)^4)/d
```

maple [A] time = 0.28, size = 220, normalized size = 1.75

$$-\frac{a^2\left(\cot^4(dx+c)\right)}{4d}+\frac{a^2\left(\cot^2(dx+c)\right)}{2d}+\frac{a^2\ln(\sin(dx+c))}{d}-\frac{2ab\left(\cos^6(dx+c)\right)}{3d\sin(dx+c)^3}+\frac{2ab\left(\cos^6(dx+c)\right)}{d\sin(dx+c)}+\frac{16ab\sin(}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/4/d*a^2*cot(d*x+c)^4+1/2/d*a^2*cot(d*x+c)^2+a^2*ln(sin(d*x+c))/d-2/3/d*a
*b/sin(d*x+c)^3*cos(d*x+c)^6+2/d*a*b/sin(d*x+c)*cos(d*x+c)^6+16/3*a*b*sin(d
*x+c)/d+2/d*a*b*sin(d*x+c)*cos(d*x+c)^4+8/3/d*a*b*cos(d*x+c)^2*sin(d*x+c)-1
/2/d*b^2/sin(d*x+c)^2*cos(d*x+c)^6-1/2/d*b^2*cos(d*x+c)^4-1/d*b^2*cos(d*x+c
)^2-2/d*b^2*ln(sin(d*x+c))
```

maxima [A] time = 0.99, size = 105, normalized size = 0.83

$$\frac{6\,b^2\sin(dx+c)^2+24\,ab\sin(dx+c)+12\left(a^2-2\,b^2\right)\log(\sin(dx+c))+\frac{48\,ab\sin(dx+c)^3-8\,ab\sin(dx+c)+6\left(2\,a^2-b^2\right)\sin(dx}{\sin(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/12*(6*b^2*sin(d*x + c)^2 + 24*a*b*sin(d*x + c) + 12*(a^2 - 2*b^2)*log(sin
(d*x + c)) + (48*a*b*sin(d*x + c)^3 - 8*a*b*sin(d*x + c) + 6*(2*a^2 - b^2)*
sin(d*x + c)^2 - 3*a^2)/sin(d*x + c)^4)/d
```

mupad [B] time = 6.44, size = 310, normalized size = 2.46

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(\frac{5\,a^2}{2}-2\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(\frac{23\,a^2}{4}-4\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(3\,a^2+30\,b^2\right)-\frac{a^2}{4}+\frac{76\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+}{d\left(16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+32\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^2*((5*a^2)/2 - 2*b^2) + tan(c/2 + (d*x)/2)^4*((23*a^2)/
4 - 4*b^2) + tan(c/2 + (d*x)/2)^6*(3*a^2 + 30*b^2) - a^2/4 + (76*a*b*tan(c/
2 + (d*x)/2)^3)/3 + (356*a*b*tan(c/2 + (d*x)/2)^5)/3 + 92*a*b*tan(c/2 + (d*
x)/2)^7 - (4*a*b*tan(c/2 + (d*x)/2))/3)/(d*(16*tan(c/2 + (d*x)/2)^4 + 32*ta
n(c/2 + (d*x)/2)^6 + 16*tan(c/2 + (d*x)/2)^8)) - (log(tan(c/2 + (d*x)/2)^2
+ 1)*(a^2 - 2*b^2))/d - (a^2*tan(c/2 + (d*x)/2)^4)/(64*d) + (log(tan(c/2 +
(d*x)/2))*(a^2 - 2*b^2))/d + (tan(c/2 + (d*x)/2)^2*((3*a^2)/16 - b^2/8))/d
- (a*b*tan(c/2 + (d*x)/2)^3)/(12*d) + (7*a*b*tan(c/2 + (d*x)/2))/(4*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin(c + dx))^2 \cot^5(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**2*cot(c + d*x)**5, x)
```

3.155 $\int (a+b\sin(c+dx))^2\tan^4(c+dx)\,dx$

Optimal. Leaf size=149

$$\frac{a^2\tan^3(c+dx)}{3d}-\frac{a^2\tan(c+dx)}{d}+a^2x-\frac{2ab\cos(c+dx)}{d}+\frac{2ab\sec^3(c+dx)}{3d}-\frac{4ab\sec(c+dx)}{d}+\frac{5b^2\tan^3(c+dx)}{6d}-\frac{5b}{}$$

```
[Out] a^2*x+5/2*b^2*x-2*a*b*cos(d*x+c)/d-4*a*b*sec(d*x+c)/d+2/3*a*b*sec(d*x+c)^3/
d-a^2*tan(d*x+c)/d-5/2*b^2*tan(d*x+c)/d+1/3*a^2*tan(d*x+c)^3/d+5/6*b^2*tan(
d*x+c)^3/d-1/2*b^2*sin(d*x+c)^2*tan(d*x+c)^3/d
```

Rubi [A] time = 0.16, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2722, 3473, 8, 2590, 270, 2591, 288, 302, 203}

$$\frac{a^2\tan^3(c+dx)}{3d}-\frac{a^2\tan(c+dx)}{d}+a^2x-\frac{2ab\cos(c+dx)}{d}+\frac{2ab\sec^3(c+dx)}{3d}-\frac{4ab\sec(c+dx)}{d}+\frac{5b^2\tan^3(c+dx)}{6d}-\frac{5b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^2*Tan[c + d*x]^4,x]
```

```
[Out] a^2*x + (5*b^2*x)/2 - (2*a*b*Cos[c + d*x])/d - (4*a*b*Sec[c + d*x])/d + (2*
a*b*Sec[c + d*x]^3)/(3*d) - (a^2*Tan[c + d*x])/d - (5*b^2*Tan[c + d*x])/(2*
d) + (a^2*Tan[c + d*x]^3)/(3*d) + (5*b^2*Tan[c + d*x]^3)/(6*d) - (b^2*Sin[c
 + d*x]^2*Tan[c + d*x]^3)/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
```

```
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int (a+b\sin(c+dx))^2\tan^4(c+dx)\,dx = \int \left(a^2\tan^4(c+dx)+2ab\sin(c+dx)\tan^4(c+dx)+b^2\sin^2(c+dx)\tan^4\right.$$

$$= a^2\int \tan^4(c+dx)\,dx+(2ab)\int \sin(c+dx)\tan^4(c+dx)\,dx+b^2\int \sin^2($$

$$= \frac{a^2\tan^3(c+dx)}{3d}-a^2\int \tan^2(c+dx)\,dx-\frac{(2ab)\,\mathrm{Subst}\left(\int \frac{(1-x^2)^2}{x^4}\,dx,x,\mathrm{co}\right.}{d}$$

$$= -\frac{a^2\tan(c+dx)}{d}+\frac{a^2\tan^3(c+dx)}{3d}-\frac{b^2\sin^2(c+dx)\tan^3(c+dx)}{2d}+a^2\int$$

$$= a^2x-\frac{2ab\cos(c+dx)}{d}-\frac{4ab\sec(c+dx)}{d}+\frac{2ab\sec^3(c+dx)}{3d}-\frac{a^2\tan(c}{d}$$

$$= a^2x-\frac{2ab\cos(c+dx)}{d}-\frac{4ab\sec(c+dx)}{d}+\frac{2ab\sec^3(c+dx)}{3d}-\frac{a^2\tan(c}{d}$$

$$= a^2x+\frac{5b^2x}{2}-\frac{2ab\cos(c+dx)}{d}-\frac{4ab\sec(c+dx)}{d}+\frac{2ab\sec^3(c+dx)}{3d}-\frac{a}{}$$

Mathematica [A] time = 0.72, size = 176, normalized size = 1.18

$$-\frac{\sec^3(c+dx)\left(-36\left(2a^2+5b^2\right)(c+dx)\cos(c+dx)+32a^2\sin(3(c+dx))-24a^2c\cos(3(c+dx))-24a^2dx\cos(3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^2*Tan[c + d*x]^4,x]
```

```
[Out] -1/96*(Sec[c + d*x]^3*(200*a*b - 36*(2*a^2 + 5*b^2)*(c + d*x)*Cos[c + d*x]
+ 288*a*b*Cos[2*(c + d*x)] - 24*a^2*c*Cos[3*(c + d*x)] - 60*b^2*c*Cos[3*(c
+ d*x)] - 24*a^2*d*x*Cos[3*(c + d*x)] - 60*b^2*d*x*Cos[3*(c + d*x)] + 24*a*
b*Cos[4*(c + d*x)] + 30*b^2*Sin[c + d*x] + 32*a^2*Sin[3*(c + d*x)] + 65*b^2
*Sin[3*(c + d*x)] + 3*b^2*Sin[5*(c + d*x)]))/d
```

fricas [A] time = 0.44, size = 118, normalized size = 0.79

$$\frac{3\left(2a^2+5b^2\right)dx\cos(dx+c)^3-12ab\cos(dx+c)^4-24ab\cos(dx+c)^2+4ab-\left(3b^2\cos(dx+c)^4+2\left(4a^2+7\right.\right.}{6d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*(2*a^2 + 5*b^2)*d*x*cos(d*x + c)^3 - 12*a*b*cos(d*x + c)^4 - 24*a*b*
cos(d*x + c)^2 + 4*a*b - (3*b^2*cos(d*x + c)^4 + 2*(4*a^2 + 7*b^2)*cos(d*x
+ c)^2 - 2*a^2 - 2*b^2)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.20, size = 185, normalized size = 1.24

$$\frac{a^2\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+dx+c\right)+2ab\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^6(dx+c)}{\cos(dx+c)}-\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^2*tan(d*x+c)^4,x)
```

```
[Out] 1/d*(a^2*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c)+2*a*b*(1/3*sin(d*x+c)^6/cos(d*
x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+
c))+b^2*(1/3*sin(d*x+c)^7/cos(d*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/3*(sin
(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c))
```

maxima [A] time = 1.25, size = 119, normalized size = 0.80

$$\frac{2\left(\tan(dx+c)^3+3\,dx+3\,c-3\,\tan(dx+c)\right)a^2+\left(2\,\tan(dx+c)^3+15\,dx+15\,c-\frac{3\,\tan(dx+c)}{\tan(dx+c)^2+1}-12\,\tan(dx+c}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a^2 + (2*tan(d*x + c
)^3 + 15*d*x + 15*c - 3*tan(d*x + c)/(tan(d*x + c)^2 + 1) - 12*tan(d*x + c)
)*b^2 - 4*a*b*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x + c)))/d
```

mupad [B] time = 10.04, size = 235, normalized size = 1.58

$$\frac{x\left(2a^2+5b^2\right)}{2}-\frac{\left(-2a^2-5b^2\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+\left(\frac{8a^2}{3}+\frac{20b^2}{3}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\left(\frac{28a^2}{3}+\frac{22b^2}{3}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\frac{64a}{}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-2\tan\left(\frac{c}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (x*(2*a^2 + 5*b^2))/2 - (tan(c/2 + (d*x)/2)^3*((8*a^2)/3 + (20*b^2)/3) - ta
n(c/2 + (d*x)/2)^9*(2*a^2 + 5*b^2) - (32*a*b)/3 + tan(c/2 + (d*x)/2)^7*((8*
a^2)/3 + (20*b^2)/3) + tan(c/2 + (d*x)/2)^5*((28*a^2)/3 + (22*b^2)/3) - tan
(c/2 + (d*x)/2)*(2*a^2 + 5*b^2) + (32*a*b*tan(c/2 + (d*x)/2)^2)/3 + (64*a*b
*tan(c/2 + (d*x)/2)^4)/3)/(d*(tan(c/2 + (d*x)/2)^2 + 2*tan(c/2 + (d*x)/2)^4
 - 2*tan(c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 -
1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin(c+dx))^2\tan^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**2*tan(d*x+c)**4,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**2*tan(c + d*x)**4, x)
```

3.156 $\int (a+b\sin(c+dx))^2\tan^2(c+dx)\,dx$

Optimal. Leaf size=94

$$\frac{a^2\tan(c+dx)}{d}+a^2(-x)+\frac{2ab\cos(c+dx)}{d}+\frac{2ab\sec(c+dx)}{d}+\frac{3b^2\tan(c+dx)}{2d}-\frac{b^2\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{3b^2x}{2}$$

```
[Out] -a^2*x-3/2*b^2*x+2*a*b*cos(d*x+c)/d+2*a*b*sec(d*x+c)/d+a^2*tan(d*x+c)/d+3/2
*b^2*tan(d*x+c)/d-1/2*b^2*sin(d*x+c)^2*tan(d*x+c)/d
```

Rubi [A] time = 0.12, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2722, 3473, 8, 2590, 14, 2591, 288, 321, 203}

$$\frac{a^2\tan(c+dx)}{d}+a^2(-x)+\frac{2ab\cos(c+dx)}{d}+\frac{2ab\sec(c+dx)}{d}+\frac{3b^2\tan(c+dx)}{2d}-\frac{b^2\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{3b^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] -(a^2*x) - (3*b^2*x)/2 + (2*a*b*Cos[c + d*x])/d + (2*a*b*Sec[c + d*x])/d +
(a^2*Tan[c + d*x])/d + (3*b^2*Tan[c + d*x])/(2*d) - (b^2*Sin[c + d*x]^2*Tan
[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int (a+b\sin(c+dx))^2\tan^2(c+dx)\,dx = \int \left(a^2\tan^2(c+dx)+2ab\sin(c+dx)\tan^2(c+dx)+b^2\sin^2(c+dx)\tan\right.$$
$$= a^2\int \tan^2(c+dx)\,dx+(2ab)\int \sin(c+dx)\tan^2(c+dx)\,dx+b^2\int \sin$$
$$= \frac{a^2\tan(c+dx)}{d}-a^2\int 1\,dx-\frac{(2ab)\,\mathrm{Subst}\left(\int \frac{1-x^2}{x^2}\,dx,x,\cos(c+dx)\right)}{d}+$$
$$= -a^2x+\frac{a^2\tan(c+dx)}{d}-\frac{b^2\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{(2ab)\,\mathrm{Subst}\left(\int\right.}{}$$
$$= -a^2x+\frac{2ab\cos(c+dx)}{d}+\frac{2ab\sec(c+dx)}{d}+\frac{a^2\tan(c+dx)}{d}+\frac{3b^2\tan(}{2}$$
$$= -a^2x-\frac{3b^2x}{2}+\frac{2ab\cos(c+dx)}{d}+\frac{2ab\sec(c+dx)}{d}+\frac{a^2\tan(c+dx)}{d}+$$

Mathematica [A] time = 0.48, size = 77, normalized size = 0.82

$$\frac{-4\left(2a^2+3b^2\right)(c+dx)+\left(8a^2+9b^2\right)\tan(c+dx)+b\sec(c+dx)(8a\cos(2(c+dx))+24a+b\sin(3(c+dx)))}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] (-4*(2*a^2 + 3*b^2)*(c + d*x) + b*Sec[c + d*x]*(24*a + 8*a*Cos[2*(c + d*x)]
 + b*Sin[3*(c + d*x)]) + (8*a^2 + 9*b^2)*Tan[c + d*x])/(8*d)
```

fricas [A] time = 0.44, size = 81, normalized size = 0.86

$$-\frac{\left(2\,a^2+3\,b^2\right)dx\cos\left(dx+c\right)-4\,ab\cos\left(dx+c\right)^2-4\,ab-\left(b^2\cos\left(dx+c\right)^2+2\,a^2+2\,b^2\right)\sin\left(dx+c\right)}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((2*a^2 + 3*b^2)*d*x*cos(d*x + c) - 4*a*b*cos(d*x + c)^2 - 4*a*b - (b^
2*cos(d*x + c)^2 + 2*a^2 + 2*b^2)*sin(d*x + c))/(d*cos(d*x + c))
```

giac [B] time = 45.20, size = 7670, normalized size = 81.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 + 3*b^2*d*x
*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 + 2*a^2*d*x*tan(d*x)^3*tan
(1/2*d*x)^4*tan(1/2*c)^4*tan(c) + 3*b^2*d*x*tan(d*x)^3*tan(1/2*d*x)^4*tan(1
/2*c)^4*tan(c) - 2*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2
- 3*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 - 8*a^2*d*x*tan
(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^3 - 12*b^2*d*x*tan(d*x)^3*tan(1/
2*d*x)^3*tan(1/2*c)^3*tan(c)^3 + 2*a^2*d*x*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*
c)^4*tan(c)^3 + 3*b^2*d*x*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 - 8
*a*b*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 + 2*a^2*tan(d*x)^3*tan
(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 + 3*b^2*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2
*c)^4*tan(c)^2 + 2*a^2*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 + 3*
b^2*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 - 2*a^2*d*x*tan(d*x)^2*
tan(1/2*d*x)^4*tan(1/2*c)^4 - 3*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c
)^4 - 8*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) - 12*b^2*d*x*
tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) + 2*a^2*d*x*tan(d*x)*tan(1/2*
d*x)^4*tan(1/2*c)^4*tan(c) + 3*b^2*d*x*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4
*tan(c) - 8*a*b*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) + 8*a^2*d*x*t
an(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 + 12*b^2*d*x*tan(d*x)^2*tan(
1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 - 2*a^2*d*x*tan(1/2*d*x)^4*tan(1/2*c)^4*ta
n(c)^2 - 3*b^2*d*x*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 + 8*a*b*tan(d*x)^2*
tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 - 2*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^4*
tan(c)^3 - 3*b^2*d*x*tan(d*x)^3*tan(1/2*d*x)^4*tan(c)^3 - 8*a^2*d*x*tan(d*x
)^3*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^3 - 12*b^2*d*x*tan(d*x)^3*tan(1/2*d*x)
^3*tan(1/2*c)*tan(c)^3 - 8*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)^3*tan
(c)^3 - 12*b^2*d*x*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^3 - 8*a^2*d*
x*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^3 - 12*b^2*d*x*tan(d*x)*tan(1
/2*d*x)^3*tan(1/2*c)^3*tan(c)^3 + 32*a*b*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*
c)^3*tan(c)^3 - 2*a^2*d*x*tan(d*x)^3*tan(1/2*c)^4*tan(c)^3 - 3*b^2*d*x*tan(
d*x)^3*tan(1/2*c)^4*tan(c)^3 - 8*a*b*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*t
an(c)^3 + 2*a^2*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2*c)^4 + 2*b^2*tan(d*x)^3*t
an(1/2*d*x)^4*tan(1/2*c)^4 + 2*a^2*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4*t
an(c) - 8*a^2*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 - 12*b^2*tan(
d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 + 2*a^2*tan(d*x)*tan(1/2*d*x)^4
*tan(1/2*c)^4*tan(c)^2 - 8*a^2*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c
)^3 - 12*b^2*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^3 + 2*a^2*tan(1/
2*d*x)^4*tan(1/2*c)^4*tan(c)^3 + 2*b^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3
 + 8*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3 + 12*b^2*d*x*tan(d*x)^2
*tan(1/2*d*x)^3*tan(1/2*c)^3 - 2*a^2*d*x*tan(1/2*d*x)^4*tan(1/2*c)^4 - 3*b^
2*d*x*tan(1/2*d*x)^4*tan(1/2*c)^4 + 8*a*b*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2
*c)^4 - 2*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^4*tan(c) - 3*b^2*d*x*tan(d*x)^3*t
an(1/2*d*x)^4*tan(c) - 8*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)*tan(c
```

) - 12*b^2*d*x*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 8*a^2*d*x*tan(
d*x)^3*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - 12*b^2*d*x*tan(d*x)^3*tan(1/2*d*x
)*tan(1/2*c)^3*tan(c) - 8*a^2*d*x*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(
c) - 12*b^2*d*x*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) + 32*a*b*tan(d*
x)^3*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) - 2*a^2*d*x*tan(d*x)^3*tan(1/2*c)^4
*tan(c) - 3*b^2*d*x*tan(d*x)^3*tan(1/2*c)^4*tan(c) - 8*a*b*tan(d*x)*tan(1/2
*d*x)^4*tan(1/2*c)^4*tan(c) + 2*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*tan(c)^2
+ 3*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*tan(c)^2 + 8*a^2*d*x*tan(d*x)^2*tan(1
/2*d*x)^3*tan(1/2*c)*tan(c)^2 + 12*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/
2*c)*tan(c)^2 + 8*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^2 + 1
2*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^2 + 8*a^2*d*x*tan(1/2
*d*x)^3*tan(1/2*c)^3*tan(c)^2 + 12*b^2*d*x*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(
c)^2 - 32*a*b*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 + 2*a^2*d*x*t
an(d*x)^2*tan(1/2*c)^4*tan(c)^2 + 3*b^2*d*x*tan(d*x)^2*tan(1/2*c)^4*tan(c)^
2 + 8*a*b*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 - 2*a^2*d*x*tan(d*x)*tan(1/2
*d*x)^4*tan(c)^3 - 3*b^2*d*x*tan(d*x)*tan(1/2*d*x)^4*tan(c)^3 - 8*a*b*tan(d
*x)^3*tan(1/2*d*x)^4*tan(c)^3 - 8*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c
)*tan(c)^3 - 12*b^2*d*x*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)*tan(c)^3 - 8*a^2
*d*x*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^3 - 12*b^2*d*x*tan(d*x)*tan(
1/2*d*x)^3*tan(1/2*c)*tan(c)^3 - 32*a*b*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c
)*tan(c)^3 - 96*a*b*tan(d*x)^3*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^3 - 8*a^2
*d*x*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^3 - 12*b^2*d*x*tan(d*x)*tan(
1/2*d*x)*tan(1/2*c)^3*tan(c)^3 - 32*a*b*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)^
3*tan(c)^3 + 32*a*b*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^3 - 2*a^2*d
*x*tan(d*x)*tan(1/2*c)^4*tan(c)^3 - 3*b^2*d*x*tan(d*x)*tan(1/2*c)^4*tan(c)^
3 - 8*a*b*tan(d*x)^3*tan(1/2*c)^4*tan(c)^3 - 8*a^2*tan(d*x)^3*tan(1/2*d*x)^
3*tan(1/2*c)^3 - 8*b^2*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)^3 + 2*a^2*tan(d
*x)*tan(1/2*d*x)^4*tan(1/2*c)^4 + 3*b^2*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^
4 - 8*a^2*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) + 2*a^2*tan(1/2*d*x
)^4*tan(1/2*c)^4*tan(c) + 3*b^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) - 2*a^2*
tan(d*x)^3*tan(1/2*d*x)^4*tan(c)^2 - 3*b^2*tan(d*x)^3*tan(1/2*d*x)^4*tan(c)
^2 - 8*a^2*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^2 - 12*b^2*tan(d*x)^
3*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^2 - 8*a^2*tan(d*x)^3*tan(1/2*d*x)*tan(1/
2*c)^3*tan(c)^2 - 12*b^2*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^2 - 8*
a^2*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 - 2*a^2*tan(d*x)^3*tan(1/
2*c)^4*tan(c)^2 - 3*b^2*tan(d*x)^3*tan(1/2*c)^4*tan(c)^2 - 2*a^2*tan(d*x)^2
*tan(1/2*d*x)^4*tan(c)^3 - 3*b^2*tan(d*x)^2*tan(1/2*d*x)^4*tan(c)^3 - 8*a^2
*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^3 - 12*b^2*tan(d*x)^2*tan(1/2*
d*x)^3*tan(1/2*c)*tan(c)^3 - 8*a^2*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^3*tan
(c)^3 - 12*b^2*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^3 - 8*a^2*tan(1/
2*d*x)^3*tan(1/2*c)^3*tan(c)^3 - 8*b^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^3
 - 2*a^2*tan(d*x)^2*tan(1/2*c)^4*tan(c)^3 - 3*b^2*tan(d*x)^2*tan(1/2*c)^4*t
an(c)^3 + 2*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4 + 3*b^2*d*x*tan(d*x)^2*tan(1/
2*d*x)^4 + 8*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c) + 12*b^2*d*x*tan(
d*x)^2*tan(1/2*d*x)^3*tan(1/2*c) + 8*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)*tan(1/

2*c)^3 + 12*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^3 + 8*a^2*d*x*tan(1/2*d*x)^3*tan(1/2*c)^3 + 12*b^2*d*x*tan(1/2*d*x)^3*tan(1/2*c)^3 - 32*a*b*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3 + 2*a^2*d*x*tan(d*x)^2*tan(1/2*c)^4 + 3*b^2*d*x*tan(d*x)^2*tan(1/2*c)^4 + 8*a*b*tan(1/2*d*x)^4*tan(1/2*c)^4 - 2*a^2*d*x*tan(d*x)*tan(1/2*d*x)^4*tan(c) - 3*b^2*d*x*tan(d*x)*tan(1/2*d*x)^4*tan(c) - 8*a*b*tan(d*x)^3*tan(1/2*d*x)^4*tan(c) - 8*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)*tan(c) - 12*b^2*d*x*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)*tan(c) - 8*a^2*d*x*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 12*b^2*d*x*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 32*a*b*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 96*a*b*tan(d*x)^3*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c) - 8*a^2*d*x*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - 12*b^2*d*x*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - 32*a*b*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) + 32*a*b*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) - 2*a^2*d*x*tan(d*x)*tan(1/2*c)^4*tan(c) - 3*b^2*d*x*tan(d*x)*tan(1/2*c)^4*tan(c) - 8*a*b*tan(d*x)^3*tan(1/2*c)^4*tan(c) + 2*a^2*d*x*tan(1/2*d*x)^4*tan(c)^2 + 3*b^2*d*x*tan(1/2*d*x)^4*tan(c)^2 + 8*a*b*tan(d*x)^2*tan(1/2*d*x)^4*tan(c)^2 + 8*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)*tan(c)^2 + 12*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)*tan(c)^2 + 8*a^2*d*x*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^2 + 12*b^2*d*x*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^2 + 32*a*b*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^2 + 96*a*b*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + 8*a^2*d*x*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^2 + 12*b^2*d*x*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^2 + 32*a*b*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^2 - 32*a*b*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 + 2*a^2*d*x*tan(1/2*c)^4*tan(c)^2 + 3*b^2*d*x*tan(1/2*c)^4*tan(c)^2 + 8*a*b*tan(d*x)^2*tan(1/2*c)^4*tan(c)^2 + 2*a^2*d*x*tan(d*x)^3*tan(c)^3 + 3*b^2*d*x*tan(d*x)^3*tan(c)^3 - 8*a*b*tan(d*x)*tan(1/2*d*x)^4*tan(c)^3 - 8*a^2*d*x*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*tan(c)^3 - 12*b^2*d*x*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*tan(c)^3 + 32*a*b*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)*tan(c)^3 - 32*a*b*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^3 - 96*a*b*tan(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^3 - 32*a*b*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^3 - 8*a*b*tan(d*x)*tan(1/2*c)^4*tan(c)^3 - 2*a^2*tan(d*x)^3*tan(1/2*d*x)^4 - 2*b^2*tan(d*x)^3*tan(1/2*d*x)^4 - 8*a^2*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c) - 8*b^2*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c) - 8*a^2*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)^3 - 8*b^2*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)^3 - 8*a^2*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3 - 12*b^2*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3 - 2*a^2*tan(d*x)^3*tan(1/2*c)^4 - 2*b^2*tan(d*x)^3*tan(1/2*c)^4 - 2*a^2*tan(d*x)^2*tan(1/2*d*x)^4*tan(c) - 8*a^2*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 8*a^2*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - 8*a^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) - 12*b^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) - 2*a^2*tan(d*x)^2*tan(1/2*c)^4*tan(c) - 2*a^2*tan(d*x)*tan(1/2*d*x)^4*tan(c)^2 - 8*a^2*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)*tan(c)^2 - 12*b^2*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)*tan(c)^2 - 8*a^2*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^2 - 8*a^2*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^2 - 2*a^2*tan(d*x)*tan(1/2*c)^4*tan(c)^2 - 2*a^2*tan(1/2*d*x)^4*tan(c)^3 - 2*b^2*tan(1/2*d*x)^4*tan(c)^3 - 8*a^2*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)*tan(c)^3 - 12*b

^2*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)*tan(c)^3 - 8*a^2*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^3 - 8*b^2*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^3 - 8*a^2*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^3 - 8*b^2*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^3 - 2*a^2*tan(1/2*c)^4*tan(c)^3 - 2*b^2*tan(1/2*c)^4*tan(c)^3 + 2*a^2*d*x*tan(1/2*d*x)^4 + 3*b^2*d*x*tan(1/2*d*x)^4 + 8*a*b*tan(d*x)^2*tan(1/2*d*x)^4 + 8*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c) + 12*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c) + 8*a^2*d*x*tan(1/2*d*x)^3*tan(1/2*c) + 12*b^2*d*x*tan(1/2*d*x)^3*tan(1/2*c) + 32*a*b*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c) + 96*a*b*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 8*a^2*d*x*tan(1/2*d*x)*tan(1/2*c)^3 + 12*b^2*d*x*tan(1/2*d*x)*tan(1/2*c)^3 + 32*a*b*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^3 - 32*a*b*tan(1/2*d*x)^3*tan(1/2*c)^3 + 2*a^2*d*x*tan(1/2*c)^4 + 3*b^2*d*x*tan(1/2*c)^4 + 8*a*b*tan(d*x)^2*tan(1/2*c)^4 + 2*a^2*d*x*tan(d*x)^3*tan(c) + 3*b^2*d*x*tan(d*x)^3*tan(c) - 8*a*b*tan(d*x)*tan(1/2*d*x)^4*tan(c) - 8*a^2*d*x*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*tan(c) - 12*b^2*d*x*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*tan(c) + 32*a*b*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)*tan(c) - 32*a*b*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 96*a*b*tan(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c) - 32*a*b*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - 8*a*b*tan(d*x)*tan(1/2*c)^4*tan(c) - 2*a^2*d*x*tan(d*x)^2*tan(c)^2 - 3*b^2*d*x*tan(d*x)^2*tan(c)^2 + 8*a*b*tan(1/2*d*x)^4*tan(c)^2 + 8*a^2*d*x*tan(1/2*d*x)*tan(1/2*c)*tan(c)^2 + 12*b^2*d*x*tan(1/2*d*x)*tan(1/2*c)*tan(c)^2 - 32*a*b*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)*tan(c)^2 + 32*a*b*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^2 + 96*a*b*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + 32*a*b*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^2 + 8*a*b*tan(1/2*c)^4*tan(c)^2 + 2*a^2*d*x*tan(d*x)*tan(c)^3 + 3*b^2*d*x*tan(d*x)*tan(c)^3 - 8*a*b*tan(d*x)^3*tan(c)^3 + 32*a*b*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*tan(c)^3 - 2*a^2*tan(d*x)*tan(1/2*d*x)^4 - 3*b^2*tan(d*x)*tan(1/2*d*x)^4 - 8*a^2*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c) - 8*b^2*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c) - 8*a^2*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c) - 12*b^2*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c) - 8*a^2*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3 - 12*b^2*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3 - 2*a^2*tan(d*x)*tan(1/2*c)^4 - 3*b^2*tan(d*x)*tan(1/2*c)^4 - 2*a^2*tan(1/2*d*x)^4*tan(c) - 3*b^2*tan(1/2*d*x)^4*tan(c) - 8*a^2*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)*tan(c) - 8*a^2*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 12*b^2*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 8*a^2*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - 12*b^2*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - 2*a^2*tan(1/2*c)^4*tan(c) - 3*b^2*tan(1/2*c)^4*tan(c) + 2*a^2*tan(d*x)^3*tan(c)^2 + 3*b^2*tan(d*x)^3*tan(c)^2 - 8*a^2*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*tan(c)^2 + 2*a^2*tan(d*x)^2*tan(c)^3 + 3*b^2*tan(d*x)^2*tan(c)^3 - 8*a^2*tan(1/2*d*x)*tan(1/2*c)*tan(c)^3 - 8*b^2*tan(1/2*d*x)*tan(1/2*c)*tan(c)^3 - 2*a^2*d*x*tan(d*x)^2 - 3*b^2*d*x*tan(d*x)^2 + 8*a*b*tan(1/2*d*x)^4 + 8*a^2*d*x*tan(1/2*d*x)*tan(1/2*c) + 12*b^2*d*x*tan(1/2*d*x)*tan(1/2*c) - 32*a*b*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c) + 32*a*b*tan(1/2*d*x)^3*tan(1/2*c) + 96*a*b*tan(1/2*d*x)^2*tan(1/2*c)^2 + 32*a*b*tan(1/2*d*x)*tan(1/2*c)^3 + 8*a*b*tan(1/2*c)^4 + 2*a^2*d*x*tan(d*x)*tan(c) + 3*b^2*d*x*tan(d*x)*tan(c) - 8*a*b*tan(d*x)^3*tan(c) + 32*a*b*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*tan(c) - 2*a^2*d*x*tan(c)^2 - 3*b^2*d*x*tan(c)^2 + 8*a*b*tan(d*x)^2*tan(c)^2 - 32*a*b*tan(1/2*d*x)*tan(1/2*c)*ta n(c)^2 - 8*a*b*tan(d*x)*tan(c)^3 + 2*a^2*tan(d*x)^3 + 2*b^2*tan(d*x)^3 - 8*a^2*tan(d*x)*tan(1/2*d*x)*tan(1/2*c) - 12*b^2*tan(d*x)*tan(1/2*d*x)*tan(1/2*c) + 2*a^2*tan(d*x)^2*tan(c) - 8*a^2*tan(1/2*d*x)*tan(1/2*c)*tan(c) - 12*b^2*tan(1/2*d*x)*tan(1/2*c)*tan(c) + 2*a^2*tan(d*x)*tan(c)^2 + 2*a^2*tan(c)^3 + 2*b^2*tan(c)^3 - 2*a^2*d*x - 3*b^2*d*x + 8*a*b*tan(d*x)^2 - 32*a*b*tan(1/2*d*x)*tan(1/2*c) - 8*a*b*tan(d*x)*tan(c) + 8*a*b*tan(c)^2 + 2*a^2*tan(d*x) + 3*b^2*tan(d*x) + 2*a^2*tan(c) + 3*b^2*tan(c) + 8*a*b)/(d*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 + d*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) - d*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 - 4*d*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^3 + d*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 - d*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*d*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) + d*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) + 4*d*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 - d*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 - d*tan(d*x)^3*tan(1/2*d*x)^4*tan(c)^3 - 4*d*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^3 - 4*d*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^3 - 4*d*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^3 - d*tan(d*x)^3*tan(1/2*c)^4*tan(c)^3 + 4*d*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3 - d*tan(1/2*d*x)^4*tan(1/2*c)^4 - d*tan(d*x)^3*tan(1/2*d*x)^4*tan(c) - 4*d*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 4*d*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - 4*d*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) - d*tan(d*x)^3*tan(1/2*c)^4*tan(c) + d*tan(d*x)^2*tan(1/2*d*x)^4*tan(c)^2 + 4*d*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^2 + 4*d*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^2 + 4*d*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 + d*tan(d*x)^2*tan(1/2*c)^4*tan(c)^2 - d*tan(d*x)*tan(1/2*d*x)^4*tan(c)^3 - 4*d*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)*tan(c)^3 - 4*d*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^3 - 4*d*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^3 - d*tan(d*x)*tan(1/2*c)^4*tan(c)^3 + d*tan(d*x)^2*tan(1/2*d*x)^4 + 4*d*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c) + 4*d*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^3 + 4*d*tan(1/2*d*x)^3*tan(1/2*c)^3 + d*tan(d*x)^2*tan(1/2*c)^4 - d*tan(d*x)*tan(1/2*d*x)^4*tan(c) - 4*d*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)*tan(c) - 4*d*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 4*d*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - d*tan(d*x)*tan(1/2*c)^4*tan(c) + d*tan(1/2*d*x)^4*tan(c)^2 + 4*d*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)*tan(c)^2 + 4*d*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^2 + 4*d*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^2 + d*tan(1/2*c)^4*tan(c)^2 + d*tan(d*x)^3*tan(c)^3 - 4*d*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*tan(c)^3 + d*tan(1/2*d*x)^4 + 4*d*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c) + 4*d*tan(1/2*d*x)^3*tan(1/2*c) + 4*d*tan(1/2*d*x)*tan(1/2*c)^3 + d*tan(1/2*c)^4 + d*tan(d*x)^3*tan(c) - 4*d*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*tan(c) - d*tan(d*x)^2*tan(c)^2 + 4*d*tan(1/2*d*x)*tan(1/2*c)*tan(c)^2 + d*tan(d*x)*tan(c)^3 - d*tan(d*x)^2 + 4*d*tan(1/2*d*x)*tan(1/2*c) + d*tan(d*x)*tan(c) - d*tan(c)^2 - d)

maple [A] time = 0.23, size = 116, normalized size = 1.23

$$\frac{a^2(\tan(dx+c)-dx-c)+2ab\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+\left(2+\sin^2(dx+c)\right)\cos(dx+c)\right)+b^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^2*tan(d*x+c)^2,x)
```

```
[Out] 1/d*(a^2*(tan(d*x+c)-d*x-c)+2*a*b*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)
*cos(d*x+c))+b^2*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos
(d*x+c)-3/2*d*x-3/2*c))
```

maxima [A] time = 1.04, size = 83, normalized size = 0.88

$$-\frac{2(dx+c-\tan(dx+c))a^2+\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\,\tan(dx+c)\right)b^2-4\,ab\left(\frac{1}{\cos(dx+c)}+\cos(dx+c)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(d*x + c - tan(d*x + c))*a^2 + (3*d*x + 3*c - tan(d*x + c)/(tan(d*x
 + c)^2 + 1) - 2*tan(d*x + c))*b^2 - 4*a*b*(1/cos(d*x + c) + cos(d*x + c)))
/d
```

mupad [B] time = 9.29, size = 147, normalized size = 1.56

$$\frac{\left(2\,a^2+3\,b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(4\,a^2+2\,b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+8\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\left(2\,a^2+3\,b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+8}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (8*a*b + tan(c/2 + (d*x)/2)^3*(4*a^2 + 2*b^2) + tan(c/2 + (d*x)/2)^5*(2*a^2
 + 3*b^2) + tan(c/2 + (d*x)/2)*(2*a^2 + 3*b^2) + 8*a*b*tan(c/2 + (d*x)/2)^2
)/(d*(tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)^6 +
1)) - (x*(2*a^2 + 3*b^2))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(a+b\sin(c+dx))^2\tan^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**2*tan(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**2*tan(c + d*x)**2, x)
```

3.157 $\int \cot^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=78

$$-\frac{a^2\cot(c+dx)}{d}+a^2(-x)+\frac{2ab\cos(c+dx)}{d}-\frac{2ab\tanh^{-1}(\cos(c+dx))}{d}+\frac{b^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{b^2x}{2}$$

```
[Out] -a^2*x+1/2*b^2*x-2*a*b*arctanh(cos(d*x+c))/d+2*a*b*cos(d*x+c)/d-a^2*cot(d*x
+c)/d+1/2*b^2*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A] time = 0.09, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2722, 2635, 8, 2592, 321, 206, 3473}

$$-\frac{a^2\cot(c+dx)}{d}+a^2(-x)+\frac{2ab\cos(c+dx)}{d}-\frac{2ab\tanh^{-1}(\cos(c+dx))}{d}+\frac{b^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{b^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*x) + (b^2*x)/2 - (2*a*b*ArcTanh[Cos[c + d*x]])/d + (2*a*b*Cos[c + d*x
])/d - (a^2*Cot[c + d*x])/d + (b^2*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
```

```
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int \cot^2(c+dx)(a+b\sin(c+dx))^2\,dx &= \int \left(b^2\cos^2(c+dx)+2ab\cos(c+dx)\cot(c+dx)+a^2\cot^2(c+dx)\right)dx \\
&= a^2\int \cot^2(c+dx)\,dx+(2ab)\int \cos(c+dx)\cot(c+dx)\,dx+b^2\int \cos^2(c \\
&= -\frac{a^2\cot(c+dx)}{d}+\frac{b^2\cos(c+dx)\sin(c+dx)}{2d}-a^2\int 1\,dx+\frac{1}{2}b^2\int 1\,dx- \\
&= -a^2x+\frac{b^2x}{2}+\frac{2ab\cos(c+dx)}{d}-\frac{a^2\cot(c+dx)}{d}+\frac{b^2\cos(c+dx)\sin(c+ \\
&= -a^2x+\frac{b^2x}{2}-\frac{2ab\tanh^{-1}(\cos(c+dx))}{d}+\frac{2ab\cos(c+dx)}{d}-\frac{a^2\cot(c+d}{d}
\end{aligned}$$

Mathematica [A] time = 0.41, size = 116, normalized size = 1.49

$$\frac{2a^2\tan\left(\frac{1}{2}(c+dx)\right)-2a^2\cot\left(\frac{1}{2}(c+dx)\right)-4a^2c-4a^2dx+8ab\cos(c+dx)+8ab\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-8ab\log}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-4*a^2*c + 2*b^2*c - 4*a^2*d*x + 2*b^2*d*x + 8*a*b*Cos[c + d*x] - 2*a^2*Co
t[(c + d*x)/2] - 8*a*b*Log[Cos[(c + d*x)/2]] + 8*a*b*Log[Sin[(c + d*x)/2]]
+ b^2*Sin[2*(c + d*x)] + 2*a^2*Tan[(c + d*x)/2])/(4*d)
```

fricas [A] time = 0.46, size = 118, normalized size = 1.51

$$-\frac{b^2\cos(dx+c)^3 + 2\,ab\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c) - 2\,ab\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c) + \left(2\right.}{2\,d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*cos(d*x + c)^3 + 2*a*b*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) -
 2*a*b*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + (2*a^2 - b^2)*cos(d*x +
c) + ((2*a^2 - b^2)*d*x - 4*a*b*cos(d*x + c))*sin(d*x + c))/(d*sin(d*x + c)
)
```

giac [A] time = 0.20, size = 148, normalized size = 1.90

$$\frac{4\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right) + a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - \left(2\,a^2-b^2\right)(dx+c) - \frac{4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)} - \frac{2\left(b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 4\right.}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*a*b*log(abs(tan(1/2*d*x + 1/2*c))) + a^2*tan(1/2*d*x + 1/2*c) - (2*a
^2 - b^2)*(d*x + c) - (4*a*b*tan(1/2*d*x + 1/2*c) + a^2)/tan(1/2*d*x + 1/2*
c) - 2*(b^2*tan(1/2*d*x + 1/2*c)^3 - 4*a*b*tan(1/2*d*x + 1/2*c)^2 - b^2*tan
(1/2*d*x + 1/2*c) - 4*a*b)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

maple [A] time = 0.13, size = 102, normalized size = 1.31

$$-a^2x - \frac{a^2\cot(dx+c)}{d} - \frac{a^2c}{d} + \frac{2ab\cos(dx+c)}{d} + \frac{2ab\ln(\csc(dx+c)-\cot(dx+c))}{d} + \frac{b^2\cos(dx+c)\sin(dx+c)}{2d} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -a^2*x-a^2*cot(d*x+c)/d-1/d*a^2*c+2*a*b*cos(d*x+c)/d+2/d*a*b*ln(csc(d*x+c)-
cot(d*x+c))+1/2*b^2*cos(d*x+c)*sin(d*x+c)/d+1/2*b^2*x+1/2/d*b^2*c
```

maxima [A] time = 0.73, size = 79, normalized size = 1.01

$$-\frac{4\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^2-(2\,dx+2\,c+\sin(2\,dx+2\,c))b^2-4\,ab\Big(2\,\cos(dx+c)-\log(\cos(dx+c)+1)+\log(c}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(4*(d*x + c + 1/tan(d*x + c))*a^2 - (2*d*x + 2*c + sin(2*d*x + 2*c))*b
^2 - 4*a*b*(2*cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1))
)/d
```

mupad [B] time = 7.27, size = 277, normalized size = 3.55

$$\frac{b^2\,\mathrm{atan}\left(\frac{-2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2+4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^2}{2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2+4\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^2}\right)-2\,a^2\,\mathrm{atan}\left(\frac{-2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2+4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^2}{2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2+4\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^2}\right)+2\,a\,b\,\mathrm{l}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (b^2*atan((b^2*cos(c/2 + (d*x)/2) - 2*a^2*cos(c/2 + (d*x)/2) + 4*a*b*sin(c/
2 + (d*x)/2))/(2*a^2*sin(c/2 + (d*x)/2) - b^2*sin(c/2 + (d*x)/2) + 4*a*b*co
s(c/2 + (d*x)/2))) - 2*a^2*atan((b^2*cos(c/2 + (d*x)/2) - 2*a^2*cos(c/2 + (
d*x)/2) + 4*a*b*sin(c/2 + (d*x)/2))/(2*a^2*sin(c/2 + (d*x)/2) - b^2*sin(c/2
 + (d*x)/2) + 4*a*b*cos(c/2 + (d*x)/2))) + 2*a*b*log(sin(c/2 + (d*x)/2)/cos
(c/2 + (d*x)/2)))/d - (a^2*cos(c + d*x) - (b^2*cos(c + d*x))/8 + (b^2*cos(3
*c + 3*d*x))/8 - a*b*sin(2*c + 2*d*x))/(d*sin(c + d*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(a+b\sin(c+dx))^2\cot^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**2*cot(c + d*x)**2, x)
```

3.158 $\int \cot^4(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=133

$$-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}+a^2x-\frac{3ab\cos(c+dx)}{d}-\frac{ab\cos(c+dx)\cot^2(c+dx)}{d}+\frac{3ab\tanh^{-1}(\cos(c+dx))}{d}$$

```
[Out] a^2*x-3/2*b^2*x+3*a*b*arctanh(cos(d*x+c))/d-3*a*b*cos(d*x+c)/d+a^2*cot(d*x+
c)/d-3/2*b^2*cot(d*x+c)/d+1/2*b^2*cos(d*x+c)^2*cot(d*x+c)/d-a*b*cos(d*x+c)*
cot(d*x+c)^2/d-1/3*a^2*cot(d*x+c)^3/d
```

Rubi [A] time = 0.15, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2722, 2591, 288, 321, 203, 2592, 206, 3473, 8}

$$-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}+a^2x-\frac{3ab\cos(c+dx)}{d}-\frac{ab\cos(c+dx)\cot^2(c+dx)}{d}+\frac{3ab\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] a^2*x - (3*b^2*x)/2 + (3*a*b*ArcTanh[Cos[c + d*x]])/d - (3*a*b*Cos[c + d*x]
)/d + (a^2*Cot[c + d*x])/d - (3*b^2*Cot[c + d*x])/(2*d) + (b^2*Cos[c + d*x]
^2*Cot[c + d*x])/(2*d) - (a*b*Cos[c + d*x]*Cot[c + d*x]^2)/d - (a^2*Cot[c +
 d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \cot^4(c+dx)(a+b\sin(c+dx))^2\,dx = \int \left(b^2\cos^2(c+dx)\cot^2(c+dx)+2ab\cos(c+dx)\cot^3(c+dx)+a^2\cot\right.$$
$$= a^2\int \cot^4(c+dx)\,dx+(2ab)\int \cos(c+dx)\cot^3(c+dx)\,dx+b^2\int \cos$$
$$= -\frac{a^2\cot^3(c+dx)}{3d}-a^2\int \cot^2(c+dx)\,dx-\frac{(2ab)\,\mathrm{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx,x,\right.}{d}$$
$$= \frac{a^2\cot(c+dx)}{d}+\frac{b^2\cos^2(c+dx)\cot(c+dx)}{2d}-\frac{ab\cos(c+dx)\cot^2(c+}{d}$$
$$= a^2x-\frac{3ab\cos(c+dx)}{d}+\frac{a^2\cot(c+dx)}{d}-\frac{3b^2\cot(c+dx)}{2d}+\frac{b^2\cos^2(c+}{}$$
$$= a^2x-\frac{3b^2x}{2}+\frac{3ab\tanh^{-1}(\cos(c+dx))}{d}-\frac{3ab\cos(c+dx)}{d}+\frac{a^2\cot(c+}{d}$$

Mathematica [B] time = 6.21, size = 293, normalized size = 2.20

$$\frac{(2a^2-3b^2)(c+dx)}{2d}+\frac{\csc\left(\frac{1}{2}(c+dx)\right)\left(4a^2\cos\left(\frac{1}{2}(c+dx)\right)-3b^2\cos\left(\frac{1}{2}(c+dx)\right)\right)}{6d}+\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(3b^2\sin\left(\frac{1}{2}(\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((2*a^2 - 3*b^2)*(c + d*x))/(2*d) - (2*a*b*Cos[c + d*x])/d + ((4*a^2*Cos[(c
 + d*x)/2] - 3*b^2*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*d) - (a*b*Csc[(c
+ d*x)/2]^2)/(4*d) - (a^2*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(24*d) + (3*
a*b*Log[Cos[(c + d*x)/2]])/d - (3*a*b*Log[Sin[(c + d*x)/2]])/d + (a*b*Sec[(
c + d*x)/2]^2)/(4*d) + (Sec[(c + d*x)/2]*(-4*a^2*Sin[(c + d*x)/2] + 3*b^2*S
in[(c + d*x)/2]))/(6*d) - (b^2*Sin[2*(c + d*x)])/(4*d) + (a^2*Sec[(c + d*x)
/2]^2*Tan[(c + d*x)/2])/(24*d)
```

fricas [A] time = 0.48, size = 218, normalized size = 1.64

$$\frac{3\,b^2\cos(dx+c)^5+4\left(2\,a^2-3\,b^2\right)\cos(dx+c)^3+9\left(ab\cos(dx+c)^2-ab\right)\log\left(\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^2*cos(d*x + c)^5 + 4*(2*a^2 - 3*b^2)*cos(d*x + c)^3 + 9*(a*b*cos(d
*x + c)^2 - a*b)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 9*(a*b*cos(d*x
+ c)^2 - a*b)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*(2*a^2 - 3*b^2)
*cos(d*x + c) + 3*((2*a^2 - 3*b^2)*d*x*cos(d*x + c)^2 - 4*a*b*cos(d*x + c)^
3 - (2*a^2 - 3*b^2)*d*x + 6*a*b*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c
)^2 - d)*sin(d*x + c))
```

giac [A] time = 0.35, size = 241, normalized size = 1.81

$$a^2 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 + 6\,ab\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 - 72\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right|\right) - 15\,a^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 12\,b^2\tan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*tan(1/2*d*x + 1/2*c)^3 + 6*a*b*tan(1/2*d*x + 1/2*c)^2 - 72*a*b*lo
g(abs(tan(1/2*d*x + 1/2*c))) - 15*a^2*tan(1/2*d*x + 1/2*c) + 12*b^2*tan(1/2
*d*x + 1/2*c) + 12*(2*a^2 - 3*b^2)*(d*x + c) + 24*(b^2*tan(1/2*d*x + 1/2*c)
^3 - 4*a*b*tan(1/2*d*x + 1/2*c)^2 - b^2*tan(1/2*d*x + 1/2*c) - 4*a*b)/(tan(
1/2*d*x + 1/2*c)^2 + 1)^2 + (132*a*b*tan(1/2*d*x + 1/2*c)^3 + 15*a^2*tan(1/
2*d*x + 1/2*c)^2 - 12*b^2*tan(1/2*d*x + 1/2*c)^2 - 6*a*b*tan(1/2*d*x + 1/2*
c) - a^2)/tan(1/2*d*x + 1/2*c)^3)/d
```

maple [A] time = 0.25, size = 199, normalized size = 1.50

$$-\frac{a^2\left(\cot^3(dx+c)\right)}{3d}+\frac{a^2\cot(dx+c)}{d}+a^2x+\frac{a^2c}{d}-\frac{ab\left(\cos^5(dx+c)\right)}{d\sin(dx+c)^2}-\frac{ab\left(\cos^3(dx+c)\right)}{d}-\frac{3ab\cos(dx+c)}{d}-\frac{3ab\ln(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/3*a^2*cot(d*x+c)^3/d+a^2*cot(d*x+c)/d+a^2*x+1/d*a^2*c-1/d*a*b/sin(d*x+c)
^2*cos(d*x+c)^5-a*b*cos(d*x+c)^3/d-3*a*b*cos(d*x+c)/d-3/d*a*b*ln(csc(d*x+c)
-cot(d*x+c))-1/d*b^2/sin(d*x+c)*cos(d*x+c)^5-1/d*b^2*sin(d*x+c)*cos(d*x+c)^
3-3/2*b^2*cos(d*x+c)*sin(d*x+c)/d-3/2*b^2*x-3/2/d*b^2*c
```

maxima [A] time = 1.03, size = 138, normalized size = 1.04

$$\frac{2\left(3\,dx + 3\,c + \frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^2 - 3\left(3\,dx + 3\,c + \frac{3\,\tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)}\right)b^2 + 3\,ab\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1} - 4\,\cos(dx+c) + 3\,\log\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^2 - 3*(3*d*x
 + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x + c)))*b^2 + 3*a*
b*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x + c
) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 9.00, size = 584, normalized size = 4.39

$$-\frac{\frac{5\,b^2\,\cos(c+d\,x)}{16}+\frac{a^2\,\cos(3\,c+3\,d\,x)}{3}-\frac{11\,b^2\,\cos(3\,c+3\,d\,x)}{32}+\frac{b^2\,\cos(5\,c+5\,d\,x)}{32}+\frac{a^2\,\mathrm{atan}\left(\frac{-2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2+6\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b+3\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^2}{2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2+6\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b-3\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^2}\right)\,\mathrm{si}}{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + b*sin(c + d*x))^2,x)
```

```
[Out] -((5*b^2*cos(c + d*x))/16 + (a^2*cos(3*c + 3*d*x))/3 - (11*b^2*cos(3*c + 3*
d*x))/32 + (b^2*cos(5*c + 5*d*x))/32 + (a^2*atan((3*b^2*cos(c/2 + (d*x)/2)
- 2*a^2*cos(c/2 + (d*x)/2) + 6*a*b*sin(c/2 + (d*x)/2))/(2*a^2*sin(c/2 + (d*
x)/2) - 3*b^2*sin(c/2 + (d*x)/2) + 6*a*b*cos(c/2 + (d*x)/2)))*sin(3*c + 3*d
*x))/2 - (3*b^2*atan((3*b^2*cos(c/2 + (d*x)/2) - 2*a^2*cos(c/2 + (d*x)/2) +
 6*a*b*sin(c/2 + (d*x)/2))/(2*a^2*sin(c/2 + (d*x)/2) - 3*b^2*sin(c/2 + (d*x
)/2) + 6*a*b*cos(c/2 + (d*x)/2)))*sin(3*c + 3*d*x))/4 + (3*a*b*sin(c + d*x)
)/2 - (3*a^2*atan((3*b^2*cos(c/2 + (d*x)/2) - 2*a^2*cos(c/2 + (d*x)/2) + 6*
a*b*sin(c/2 + (d*x)/2))/(2*a^2*sin(c/2 + (d*x)/2) - 3*b^2*sin(c/2 + (d*x)/2
) + 6*a*b*cos(c/2 + (d*x)/2)))*sin(c + d*x))/2 + (9*b^2*atan((3*b^2*cos(c/2
 + (d*x)/2) - 2*a^2*cos(c/2 + (d*x)/2) + 6*a*b*sin(c/2 + (d*x)/2))/(2*a^2*s
in(c/2 + (d*x)/2) - 3*b^2*sin(c/2 + (d*x)/2) + 6*a*b*cos(c/2 + (d*x)/2)))*s
in(c + d*x))/4 + a*b*sin(2*c + 2*d*x) - (a*b*sin(3*c + 3*d*x))/2 - (a*b*sin
(4*c + 4*d*x))/4 + (9*a*b*sin(c + d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*
x)/2)))/4 - (3*a*b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(3*c + 3*d
*x))/4)/(d*sin(c + d*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin(c + dx))^2 \cot^4(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**2*cot(c + d*x)**4, x)
```

3.159 $\int \cot^6(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=202

$$-\frac{a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)}{d}-a^2x+\frac{15ab\cos(c+dx)}{4d}-\frac{ab\cos(c+dx)\cot^4(c+dx)}{2d}+\frac{5ab\cos(}{}$$

```
[Out] -a^2*x+5/2*b^2*x-15/4*a*b*arctanh(cos(d*x+c))/d+15/4*a*b*cos(d*x+c)/d-a^2*c
ot(d*x+c)/d+5/2*b^2*cot(d*x+c)/d+5/4*a*b*cos(d*x+c)*cot(d*x+c)^2/d+1/3*a^2*
cot(d*x+c)^3/d-5/6*b^2*cot(d*x+c)^3/d+1/2*b^2*cos(d*x+c)^2*cot(d*x+c)^3/d-1
/2*a*b*cos(d*x+c)*cot(d*x+c)^4/d-1/5*a^2*cot(d*x+c)^5/d
```

Rubi [A] time = 0.17, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {2722, 2591, 288, 302, 203, 2592, 321, 206, 3473, 8}

$$-\frac{a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)}{d}-a^2x+\frac{15ab\cos(c+dx)}{4d}-\frac{ab\cos(c+dx)\cot^4(c+dx)}{2d}+\frac{5ab\cos(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*x) + (5*b^2*x)/2 - (15*a*b*ArcTanh[Cos[c + d*x]])/(4*d) + (15*a*b*Cos
[c + d*x])/(4*d) - (a^2*Cot[c + d*x])/d + (5*b^2*Cot[c + d*x])/(2*d) + (5*a
*b*Cos[c + d*x]*Cot[c + d*x]^2)/(4*d) + (a^2*Cot[c + d*x]^3)/(3*d) - (5*b^2
*Cot[c + d*x]^3)/(6*d) + (b^2*Cos[c + d*x]^2*Cot[c + d*x]^3)/(2*d) - (a*b*C
os[c + d*x]*Cot[c + d*x]^4)/(2*d) - (a^2*Cot[c + d*x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \cot^6(c+dx)(a+b\sin(c+dx))^2\,dx = \int \left(b^2\cos^2(c+dx)\cot^4(c+dx)+2ab\cos(c+dx)\cot^5(c+dx)+a^2\cot^6\right.$$

$$= a^2\int \cot^6(c+dx)\,dx+(2ab)\int \cos(c+dx)\cot^5(c+dx)\,dx+b^2\int \cos^2($$

$$= -\frac{a^2\cot^5(c+dx)}{5d}-a^2\int \cot^4(c+dx)\,dx-\frac{(2ab)\,\mathrm{Subst}\left(\int \frac{x^6}{\left(1-x^2\right)^3}\,dx,x,c\right.}{d}$$

$$= \frac{a^2\cot^3(c+dx)}{3d}+\frac{b^2\cos^2(c+dx)\cot^3(c+dx)}{2d}-\frac{ab\cos(c+dx)\cot^4(c+}{2d}$$

$$= -\frac{a^2\cot(c+dx)}{d}+\frac{5ab\cos(c+dx)\cot^2(c+dx)}{4d}+\frac{a^2\cot^3(c+dx)}{3d}+\frac{b^2\,\mathrm{c}}{}$$

$$= -a^2x+\frac{15ab\cos(c+dx)}{4d}-\frac{a^2\cot(c+dx)}{d}+\frac{5b^2\cot(c+dx)}{2d}+\frac{5ab\cos(c}{}$$

$$= -a^2x+\frac{5b^2x}{2}-\frac{15ab\tanh^{-1}(\cos(c+dx))}{4d}+\frac{15ab\cos(c+dx)}{4d}-\frac{a^2\cot(c}{d}$$

Mathematica [A] time = 1.12, size = 351, normalized size = 1.74

$$\frac{\left(560b^2-368a^2\right)\cot\left(\frac{1}{2}(c+dx)\right)+368a^2\tan\left(\frac{1}{2}(c+dx)\right)-\frac{3}{2}a^2\sin(c+dx)\csc^6\left(\frac{1}{2}(c+dx)\right)+96a^2\sin^6\left(\frac{1}{2}(c+dx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-480*a^2*c + 1200*b^2*c - 480*a^2*d*x + 1200*b^2*d*x + 960*a*b*Cos[c + d*x
] + (-368*a^2 + 560*b^2)*Cot[(c + d*x)/2] + 270*a*b*Csc[(c + d*x)/2]^2 - 15
*a*b*Csc[(c + d*x)/2]^4 - 1800*a*b*Log[Cos[(c + d*x)/2]] + 1800*a*b*Log[Sin
[(c + d*x)/2]] - 270*a*b*Sec[(c + d*x)/2]^2 + 15*a*b*Sec[(c + d*x)/2]^4 - 3
28*a^2*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 160*b^2*Csc[c + d*x]^3*Sin[(c +
d*x)/2]^4 + 96*a^2*Csc[c + d*x]^5*Sin[(c + d*x)/2]^6 + (41*a^2*Csc[(c + d*x
)/2]^4*Sin[c + d*x])/2 - 10*b^2*Csc[(c + d*x)/2]^4*Sin[c + d*x] - (3*a^2*Cs
c[(c + d*x)/2]^6*Sin[c + d*x])/2 + 120*b^2*Sin[2*(c + d*x)] + 368*a^2*Tan[(
c + d*x)/2] - 560*b^2*Tan[(c + d*x)/2])/(480*d)
```

fricas [A] time = 0.52, size = 306, normalized size = 1.51

$$-\frac{60\,b^2\cos\left(dx+c\right)^7+92\left(2\,a^2-5\,b^2\right)\cos\left(dx+c\right)^5-140\left(2\,a^2-5\,b^2\right)\cos\left(dx+c\right)^3+225\left(ab\cos\left(dx+c\right)^4-2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/120*(60*b^2*cos(d*x + c)^7 + 92*(2*a^2 - 5*b^2)*cos(d*x + c)^5 - 140*(2*
a^2 - 5*b^2)*cos(d*x + c)^3 + 225*(a*b*cos(d*x + c)^4 - 2*a*b*cos(d*x + c)^
2 + a*b)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 225*(a*b*cos(d*x + c)^4
 - 2*a*b*cos(d*x + c)^2 + a*b)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) +
60*(2*a^2 - 5*b^2)*cos(d*x + c) + 30*(2*(2*a^2 - 5*b^2)*d*x*cos(d*x + c)^4
- 8*a*b*cos(d*x + c)^5 - 4*(2*a^2 - 5*b^2)*d*x*cos(d*x + c)^2 + 25*a*b*cos(
d*x + c)^3 + 2*(2*a^2 - 5*b^2)*d*x - 15*a*b*cos(d*x + c))*sin(d*x + c))/((d
*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

giac [A] time = 0.50, size = 337, normalized size = 1.67

$$\frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+15\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-35\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+20\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-240\,ab\tan\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/480*(3*a^2*tan(1/2*d*x + 1/2*c)^5 + 15*a*b*tan(1/2*d*x + 1/2*c)^4 - 35*a^
2*tan(1/2*d*x + 1/2*c)^3 + 20*b^2*tan(1/2*d*x + 1/2*c)^3 - 240*a*b*tan(1/2*
d*x + 1/2*c)^2 + 1800*a*b*log(abs(tan(1/2*d*x + 1/2*c))) + 330*a^2*tan(1/2*
d*x + 1/2*c) - 540*b^2*tan(1/2*d*x + 1/2*c) - 240*(2*a^2 - 5*b^2)*(d*x + c)
 - 480*(b^2*tan(1/2*d*x + 1/2*c)^3 - 4*a*b*tan(1/2*d*x + 1/2*c)^2 - b^2*tan
(1/2*d*x + 1/2*c) - 4*a*b)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2 - (4110*a*b*tan(1
/2*d*x + 1/2*c)^5 + 330*a^2*tan(1/2*d*x + 1/2*c)^4 - 540*b^2*tan(1/2*d*x +
1/2*c)^4 - 240*a*b*tan(1/2*d*x + 1/2*c)^3 - 35*a^2*tan(1/2*d*x + 1/2*c)^2 +
 20*b^2*tan(1/2*d*x + 1/2*c)^2 + 15*a*b*tan(1/2*d*x + 1/2*c) + 3*a^2)/tan(1
/2*d*x + 1/2*c)^5)/d
```

maple [A] time = 0.23, size = 302, normalized size = 1.50

$$-\frac{a^2\left(\cot^5\left(dx+c\right)\right)}{5d}+\frac{a^2\left(\cot^3\left(dx+c\right)\right)}{3d}-\frac{a^2\cot\left(dx+c\right)}{d}-a^2x-\frac{a^2c}{d}-\frac{ab\left(\cos^7\left(dx+c\right)\right)}{2d\sin\left(dx+c\right)^4}+\frac{3ab\left(\cos^7\left(dx+c\right)\right)}{4d\sin\left(dx+c\right)^2}+\frac{3ab}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/5*a^2*cot(d*x+c)^5/d+1/3*a^2*cot(d*x+c)^3/d-a^2*cot(d*x+c)/d-a^2*x-1/d*a
^2*c-1/2/d*a*b/sin(d*x+c)^4*cos(d*x+c)^7+3/4/d*a*b/sin(d*x+c)^2*cos(d*x+c)^
7+3/4*a*b*cos(d*x+c)^5/d+5/4*a*b*cos(d*x+c)^3/d+15/4*a*b*cos(d*x+c)/d+15/4/
d*a*b*ln(csc(d*x+c)-cot(d*x+c))-1/3/d*b^2/sin(d*x+c)^3*cos(d*x+c)^7+4/3/d*b
^2/sin(d*x+c)*cos(d*x+c)^7+4/3/d*b^2*sin(d*x+c)*cos(d*x+c)^5+5/3/d*b^2*sin(
d*x+c)*cos(d*x+c)^3+5/2*b^2*cos(d*x+c)*sin(d*x+c)/d+5/2*b^2*x+5/2/d*b^2*c
```

maxima [A] time = 2.12, size = 183, normalized size = 0.91

$$-\frac{8\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^{4}-5\,\tan(dx+c)^{2}+3}{\tan(dx+c)^{5}}\right)a^{2}-20\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^{4}+10\,\tan(dx+c)^{2}-2}{\tan(dx+c)^{5}+\tan(dx+c)^{3}}\right)b^{2}+15\,ab\left(\frac{2\left(9\,\cos(\ldots\right.}{\cos(dx+\ldots}\right.}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/120*(8*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(d
*x + c)^5)*a^2 - 20*(15*d*x + 15*c + (15*tan(d*x + c)^4 + 10*tan(d*x + c)^2
 - 2)/(tan(d*x + c)^5 + tan(d*x + c)^3))*b^2 + 15*a*b*(2*(9*cos(d*x + c)^3
- 7*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - 16*cos(d*x + c)
 + 15*log(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 11.28, size = 888, normalized size = 4.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6*(a + b*sin(c + d*x))^2,x)
```

```
[Out] ((95*b^2*cos(c + d*x))/384 - (5*a^2*cos(c + d*x))/24 + (5*a^2*cos(3*c + 3*d
*x))/48 - (23*a^2*cos(5*c + 5*d*x))/240 - (163*b^2*cos(3*c + 3*d*x))/384 +
(71*b^2*cos(5*c + 5*d*x))/384 - (b^2*cos(7*c + 7*d*x))/128 + (5*a^2*atan((1
0*b^2*cos(c/2 + (d*x)/2) - 4*a^2*cos(c/2 + (d*x)/2) + 15*a*b*sin(c/2 + (d*x
)/2))/(4*a^2*sin(c/2 + (d*x)/2) - 10*b^2*sin(c/2 + (d*x)/2) + 15*a*b*cos(c/
2 + (d*x)/2)))*sin(3*c + 3*d*x))/8 - (a^2*atan((10*b^2*cos(c/2 + (d*x)/2) -
 4*a^2*cos(c/2 + (d*x)/2) + 15*a*b*sin(c/2 + (d*x)/2))/(4*a^2*sin(c/2 + (d*
x)/2) - 10*b^2*sin(c/2 + (d*x)/2) + 15*a*b*cos(c/2 + (d*x)/2)))*sin(5*c + 5
*d*x))/8 - (25*b^2*atan((10*b^2*cos(c/2 + (d*x)/2) - 4*a^2*cos(c/2 + (d*x)/
2) + 15*a*b*sin(c/2 + (d*x)/2))/(4*a^2*sin(c/2 + (d*x)/2) - 10*b^2*sin(c/2
+ (d*x)/2) + 15*a*b*cos(c/2 + (d*x)/2)))*sin(3*c + 3*d*x))/16 + (5*b^2*atan
((10*b^2*cos(c/2 + (d*x)/2) - 4*a^2*cos(c/2 + (d*x)/2) + 15*a*b*sin(c/2 + (
```

```
d*x)/2))/(4*a^2*sin(c/2 + (d*x)/2) - 10*b^2*sin(c/2 + (d*x)/2) + 15*a*b*cos
(c/2 + (d*x)/2)))*sin(5*c + 5*d*x))/16 + (5*a*b*sin(c + d*x))/4 - (5*a^2*at
an((10*b^2*cos(c/2 + (d*x)/2) - 4*a^2*cos(c/2 + (d*x)/2) + 15*a*b*sin(c/2 +
 (d*x)/2))/(4*a^2*sin(c/2 + (d*x)/2) - 10*b^2*sin(c/2 + (d*x)/2) + 15*a*b*c
os(c/2 + (d*x)/2)))*sin(c + d*x))/4 + (25*b^2*atan((10*b^2*cos(c/2 + (d*x)/
2) - 4*a^2*cos(c/2 + (d*x)/2) + 15*a*b*sin(c/2 + (d*x)/2))/(4*a^2*sin(c/2 +
 (d*x)/2) - 10*b^2*sin(c/2 + (d*x)/2) + 15*a*b*cos(c/2 + (d*x)/2)))*sin(c +
 d*x))/8 + (5*a*b*sin(2*c + 2*d*x))/8 - (5*a*b*sin(3*c + 3*d*x))/8 - (17*a*
b*sin(4*c + 4*d*x))/32 + (a*b*sin(5*c + 5*d*x))/8 + (a*b*sin(6*c + 6*d*x))/
16 + (75*a*b*sin(c + d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/32 -
(75*a*b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(3*c + 3*d*x))/64 + (
15*a*b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(5*c + 5*d*x))/64)/(d*
sin(c + d*x)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin(c + dx))^2 \cot^6(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**2*cot(c + d*x)**6, x)
```

3.160 $\int (a+b\sin(c+dx))^3\tan^3(c+dx)\,dx$

Optimal. Leaf size=150

$$\frac{b(6a^2+5b^2)\sin(c+dx)}{2d}+\frac{3ab^2\sin^2(c+dx)}{2d}+\frac{(a+b)^2(2a+5b)\log(1-\sin(c+dx))}{4d}+\frac{(2a-5b)(a-b)^2\log(\sin(c}{4d}$$

```
[Out] 1/4*(a+b)^2*(2*a+5*b)*ln(1-sin(d*x+c))/d+1/4*(2*a-5*b)*(a-b)^2*ln(1+sin(d*x
+c))/d+1/2*b*(6*a^2+5*b^2)*sin(d*x+c)/d+3/2*a*b^2*sin(d*x+c)^2/d+1/3*b^3*si
n(d*x+c)^3/d+1/2*sec(d*x+c)^2*(a+b*sin(d*x+c))^3/d
```

Rubi [A] time = 0.24, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2721, 1645, 1629, 633, 31}

$$\frac{b(6a^2+5b^2)\sin(c+dx)}{2d}+\frac{3ab^2\sin^2(c+dx)}{2d}+\frac{(a+b)^2(2a+5b)\log(1-\sin(c+dx))}{4d}+\frac{(2a-5b)(a-b)^2\log(\sin(c}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^3*Tan[c + d*x]^3,x]
```

```
[Out] ((a + b)^2*(2*a + 5*b)*Log[1 - Sin[c + d*x]])/(4*d) + ((2*a - 5*b)*(a - b)^
2*Log[1 + Sin[c + d*x]])/(4*d) + (b*(6*a^2 + 5*b^2)*Sin[c + d*x])/(2*d) + (
3*a*b^2*Sin[c + d*x]^2)/(2*d) + (b^3*Sin[c + d*x]^3)/(3*d) + (Sec[c + d*x]^
2*(a + b*Sin[c + d*x])^3)/(2*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int (a+b\sin(c+dx))^3\tan^3(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^3(a+x)^3}{\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^2(c+dx)(a+b\sin(c+dx))^3}{2d} + \frac{\text{Subst}\left(\int \frac{(a+x)^2\left(-3b^4-2ab^2x-2b^2x^2\right)}{b^2-x^2}\,dx, \ldots\right.}{2b^2d}$$

$$= \frac{\sec^2(c+dx)(a+b\sin(c+dx))^3}{2d} + \frac{\text{Subst}\left(\int \left(6a^2b^2+5b^4+6ab^2x+2\ldots\right.\right.}{}$$

$$= \frac{b\left(6a^2+5b^2\right)\sin(c+dx)}{2d} + \frac{3ab^2\sin^2(c+dx)}{2d} + \frac{b^3\sin^3(c+dx)}{3d} + \frac{\sec^2\ldots}{}$$

$$= \frac{b\left(6a^2+5b^2\right)\sin(c+dx)}{2d} + \frac{3ab^2\sin^2(c+dx)}{2d} + \frac{b^3\sin^3(c+dx)}{3d} + \frac{\sec^2\ldots}{}$$

$$= \frac{(a+b)^2(2a+5b)\log(1-\sin(c+dx))}{4d} + \frac{(2a-5b)(a-b)^2\log(1+\sin(\ldots}{4d}$$

Mathematica [A] time = 0.25, size = 141, normalized size = 0.94

$$\frac{12b\left(3a^2+2b^2\right)\sin(c+dx)+18ab^2\sin^2(c+dx)+\frac{3(a-b)^3}{\sin(c+dx)+1}-\frac{3(a+b)^3}{\sin(c+dx)-1}+3(2a-5b)(a-b)^2\log(\sin(c+dx)+}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^3*Tan[c + d*x]^3,x]
```

```
[Out] (3*(a + b)^2*(2*a + 5*b)*Log[1 - Sin[c + d*x]] + 3*(2*a - 5*b)*(a - b)^2*Lo
g[1 + Sin[c + d*x]] - (3*(a + b)^3)/(-1 + Sin[c + d*x]) + 12*b*(3*a^2 + 2*b
^2)*Sin[c + d*x] + 18*a*b^2*Sin[c + d*x]^2 + 4*b^3*Sin[c + d*x]^3 + (3*(a -
 b)^3)/(1 + Sin[c + d*x]))/(12*d)
```

fricas [A] time = 0.49, size = 194, normalized size = 1.29

$$-\frac{18\,ab^2\cos(dx+c)^4-9\,ab^2\cos(dx+c)^2-3\left(2\,a^3-9\,a^2b+12\,ab^2-5\,b^3\right)\cos(dx+c)^2\log(\sin(dx+c)+1)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/12*(18*a*b^2*cos(d*x + c)^4 - 9*a*b^2*cos(d*x + c)^2 - 3*(2*a^3 - 9*a^2*
b + 12*a*b^2 - 5*b^3)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - 3*(2*a^3 + 9*a
^2*b + 12*a*b^2 + 5*b^3)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - 6*a^3 - 18
*a*b^2 + 2*(2*b^3*cos(d*x + c)^4 - 9*a^2*b - 3*b^3 - 2*(9*a^2*b + 7*b^3)*co
s(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.16, size = 279, normalized size = 1.86

$$\frac{a^3\left(\tan^2(dx+c)\right)}{2d}+\frac{a^3\ln(\cos(dx+c))}{d}+\frac{3a^2b\left(\sin^5(dx+c)\right)}{2d\cos(dx+c)^2}+\frac{3a^2b\left(\sin^3(dx+c)\right)}{2d}+\frac{9a^2b\sin(dx+c)}{2d}-\frac{9a^2b\ln(\mathrm{s}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^3*tan(d*x+c)^3,x)
```

```
[Out] 1/2/d*a^3*tan(d*x+c)^2+1/d*a^3*ln(cos(d*x+c))+3/2/d*a^2*b*sin(d*x+c)^5/cos(
d*x+c)^2+3/2/d*a^2*b*sin(d*x+c)^3+9/2*a^2*b*sin(d*x+c)/d-9/2/d*a^2*b*ln(sec
(d*x+c)+tan(d*x+c))+3/2/d*a*b^2*sin(d*x+c)^6/cos(d*x+c)^2+3/2/d*a*b^2*sin(d
*x+c)^4+3*a*b^2*sin(d*x+c)^2/d+6/d*a*b^2*ln(cos(d*x+c))+1/2/d*b^3*sin(d*x+c
)^7/cos(d*x+c)^2+1/2/d*b^3*sin(d*x+c)^5+5/6*b^3*sin(d*x+c)^3/d+5/2/d*b^3*si
n(d*x+c)-5/2/d*b^3*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.40, size = 162, normalized size = 1.08

$$\frac{4\,b^3\sin\left(dx+c\right)^3+18\,ab^2\sin\left(dx+c\right)^2+3\left(2\,a^3-9\,a^2b+12\,ab^2-5\,b^3\right)\log\left(\sin\left(dx+c\right)+1\right)+3\left(2\,a^3+9\,a^2b\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(4*b^3*sin(d*x + c)^3 + 18*a*b^2*sin(d*x + c)^2 + 3*(2*a^3 - 9*a^2*b +
 12*a*b^2 - 5*b^3)*log(sin(d*x + c) + 1) + 3*(2*a^3 + 9*a^2*b + 12*a*b^2 +
5*b^3)*log(sin(d*x + c) - 1) + 12*(3*a^2*b + 2*b^3)*sin(d*x + c) - 6*(a^3 +
 3*a*b^2 + (3*a^2*b + b^3)*sin(d*x + c))/(sin(d*x + c)^2 - 1))/d
```

mupad [B] time = 6.98, size = 366, normalized size = 2.44

$$\frac{\left(9\,a^2\,b+5\,b^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(2\,a^3+12\,a\,b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\left(12\,a^2\,b+\frac{20\,b^3}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(6\,a^3+12\,a\,b^2\right.}{d\,\left(\tan\left(\frac{c}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + b*sin(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(9*a^2*b + 5*b^3) + tan(c/2 + (d*x)/2)^2*(12*a*b^2 + 2*
a^3) + tan(c/2 + (d*x)/2)^4*(12*a*b^2 + 6*a^3) + tan(c/2 + (d*x)/2)^8*(12*a
*b^2 + 2*a^3) + tan(c/2 + (d*x)/2)^6*(12*a*b^2 + 6*a^3) + tan(c/2 + (d*x)/2
)^9*(9*a^2*b + 5*b^3) + tan(c/2 + (d*x)/2)^5*(6*a^2*b - (22*b^3)/3) + tan(c
/2 + (d*x)/2)^3*(12*a^2*b + (20*b^3)/3) + tan(c/2 + (d*x)/2)^7*(12*a^2*b +
(20*b^3)/3))/(d*(tan(c/2 + (d*x)/2)^2 - 2*tan(c/2 + (d*x)/2)^4 - 2*tan(c/2
+ (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 + 1)) - (log(ta
n(c/2 + (d*x)/2)^2 + 1)*(6*a*b^2 + a^3))/d + (log(tan(c/2 + (d*x)/2) + 1)*(
a - b)^2*(a - (5*b)/2))/d + (log(tan(c/2 + (d*x)/2) - 1)*(a + b)^2*(a + (5*
b)/2))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin(c+dx))^3\tan^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**3*tan(d*x+c)**3,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**3*tan(c + d*x)**3, x)
```

3.161 $\int (a+b\sin(c+dx))^3\tan(c+dx)\,dx$

Optimal. Leaf size=105

$$-\frac{b\left(3a^2+b^2\right)\sin(c+dx)}{d}-\frac{3ab^2\sin^2(c+dx)}{2d}-\frac{(a-b)^3\log(\sin(c+dx)+1)}{2d}-\frac{(a+b)^3\log(1-\sin(c+dx))}{2d}-\frac{b^3\text{si}}{}$$

```
[Out] -1/2*(a+b)^3*ln(1-sin(d*x+c))/d-1/2*(a-b)^3*ln(1+sin(d*x+c))/d-b*(3*a^2+b^2
)*sin(d*x+c)/d-3/2*a*b^2*sin(d*x+c)^2/d-1/3*b^3*sin(d*x+c)^3/d
```

Rubi [A] time = 0.11, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2721, 801, 633, 31}

$$-\frac{b\left(3a^2+b^2\right)\sin(c+dx)}{d}-\frac{3ab^2\sin^2(c+dx)}{2d}-\frac{(a-b)^3\log(\sin(c+dx)+1)}{2d}-\frac{(a+b)^3\log(1-\sin(c+dx))}{2d}-\frac{b^3\text{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^3*Tan[c + d*x],x]
```

```
[Out] -((a + b)^3*Log[1 - Sin[c + d*x]])/(2*d) - ((a - b)^3*Log[1 + Sin[c + d*x]]
)/(2*d) - (b*(3*a^2 + b^2)*Sin[c + d*x])/d - (3*a*b^2*Sin[c + d*x]^2)/(2*d)
 - (b^3*Sin[c + d*x]^3)/(3*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int (a+b\sin(c+dx))^3\tan(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x(a+x)^3}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-3a^2-b^2-3ax-x^2+\frac{3a^2b^2+b^4+a\left(a^2+3b^2\right)x}{b^2-x^2}\right) dx, x, b\sin(c+dx)\right)}{d}$$
$$= -\frac{b\left(3a^2+b^2\right)\sin(c+dx)}{d}-\frac{3ab^2\sin^2(c+dx)}{2d}-\frac{b^3\sin^3(c+dx)}{3d}+\frac{\text{Subst}}{}$$
$$= -\frac{b\left(3a^2+b^2\right)\sin(c+dx)}{d}-\frac{3ab^2\sin^2(c+dx)}{2d}-\frac{b^3\sin^3(c+dx)}{3d}+\frac{(a-b)}{}$$
$$= -\frac{(a+b)^3\log(1-\sin(c+dx))}{2d}-\frac{(a-b)^3\log(1+\sin(c+dx))}{2d}-\frac{b\left(3a^2+\right.}{}$$

Mathematica [A] time = 0.20, size = 90, normalized size = 0.86

$$-\frac{6b\left(3a^2+b^2\right)\sin(c+dx)+9ab^2\sin^2(c+dx)+3\left((a-b)^3\log(\sin(c+dx)+1)+(a+b)^3\log(1-\sin(c+dx))\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^3*Tan[c + d*x],x]
```

```
[Out] -1/6*(3*((a + b)^3*Log[1 - Sin[c + d*x]] + (a - b)^3*Log[1 + Sin[c + d*x]])
 + 6*b*(3*a^2 + b^2)*Sin[c + d*x] + 9*a*b^2*Sin[c + d*x]^2 + 2*b^3*Sin[c +
d*x]^3)/d
```

fricas [A] time = 0.47, size = 116, normalized size = 1.10

$$\frac{9\,ab^2\cos\left(dx+c\right)^2-3\left(a^3-3\,a^2b+3\,ab^2-b^3\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\log\left(-\sin\left(dx+\right.\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/6*(9*a*b^2*cos(d*x + c)^2 - 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*log(sin(d*x
 + c) + 1) - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(-sin(d*x + c) + 1) + 2*(
b^3*cos(d*x + c)^2 - 9*a^2*b - 4*b^3)*sin(d*x + c))/d
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.14, size = 139, normalized size = 1.32

$$-\frac{a^3\ln(\cos(dx+c))}{d}+\frac{3a^2b\ln(\sec(dx+c)+\tan(dx+c))}{d}-\frac{3a^2b\sin(dx+c)}{d}-\frac{3a\,b^2\left(\sin^2(dx+c)\right)}{2d}-\frac{3a\,b^2\ln(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^3*tan(d*x+c),x)
```

```
[Out] -1/d*a^3*ln(cos(d*x+c))+3/d*a^2*b*ln(sec(d*x+c)+tan(d*x+c))-3*a^2*b*sin(d*x
+c)/d-3/2*a*b^2*sin(d*x+c)^2/d-3/d*a*b^2*ln(cos(d*x+c))-1/3*b^3*sin(d*x+c)^
3/d-1/d*b^3*sin(d*x+c)+1/d*b^3*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.30, size = 113, normalized size = 1.08

$$-\frac{2\,b^3\sin(dx+c)^3+9\,ab^2\sin(dx+c)^2+3\left(a^3-3\,a^2b+3\,ab^2-b^3\right)\log(\sin(dx+c)+1)+3\left(a^3+3\,a^2b+3\,ab\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/6*(2*b^3*sin(d*x + c)^3 + 9*a*b^2*sin(d*x + c)^2 + 3*(a^3 - 3*a^2*b + 3*
a*b^2 - b^3)*log(sin(d*x + c) + 1) + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(
sin(d*x + c) - 1) + 6*(3*a^2*b + b^3)*sin(d*x + c))/d
```

mupad [B] time = 6.75, size = 226, normalized size = 2.15

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)\left(a^3+3\,a\,b^2\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(6\,a^2\,b+2\,b^3\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\left(6\,a^2\,b+2\,b^3\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + b*sin(c + d*x))^3,x)
```

```
[Out] (log(tan(c/2 + (d*x)/2)^2 + 1)*(3*a*b^2 + a^3))/d - (tan(c/2 + (d*x)/2)*(6*
a^2*b + 2*b^3) + tan(c/2 + (d*x)/2)^5*(6*a^2*b + 2*b^3) + tan(c/2 + (d*x)/2
)^3*(12*a^2*b + (20*b^3)/3) + 6*a*b^2*tan(c/2 + (d*x)/2)^2 + 6*a*b^2*tan(c/
2 + (d*x)/2)^4)/(d*(3*tan(c/2 + (d*x)/2)^2 + 3*tan(c/2 + (d*x)/2)^4 + tan(c
/2 + (d*x)/2)^6 + 1)) - (log(tan(c/2 + (d*x)/2) + 1)*(a - b)^3)/d - (log(ta
n(c/2 + (d*x)/2) - 1)*(a + b)^3)/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin(c + dx))^3 \tan(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**3*tan(d*x+c),x)
```

```
[Out] Integral((a + b*sin(c + d*x))**3*tan(c + d*x), x)
```

3.162 $\int \cot(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=67

$$\frac{a^3\log(\sin(c+dx))}{d}+\frac{3a^2b\sin(c+dx)}{d}+\frac{3ab^2\sin^2(c+dx)}{2d}+\frac{b^3\sin^3(c+dx)}{3d}$$

```
[Out] a^3*ln(sin(d*x+c))/d+3*a^2*b*sin(d*x+c)/d+3/2*a*b^2*sin(d*x+c)^2/d+1/3*b^3*
sin(d*x+c)^3/d
```

Rubi [A] time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2721, 43}

$$\frac{3a^2b\sin(c+dx)}{d}+\frac{a^3\log(\sin(c+dx))}{d}+\frac{3ab^2\sin^2(c+dx)}{2d}+\frac{b^3\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Log[Sin[c + d*x]])/d + (3*a^2*b*Sin[c + d*x])/d + (3*a*b^2*Sin[c + d*x
]^2)/(2*d) + (b^3*Sin[c + d*x]^3)/(3*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \cot(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^3}{x}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(3a^2+\frac{a^3}{x}+3ax+x^2\right)dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{a^3\log(\sin(c+dx))}{d}+\frac{3a^2b\sin(c+dx)}{d}+\frac{3ab^2\sin^2(c+dx)}{2d}+\frac{b^3\sin^3(c+}{3d}$$

Mathematica [A] time = 0.03, size = 67, normalized size = 1.00

$$\frac{a^3\log(\sin(c+dx))}{d}+\frac{3a^2b\sin(c+dx)}{d}+\frac{3ab^2\sin^2(c+dx)}{2d}+\frac{b^3\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Log[Sin[c + d*x]])/d + (3*a^2*b*Sin[c + d*x])/d + (3*a*b^2*Sin[c + d*x
]^2)/(2*d) + (b^3*Sin[c + d*x]^3)/(3*d)
```

fricas [A] time = 0.47, size = 66, normalized size = 0.99

$$-\frac{9\,ab^2\cos(dx+c)^2-6\,a^3\log\left(\frac{1}{2}\sin(dx+c)\right)+2\left(b^3\cos(dx+c)^2-9\,a^2b-b^3\right)\sin(dx+c)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/6*(9*a*b^2*cos(d*x + c)^2 - 6*a^3*log(1/2*sin(d*x + c)) + 2*(b^3*cos(d*x
 + c)^2 - 9*a^2*b - b^3)*sin(d*x + c))/d
```

giac [A] time = 0.92, size = 58, normalized size = 0.87

$$\frac{2\,b^3\sin(dx+c)^3+9\,ab^2\sin(dx+c)^2+6\,a^3\log(|\sin(dx+c)|)+18\,a^2b\sin(dx+c)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(2*b^3*sin(d*x + c)^3 + 9*a*b^2*sin(d*x + c)^2 + 6*a^3*log(abs(sin(d*x
+ c))) + 18*a^2*b*sin(d*x + c))/d
```

maple [A] time = 0.10, size = 64, normalized size = 0.96

$$\frac{a^3\ln(\sin(dx+c))}{d}+\frac{3a^2b\sin(dx+c)}{d}+\frac{3ab^2\left(\sin^2(dx+c)\right)}{2d}+\frac{b^3\left(\sin^3(dx+c)\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+b*sin(d*x+c))^3,x)
```

```
[Out] a^3*ln(sin(d*x+c))/d+3*a^2*b*sin(d*x+c)/d+3/2*a*b^2*sin(d*x+c)^2/d+1/3*b^3*
sin(d*x+c)^3/d
```

maxima [A] time = 1.20, size = 57, normalized size = 0.85

$$\frac{2\,b^3\sin(dx+c)^3+9\,ab^2\sin(dx+c)^2+6\,a^3\log(\sin(dx+c))+18\,a^2b\sin(dx+c)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*b^3*sin(d*x + c)^3 + 9*a*b^2*sin(d*x + c)^2 + 6*a^3*log(sin(d*x + c)
) + 18*a^2*b*sin(d*x + c))/d
```

mupad [B] time = 6.68, size = 118, normalized size = 1.76

$$\frac{b^3\,\sin(c+d\,x)}{3\,d}-\frac{a^3\,\ln\left(\frac{1}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}\right)}{d}+\frac{a^3\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{3\,a\,b^2\cos(c+d\,x)^2}{2\,d}-\frac{b^3\cos(c+d\,x)^2\,\sin(c+d\,x)}{3\,d}+\frac{3\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + b*sin(c + d*x))^3,x)
```

```
[Out] (b^3*sin(c + d*x))/(3*d) - (a^3*log(1/cos(c/2 + (d*x)/2)^2))/d + (a^3*log(s
in(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (3*a*b^2*cos(c + d*x)^2)/(2*d) -
 (b^3*cos(c + d*x)^2*sin(c + d*x))/(3*d) + (3*a^2*b*sin(c + d*x))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(a+b\sin(c+dx))^3\cot(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**3*cot(c + d*x), x)
```

3.163 $\int \cot^3(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=116

$$-\frac{a^3\csc^2(c+dx)}{2d}-\frac{b\left(3a^2-b^2\right)\sin(c+dx)}{d}-\frac{a\left(a^2-3b^2\right)\log(\sin(c+dx))}{d}-\frac{3a^2b\csc(c+dx)}{d}-\frac{3ab^2\sin^2(c+dx)}{2d}-b$$

```
[Out] -3*a^2*b*csc(d*x+c)/d-1/2*a^3*csc(d*x+c)^2/d-a*(a^2-3*b^2)*ln(sin(d*x+c))/d
-b*(3*a^2-b^2)*sin(d*x+c)/d-3/2*a*b^2*sin(d*x+c)^2/d-1/3*b^3*sin(d*x+c)^3/d
```

Rubi [A] time = 0.09, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2721, 894}

$$-\frac{b\left(3a^2-b^2\right)\sin(c+dx)}{d}-\frac{a\left(a^2-3b^2\right)\log(\sin(c+dx))}{d}-\frac{3a^2b\csc(c+dx)}{d}-\frac{a^3\csc^2(c+dx)}{2d}-\frac{3ab^2\sin^2(c+dx)}{2d}-b$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a^2*b*Csc[c + d*x])/d - (a^3*Csc[c + d*x]^2)/(2*d) - (a*(a^2 - 3*b^2)*L
og[Sin[c + d*x]])/d - (b*(3*a^2 - b^2)*Sin[c + d*x])/d - (3*a*b^2*Sin[c + d
*x]^2)/(2*d) - (b^3*Sin[c + d*x]^3)/(3*d)
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \cot^3(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^3\left(b^2-x^2\right)}{x^3}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-3a^2\left(1-\frac{b^2}{3a^2}\right)+\frac{a^3b^2}{x^3}+\frac{3a^2b^2}{x^2}+\frac{-a^3+3ab^2}{x}-3ax-x^2\right)dx, x, b$$
$$= -\frac{3a^2b\csc(c+dx)}{d}-\frac{a^3\csc^2(c+dx)}{2d}-\frac{a\left(a^2-3b^2\right)\log(\sin(c+dx))}{d}-$$

Mathematica [A] time = 0.29, size = 97, normalized size = 0.84

$$-\frac{3a^3\csc^2(c+dx)-6b\left(b^2-3a^2\right)\sin(c+dx)+6a\left(a^2-3b^2\right)\log(\sin(c+dx))+18a^2b\csc(c+dx)+9ab^2\sin^2(c}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/6*(18*a^2*b*Csc[c + d*x] + 3*a^3*Csc[c + d*x]^2 + 6*a*(a^2 - 3*b^2)*Log[
Sin[c + d*x]] - 6*b*(-3*a^2 + b^2)*Sin[c + d*x] + 9*a*b^2*Sin[c + d*x]^2 +
2*b^3*Sin[c + d*x]^3)/d
```

fricas [A] time = 0.47, size = 153, normalized size = 1.32

$$\frac{18\,ab^2\cos\left(dx+c\right)^4-27\,ab^2\cos\left(dx+c\right)^2+6\,a^3+9\,ab^2+12\left(a^3-3\,ab^2-\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)^2\right)\log\left(\frac{1}{2}\sin\right.}{12\left(d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(18*a*b^2*cos(d*x + c)^4 - 27*a*b^2*cos(d*x + c)^2 + 6*a^3 + 9*a*b^2 +
 12*(a^3 - 3*a*b^2 - (a^3 - 3*a*b^2)*cos(d*x + c)^2)*log(1/2*sin(d*x + c))
+ 4*(b^3*cos(d*x + c)^4 + 18*a^2*b - 2*b^3 - (9*a^2*b - b^3)*cos(d*x + c)^2
)*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

giac [A] time = 0.94, size = 131, normalized size = 1.13

$$-\frac{2\,b^3\sin\left(dx+c\right)^3+9\,ab^2\sin\left(dx+c\right)^2+18\,a^2b\sin\left(dx+c\right)-6\,b^3\sin\left(dx+c\right)+6\left(a^3-3\,ab^2\right)\log\left(\left|\sin\left(dx+c\right.\right.\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*(2*b^3*sin(d*x + c)^3 + 9*a*b^2*sin(d*x + c)^2 + 18*a^2*b*sin(d*x + c)
 - 6*b^3*sin(d*x + c) + 6*(a^3 - 3*a*b^2)*log(abs(sin(d*x + c))) - 3*(3*a^3
*sin(d*x + c)^2 - 9*a*b^2*sin(d*x + c)^2 - 6*a^2*b*sin(d*x + c) - a^3)/sin(
d*x + c)^2)/d
```

maple [A] time = 0.26, size = 165, normalized size = 1.42

$$-\frac{a^3\left(\cot^2(dx+c)\right)}{2d}-\frac{a^3\ln(\sin(dx+c))}{d}-\frac{3a^2b\left(\cos^4(dx+c)\right)}{d\sin(dx+c)}-\frac{3a^2b\left(\cos^2(dx+c)\right)\sin(dx+c)}{d}-\frac{6a^2b\sin(dx+c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/2/d*a^3*cot(d*x+c)^2-a^3*ln(sin(d*x+c))/d-3/d*a^2*b/sin(d*x+c)*cos(d*x+c
)^4-3/d*a^2*b*cos(d*x+c)^2*sin(d*x+c)-6*a^2*b*sin(d*x+c)/d+3/2/d*a*b^2*cos(
d*x+c)^2+3/d*a*b^2*ln(sin(d*x+c))+1/3/d*b^3*cos(d*x+c)^2*sin(d*x+c)+2/3/d*b
^3*sin(d*x+c)
```

maxima [A] time = 1.14, size = 98, normalized size = 0.84

$$-\frac{2\,b^3\sin(dx+c)^3+9\,ab^2\sin(dx+c)^2+6\left(a^3-3\,ab^2\right)\log(\sin(dx+c))+6\left(3\,a^2b-b^3\right)\sin(dx+c)+\frac{3\left(6\,a^2b\sin\right.}{\sin(d}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/6*(2*b^3*sin(d*x + c)^3 + 9*a*b^2*sin(d*x + c)^2 + 6*(a^3 - 3*a*b^2)*log
(sin(d*x + c)) + 6*(3*a^2*b - b^3)*sin(d*x + c) + 3*(6*a^2*b*sin(d*x + c) +
 a^3)/sin(d*x + c)^2)/d
```

mupad [B] time = 6.95, size = 312, normalized size = 2.69

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\left(3\,a\,b^2-a^3\right)}{d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)\left(3\,a\,b^2-a^3\right)}{d}-\frac{\frac{3\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(\frac{3\,a^3}{2}+24\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + b*sin(c + d*x))^3,x)
```

```
[Out] (log(tan(c/2 + (d*x)/2))*(3*a*b^2 - a^3))/d - (log(tan(c/2 + (d*x)/2)^2 + 1
)*(3*a*b^2 - a^3))/d - ((3*a^3*tan(c/2 + (d*x)/2)^2)/2 + tan(c/2 + (d*x)/2)
^4*(24*a*b^2 + (3*a^3)/2) + tan(c/2 + (d*x)/2)^6*(24*a*b^2 + a^3/2) + tan(c
/2 + (d*x)/2)^7*(30*a^2*b - 8*b^3) + tan(c/2 + (d*x)/2)^3*(42*a^2*b - 8*b^3
) + tan(c/2 + (d*x)/2)^5*(66*a^2*b - (16*b^3)/3) + a^3/2 + 6*a^2*b*tan(c/2
+ (d*x)/2))/(d*(4*tan(c/2 + (d*x)/2)^2 + 12*tan(c/2 + (d*x)/2)^4 + 12*tan(c
/2 + (d*x)/2)^6 + 4*tan(c/2 + (d*x)/2)^8)) - (a^3*tan(c/2 + (d*x)/2)^2)/(8*
d) - (3*a^2*b*tan(c/2 + (d*x)/2))/(2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin(c + dx))^3 \cot^3(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**3*cot(c + d*x)**3, x)
```

3.164 $\int \cot^5(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=165

$$-\frac{a^3\csc^4(c+dx)}{4d}+\frac{b\left(3a^2-2b^2\right)\sin(c+dx)}{d}+\frac{a\left(2a^2-3b^2\right)\csc^2(c+dx)}{2d}+\frac{b\left(6a^2-b^2\right)\csc(c+dx)}{d}+\frac{a\left(a^2-6b^2\right)\text{lo}}{}$$

```
[Out] b*(6*a^2-b^2)*csc(d*x+c)/d+1/2*a*(2*a^2-3*b^2)*csc(d*x+c)^2/d-a^2*b*csc(d*x
+c)^3/d-1/4*a^3*csc(d*x+c)^4/d+a*(a^2-6*b^2)*ln(sin(d*x+c))/d+b*(3*a^2-2*b^
2)*sin(d*x+c)/d+3/2*a*b^2*sin(d*x+c)^2/d+1/3*b^3*sin(d*x+c)^3/d
```

Rubi [A] time = 0.14, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2721, 948}

$$\frac{b\left(3a^2-2b^2\right)\sin(c+dx)}{d}+\frac{a\left(2a^2-3b^2\right)\csc^2(c+dx)}{2d}+\frac{b\left(6a^2-b^2\right)\csc(c+dx)}{d}+\frac{a\left(a^2-6b^2\right)\log(\sin(c+dx))}{d}-\frac{a^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (b*(6*a^2 - b^2)*Csc[c + d*x])/d + (a*(2*a^2 - 3*b^2)*Csc[c + d*x]^2)/(2*d)
 - (a^2*b*Csc[c + d*x]^3)/d - (a^3*Csc[c + d*x]^4)/(4*d) + (a*(a^2 - 6*b^2)
*Log[Sin[c + d*x]])/d + (b*(3*a^2 - 2*b^2)*Sin[c + d*x])/d + (3*a*b^2*Sin[c
 + d*x]^2)/(2*d) + (b^3*Sin[c + d*x]^3)/(3*d)
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \cot^5(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^3\left(b^2-x^2\right)^2}{x^5}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(3a^2\left(1-\frac{2b^2}{3a^2}\right)+\frac{a^3b^4}{x^5}+\frac{3a^2b^4}{x^4}+\frac{-2a^3b^2+3ab^4}{x^3}+\frac{-6a^2b^2+b^4}{x^2}+\frac{a^3-6al}{x}\right.\right.}{d}$$
$$= \frac{b\left(6a^2-b^2\right)\csc(c+dx)}{d}+\frac{a\left(2a^2-3b^2\right)\csc^2(c+dx)}{2d}-\frac{a^2b\csc^3(c+dx}{d}$$

Mathematica [A] time = 1.04, size = 144, normalized size = 0.87

$$\frac{-3a^3\csc^4(c+dx)+6a\left(2a^2-3b^2\right)\csc^2(c+dx)-12b\left(b^2-6a^2\right)\csc(c+dx)+2\left(6b\left(3a^2-2b^2\right)\sin(c+dx)+6\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-12*b*(-6*a^2 + b^2)*Csc[c + d*x] + 6*a*(2*a^2 - 3*b^2)*Csc[c + d*x]^2 - 1
2*a^2*b*Csc[c + d*x]^3 - 3*a^3*Csc[c + d*x]^4 + 2*(6*a*(a^2 - 6*b^2)*Log[Si
n[c + d*x]] + 6*b*(3*a^2 - 2*b^2)*Sin[c + d*x] + 9*a*b^2*Sin[c + d*x]^2 + 2
*b^3*Sin[c + d*x]^3))/(12*d)
```

fricas [A] time = 0.50, size = 225, normalized size = 1.36

$$-\frac{18\,ab^2\cos(dx+c)^6-45\,ab^2\cos(dx+c)^4-9\,a^3+9\,ab^2+6\left(2\,a^3+3\,ab^2\right)\cos(dx+c)^2-12\left(\left(a^3-6\,ab^2\right)\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/12*(18*a*b^2*cos(d*x + c)^6 - 45*a*b^2*cos(d*x + c)^4 - 9*a^3 + 9*a*b^2
+ 6*(2*a^3 + 3*a*b^2)*cos(d*x + c)^2 - 12*((a^3 - 6*a*b^2)*cos(d*x + c)^4 +
 a^3 - 6*a*b^2 - 2*(a^3 - 6*a*b^2)*cos(d*x + c)^2)*log(1/2*sin(d*x + c)) +
4*(b^3*cos(d*x + c)^6 - 3*(3*a^2*b - b^3)*cos(d*x + c)^4 - 24*a^2*b + 8*b^3
 + 12*(3*a^2*b - b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^4 - 2*d
*cos(d*x + c)^2 + d)
```

giac [A] time = 2.45, size = 185, normalized size = 1.12

$$\frac{4\,b^3\sin(dx+c)^3+18\,ab^2\sin(dx+c)^2+36\,a^2b\sin(dx+c)-24\,b^3\sin(dx+c)+12\left(a^3-6\,ab^2\right)\log\left(|\sin(dx+\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/12*(4*b^3*sin(d*x + c)^3 + 18*a*b^2*sin(d*x + c)^2 + 36*a^2*b*sin(d*x + c
) - 24*b^3*sin(d*x + c) + 12*(a^3 - 6*a*b^2)*log(abs(sin(d*x + c))) - (25*a
^3*sin(d*x + c)^4 - 150*a*b^2*sin(d*x + c)^4 - 72*a^2*b*sin(d*x + c)^3 + 12
*b^3*sin(d*x + c)^3 - 12*a^3*sin(d*x + c)^2 + 18*a*b^2*sin(d*x + c)^2 + 12*
a^2*b*sin(d*x + c) + 3*a^3)/sin(d*x + c)^4)/d
```

maple [B] time = 0.23, size = 316, normalized size = 1.92

$$-\frac{a^3\left(\cot^4(dx+c)\right)}{4d}+\frac{a^3\left(\cot^2(dx+c)\right)}{2d}+\frac{a^3\ln(\sin(dx+c))}{d}-\frac{a^2b\left(\cos^6(dx+c)\right)}{d\sin(dx+c)^3}+\frac{3a^2b\left(\cos^6(dx+c)\right)}{d\sin(dx+c)}+\frac{8a^2b\sin}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/4/d*a^3*cot(d*x+c)^4+1/2/d*a^3*cot(d*x+c)^2+a^3*ln(sin(d*x+c))/d-1/d*a^2
*b/sin(d*x+c)^3*cos(d*x+c)^6+3/d*a^2*b/sin(d*x+c)*cos(d*x+c)^6+8*a^2*b*sin(
d*x+c)/d+3/d*a^2*b*sin(d*x+c)*cos(d*x+c)^4+4/d*a^2*b*cos(d*x+c)^2*sin(d*x+c
)-3/2/d*a*b^2/sin(d*x+c)^2*cos(d*x+c)^6-3/2/d*a*b^2*cos(d*x+c)^4-3/d*a*b^2*
cos(d*x+c)^2-6/d*a*b^2*ln(sin(d*x+c))-1/d*b^3/sin(d*x+c)*cos(d*x+c)^6-8/3/d
*b^3*sin(d*x+c)-1/d*b^3*sin(d*x+c)*cos(d*x+c)^4-4/3/d*b^3*cos(d*x+c)^2*sin(
d*x+c)
```

maxima [A] time = 0.64, size = 142, normalized size = 0.86

$$\frac{4\,b^3\sin(dx+c)^3+18\,ab^2\sin(dx+c)^2+12\left(a^3-6\,ab^2\right)\log(\sin(dx+c))+12\left(3\,a^2b-2\,b^3\right)\sin(dx+c)-\frac{3\left(4\,a^2\cdots\right.}{\cdots}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/12*(4*b^3*sin(d*x + c)^3 + 18*a*b^2*sin(d*x + c)^2 + 12*(a^3 - 6*a*b^2)*l
og(sin(d*x + c)) + 12*(3*a^2*b - 2*b^3)*sin(d*x + c) - 3*(4*a^2*b*sin(d*x +
 c) - 4*(6*a^2*b - b^3)*sin(d*x + c)^3 + a^3 - 2*(2*a^3 - 3*a*b^2)*sin(d*x
+ c)^2)/sin(d*x + c)^4)/d
```

mupad [B] time = 6.97, size = 424, normalized size = 2.57

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)\left(6\,a\,b^2-a^3\right)}{d}-\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(\frac{3\,a\,b^2}{8}-\frac{3\,a^3}{16}\right)}{d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\left(6\,a\,b^2-\cdots\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5*(a + b*sin(c + d*x))^3,x)
```

```
[Out] (log(tan(c/2 + (d*x)/2)^2 + 1)*(6*a*b^2 - a^3))/d - (a^3*tan(c/2 + (d*x)/2)
^4)/(64*d) - (tan(c/2 + (d*x)/2)^2*((3*a*b^2)/8 - (3*a^3)/16))/d - (log(tan
(c/2 + (d*x)/2))*(6*a*b^2 - a^3))/d + (tan(c/2 + (d*x)/2)^8*(90*a*b^2 + 3*a
^3) - tan(c/2 + (d*x)/2)^4*(18*a*b^2 - (33*a^3)/4) - tan(c/2 + (d*x)/2)^2*(
6*a*b^2 - (9*a^3)/4) + tan(c/2 + (d*x)/2)^6*(78*a*b^2 + (35*a^3)/4) + tan(c
/2 + (d*x)/2)^3*(36*a^2*b - 8*b^3) + tan(c/2 + (d*x)/2)^9*(138*a^2*b - 72*b
^3) + tan(c/2 + (d*x)/2)^5*(216*a^2*b - 88*b^3) + tan(c/2 + (d*x)/2)^7*(316
*a^2*b - (328*b^3)/3) - a^3/4 - 2*a^2*b*tan(c/2 + (d*x)/2))/(d*(16*tan(c/2
+ (d*x)/2)^4 + 48*tan(c/2 + (d*x)/2)^6 + 48*tan(c/2 + (d*x)/2)^8 + 16*tan(c
/2 + (d*x)/2)^10)) + (tan(c/2 + (d*x)/2)*((21*a^2*b)/8 - b^3/2))/d - (a^2*b
*tan(c/2 + (d*x)/2)^3)/(8*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin(c + dx))^3 \cot^5(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**3*cot(c + d*x)**5, x)
```

3.165 $\int (a+b\sin(c+dx))^3\tan^4(c+dx)\,dx$

Optimal. Leaf size=220

$$\frac{a^3\tan^3(c+dx)}{3d}-\frac{a^3\tan(c+dx)}{d}+a^3x-\frac{3a^2b\cos(c+dx)}{d}+\frac{a^2b\sec^3(c+dx)}{d}-\frac{6a^2b\sec(c+dx)}{d}+\frac{5ab^2\tan^3(c+dx)}{2d}-$$

```
[Out] a^3*x+15/2*a*b^2*x-3*a^2*b*cos(d*x+c)/d-3*b^3*cos(d*x+c)/d+1/3*b^3*cos(d*x+
c)^3/d-6*a^2*b*sec(d*x+c)/d-3*b^3*sec(d*x+c)/d+a^2*b*sec(d*x+c)^3/d+1/3*b^3
*sec(d*x+c)^3/d-a^3*tan(d*x+c)/d-15/2*a*b^2*tan(d*x+c)/d+1/3*a^3*tan(d*x+c)
^3/d+5/2*a*b^2*tan(d*x+c)^3/d-3/2*a*b^2*sin(d*x+c)^2*tan(d*x+c)^3/d
```

Rubi [A] time = 0.22, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2722, 3473, 8, 2590, 270, 2591, 288, 302, 203}

$$-\frac{3a^2b\cos(c+dx)}{d}+\frac{a^2b\sec^3(c+dx)}{d}-\frac{6a^2b\sec(c+dx)}{d}+\frac{a^3\tan^3(c+dx)}{3d}-\frac{a^3\tan(c+dx)}{d}+a^3x+\frac{5ab^2\tan^3(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^3*Tan[c + d*x]^4,x]
```

```
[Out] a^3*x + (15*a*b^2*x)/2 - (3*a^2*b*Cos[c + d*x])/d - (3*b^3*Cos[c + d*x])/d
+ (b^3*Cos[c + d*x]^3)/(3*d) - (6*a^2*b*Sec[c + d*x])/d - (3*b^3*Sec[c + d*
x])/d + (a^2*b*Sec[c + d*x]^3)/d + (b^3*Sec[c + d*x]^3)/(3*d) - (a^3*Tan[c
+ d*x])/d - (15*a*b^2*Tan[c + d*x])/(2*d) + (a^3*Tan[c + d*x]^3)/(3*d) + (5
*a*b^2*Tan[c + d*x]^3)/(2*d) - (3*a*b^2*Sin[c + d*x]^2*Tan[c + d*x]^3)/(2*d
)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int (a+b\sin(c+dx))^3\tan^4(c+dx)\,dx = \int \left(a^3\tan^4(c+dx)+3a^2b\sin(c+dx)\tan^4(c+dx)+3ab^2\sin^2(c+dx)\,\mathrm{t}\right.$$

$$= a^3\int \tan^4(c+dx)\,dx+\left(3a^2b\right)\int \sin(c+dx)\tan^4(c+dx)\,dx+\left(3ab^2\right)\int$$

$$= \frac{a^3\tan^3(c+dx)}{3d}-a^3\int \tan^2(c+dx)\,dx-\frac{\left(3a^2b\right)\mathrm{Subst}\left(\int \frac{(1-x^2)^2}{x^4}\,dx,x,\mathrm{c}\right.}{d}$$

$$= -\frac{a^3\tan(c+dx)}{d}+\frac{a^3\tan^3(c+dx)}{3d}-\frac{3ab^2\sin^2(c+dx)\tan^3(c+dx)}{2d}+a^3$$

$$= a^3x-\frac{3a^2b\cos(c+dx)}{d}-\frac{3b^3\cos(c+dx)}{d}+\frac{b^3\cos^3(c+dx)}{3d}-\frac{6a^2b\sec(}{d}$$

$$= a^3x-\frac{3a^2b\cos(c+dx)}{d}-\frac{3b^3\cos(c+dx)}{d}+\frac{b^3\cos^3(c+dx)}{3d}-\frac{6a^2b\sec(}{d}$$

$$= a^3x+\frac{15}{2}ab^2x-\frac{3a^2b\cos(c+dx)}{d}-\frac{3b^3\cos(c+dx)}{d}+\frac{b^3\cos^3(c+dx)}{3d}-$$

Mathematica [A] time = 0.71, size = 226, normalized size = 1.03

$$\frac{\sec^3(c+dx)\left(-32a^3\sin(3(c+dx))+24a^3c\cos(3(c+dx))+24a^3dx\cos(3(c+dx))-3\left(144a^2b+91b^3\right)\cos(2(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^3*Tan[c + d*x]^4,x]
```

```
[Out] (Sec[c + d*x]^3*(-300*a^2*b - 210*b^3 + 36*a*(2*a^2 + 15*b^2)*(c + d*x)*Cos
[c + d*x] - 3*(144*a^2*b + 91*b^3)*Cos[2*(c + d*x)] + 24*a^3*c*Cos[3*(c + d
*x)] + 180*a*b^2*c*Cos[3*(c + d*x)] + 24*a^3*d*x*Cos[3*(c + d*x)] + 180*a*b
^2*d*x*Cos[3*(c + d*x)] - 36*a^2*b*Cos[4*(c + d*x)] - 30*b^3*Cos[4*(c + d*x
)] + b^3*Cos[6*(c + d*x)] - 90*a*b^2*Sin[c + d*x] - 32*a^3*Sin[3*(c + d*x)]
 - 195*a*b^2*Sin[3*(c + d*x)] - 9*a*b^2*Sin[5*(c + d*x)]))/(96*d)
```

fricas [A] time = 0.44, size = 157, normalized size = 0.71

$$\frac{2\,b^3\cos(dx+c)^6+3\left(2\,a^3+15\,ab^2\right)dx\cos(dx+c)^3-18\left(a^2b+b^3\right)\cos(dx+c)^4+6\,a^2b+2\,b^3-18\left(2\,a^2b+b^3\right)}{6\,d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*cos(d*x + c)^6 + 3*(2*a^3 + 15*a*b^2)*d*x*cos(d*x + c)^3 - 18*(a
^2*b + b^3)*cos(d*x + c)^4 + 6*a^2*b + 2*b^3 - 18*(2*a^2*b + b^3)*cos(d*x +
 c)^2 - (9*a*b^2*cos(d*x + c)^4 - 2*a^3 - 6*a*b^2 + 2*(4*a^3 + 21*a*b^2)*co
s(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.25, size = 268, normalized size = 1.22

$$a^3\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+dx+c\right)+3a^2b\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^6(dx+c)}{\cos(dx+c)}-\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^3*tan(d*x+c)^4,x)
```

```
[Out] 1/d*(a^3*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c)+3*a^2*b*(1/3*sin(d*x+c)^6/cos(
d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*
x+c))+3*a*b^2*(1/3*sin(d*x+c)^7/cos(d*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/
3*(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c)
+b^3*(1/3*sin(d*x+c)^8/cos(d*x+c)^3-5/3*sin(d*x+c)^8/cos(d*x+c)-5/3*(16/5+s
in(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c)))
```

maxima [A] time = 1.26, size = 167, normalized size = 0.76

$$\frac{2\left(\tan(dx+c)^3+3\,dx+3\,c-3\,\tan(dx+c)\right)a^3+3\left(2\,\tan(dx+c)^3+15\,dx+15\,c-\frac{3\,\tan(dx+c)}{\tan(dx+c)^2+1}-12\,\tan(dx+\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a^3 + 3*(2*tan(d*x +
 c)^3 + 15*d*x + 15*c - 3*tan(d*x + c)/(tan(d*x + c)^2 + 1) - 12*tan(d*x +
```

```
c))*a*b^2 + 2*(cos(d*x + c)^3 - (9*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 - 9*c
os(d*x + c))*b^3 - 6*a^2*b*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d
*x + c)))/d
```

mupad [B] time = 9.20, size = 297, normalized size = 1.35

$$\frac{a\,\mathrm{atan}\left(\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(2\,a^2+15\,b^2\right)}{2\,a^3+15\,a\,b^2}\right)\left(2\,a^2+15\,b^2\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{2\,a^3}{3}+5\,a\,b^2\right)-16\,a^2\,b-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^3+15\,a\,b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + b*sin(c + d*x))^3,x)
```

```
[Out] (a*atan((a*tan(c/2 + (d*x)/2)*(2*a^2 + 15*b^2))/(15*a*b^2 + 2*a^3))*(2*a^2
+ 15*b^2))/d - (tan(c/2 + (d*x)/2)^3*(5*a*b^2 + (2*a^3)/3) - 16*a^2*b - tan
(c/2 + (d*x)/2)*(15*a*b^2 + 2*a^3) + tan(c/2 + (d*x)/2)^9*(5*a*b^2 + (2*a^3
)/3) - tan(c/2 + (d*x)/2)^11*(15*a*b^2 + 2*a^3) + tan(c/2 + (d*x)/2)^5*(42*
a*b^2 + 12*a^3) + tan(c/2 + (d*x)/2)^7*(42*a*b^2 + 12*a^3) + tan(c/2 + (d*x
)/2)^4*(48*a^2*b + 32*b^3) - (32*b^3)/3 + 32*a^2*b*tan(c/2 + (d*x)/2)^6)/(d
*(3*tan(c/2 + (d*x)/2)^4 - 3*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^12 -
 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin(c+dx))^3\tan^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**3*tan(d*x+c)**4,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**3*tan(c + d*x)**4, x)
```

3.166 $\int (a+b\sin(c+dx))^3\tan^2(c+dx)\,dx$

Optimal. Leaf size=146

$$\frac{a^3\tan(c+dx)}{d}+a^3(-x)+\frac{3a^2b\cos(c+dx)}{d}+\frac{3a^2b\sec(c+dx)}{d}+\frac{9ab^2\tan(c+dx)}{2d}-\frac{3ab^2\sin^2(c+dx)\tan(c+dx)}{2d}-$$

```
[Out] -a^3*x-9/2*a*b^2*x+3*a^2*b*cos(d*x+c)/d+2*b^3*cos(d*x+c)/d-1/3*b^3*cos(d*x+
c)^3/d+3*a^2*b*sec(d*x+c)/d+b^3*sec(d*x+c)/d+a^3*tan(d*x+c)/d+9/2*a*b^2*tan
(d*x+c)/d-3/2*a*b^2*sin(d*x+c)^2*tan(d*x+c)/d
```

Rubi [A] time = 0.17, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {2722, 3473, 8, 2590, 14, 2591, 288, 321, 203, 270}

$$\frac{3a^2b\cos(c+dx)}{d}+\frac{3a^2b\sec(c+dx)}{d}+\frac{a^3\tan(c+dx)}{d}+a^3(-x)+\frac{9ab^2\tan(c+dx)}{2d}-\frac{3ab^2\sin^2(c+dx)\tan(c+dx)}{2d}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] -(a^3*x) - (9*a*b^2*x)/2 + (3*a^2*b*Cos[c + d*x])/d + (2*b^3*Cos[c + d*x])/
d - (b^3*Cos[c + d*x]^3)/(3*d) + (3*a^2*b*Sec[c + d*x])/d + (b^3*Sec[c + d*
x])/d + (a^3*Tan[c + d*x])/d + (9*a*b^2*Tan[c + d*x])/(2*d) - (3*a*b^2*Sin[
c + d*x]^2*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int (a+b\sin(c+dx))^3\tan^2(c+dx)\,dx = \int \left(a^3\tan^2(c+dx)+3a^2b\sin(c+dx)\tan^2(c+dx)+3ab^2\sin^2(c+dx)\right.$$
$$= a^3\int \tan^2(c+dx)\,dx+\left(3a^2b\right)\int \sin(c+dx)\tan^2(c+dx)\,dx+\left(3ab^2\right)$$
$$= \frac{a^3\tan(c+dx)}{d}-a^3\int 1\,dx-\frac{\left(3a^2b\right)\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx,x,\cos(c+dx)\right)}{d}$$
$$= -a^3x+\frac{a^3\tan(c+dx)}{d}-\frac{3ab^2\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{\left(3a^2b\right)\text{Subst}}{}$$
$$= -a^3x+\frac{3a^2b\cos(c+dx)}{d}+\frac{2b^3\cos(c+dx)}{d}-\frac{b^3\cos^3(c+dx)}{3d}+\frac{3a^2b\,\text{s}}{}$$
$$= -a^3x-\frac{9}{2}ab^2x+\frac{3a^2b\cos(c+dx)}{d}+\frac{2b^3\cos(c+dx)}{d}-\frac{b^3\cos^3(c+dx)}{3d}$$

Mathematica [A] time = 0.77, size = 113, normalized size = 0.77

$$\frac{3a\left(\left(8a^2+27b^2\right)\tan(c+dx)-4\left(2a^2+9b^2\right)(c+dx)\right)+b\sec(c+dx)\left(4\left(9a^2+5b^2\right)\cos(2(c+dx))+108a^2+9\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] (b*Sec[c + d*x]*(108*a^2 + 45*b^2 + 4*(9*a^2 + 5*b^2)*Cos[2*(c + d*x)] - b^
2*Cos[4*(c + d*x)] + 9*a*b*Sin[3*(c + d*x)]) + 3*a*(-4*(2*a^2 + 9*b^2)*(c +
 d*x) + (8*a^2 + 27*b^2)*Tan[c + d*x]))/(24*d)
```

fricas [A] time = 0.42, size = 116, normalized size = 0.79

$$-\frac{2\,b^3\cos\left(dx+c\right)^4+3\left(2\,a^3+9\,ab^2\right)dx\cos\left(dx+c\right)-18\,a^2b-6\,b^3-6\left(3\,a^2b+2\,b^3\right)\cos\left(dx+c\right)^2-3\left(3\,ab^2\,\text{co}\right.}{6\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*b^3*cos(d*x + c)^4 + 3*(2*a^3 + 9*a*b^2)*d*x*cos(d*x + c) - 18*a^2*
b - 6*b^3 - 6*(3*a^2*b + 2*b^3)*cos(d*x + c)^2 - 3*(3*a*b^2*cos(d*x + c)^2
+ 2*a^3 + 6*a*b^2)*sin(d*x + c))/(d*cos(d*x + c))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.29, size = 169, normalized size = 1.16

$$\frac{a^3(\tan(dx+c)-dx-c)+3a^2b\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+\left(2+\sin^2(dx+c)\right)\cos(dx+c)\right)+3a\,b^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^3*tan(d*x+c)^2,x)
```

```
[Out] 1/d*(a^3*(tan(d*x+c)-d*x-c)+3*a^2*b*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^
2)*cos(d*x+c))+3*a*b^2*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c
))*cos(d*x+c)-3/2*d*x-3/2*c)+b^3*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d*x+c)^4
+4/3*sin(d*x+c)^2)*cos(d*x+c)))
```

maxima [A] time = 2.15, size = 119, normalized size = 0.82

$$-\frac{6(dx+c-\tan(dx+c))a^3+9\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\,\tan(dx+c)\right)ab^2+2\left(\cos(dx+c)^3-\frac{3}{\cos(dx+c)}-6\,\mathrm{c}\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(6*(d*x + c - tan(d*x + c))*a^3 + 9*(3*d*x + 3*c - tan(d*x + c)/(tan(d
*x + c)^2 + 1) - 2*tan(d*x + c))*a*b^2 + 2*(cos(d*x + c)^3 - 3/cos(d*x + c)
 - 6*cos(d*x + c))*b^3 - 18*a^2*b*(1/cos(d*x + c) + cos(d*x + c)))/d
```

mupad [B] time = 9.15, size = 249, normalized size = 1.71

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(2\,a^3+9\,a\,b^2\right)+12\,a^2\,b+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\left(2\,a^3+9\,a\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(6\,a^3+15\,a\,b^2\right)+\tan\left(\frac{c}{2}+\right.}{d\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+2\,\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + b*sin(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(9*a*b^2 + 2*a^3) + 12*a^2*b + tan(c/2 + (d*x)/2)^7*(9*
a*b^2 + 2*a^3) + tan(c/2 + (d*x)/2)^3*(15*a*b^2 + 6*a^3) + tan(c/2 + (d*x)/
2)^5*(15*a*b^2 + 6*a^3) + tan(c/2 + (d*x)/2)^2*(24*a^2*b + (32*b^3)/3) + (1
6*b^3)/3 + 12*a^2*b*tan(c/2 + (d*x)/2)^4)/(d*(2*tan(c/2 + (d*x)/2)^2 - 2*ta
n(c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^8 + 1)) - (a*atan((a*tan(c/2 + (d*x
)/2)*(2*a^2 + 9*b^2))/(9*a*b^2 + 2*a^3))*(2*a^2 + 9*b^2))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin(c+dx))^3 \tan^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**3*tan(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**3*tan(c + d*x)**2, x)
```

3.167 $\int \cot^2(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=102

$$-\frac{a^3\cot(c+dx)}{d}+a^3(-x)+\frac{3a^2b\cos(c+dx)}{d}-\frac{3a^2b\tanh^{-1}(\cos(c+dx))}{d}+\frac{3ab^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{3}{2}ab^2x-\frac{b^3 \cdots}{}$$

```
[Out] -a^3*x+3/2*a*b^2*x-3*a^2*b*arctanh(cos(d*x+c))/d+3*a^2*b*cos(d*x+c)/d-1/3*b
^3*cos(d*x+c)^3/d-a^3*cot(d*x+c)/d+3/2*a*b^2*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A] time = 0.11, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2722, 2635, 8, 2592, 321, 206, 3473, 2565, 30}

$$\frac{3a^2b\cos(c+dx)}{d}-\frac{3a^2b\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^3\cot(c+dx)}{d}+a^3(-x)+\frac{3ab^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{3}{2}ab^2x-\frac{b^3 \cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -(a^3*x) + (3*a*b^2*x)/2 - (3*a^2*b*ArcTanh[Cos[c + d*x]])/d + (3*a^2*b*Cos
[c + d*x])/d - (b^3*Cos[c + d*x]^3)/(3*d) - (a^3*Cot[c + d*x])/d + (3*a*b^2
*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \cot^2(c+dx)(a+b\sin(c+dx))^3\,dx = \int \left(3ab^2\cos^2(c+dx)+3a^2b\cos(c+dx)\cot(c+dx)+a^3\cot^2(c+dx)+\right.$$
$$= a^3\int \cot^2(c+dx)\,dx + \left(3a^2b\right)\int \cos(c+dx)\cot(c+dx)\,dx + \left(3ab^2\right)\int$$
$$= -\frac{a^3\cot(c+dx)}{d} + \frac{3ab^2\cos(c+dx)\sin(c+dx)}{2d} - a^3\int 1\,dx + \frac{1}{2}\left(3ab^2\right)$$
$$= -a^3x + \frac{3}{2}ab^2x + \frac{3a^2b\cos(c+dx)}{d} - \frac{b^3\cos^3(c+dx)}{3d} - \frac{a^3\cot(c+dx)}{d} +$$
$$= -a^3x + \frac{3}{2}ab^2x - \frac{3a^2b\tanh^{-1}(\cos(c+dx))}{d} + \frac{3a^2b\cos(c+dx)}{d} - \frac{b^3\cos^3}{3}$$

Mathematica [A] time = 1.32, size = 143, normalized size = 1.40

$$\frac{-6a^3\cot\left(\frac{1}{2}(c+dx)\right)+\left(36a^2b-3b^3\right)\cos(c+dx)+6a\left(a^2\tan\left(\frac{1}{2}(c+dx)\right)-2a^2c-2a^2dx+6ab\log\left(\sin\left(\frac{1}{2}(c+\right.\right.\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((36*a^2*b - 3*b^3)*Cos[c + d*x] - b^3*Cos[3*(c + d*x)] - 6*a^3*Cot[(c + d*
x)/2] + 9*a*b^2*Sin[2*(c + d*x)] + 6*a*(-2*a^2*c + 3*b^2*c - 2*a^2*d*x + 3*
b^2*d*x - 6*a*b*Log[Cos[(c + d*x)/2]] + 6*a*b*Log[Sin[(c + d*x)/2]] + a^2*T
an[(c + d*x)/2]))/(12*d)
```

fricas [A] time = 0.47, size = 143, normalized size = 1.40

$$-\frac{9\,ab^2\cos(dx+c)^3+9\,a^2b\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-9\,a^2b\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+}{6\,d\sin(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/6*(9*a*b^2*cos(d*x + c)^3 + 9*a^2*b*log(1/2*cos(d*x + c) + 1/2)*sin(d*x
+ c) - 9*a^2*b*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*(2*a^3 - 3*a*b
^2)*cos(d*x + c) + (2*b^3*cos(d*x + c)^3 - 18*a^2*b*cos(d*x + c) + 3*(2*a^3
 - 3*a*b^2)*d*x)*sin(d*x + c))/(d*sin(d*x + c))
```

giac [B] time = 1.62, size = 199, normalized size = 1.95

$$\frac{18\,a^2b\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right|\right) + 3\,a^3\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 3\left(2\,a^3 - 3\,ab^2\right)(dx + c) - \frac{3\left(6\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)} - \frac{2\left(9\,ab^2\right.}{}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(18*a^2*b*log(abs(tan(1/2*d*x + 1/2*c))) + 3*a^3*tan(1/2*d*x + 1/2*c) -
 3*(2*a^3 - 3*a*b^2)*(d*x + c) - 3*(6*a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/tan
(1/2*d*x + 1/2*c) - 2*(9*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 18*a^2*b*tan(1/2*d*
x + 1/2*c)^4 + 6*b^3*tan(1/2*d*x + 1/2*c)^4 - 36*a^2*b*tan(1/2*d*x + 1/2*c)
^2 - 9*a*b^2*tan(1/2*d*x + 1/2*c) - 18*a^2*b + 2*b^3)/(tan(1/2*d*x + 1/2*c)
^2 + 1)^3)/d
```

maple [A] time = 0.14, size = 125, normalized size = 1.23

$$-a^3x - \frac{a^3\cot(dx+c)}{d} - \frac{a^3c}{d} + \frac{3a^2b\cos(dx+c)}{d} + \frac{3a^2b\ln(\csc(dx+c) - \cot(dx+c))}{d} + \frac{3a\,b^2\cos(dx+c)\sin(dx+}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+b*sin(d*x+c))^3,x)
```

```
[Out] -a^3*x-a^3*cot(d*x+c)/d-1/d*a^3*c+3*a^2*b*cos(d*x+c)/d+3/d*a^2*b*ln(csc(d*x
+c)-cot(d*x+c))+3/2*a*b^2*cos(d*x+c)*sin(d*x+c)/d+3/2*a*b^2*x+3/2/d*a*b^2*c
-1/3*b^3*cos(d*x+c)^3/d
```

maxima [A] time = 0.90, size = 95, normalized size = 0.93

$$-\frac{4\,b^3\cos(dx+c)^3 + 12\left(dx + c + \frac{1}{\tan(dx+c)}\right)a^3 - 9\,(2\,dx + 2\,c + \sin(2\,dx + 2\,c))ab^2 - 18\,a^2b\left(2\,\cos(dx+c) - \mathrm{log}\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/12*(4*b^3*cos(d*x + c)^3 + 12*(d*x + c + 1/tan(d*x + c))*a^3 - 9*(2*d*x
+ 2*c + sin(2*d*x + 2*c))*a*b^2 - 18*a^2*b*(2*cos(d*x + c) - log(cos(d*x +
c) + 1) + log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 6.86, size = 289, normalized size = 2.83

$$\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\mathrm{i}\right)\left(\frac{a\,b^2\,3\mathrm{i}}{2}-a^3\,1\mathrm{i}\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(12\,a^2\,b-\frac{4\,b^3}{3}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(a^3+6\,a\,b^2\right.}{d\,\left(2\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + b*sin(c + d*x))^3,x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2))/(2*d) - (log(tan(c/2 + (d*x)/2) - 1i)*((a*b^2*3i)/
2 - a^3*1i))/d + (tan(c/2 + (d*x)/2)*(12*a^2*b - (4*b^3)/3) - tan(c/2 + (d*
x)/2)^6*(6*a*b^2 + a^3) - 3*a^3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^2
*(6*a*b^2 - 3*a^3) + tan(c/2 + (d*x)/2)^5*(12*a^2*b - 4*b^3) - a^3 + 24*a^2
*b*tan(c/2 + (d*x)/2)^3)/(d*(2*tan(c/2 + (d*x)/2) + 6*tan(c/2 + (d*x)/2)^3
+ 6*tan(c/2 + (d*x)/2)^5 + 2*tan(c/2 + (d*x)/2)^7)) + (3*a^2*b*log(tan(c/2
+ (d*x)/2)))/d - (a*log(tan(c/2 + (d*x)/2) + 1i)*(2*a^2 - 3*b^2)*1i)/(2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin{(c+dx)})^3\cot^2{(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**3*cot(c + d*x)**2, x)
```

3.168 $\int \cot^4(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=194

$$-\frac{a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot(c+dx)}{d}+a^3x-\frac{9a^2b\cos(c+dx)}{2d}-\frac{3a^2b\cos(c+dx)\cot^2(c+dx)}{2d}+\frac{9a^2b\tanh^{-1}(\cos(c+a$$

```
[Out] a^3*x-9/2*a*b^2*x+9/2*a^2*b*arctanh(cos(d*x+c))/d-b^3*arctanh(cos(d*x+c))/d
-9/2*a^2*b*cos(d*x+c)/d+b^3*cos(d*x+c)/d+1/3*b^3*cos(d*x+c)^3/d+a^3*cot(d*x
+c)/d-9/2*a*b^2*cot(d*x+c)/d+3/2*a*b^2*cos(d*x+c)^2*cot(d*x+c)/d-3/2*a^2*b*
cos(d*x+c)*cot(d*x+c)^2/d-1/3*a^3*cot(d*x+c)^3/d
```

Rubi [A] time = 0.18, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {2722, 2592, 302, 206, 2591, 288, 321, 203, 3473, 8}

$$-\frac{9a^2b\cos(c+dx)}{2d}-\frac{3a^2b\cos(c+dx)\cot^2(c+dx)}{2d}+\frac{9a^2b\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot(c+dx)}{d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] a^3*x - (9*a*b^2*x)/2 + (9*a^2*b*ArcTanh[Cos[c + d*x]])/(2*d) - (b^3*ArcTan
h[Cos[c + d*x]])/d - (9*a^2*b*Cos[c + d*x])/(2*d) + (b^3*Cos[c + d*x])/d +
(b^3*Cos[c + d*x]^3)/(3*d) + (a^3*Cot[c + d*x])/d - (9*a*b^2*Cot[c + d*x])/
(2*d) + (3*a*b^2*Cos[c + d*x]^2*Cot[c + d*x])/(2*d) - (3*a^2*b*Cos[c + d*x]
*Cot[c + d*x]^2)/(2*d) - (a^3*Cot[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \cot^4(c+dx)(a+b\sin(c+dx))^3\,dx = \int \left(b^3\cos^3(c+dx)\cot(c+dx)+3ab^2\cos^2(c+dx)\cot^2(c+dx)+3a^2b\right.$$
$$= a^3\int \cot^4(c+dx)\,dx+\left(3a^2b\right)\int \cos(c+dx)\cot^3(c+dx)\,dx+\left(3ab^2\right)$$
$$= -\frac{a^3\cot^3(c+dx)}{3d}-a^3\int \cot^2(c+dx)\,dx-\frac{\left(3a^2b\right)\text{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)^2}\,dx,x\right.}{d}$$
$$= \frac{a^3\cot(c+dx)}{d}+\frac{3ab^2\cos^2(c+dx)\cot(c+dx)}{2d}-\frac{3a^2b\cos(c+dx)\cot^2}{2d}$$
$$= a^3x-\frac{9a^2b\cos(c+dx)}{2d}+\frac{b^3\cos(c+dx)}{d}+\frac{b^3\cos^3(c+dx)}{3d}+\frac{a^3\cot(c+}{d}$$
$$= a^3x-\frac{9}{2}ab^2x+\frac{9a^2b\tanh^{-1}(\cos(c+dx))}{2d}-\frac{b^3\tanh^{-1}(\cos(c+dx))}{d}-\frac{9}{}$$

Mathematica [A] time = 6.23, size = 355, normalized size = 1.83

$$\frac{\csc\left(\frac{1}{2}(c+dx)\right)\left(4a^3\cos\left(\frac{1}{2}(c+dx)\right)-9ab^2\cos\left(\frac{1}{2}(c+dx)\right)\right)}{6d}+\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(9ab^2\sin\left(\frac{1}{2}(c+dx)\right)-4a^3\sin\left(\frac{1}{2}\right.\right.}{6d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (a*(2*a^2 - 9*b^2)*(c + d*x))/(2*d) + (b*(-12*a^2 + 5*b^2)*Cos[c + d*x])/(4
*d) + (b^3*Cos[3*(c + d*x)])/(12*d) + ((4*a^3*Cos[(c + d*x)/2] - 9*a*b^2*Co
s[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*d) - (3*a^2*b*Csc[(c + d*x)/2]^2)/(8*d
) - (a^3*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(24*d) + ((9*a^2*b - 2*b^3)*L
og[Cos[(c + d*x)/2]])/(2*d) + ((-9*a^2*b + 2*b^3)*Log[Sin[(c + d*x)/2]])/(2
*d) + (3*a^2*b*Sec[(c + d*x)/2]^2)/(8*d) + (Sec[(c + d*x)/2]*(-4*a^3*Sin[(c
 + d*x)/2] + 9*a*b^2*Sin[(c + d*x)/2]))/(6*d) - (3*a*b^2*Sin[2*(c + d*x)])/
(4*d) + (a^3*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24*d)
```

fricas [A] time = 0.49, size = 293, normalized size = 1.51

$$18\,ab^2\cos(dx+c)^5+8\left(2\,a^3-9\,ab^2\right)\cos(dx+c)^3-3\left(9\,a^2b-2\,b^3-\left(9\,a^2b-2\,b^3\right)\cos(dx+c)^2\right)\log\left(\frac{1}{2}\,\cos(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(18*a*b^2*cos(d*x + c)^5 + 8*(2*a^3 - 9*a*b^2)*cos(d*x + c)^3 - 3*(9*a
^2*b - 2*b^3 - (9*a^2*b - 2*b^3)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2
)*sin(d*x + c) + 3*(9*a^2*b - 2*b^3 - (9*a^2*b - 2*b^3)*cos(d*x + c)^2)*log
(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 6*(2*a^3 - 9*a*b^2)*cos(d*x + c) +
 2*(2*b^3*cos(d*x + c)^5 + 3*(2*a^3 - 9*a*b^2)*d*x*cos(d*x + c)^2 - 2*(9*a^
2*b - 2*b^3)*cos(d*x + c)^3 - 3*(2*a^3 - 9*a*b^2)*d*x + 3*(9*a^2*b - 2*b^3)
*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

giac [B] time = 0.69, size = 421, normalized size = 2.17

$$3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+27\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-45\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+108\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+36\left(2\,a^3-9\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/72*(3*a^3*tan(1/2*d*x + 1/2*c)^3 + 27*a^2*b*tan(1/2*d*x + 1/2*c)^2 - 45*a
^3*tan(1/2*d*x + 1/2*c) + 108*a*b^2*tan(1/2*d*x + 1/2*c) + 36*(2*a^3 - 9*a*
b^2)*(d*x + c) - 36*(9*a^2*b - 2*b^3)*log(abs(tan(1/2*d*x + 1/2*c))) + (198
*a^2*b*tan(1/2*d*x + 1/2*c)^9 - 44*b^3*tan(1/2*d*x + 1/2*c)^9 + 45*a^3*tan(
1/2*d*x + 1/2*c)^8 + 108*a*b^2*tan(1/2*d*x + 1/2*c)^8 + 135*a^2*b*tan(1/2*d
*x + 1/2*c)^7 + 156*b^3*tan(1/2*d*x + 1/2*c)^7 + 132*a^3*tan(1/2*d*x + 1/2*
c)^6 - 324*a*b^2*tan(1/2*d*x + 1/2*c)^6 - 351*a^2*b*tan(1/2*d*x + 1/2*c)^5
+ 156*b^3*tan(1/2*d*x + 1/2*c)^5 + 126*a^3*tan(1/2*d*x + 1/2*c)^4 - 540*a*b
^2*tan(1/2*d*x + 1/2*c)^4 - 315*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 148*b^3*tan(
1/2*d*x + 1/2*c)^3 + 36*a^3*tan(1/2*d*x + 1/2*c)^2 - 108*a*b^2*tan(1/2*d*x
+ 1/2*c)^2 - 27*a^2*b*tan(1/2*d*x + 1/2*c) - 3*a^3)/(tan(1/2*d*x + 1/2*c)^3
 + tan(1/2*d*x + 1/2*c))^3)/d
```

maple [A] time = 0.23, size = 264, normalized size = 1.36

$$-\frac{a^3\left(\cot^3(dx+c)\right)}{3d}+\frac{a^3\cot(dx+c)}{d}+a^3x+\frac{a^3c}{d}-\frac{3a^2b\left(\cos^5(dx+c)\right)}{2d\sin(dx+c)^2}-\frac{3a^2b\left(\cos^3(dx+c)\right)}{2d}-\frac{9a^2b\cos(dx+c)}{2d}-\frac{9a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/3*a^3*cot(d*x+c)^3/d+a^3*cot(d*x+c)/d+a^3*x+1/d*a^3*c-3/2/d*a^2*b/sin(d*
x+c)^2*cos(d*x+c)^5-3/2/d*a^2*b*cos(d*x+c)^3-9/2*a^2*b*cos(d*x+c)/d-9/2/d*a
```

```
^2*b*ln(csc(d*x+c)-cot(d*x+c))-3/d*a*b^2/sin(d*x+c)*cos(d*x+c)^5-3/d*a*b^2*
sin(d*x+c)*cos(d*x+c)^3-9/2*a*b^2*cos(d*x+c)*sin(d*x+c)/d-9/2*a*b^2*x-9/2/d
*a*b^2*c+1/3*b^3*cos(d*x+c)^3/d+b^3*cos(d*x+c)/d+1/d*b^3*ln(csc(d*x+c)-cot(
d*x+c))
```

maxima [A] time = 1.84, size = 187, normalized size = 0.96

$$\frac{4\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^3-18\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)}\right)ab^2+2\left(2\,\cos\left(dx+c\right)^3+6\,\cos\left(dx+c\right)-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/12*(4*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^3 - 18*(3*d
*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x + c)))*a*b^2 +
2*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) + 1) + 3*log(cos(
d*x + c) - 1))*b^3 + 9*a^2*b*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d
*x + c) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 6.81, size = 405, normalized size = 2.09

$$\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\left(\frac{9\,a^2\,b}{2}-b^3\right)}{d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)\left(\frac{a\,b^2\,9\mathrm{i}}{2}-a^3\,1\mathrm{i}\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{3\,a\,b^2}{2}-\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + b*sin(c + d*x))^3,x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2)^3)/(24*d) - (log(tan(c/2 + (d*x)/2))*((9*a^2*b)/2 -
 b^3))/d - (log(tan(c/2 + (d*x)/2) + 1i)*((a*b^2*9i)/2 - a^3*1i))/d + (tan(
c/2 + (d*x)/2)*((3*a*b^2)/2 - (5*a^3)/8))/d - (tan(c/2 + (d*x)/2)^2*(12*a*b
^2 - 4*a^3) - tan(c/2 + (d*x)/2)^8*(12*a*b^2 + 5*a^3) + tan(c/2 + (d*x)/2)^
4*(60*a*b^2 - 14*a^3) + tan(c/2 + (d*x)/2)^6*(36*a*b^2 - (44*a^3)/3) + tan(
c/2 + (d*x)/2)^7*(51*a^2*b - 32*b^3) + tan(c/2 + (d*x)/2)^3*(57*a^2*b - (64
*b^3)/3) + tan(c/2 + (d*x)/2)^5*(105*a^2*b - 32*b^3) + a^3/3 + 3*a^2*b*tan(
c/2 + (d*x)/2))/(d*(8*tan(c/2 + (d*x)/2)^3 + 24*tan(c/2 + (d*x)/2)^5 + 24*t
an(c/2 + (d*x)/2)^7 + 8*tan(c/2 + (d*x)/2)^9)) + (3*a^2*b*tan(c/2 + (d*x)/2
)^2)/(8*d) - (a*log(tan(c/2 + (d*x)/2) - 1i)*(2*a^2 - 9*b^2)*1i)/(2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sin\left(c+dx\right)\right)^3\cot^4\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**3*cot(c + d*x)**4, x)
```

3.169 $\int \cot^6(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=291

$$-\frac{a^3\cot^5(c+dx)}{5d}+\frac{a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}-a^3x+\frac{45a^2b\cos(c+dx)}{8d}-\frac{3a^2b\cos(c+dx)\cot^4(c+dx)}{4d}+\frac{15a^2}{}$$

```
[Out] -a^3*x+15/2*a*b^2*x-45/8*a^2*b*arctanh(cos(d*x+c))/d+5/2*b^3*arctanh(cos(d*
x+c))/d+45/8*a^2*b*cos(d*x+c)/d-5/2*b^3*cos(d*x+c)/d-5/6*b^3*cos(d*x+c)^3/d
-a^3*cot(d*x+c)/d+15/2*a*b^2*cot(d*x+c)/d+15/8*a^2*b*cos(d*x+c)*cot(d*x+c)^
2/d-1/2*b^3*cos(d*x+c)^3*cot(d*x+c)^2/d+1/3*a^3*cot(d*x+c)^3/d-5/2*a*b^2*co
t(d*x+c)^3/d+3/2*a*b^2*cos(d*x+c)^2*cot(d*x+c)^3/d-3/4*a^2*b*cos(d*x+c)*cot
(d*x+c)^4/d-1/5*a^3*cot(d*x+c)^5/d
```

Rubi [A] time = 0.23, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {2722, 2592, 288, 302, 206, 2591, 203, 321, 3473, 8}

$$\frac{45a^2b\cos(c+dx)}{8d}-\frac{3a^2b\cos(c+dx)\cot^4(c+dx)}{4d}+\frac{15a^2b\cos(c+dx)\cot^2(c+dx)}{8d}-\frac{45a^2b\tanh^{-1}(\cos(c+dx))}{8d}-$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -(a^3*x) + (15*a*b^2*x)/2 - (45*a^2*b*ArcTanh[Cos[c + d*x]])/(8*d) + (5*b^3
*ArcTanh[Cos[c + d*x]])/(2*d) + (45*a^2*b*Cos[c + d*x])/(8*d) - (5*b^3*Cos[
c + d*x])/(2*d) - (5*b^3*Cos[c + d*x]^3)/(6*d) - (a^3*Cot[c + d*x])/d + (15
*a*b^2*Cot[c + d*x])/(2*d) + (15*a^2*b*Cos[c + d*x]*Cot[c + d*x]^2)/(8*d) -
 (b^3*Cos[c + d*x]^3*Cot[c + d*x]^2)/(2*d) + (a^3*Cot[c + d*x]^3)/(3*d) - (
5*a*b^2*Cot[c + d*x]^3)/(2*d) + (3*a*b^2*Cos[c + d*x]^2*Cot[c + d*x]^3)/(2*
d) - (3*a^2*b*Cos[c + d*x]*Cot[c + d*x]^4)/(4*d) - (a^3*Cot[c + d*x]^5)/(5*
d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
```

```
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \cot^6(c+dx)(a+b\sin(c+dx))^3\,dx = \int \left(b^3\cos^3(c+dx)\cot^3(c+dx)+3ab^2\cos^2(c+dx)\cot^4(c+dx)+3a^2\right.$$

$$= a^3\int \cot^6(c+dx)\,dx+\left(3a^2b\right)\int \cos(c+dx)\cot^5(c+dx)\,dx+\left(3ab^2\right)$$

$$= -\frac{a^3\cot^5(c+dx)}{5d}-a^3\int \cot^4(c+dx)\,dx-\frac{\left(3a^2b\right)\operatorname{Subst}\left(\int \frac{x^6}{\left(1-x^2\right)^3}\,dx,x\right.}{d}$$

$$= -\frac{b^3\cos^3(c+dx)\cot^2(c+dx)}{2d}+\frac{a^3\cot^3(c+dx)}{3d}+\frac{3ab^2\cos^2(c+dx)\cot}{2d}$$

$$= -\frac{a^3\cot(c+dx)}{d}+\frac{15a^2b\cos(c+dx)\cot^2(c+dx)}{8d}-\frac{b^3\cos^3(c+dx)\cot}{2d}$$

$$= -a^3x+\frac{45a^2b\cos(c+dx)}{8d}-\frac{5b^3\cos(c+dx)}{2d}-\frac{5b^3\cos^3(c+dx)}{6d}-\frac{a^3\cot}{}$$

$$= -a^3x+\frac{15}{2}ab^2x-\frac{45a^2b\tanh^{-1}(\cos(c+dx))}{8d}+\frac{5b^3\tanh^{-1}(\cos(c+dx)}{2d}$$

Mathematica [A] time = 2.57, size = 346, normalized size = 1.19

$$\frac{-600a\left(2a^2-15b^2\right)(c+dx)\csc^4(c+dx)+1200b\left(4b^2-9a^2\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-600*a*(2*a^2 - 15*b^2)*(c + d*x)*Csc[c + d*x]^4 + 1200*b*(-9*a^2 + 4*b^2)
*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]]) + 5*Cot[c + d*x]*Csc[c + d
*x]^4*(-80*a^3 + 285*a*b^2 + 12*b*(60*a^2 - 29*b^2)*Sin[c + d*x]) + Csc[c +
```

```
d*x]^5*(5*(40*a^3 - 489*a*b^2)*Cos[3*(c + d*x)] + (-184*a^3 + 1065*a*b^2)*
Cos[5*(c + d*x)] + 5*(-9*a*b^2*Cos[7*(c + d*x)] + 60*a*(2*a^2 - 15*b^2)*(c
+ d*x)*Sin[3*(c + d*x)] - 306*a^2*b*Sin[4*(c + d*x)] + 122*b^3*Sin[4*(c + d
*x)] - 24*a^3*c*Sin[5*(c + d*x)] + 180*a*b^2*c*Sin[5*(c + d*x)] - 24*a^3*d*
x*Sin[5*(c + d*x)] + 180*a*b^2*d*x*Sin[5*(c + d*x)] + 36*a^2*b*Sin[6*(c + d
*x)] - 22*b^3*Sin[6*(c + d*x)] - b^3*Sin[8*(c + d*x)])))/(1920*d)
```

fricas [A] time = 0.57, size = 412, normalized size = 1.42

$$-\frac{360\,ab^2\cos(dx+c)^7+184\left(2\,a^3-15\,ab^2\right)\cos(dx+c)^5-280\left(2\,a^3-15\,ab^2\right)\cos(dx+c)^3+75\left(\left(9\,a^2b-4\,b^3\right)\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/240*(360*a*b^2*cos(d*x + c)^7 + 184*(2*a^3 - 15*a*b^2)*cos(d*x + c)^5 -
280*(2*a^3 - 15*a*b^2)*cos(d*x + c)^3 + 75*((9*a^2*b - 4*b^3)*cos(d*x + c)^
4 + 9*a^2*b - 4*b^3 - 2*(9*a^2*b - 4*b^3)*cos(d*x + c)^2)*log(1/2*cos(d*x +
 c) + 1/2)*sin(d*x + c) - 75*((9*a^2*b - 4*b^3)*cos(d*x + c)^4 + 9*a^2*b -
4*b^3 - 2*(9*a^2*b - 4*b^3)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2)*si
n(d*x + c) + 120*(2*a^3 - 15*a*b^2)*cos(d*x + c) + 10*(8*b^3*cos(d*x + c)^7
 + 12*(2*a^3 - 15*a*b^2)*d*x*cos(d*x + c)^4 - 8*(9*a^2*b - 4*b^3)*cos(d*x +
 c)^5 - 24*(2*a^3 - 15*a*b^2)*d*x*cos(d*x + c)^2 + 25*(9*a^2*b - 4*b^3)*cos
(d*x + c)^3 + 12*(2*a^3 - 15*a*b^2)*d*x - 15*(9*a^2*b - 4*b^3)*cos(d*x + c)
)*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

giac [A] time = 1.05, size = 471, normalized size = 1.62

$$\frac{6\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+45\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-70\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+120\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-720\,a^2b\,\mathrm{ta}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/960*(6*a^3*tan(1/2*d*x + 1/2*c)^5 + 45*a^2*b*tan(1/2*d*x + 1/2*c)^4 - 70*
a^3*tan(1/2*d*x + 1/2*c)^3 + 120*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 720*a^2*b*t
an(1/2*d*x + 1/2*c)^2 + 120*b^3*tan(1/2*d*x + 1/2*c)^2 + 660*a^3*tan(1/2*d*
x + 1/2*c) - 3240*a*b^2*tan(1/2*d*x + 1/2*c) - 480*(2*a^3 - 15*a*b^2)*(d*x
+ c) + 600*(9*a^2*b - 4*b^3)*log(abs(tan(1/2*d*x + 1/2*c))) - 320*(9*a*b^2*
tan(1/2*d*x + 1/2*c)^5 - 18*a^2*b*tan(1/2*d*x + 1/2*c)^4 + 18*b^3*tan(1/2*d
*x + 1/2*c)^4 - 36*a^2*b*tan(1/2*d*x + 1/2*c)^2 + 24*b^3*tan(1/2*d*x + 1/2*
```

```
c)^2 - 9*a*b^2*tan(1/2*d*x + 1/2*c) - 18*a^2*b + 14*b^3)/(tan(1/2*d*x + 1/2
*c)^2 + 1)^3 - (12330*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 5480*b^3*tan(1/2*d*x +
 1/2*c)^5 + 660*a^3*tan(1/2*d*x + 1/2*c)^4 - 3240*a*b^2*tan(1/2*d*x + 1/2*c
)^4 - 720*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 120*b^3*tan(1/2*d*x + 1/2*c)^3 - 7
0*a^3*tan(1/2*d*x + 1/2*c)^2 + 120*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 45*a^2*b*
tan(1/2*d*x + 1/2*c) + 6*a^3)/tan(1/2*d*x + 1/2*c)^5)/d
```

maple [A] time = 0.26, size = 415, normalized size = 1.43

$$-\frac{a^3\left(\cot^5(dx+c)\right)}{5d}+\frac{a^3\left(\cot^3(dx+c)\right)}{3d}-\frac{a^3\cot(dx+c)}{d}-a^3x-\frac{a^3c}{d}-\frac{3a^2b\left(\cos^7(dx+c)\right)}{4d\sin(dx+c)^4}+\frac{9a^2b\left(\cos^7(dx+c)\right)}{8d\sin(dx+c)^2}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/5*a^3*cot(d*x+c)^5/d+1/3*a^3*cot(d*x+c)^3/d-a^3*cot(d*x+c)/d-a^3*x-1/d*a
^3*c-3/4/d*a^2*b/sin(d*x+c)^4*cos(d*x+c)^7+9/8/d*a^2*b/sin(d*x+c)^2*cos(d*x
+c)^7+9/8/d*a^2*b*cos(d*x+c)^5+15/8/d*a^2*b*cos(d*x+c)^3+45/8*a^2*b*cos(d*x
+c)/d+45/8/d*a^2*b*ln(csc(d*x+c)-cot(d*x+c))-1/d*a*b^2/sin(d*x+c)^3*cos(d*x
+c)^7+4/d*a*b^2/sin(d*x+c)*cos(d*x+c)^7+4/d*a*b^2*sin(d*x+c)*cos(d*x+c)^5+5
/d*a*b^2*sin(d*x+c)*cos(d*x+c)^3+15/2*a*b^2*cos(d*x+c)*sin(d*x+c)/d+15/2*a*
b^2*x+15/2/d*a*b^2*c-1/2/d*b^3/sin(d*x+c)^2*cos(d*x+c)^7-1/2/d*b^3*cos(d*x+
c)^5-5/6*b^3*cos(d*x+c)^3/d-5/2*b^3*cos(d*x+c)/d-5/2/d*b^3*ln(csc(d*x+c)-co
t(d*x+c))
```

maxima [A] time = 1.89, size = 252, normalized size = 0.87

$$-\frac{16\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a^3-120\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4+10\,\tan(dx+c)^2-2}{\tan(dx+c)^5+\tan(dx+c)^3}\right)ab^2+20\left(4\,\mathrm{co}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/240*(16*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(
d*x + c)^5)*a^3 - 120*(15*d*x + 15*c + (15*tan(d*x + c)^4 + 10*tan(d*x + c)
^2 - 2)/(tan(d*x + c)^5 + tan(d*x + c)^3))*a*b^2 + 20*(4*cos(d*x + c)^3 - 6
*cos(d*x + c)/(cos(d*x + c)^2 - 1) + 24*cos(d*x + c) - 15*log(cos(d*x + c)
+ 1) + 15*log(cos(d*x + c) - 1))*b^3 + 45*a^2*b*(2*(9*cos(d*x + c)^3 - 7*co
s(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - 16*cos(d*x + c) + 15*
log(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)))/d
```

mupad [B] time = 7.06, size = 507, normalized size = 1.74

$$\frac{a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{160\,d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\left(12\,a\,b^2-22\,a^3\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(4\,a\,b^2-\frac{26\,a^3}{15}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(96\,a\,b^2-\frac{78}{5}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6*(a + b*sin(c + d*x))^3,x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2)^5)/(160*d) + (tan(c/2 + (d*x)/2)^10*(12*a*b^2 - 22*
a^3) - tan(c/2 + (d*x)/2)^2*(4*a*b^2 - (26*a^3)/15) + tan(c/2 + (d*x)/2)^4*
(96*a*b^2 - (78*a^3)/5) + tan(c/2 + (d*x)/2)^8*(320*a*b^2 - (191*a^3)/3) +
tan(c/2 + (d*x)/2)^6*(408*a*b^2 - (296*a^3)/5) + tan(c/2 + (d*x)/2)^3*((39*
a^2*b)/2 - 4*b^3) + tan(c/2 + (d*x)/2)^9*(216*a^2*b - 196*b^3) + tan(c/2 +
(d*x)/2)^5*((519*a^2*b)/2 - (484*b^3)/3) + tan(c/2 + (d*x)/2)^7*((909*a^2*b
)/2 - 268*b^3) - a^3/5 - (3*a^2*b*tan(c/2 + (d*x)/2))/2)/(d*(32*tan(c/2 + (
d*x)/2)^5 + 96*tan(c/2 + (d*x)/2)^7 + 96*tan(c/2 + (d*x)/2)^9 + 32*tan(c/2
+ (d*x)/2)^11)) + (tan(c/2 + (d*x)/2)^3*((a*b^2)/8 - (7*a^3)/96))/d - (tan(
c/2 + (d*x)/2)^2*((3*a^2*b)/4 - b^3/8))/d + (log(tan(c/2 + (d*x)/2))*((45*a
^2*b)/8 - (5*b^3)/2))/d - (log(tan(c/2 + (d*x)/2) - 1i)*((a*b^2*15i)/2 - a^
3*1i))/d - (tan(c/2 + (d*x)/2)*((27*a*b^2)/8 - (11*a^3)/16))/d + (3*a^2*b*t
an(c/2 + (d*x)/2)^4)/(64*d) - (a*log(tan(c/2 + (d*x)/2) + 1i)*(2*a^2 - 15*b
^2)*1i)/(2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.170 $\int \frac{\tan^5(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=204

$$-\frac{\left(8a^2+9ab+3b^2\right)\log(1-\sin(c+dx))}{16d(a+b)^3}-\frac{\left(8a^2-9ab+3b^2\right)\log(\sin(c+dx)+1)}{16d(a-b)^3}+\frac{\sec^4(c+dx)(a-b\sin(c+dx$$

```
[Out] -1/16*(8*a^2+9*a*b+3*b^2)*ln(1-sin(d*x+c))/(a+b)^3/d-1/16*(8*a^2-9*a*b+3*b^
2)*ln(1+sin(d*x+c))/(a-b)^3/d+a^5*ln(a+b*sin(d*x+c))/(a^2-b^2)^3/d+1/4*sec(
d*x+c)^4*(a-b*sin(d*x+c))/(a^2-b^2)/d-1/8*sec(d*x+c)^2*(4*a*(2*a^2-b^2)-b*(
9*a^2-5*b^2)*sin(d*x+c))/(a^2-b^2)^2/d
```

Rubi [A] time = 0.36, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2721, 1647, 801}

$$\frac{a^5\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^3}-\frac{\left(8a^2+9ab+3b^2\right)\log(1-\sin(c+dx))}{16d(a+b)^3}-\frac{\left(8a^2-9ab+3b^2\right)\log(\sin(c+dx)+1)}{16d(a-b)^3}+\sec$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((8*a^2 + 9*a*b + 3*b^2)*Log[1 - Sin[c + d*x]])/(16*(a + b)^3*d) - ((8*a^2
 - 9*a*b + 3*b^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^3*d) + (a^5*Log[a + b*
Sin[c + d*x]])/((a^2 - b^2)^3*d) + (Sec[c + d*x]^4*(a - b*Sin[c + d*x]))/(4
*(a^2 - b^2)*d) - (Sec[c + d*x]^2*(4*a*(2*a^2 - b^2) - b*(9*a^2 - 5*b^2)*Si
n[c + d*x]))/(8*(a^2 - b^2)^2*d)
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
```

```
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4\left(a^2-b^2\right)d} + \frac{\text{Subst}\left(\int \frac{\frac{ab^6}{a^2-b^2}-\frac{b^4\left(4a^2-b^2\right)x}{a^2-b^2}-4b^2x^3}{(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{4b^2d}$$

$$= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4\left(a^2-b^2\right)d} - \frac{\sec^2(c+dx)\left(4a\left(2a^2-b^2\right)-b\left(9a^2-5b^2\right)\sin(c+dx\right.}{8\left(a^2-b^2\right)^2d}$$

$$= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4\left(a^2-b^2\right)d} - \frac{\sec^2(c+dx)\left(4a\left(2a^2-b^2\right)-b\left(9a^2-5b^2\right)\sin(c+dx\right.}{8\left(a^2-b^2\right)^2d}$$

$$= -\frac{\left(8a^2+9ab+3b^2\right)\log(1-\sin(c+dx))}{16(a+b)^3d} - \frac{\left(8a^2-9ab+3b^2\right)\log(1+\sin(c+dx))}{16(a-b)^3d} + \frac{a^5}{}$$

Mathematica [A] time = 1.30, size = 184, normalized size = 0.90

$$\frac{\frac{16a^5\log(a+b\sin(c+dx))}{(a-b)^3(a+b)^3} - \frac{\left(8a^2+9ab+3b^2\right)\log(1-\sin(c+dx))}{(a+b)^3} - \frac{\left(8a^2-9ab+3b^2\right)\log(\sin(c+dx)+1)}{(a-b)^3} + \frac{7a+5b}{(a+b)^2(\sin(c+dx)-1)} + \frac{5b-7a}{(a-b)^2(\sin(c+dx)+1)}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-(((8*a^2 + 9*a*b + 3*b^2)*Log[1 - Sin[c + d*x]])/(a + b)^3) - ((8*a^2 - 9
*a*b + 3*b^2)*Log[1 + Sin[c + d*x]])/(a - b)^3 + (16*a^5*Log[a + b*Sin[c +
d*x]])/((a - b)^3*(a + b)^3) + 1/((a + b)*(-1 + Sin[c + d*x])^2) + (7*a + 5
*b)/((a + b)^2*(-1 + Sin[c + d*x])) + 1/((a - b)*(1 + Sin[c + d*x])^2) + (-
7*a + 5*b)/((a - b)^2*(1 + Sin[c + d*x])))/(16*d)
```

fricas [A] time = 0.63, size = 261, normalized size = 1.28

$$\frac{16\,a^{5}\cos\left(dx+c\right)^{4}\log\left(b\sin\left(dx+c\right)+a\right)-\left(8\,a^{5}+15\,a^{4}b-10\,a^{2}b^{3}+3\,b^{5}\right)\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(16*a^5*cos(d*x + c)^4*log(b*sin(d*x + c) + a) - (8*a^5 + 15*a^4*b - 1
0*a^2*b^3 + 3*b^5)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (8*a^5 - 15*a^4*b
 + 10*a^2*b^3 - 3*b^5)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 4*a^5 - 8*a^
3*b^2 + 4*a*b^4 - 8*(2*a^5 - 3*a^3*b^2 + a*b^4)*cos(d*x + c)^2 - 2*(2*a^4*b
 - 4*a^2*b^3 + 2*b^5 - (9*a^4*b - 14*a^2*b^3 + 5*b^5)*cos(d*x + c)^2)*sin(d
*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^4)
```

giac [A] time = 10.46, size = 343, normalized size = 1.68

$$\frac{\frac{16\,a^{5}b\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{a^{6}b-3\,a^{4}b^{3}+3\,a^{2}b^{5}-b^{7}}-\frac{\left(8\,a^{2}-9\,ab+3\,b^{2}\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)}{a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3}}-\frac{\left(8\,a^{2}+9\,ab+3\,b^{2}\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}}+\frac{2\left(6\,a^{5}\sin\left(dx+c\right)^{4}-9\,a^{4}b\sin\left(dx+c\right)^{3}+\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*(16*a^5*b*log(abs(b*sin(d*x + c) + a))/(a^6*b - 3*a^4*b^3 + 3*a^2*b^5
- b^7) - (8*a^2 - 9*a*b + 3*b^2)*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2*b
+ 3*a*b^2 - b^3) - (8*a^2 + 9*a*b + 3*b^2)*log(abs(sin(d*x + c) - 1))/(a^3
+ 3*a^2*b + 3*a*b^2 + b^3) + 2*(6*a^5*sin(d*x + c)^4 - 9*a^4*b*sin(d*x + c)
^3 + 14*a^2*b^3*sin(d*x + c)^3 - 5*b^5*sin(d*x + c)^3 - 4*a^5*sin(d*x + c)^
2 - 12*a^3*b^2*sin(d*x + c)^2 + 4*a*b^4*sin(d*x + c)^2 + 7*a^4*b*sin(d*x +
c) - 10*a^2*b^3*sin(d*x + c) + 3*b^5*sin(d*x + c) + 8*a^3*b^2 - 2*a*b^4)/((
a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(sin(d*x + c)^2 - 1)^2))/d
```

maple [A] time = 0.18, size = 304, normalized size = 1.49

$$\frac{1}{2d\left(8a+8b\right)\left(\sin\left(dx+c\right)-1\right)^{2}}+\frac{7a}{16d\left(a+b\right)^{2}\left(\sin\left(dx+c\right)-1\right)}+\frac{5b}{16d\left(a+b\right)^{2}\left(\sin\left(dx+c\right)-1\right)}-\frac{\ln\left(\sin\left(dx+c\right)\right.}{2d\left(a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/2/d/(8*a+8*b)/(sin(d*x+c)-1)^2+7/16/d/(a+b)^2/(sin(d*x+c)-1)*a+5/16/d/(a+
b)^2/(sin(d*x+c)-1)*b-1/2/d/(a+b)^3*ln(sin(d*x+c)-1)*a^2-9/16/d/(a+b)^3*ln(
sin(d*x+c)-1)*a*b-3/16/d/(a+b)^3*ln(sin(d*x+c)-1)*b^2+1/d*a^5/(a+b)^3/(a-b)
^3*ln(a+b*sin(d*x+c))+1/2/d/(8*a-8*b)/(1+sin(d*x+c))^2-7/16/d/(a-b)^2/(1+si
n(d*x+c))*a+5/16/d/(a-b)^2/(1+sin(d*x+c))*b-1/2/d/(a-b)^3*ln(1+sin(d*x+c))*
a^2+9/16/d/(a-b)^3*ln(1+sin(d*x+c))*a*b-3/16/d/(a-b)^3*ln(1+sin(d*x+c))*b^2
```

maxima [A] time = 1.90, size = 288, normalized size = 1.41

$$\frac{\frac{16\,a^5\log(b\sin(dx+c)+a)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}-\frac{\left(8\,a^2-9\,ab+3\,b^2\right)\log(\sin(dx+c)+1)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{\left(8\,a^2+9\,ab+3\,b^2\right)\log(\sin(dx+c)-1)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{2\left(\left(9\,a^2b-5\,b^3\right)\sin(dx+c)^3+6\,a^3-2\,ab^2-4\left(2\right.\right.}{\left(a^4-2\,a^2b^2+b^4\right)\sin(dx+c)^4+a^4-2\,a^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(16*a^5*log(b*sin(d*x + c) + a)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) -
(8*a^2 - 9*a*b + 3*b^2)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^
3) - (8*a^2 + 9*a*b + 3*b^2)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2
 + b^3) - 2*((9*a^2*b - 5*b^3)*sin(d*x + c)^3 + 6*a^3 - 2*a*b^2 - 4*(2*a^3
- a*b^2)*sin(d*x + c)^2 - (7*a^2*b - 3*b^3)*sin(d*x + c))/((a^4 - 2*a^2*b^2
 + b^4)*sin(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*
sin(d*x + c)^2))/d
```

mupad [B] time = 7.43, size = 498, normalized size = 2.44

$$\frac{a^5\,\ln\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}{d\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(\frac{1}{a+b}-\frac{7\,b}{8\,(a+b)^2}+\frac{b^2}{4\,(a+b)^3}\right)}{d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + b*sin(c + d*x)),x)
```

```
[Out] (a^5*log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2))/(d*(a^6 - b^
6 + 3*a^2*b^4 - 3*a^4*b^2)) - (log(tan(c/2 + (d*x)/2) - 1)*(1/(a + b) - (7*
b)/(8*(a + b)^2) + b^2/(4*(a + b)^3)))/d - (log(tan(c/2 + (d*x)/2) + 1)*(b^
2/(4*(a - b)^3) + (7*b)/(8*(a - b)^2) + 1/(a - b)))/d - ((2*a^3*tan(c/2 + (
d*x)/2)^2)/(a^4 + b^4 - 2*a^2*b^2) + (2*a^3*tan(c/2 + (d*x)/2)^6)/(a^4 + b^
4 - 2*a^2*b^2) + (4*tan(c/2 + (d*x)/2)^4*(a*b^2 - 2*a^3))/(a^4 + b^4 - 2*a^
```

```
2*b^2) - (tan(c/2 + (d*x)/2)^7*(7*a^2*b - 3*b^3))/(4*(a^4 + b^4 - 2*a^2*b^2
)) + (tan(c/2 + (d*x)/2)^3*(15*a^2*b - 11*b^3))/(4*(a^4 + b^4 - 2*a^2*b^2))
 + (tan(c/2 + (d*x)/2)^5*(15*a^2*b - 11*b^3))/(4*(a^4 + b^4 - 2*a^2*b^2)) -
 (b*tan(c/2 + (d*x)/2)*(7*a^2 - 3*b^2))/(4*(a^4 + b^4 - 2*a^2*b^2)))/(d*(6*
tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + ta
n(c/2 + (d*x)/2)^8 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^5(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**5/(a + b*sin(c + d*x)), x)
```

3.171 $\int \frac{\tan^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=126

$$\frac{\sec^2(c+dx)(a-b\sin(c+dx))}{2d\left(a^2-b^2\right)}-\frac{a^3\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^2}+\frac{(2a+b)\log(1-\sin(c+dx))}{4d(a+b)^2}+\frac{(2a-b)\log(\sin(c+dx}{4d(a-b)^2}$$

```
[Out] 1/4*(2*a+b)*ln(1-sin(d*x+c))/(a+b)^2/d+1/4*(2*a-b)*ln(1+sin(d*x+c))/(a-b)^2
/d-a^3*ln(a+b*sin(d*x+c))/(a^2-b^2)^2/d+1/2*sec(d*x+c)^2*(a-b*sin(d*x+c))/(
a^2-b^2)/d
```

Rubi [A] time = 0.19, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2721, 1647, 801}

$$-\frac{a^3\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^2}+\frac{\sec^2(c+dx)(a-b\sin(c+dx))}{2d\left(a^2-b^2\right)}+\frac{(2a+b)\log(1-\sin(c+dx))}{4d(a+b)^2}+\frac{(2a-b)\log(\sin(c+}{4d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((2*a + b)*Log[1 - Sin[c + d*x]])/(4*(a + b)^2*d) + ((2*a - b)*Log[1 + Sin[
c + d*x]])/(4*(a - b)^2*d) - (a^3*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^2*d
) + (Sec[c + d*x]^2*(a - b*Sin[c + d*x]))/(2*(a^2 - b^2)*d)
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^2(c+dx)(a-b\sin(c+dx))}{2\left(a^2-b^2\right)d} + \frac{\text{Subst}\left(\int \frac{\frac{ab^4}{a^2-b^2}-\frac{b^2\left(2a^2-b^2\right)x}{a^2-b^2}}{(a+x)\left(b^2-x^2\right)}\,dx, x, b\sin(c+dx)\right)}{2b^2d}$$

$$= \frac{\sec^2(c+dx)(a-b\sin(c+dx))}{2\left(a^2-b^2\right)d} + \frac{\text{Subst}\left(\int \left(-\frac{b^2(2a+b)}{2(a+b)^2(b-x)} - \frac{2a^3b^2}{(a-b)^2(a+b)^2(a+x)} + \frac{(2a-b)b^2}{2(a-b)^2(b+}\right.\right.}{2b^2d}$$

$$= \frac{(2a+b)\log(1-\sin(c+dx))}{4(a+b)^2d} + \frac{(2a-b)\log(1+\sin(c+dx))}{4(a-b)^2d} - \frac{a^3\log(a+b\sin(c+d}{\left(a^2-b^2\right)^2d}$$

Mathematica [A] time = 0.49, size = 117, normalized size = 0.93

$$\frac{-\frac{4a^3\log(a+b\sin(c+dx))}{(a-b)^2(a+b)^2} - \frac{1}{(a+b)(\sin(c+dx)-1)} + \frac{1}{(a-b)(\sin(c+dx)+1)} + \frac{(2a+b)\log(1-\sin(c+dx))}{(a+b)^2} + \frac{(2a-b)\log(\sin(c+dx)+1)}{(a-b)^2}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

```
[Out] (((2*a + b)*Log[1 - Sin[c + d*x]])/(a + b)^2 + ((2*a - b)*Log[1 + Sin[c + d
*x]])/(a - b)^2 - (4*a^3*Log[a + b*Sin[c + d*x]])/((a - b)^2*(a + b)^2) - 1
/((a + b)*(-1 + Sin[c + d*x])) + 1/((a - b)*(1 + Sin[c + d*x])))/(4*d)
```

fricas [A] time = 0.54, size = 157, normalized size = 1.25

$$-\frac{4a^3\cos(dx+c)^2\log(b\sin(dx+c)+a) - \left(2a^3+3a^2b-b^3\right)\cos(dx+c)^2\log(\sin(dx+c)+1) - \left(2a^3-3a^2\right.}{4\left(a^4-2a^2b^2+b^4\right)d\cos(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(4*a^3*cos(d*x + c)^2*log(b*sin(d*x + c) + a) - (2*a^3 + 3*a^2*b - b^3
)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*a^3 - 3*a^2*b + b^3)*cos(d*x +
c)^2*log(-sin(d*x + c) + 1) - 2*a^3 + 2*a*b^2 + 2*(a^2*b - b^3)*sin(d*x + c
))/((a^4 - 2*a^2*b^2 + b^4)*d*cos(d*x + c)^2)
```

giac [A] time = 2.10, size = 177, normalized size = 1.40

$$-\frac{\frac{4\,a^3 b\log(|b\sin(dx+c)+a|)}{a^4 b-2\,a^2b^3+b^5}-\frac{(2\,a-b)\log(|\sin(dx+c)+1|)}{a^2-2\,ab+b^2}-\frac{(2\,a+b)\log(|\sin(dx+c)-1|)}{a^2+2\,ab+b^2}+\frac{2\left(a^3\sin(dx+c)^2-a^2b\sin(dx+c)+b^3\sin(dx+c)-ab^2\right)}{\left(a^4-2\,a^2b^2+b^4\right)\left(\sin(dx+c)^2-1\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(4*a^3*b*log(abs(b*sin(d*x + c) + a))/(a^4*b - 2*a^2*b^3 + b^5) - (2*a
 - b)*log(abs(sin(d*x + c) + 1))/(a^2 - 2*a*b + b^2) - (2*a + b)*log(abs(si
n(d*x + c) - 1))/(a^2 + 2*a*b + b^2) + 2*(a^3*sin(d*x + c)^2 - a^2*b*sin(d*
x + c) + b^3*sin(d*x + c) - a*b^2)/((a^4 - 2*a^2*b^2 + b^4)*(sin(d*x + c)^2
 - 1)))/d
```

maple [A] time = 0.18, size = 164, normalized size = 1.30

$$-\frac{1}{d\left(4a+4b\right)\left(\sin\left(dx+c\right)-1\right)}+\frac{\ln\left(\sin\left(dx+c\right)-1\right)a}{2d\left(a+b\right)^{2}}+\frac{\ln\left(\sin\left(dx+c\right)-1\right)b}{4d\left(a+b\right)^{2}}-\frac{a^{3}\ln\left(a+b\sin\left(dx+c\right)\right)}{d\left(a+b\right)^{2}\left(a-b\right)^{2}}+\frac{}{d\left(4a-4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d/(4*a+4*b)/(sin(d*x+c)-1)+1/2/d/(a+b)^2*ln(sin(d*x+c)-1)*a+1/4/d/(a+b)^
2*ln(sin(d*x+c)-1)*b-1/d*a^3/(a+b)^2/(a-b)^2*ln(a+b*sin(d*x+c))+1/d/(4*a-4*
b)/(1+sin(d*x+c))+1/2/d/(a-b)^2*ln(1+sin(d*x+c))*a-1/4/d/(a-b)^2*ln(1+sin(d
*x+c))*b
```

maxima [A] time = 0.69, size = 142, normalized size = 1.13

$$-\frac{\frac{4\,a^3\log(b\sin(dx+c)+a)}{a^4-2\,a^2b^2+b^4}-\frac{(2\,a-b)\log(\sin(dx+c)+1)}{a^2-2\,ab+b^2}-\frac{(2\,a+b)\log(\sin(dx+c)-1)}{a^2+2\,ab+b^2}-\frac{2\,(b\sin(dx+c)-a)}{\left(a^2-b^2\right)\sin(dx+c)^2-a^2+b^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(4*a^3*log(b*sin(d*x + c) + a)/(a^4 - 2*a^2*b^2 + b^4) - (2*a - b)*log
(sin(d*x + c) + 1)/(a^2 - 2*a*b + b^2) - (2*a + b)*log(sin(d*x + c) - 1)/(a
^2 + 2*a*b + b^2) - 2*(b*sin(d*x + c) - a)/((a^2 - b^2)*sin(d*x + c)^2 - a^
2 + b^2))/d
```

mupad [B] time = 7.04, size = 217, normalized size = 1.72

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)(2a-b)}{2d(a-b)^2}-\frac{\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^2-b^2}-\frac{2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{a^2-b^2}+\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{a^2-b^2}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}-\frac{a^3\ln\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2b\tan\left(\frac{c}{2}+\right.\right.}{d\left(a^4-2a^2b^2+b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + b*sin(c + d*x)),x)
```

```
[Out] (log(tan(c/2 + (d*x)/2) + 1)*(2*a - b))/(2*d*(a - b)^2) - ((b*tan(c/2 + (d*
x)/2))/(a^2 - b^2) - (2*a*tan(c/2 + (d*x)/2)^2)/(a^2 - b^2) + (b*tan(c/2 +
(d*x)/2)^3)/(a^2 - b^2))/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2
+ 1)) - (a^3*log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2))/(d*(
a^4 + b^4 - 2*a^2*b^2)) + (log(tan(c/2 + (d*x)/2) - 1)*(2*a + b))/(2*d*(a +
 b)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**3/(a + b*sin(c + d*x)), x)
```

3.172 $\int \frac{\tan(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=74

$$\frac{a\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)}-\frac{\log(1-\sin(c+dx))}{2d(a+b)}-\frac{\log(\sin(c+dx)+1)}{2d(a-b)}$$

```
[Out] -1/2*ln(1-sin(d*x+c))/(a+b)/d-1/2*ln(1+sin(d*x+c))/(a-b)/d+a*ln(a+b*sin(d*x
+c))/(a^2-b^2)/d
```

Rubi [A] time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2721, 801}

$$\frac{a\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)}-\frac{\log(1-\sin(c+dx))}{2d(a+b)}-\frac{\log(\sin(c+dx)+1)}{2d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + b*Sin[c + d*x]),x]
```

```
[Out] -Log[1 - Sin[c + d*x]]/(2*(a + b)*d) - Log[1 + Sin[c + d*x]]/(2*(a - b)*d)
+ (a*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)*d)
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tan(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(a+x)\left(b^2-x^2\right)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{2(a+b)(b-x)}+\frac{a}{(a-b)(a+b)(a+x)}-\frac{1}{2(a-b)(b+x)}\right)dx, x, b\sin(c+dx)\right)}{d}$$
$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)d}-\frac{\log(1+\sin(c+dx))}{2(a-b)d}+\frac{a\log(a+b\sin(c+dx))}{\left(a^2-b^2\right)d}$$

Mathematica [A] time = 0.08, size = 87, normalized size = 1.18

$$\frac{a\log(a+b\sin(c+dx))+(b-a)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-(a+b)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}\right.\right.}{d(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-a + b)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - (a + b)*Log[Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]] + a*Log[a + b*Sin[c + d*x]])/((a - b)*(a + b)*d)
```

fricas [A] time = 0.46, size = 63, normalized size = 0.85

$$\frac{2\,a\log\left(b\sin\left(dx+c\right)+a\right)-(a+b)\log\left(\sin\left(dx+c\right)+1\right)-(a-b)\log\left(-\sin\left(dx+c\right)+1\right)}{2\left(a^2-b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*log(b*sin(d*x + c) + a) - (a + b)*log(sin(d*x + c) + 1) - (a - b)*
log(-sin(d*x + c) + 1))/((a^2 - b^2)*d)
```

giac [A] time = 0.41, size = 71, normalized size = 0.96

$$\frac{\frac{2\,ab\log(|b\sin(dx+c)+a|)}{a^2b-b^3}-\frac{\log(|\sin(dx+c)+1|)}{a-b}-\frac{\log(|\sin(dx+c)-1|)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*a*b*log(abs(b*sin(d*x + c) + a))/(a^2*b - b^3) - log(abs(sin(d*x + c
) + 1))/(a - b) - log(abs(sin(d*x + c) - 1))/(a + b))/d
```

maple [A] time = 0.18, size = 76, normalized size = 1.03

$$-\frac{\ln(\sin(dx+c)-1)}{d(2a+2b)}+\frac{a\ln(a+b\sin(dx+c))}{d(a+b)(a-b)}-\frac{\ln(1+\sin(dx+c))}{d(2a-2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d/(2*a+2*b)*ln(sin(d*x+c)-1)+1/d*a/(a+b)/(a-b)*ln(a+b*sin(d*x+c))-1/d/(2
*a-2*b)*ln(1+sin(d*x+c))
```

maxima [A] time = 0.45, size = 65, normalized size = 0.88

$$\frac{\frac{2a\log(b\sin(dx+c)+a)}{a^2-b^2}-\frac{\log(\sin(dx+c)+1)}{a-b}-\frac{\log(\sin(dx+c)-1)}{a+b}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a*log(b*sin(d*x + c) + a)/(a^2 - b^2) - log(sin(d*x + c) + 1)/(a - b
) - log(sin(d*x + c) - 1)/(a + b))/d
```

mupad [B] time = 6.73, size = 91, normalized size = 1.23

$$\frac{a\ln\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)}{d\left(a^2-b^2\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{d(a-b)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)}{d(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + b*sin(c + d*x)),x)
```

```
[Out] (a*log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2))/(d*(a^2 - b^2)
) - log(tan(c/2 + (d*x)/2) + 1)/(d*(a - b)) - log(tan(c/2 + (d*x)/2) - 1)/(
d*(a + b))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c)),x)
[Out] Integral(tan(c + d*x)/(a + b*sin(c + d*x)), x)
```

3.173 $\int \frac{\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=34

$$\frac{\log(\sin(c+dx))}{ad}-\frac{\log(a+b\sin(c+dx))}{ad}$$

```
[Out] ln(sin(d*x+c))/a/d-ln(a+b*sin(d*x+c))/a/d
```

Rubi [A] time = 0.04, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2721, 36, 29, 31}

$$\frac{\log(\sin(c+dx))}{ad}-\frac{\log(a+b\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + b*Sin[c + d*x]),x]
```

```
[Out] Log[Sin[c + d*x]]/(a*d) - Log[a + b*Sin[c + d*x]]/(a*d)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+x)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, b\sin(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sin(c+dx)\right)}{ad}$$
$$= \frac{\log(\sin(c+dx))}{ad} - \frac{\log(a+b\sin(c+dx))}{ad}$$

Mathematica [A] time = 0.02, size = 34, normalized size = 1.00

$$\frac{\log(\sin(c+dx))}{ad} - \frac{\log(a+b\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + b*Sin[c + d*x]),x]
```

```
[Out] Log[Sin[c + d*x]]/(a*d) - Log[a + b*Sin[c + d*x]]/(a*d)
```

fricas [A] time = 0.48, size = 31, normalized size = 0.91

$$-\frac{\log\left(b\sin\left(dx+c\right)+a\right) - \log\left(-\frac{1}{2}\,\sin\left(dx+c\right)\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(log(b*sin(d*x + c) + a) - log(-1/2*sin(d*x + c)))/(a*d)
```

giac [A] time = 0.72, size = 35, normalized size = 1.03

$$-\frac{\frac{\log(|b\sin(dx+c)+a|)}{a} - \frac{\log(|\sin(dx+c)|)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -(log(abs(b*sin(d*x + c) + a))/a - log(abs(sin(d*x + c)))/a)/d
```

maple [A] time = 0.11, size = 35, normalized size = 1.03

$$\frac{\ln\left(\sin\left(dx+c\right)\right)}{ad} - \frac{\ln\left(a+b\sin\left(dx+c\right)\right)}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] ln(sin(d*x+c))/a/d-1/d/a*ln(a+b*sin(d*x+c))
```

maxima [A] time = 0.65, size = 33, normalized size = 0.97

$$-\frac{\frac{\log(b\sin(dx+c)+a)}{a}-\frac{\log(\sin(dx+c))}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(log(b*sin(d*x + c) + a)/a - log(sin(d*x + c))/a)/d
```

mupad [B] time = 6.36, size = 48, normalized size = 1.41

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\ln\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + b*sin(c + d*x)),x)
```

```
[Out] (log(tan(c/2 + (d*x)/2)) - log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*
x)/2)^2))/(a*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)/(a + b*sin(c + d*x)), x)
```

3.174 $\int \frac{\cot^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=84

$$\frac{b\csc(c+dx)}{a^2d}-\frac{\left(a^2-b^2\right)\log(\sin(c+dx))}{a^3d}+\frac{\left(a^2-b^2\right)\log(a+b\sin(c+dx))}{a^3d}-\frac{\csc^2(c+dx)}{2ad}$$

```
[Out] b*csc(d*x+c)/a^2/d-1/2*csc(d*x+c)^2/a/d-(a^2-b^2)*ln(sin(d*x+c))/a^3/d+(a^2
-b^2)*ln(a+b*sin(d*x+c))/a^3/d
```

Rubi [A] time = 0.09, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2721, 894}

$$-\frac{\left(a^2-b^2\right)\log(\sin(c+dx))}{a^3d}+\frac{\left(a^2-b^2\right)\log(a+b\sin(c+dx))}{a^3d}+\frac{b\csc(c+dx)}{a^2d}-\frac{\csc^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Csc[c + d*x])/(a^2*d) - Csc[c + d*x]^2/(2*a*d) - ((a^2 - b^2)*Log[Sin[c
+ d*x]])/(a^3*d) + ((a^2 - b^2)*Log[a + b*Sin[c + d*x]])/(a^3*d)
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{b^2-x^2}{x^3(a+x)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{b^2}{ax^3}-\frac{b^2}{a^2x^2}+\frac{-a^2+b^2}{a^3x}+\frac{a^2-b^2}{a^3(a+x)}\right)dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b\csc(c+dx)}{a^2d}-\frac{\csc^2(c+dx)}{2ad}-\frac{\left(a^2-b^2\right)\log(\sin(c+dx))}{a^3d}+\frac{\left(a^2-b^2\right)\log(a+b\sin(c+}{a^3d}$$

Mathematica [A] time = 0.16, size = 65, normalized size = 0.77

$$-\frac{2\left(a^2-b^2\right)(\log(\sin(c+dx))-\log(a+b\sin(c+dx)))+a^2\csc^2(c+dx)-2ab\csc(c+dx)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/2*(-2*a*b*Csc[c + d*x] + a^2*Csc[c + d*x]^2 + 2*(a^2 - b^2)*(Log[Sin[c +
 d*x]] - Log[a + b*Sin[c + d*x]]))/(a^3*d)
```

fricas [A] time = 0.48, size = 118, normalized size = 1.40

$$-\frac{2\,ab\sin(dx+c)-a^2-2\left(\left(a^2-b^2\right)\cos(dx+c)^2-a^2+b^2\right)\log(b\sin(dx+c)+a)+2\left(\left(a^2-b^2\right)\cos(dx+c)^2\right.}{2\left(a^3d\cos(dx+c)^2-a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*sin(d*x + c) - a^2 - 2*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)
*log(b*sin(d*x + c) + a) + 2*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)*log(-
1/2*sin(d*x + c)))/(a^3*d*cos(d*x + c)^2 - a^3*d)
```

giac [A] time = 0.38, size = 114, normalized size = 1.36

$$-\frac{\frac{2\left(a^2-b^2\right)\log(|\sin(dx+c)|)}{a^3}-\frac{2\left(a^2b-b^3\right)\log(|b\sin(dx+c)+a|)}{a^3b}-\frac{3\,a^2\sin(dx+c)^2-3\,b^2\sin(dx+c)^2+2\,ab\sin(dx+c)-a^2}{a^3\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*(a^2 - b^2)*log(abs(sin(d*x + c)))/a^3 - 2*(a^2*b - b^3)*log(abs(b*
sin(d*x + c) + a))/(a^3*b) - (3*a^2*sin(d*x + c)^2 - 3*b^2*sin(d*x + c)^2 +
 2*a*b*sin(d*x + c) - a^2)/(a^3*sin(d*x + c)^2))/d
```

maple [A] time = 0.22, size = 106, normalized size = 1.26

$$\frac{\ln \left(a+b\sin \left(dx+c\right)\right)}{da}-\frac{\ln \left(a+b\sin \left(dx+c\right)\right) b^{2}}{d\,a^{3}}-\frac{1}{2da\sin \left(dx+c\right)^{2}}-\frac{\ln \left(\sin \left(dx+c\right)\right)}{ad}+\frac{\ln \left(\sin \left(dx+c\right)\right) b^{2}}{d\,a^{3}}+\frac{}{d\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d/a*ln(a+b*sin(d*x+c))-1/d/a^3*ln(a+b*sin(d*x+c))*b^2-1/2/d/a/sin(d*x+c)^
2-ln(sin(d*x+c))/a/d+1/d/a^3*ln(sin(d*x+c))*b^2+1/d/a^2*b/sin(d*x+c)
```

maxima [A] time = 2.08, size = 77, normalized size = 0.92

$$\frac{\frac{2\left(a^{2}-b^{2}\right)\log\left(b\sin\left(dx+c\right)+a\right)}{a^{3}}-\frac{2\left(a^{2}-b^{2}\right)\log\left(\sin\left(dx+c\right)\right)}{a^{3}}+\frac{2\,b\sin\left(dx+c\right)-a}{a^{2}\sin\left(dx+c\right)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a^2 - b^2)*log(b*sin(d*x + c) + a)/a^3 - 2*(a^2 - b^2)*log(sin(d*x
+ c))/a^3 + (2*b*sin(d*x + c) - a)/(a^2*sin(d*x + c)^2))/d
```

mupad [B] time = 6.63, size = 144, normalized size = 1.71

$$\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^{2}\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{8\,a\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\left(a^{2}-b^{2}\right)}{a^{3}\,d}-\frac{\frac{a}{2}-2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,a^{2}\,d\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}+\frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+2\,b\,\mathrm{ta}\right.}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3/(a + b*sin(c + d*x)),x)
```

```
[Out] (b*tan(c/2 + (d*x)/2))/(2*a^2*d) - tan(c/2 + (d*x)/2)^2/(8*a*d) - (log(tan(
c/2 + (d*x)/2))*(a^2 - b^2))/(a^3*d) - (a/2 - 2*b*tan(c/2 + (d*x)/2))/(4*a^
2*d*tan(c/2 + (d*x)/2)^2) + (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (
d*x)/2)^2)*(a^2 - b^2))/(a^3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**3/(a + b*sin(c + d*x)), x)
```

3.175 $\int \frac{\cot^5(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=148

$$\frac{b\csc^3(c+dx)}{3a^2d}+\frac{\left(a^2-b^2\right)^2\log(\sin(c+dx))}{a^5d}-\frac{\left(a^2-b^2\right)^2\log(a+b\sin(c+dx))}{a^5d}-\frac{b\left(2a^2-b^2\right)\csc(c+dx)}{a^4d}+\frac{\left(2a^2-\right.}{}$$

```
[Out] -b*(2*a^2-b^2)*csc(d*x+c)/a^4/d+1/2*(2*a^2-b^2)*csc(d*x+c)^2/a^3/d+1/3*b*cs
c(d*x+c)^3/a^2/d-1/4*csc(d*x+c)^4/a/d+(a^2-b^2)^2*ln(sin(d*x+c))/a^5/d-(a^2
-b^2)^2*ln(a+b*sin(d*x+c))/a^5/d
```

Rubi [**A**] time = 0.14, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2721, 894}

$$\frac{\left(2a^2-b^2\right)\csc^2(c+dx)}{2a^3d}-\frac{b\left(2a^2-b^2\right)\csc(c+dx)}{a^4d}+\frac{\left(a^2-b^2\right)^2\log(\sin(c+dx))}{a^5d}-\frac{\left(a^2-b^2\right)^2\log(a+b\sin(c+dx))}{a^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((b*(2*a^2 - b^2)*Csc[c + d*x])/(a^4*d)) + ((2*a^2 - b^2)*Csc[c + d*x]^2)/
(2*a^3*d) + (b*Csc[c + d*x]^3)/(3*a^2*d) - Csc[c + d*x]^4/(4*a*d) + ((a^2 -
 b^2)^2*Log[Sin[c + d*x]])/(a^5*d) - ((a^2 - b^2)^2*Log[a + b*Sin[c + d*x]]
)/(a^5*d)
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^2}{x^5(a+x)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{b^4}{ax^5} - \frac{b^4}{a^2x^4} + \frac{-2a^2b^2+b^4}{a^3x^3} + \frac{2a^2b^2-b^4}{a^4x^2} + \frac{\left(a^2-b^2\right)^2}{a^5x} - \frac{\left(a^2-b^2\right)^2}{a^5(a+x)}\right) dx, x, b\sin(c+dx)\right)}{d}$$
$$= -\frac{b\left(2a^2-b^2\right)\csc(c+dx)}{a^4d} + \frac{\left(2a^2-b^2\right)\csc^2(c+dx)}{2a^3d} + \frac{b\csc^3(c+dx)}{3a^2d} - \frac{\csc^4(c+dx)}{4ad} +$$

Mathematica [A] time = 1.06, size = 115, normalized size = 0.78

$$\frac{-3a^4\csc^4(c+dx) + 4a^3b\csc^3(c+dx) + 6a^2\left(2a^2-b^2\right)\csc^2(c+dx) + 12ab\left(b^2-2a^2\right)\csc(c+dx) + 12\left(a^2-b^2\right)^2}{12a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + b*Sin[c + d*x]),x]
```

```
[Out] (12*a*b*(-2*a^2 + b^2)*Csc[c + d*x] + 6*a^2*(2*a^2 - b^2)*Csc[c + d*x]^2 +
4*a^3*b*Csc[c + d*x]^3 - 3*a^4*Csc[c + d*x]^4 + 12*(a^2 - b^2)^2*(Log[Sin[c
 + d*x]] - Log[a + b*Sin[c + d*x]]))/(12*a^5*d)
```

fricas [A] time = 0.47, size = 271, normalized size = 1.83

$$\frac{9\,a^4 - 6\,a^2b^2 - 6\left(2\,a^4 - a^2b^2\right)\cos\left(dx+c\right)^2 - 12\left(\left(a^4 - 2\,a^2b^2 + b^4\right)\cos\left(dx+c\right)^4 + a^4 - 2\,a^2b^2 + b^4 - 2\left(a^4 - 2\,a^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(9*a^4 - 6*a^2*b^2 - 6*(2*a^4 - a^2*b^2)*cos(d*x + c)^2 - 12*((a^4 - 2
*a^2*b^2 + b^4)*cos(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2
 + b^4)*cos(d*x + c)^2)*log(b*sin(d*x + c) + a) + 12*((a^4 - 2*a^2*b^2 + b^
4)*cos(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*cos(d
*x + c)^2)*log(-1/2*sin(d*x + c)) - 4*(5*a^3*b - 3*a*b^3 - 3*(2*a^3*b - a*b
^3)*cos(d*x + c)^2)*sin(d*x + c))/(a^5*d*cos(d*x + c)^4 - 2*a^5*d*cos(d*x +
 c)^2 + a^5*d)
```

giac [A] time = 0.45, size = 201, normalized size = 1.36

$$\frac{\frac{12\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\log(|\sin(dx+c)|)}{a^{5}}-\frac{12\left(a^{4}b-2\,a^{2}b^{3}+b^{5}\right)\log(|b\sin(dx+c)+a|)}{a^{5}b}-\frac{25\,a^{4}\sin(dx+c)^{4}-50\,a^{2}b^{2}\sin(dx+c)^{4}+25\,b^{4}\sin(dx+c)^{4}+24\,a^{3}b\sin\cdots}{\cdots}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(12*(a^4 - 2*a^2*b^2 + b^4)*log(abs(sin(d*x + c)))/a^5 - 12*(a^4*b - 2
*a^2*b^3 + b^5)*log(abs(b*sin(d*x + c) + a))/(a^5*b) - (25*a^4*sin(d*x + c)
^4 - 50*a^2*b^2*sin(d*x + c)^4 + 25*b^4*sin(d*x + c)^4 + 24*a^3*b*sin(d*x +
 c)^3 - 12*a*b^3*sin(d*x + c)^3 - 12*a^4*sin(d*x + c)^2 + 6*a^2*b^2*sin(d*x
 + c)^2 - 4*a^3*b*sin(d*x + c) + 3*a^4)/(a^5*sin(d*x + c)^4))/d
```

maple [A] time = 0.20, size = 216, normalized size = 1.46

$$-\frac{\ln\left(a+b\sin\left(dx+c\right)\right)}{da}+\frac{2\ln\left(a+b\sin\left(dx+c\right)\right)b^{2}}{d\,a^{3}}-\frac{\ln\left(a+b\sin\left(dx+c\right)\right)b^{4}}{d\,a^{5}}-\frac{1}{4da\sin\left(dx+c\right)^{4}}+\frac{1}{da\sin\left(dx+c\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d/a*ln(a+b*sin(d*x+c))+2/d/a^3*ln(a+b*sin(d*x+c))*b^2-1/d/a^5*ln(a+b*sin
(d*x+c))*b^4-1/4/d/a/sin(d*x+c)^4+1/d/a/sin(d*x+c)^2-1/2/d/a^3/sin(d*x+c)^2
*b^2+ln(sin(d*x+c))/a/d-2/d/a^3*ln(sin(d*x+c))*b^2+1/d/a^5*ln(sin(d*x+c))*b
^4-2/d/a^2*b/sin(d*x+c)+1/d/a^4*b^3/sin(d*x+c)+1/3/d/a^2*b/sin(d*x+c)^3
```

maxima [A] time = 0.61, size = 139, normalized size = 0.94

$$-\frac{\frac{12\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\log(b\sin(dx+c)+a)}{a^{5}}-\frac{12\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\log(\sin(dx+c))}{a^{5}}-\frac{4\,a^{2}b\sin(dx+c)-12\left(2\,a^{2}b-b^{3}\right)\sin(dx+c)^{3}-3\,a^{3}+6\left(2\,a^{3}-ab^{2}\right)\sin(\cdots}{a^{4}\sin(dx+c)^{4}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(12*(a^4 - 2*a^2*b^2 + b^4)*log(b*sin(d*x + c) + a)/a^5 - 12*(a^4 - 2
*a^2*b^2 + b^4)*log(sin(d*x + c))/a^5 - (4*a^2*b*sin(d*x + c) - 12*(2*a^2*b
 - b^3)*sin(d*x + c)^3 - 3*a^3 + 6*(2*a^3 - a*b^2)*sin(d*x + c)^2)/(a^4*sin
(d*x + c)^4))/d
```

mupad [B] time = 6.41, size = 281, normalized size = 1.90

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{3}{16\,a}-\frac{b^2}{8\,a^3}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{b}{8\,a^2}+\frac{2\,b\,\left(\frac{3}{8\,a}-\frac{b^2}{4\,a^3}\right)}{a}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(2\,a\,b^2-3\,a^3\right)+\mathrm{ta}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5/(a + b*sin(c + d*x)),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^2*(3/(16*a) - b^2/(8*a^3)))/d - tan(c/2 + (d*x)/2)^4/(6
4*a*d) - (tan(c/2 + (d*x)/2)*(b/(8*a^2) + (2*b*(3/(8*a) - b^2/(4*a^3)))/a))
/d - (tan(c/2 + (d*x)/2)^2*(2*a*b^2 - 3*a^3) + tan(c/2 + (d*x)/2)^3*(14*a^2
*b - 8*b^3) + a^3/4 - (2*a^2*b*tan(c/2 + (d*x)/2))/3)/(16*a^4*d*tan(c/2 + (
d*x)/2)^4) - (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)*(a^4
 + b^4 - 2*a^2*b^2))/(a^5*d) + (b*tan(c/2 + (d*x)/2)^3)/(24*a^2*d) + (log(t
an(c/2 + (d*x)/2))*(a^4 + b^4 - 2*a^2*b^2))/(a^5*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^5(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**5/(a + b*sin(c + d*x)), x)
```

3.176 $\int \frac{\tan^4(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=177

$$\frac{a\tan^3(c+dx)}{3d\left(a^2-b^2\right)}-\frac{b\sec^3(c+dx)}{3d\left(a^2-b^2\right)}+\frac{a^2b\sec(c+dx)}{d\left(a^2-b^2\right)^2}+\frac{b\sec(c+dx)}{d\left(a^2-b^2\right)}+\frac{2a^4\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}-\frac{a^3\tan(c+dx)}{d\left(a^2-b^2\right)^2}$$

```
[Out] 2*a^4*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)/d+a^
2*b*sec(d*x+c)/(a^2-b^2)^2/d+b*sec(d*x+c)/(a^2-b^2)/d-1/3*b*sec(d*x+c)^3/(a
^2-b^2)/d-a^3*tan(d*x+c)/(a^2-b^2)^2/d+1/3*a*tan(d*x+c)^3/(a^2-b^2)/d
```

Rubi [A] time = 0.24, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2727, 2607, 30, 2606, 3767, 8, 2660, 618, 204}

$$\frac{2a^4\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}+\frac{a\tan^3(c+dx)}{3d\left(a^2-b^2\right)}-\frac{a^3\tan(c+dx)}{d\left(a^2-b^2\right)^2}-\frac{b\sec^3(c+dx)}{3d\left(a^2-b^2\right)}+\frac{a^2b\sec(c+dx)}{d\left(a^2-b^2\right)^2}+\frac{b\sec(c+dx)}{d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*a^4*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5/2)
*d) + (a^2*b*Sec[c + d*x])/((a^2 - b^2)^2*d) + (b*Sec[c + d*x])/((a^2 - b^2
)*d) - (b*Sec[c + d*x]^3)/(3*(a^2 - b^2)*d) - (a^3*Tan[c + d*x])/((a^2 - b^
2)^2*d) + (a*Tan[c + d*x]^3)/(3*(a^2 - b^2)*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2727

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[a/(a^2 - b^2), Int[(g*Tan[e + f*x])^p/Sin[e + f*x]^
2, x], x] + (-Dist[(b*g)/(a^2 - b^2), Int[(g*Tan[e + f*x])^(p - 1)/Cos[e +
f*x], x], x] - Dist[(a^2*g^2)/(a^2 - b^2), Int[(g*Tan[e + f*x])^(p - 2)/(a
+ b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0
] && IntegersQ[2*p] && GtQ[p, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\int \sec^2(c+dx)\tan^2(c+dx)\,dx}{a^2-b^2} - \frac{a^2\int \frac{\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx}{a^2-b^2} - \frac{b\int \sec(c+dx)\tan^3(c+dx)}{a^2-b^2}$$

$$= -\frac{a^3\int \sec^2(c+dx)\,dx}{\left(a^2-b^2\right)^2} + \frac{a^4\int \frac{1}{a+b\sin(c+dx)}\,dx}{\left(a^2-b^2\right)^2} + \frac{\left(a^2b\right)\int \sec(c+dx)\tan(c+dx)\,dx}{\left(a^2-b^2\right)^2} + $$

$$= \frac{b\sec(c+dx)}{\left(a^2-b^2\right)d} - \frac{b\sec^3(c+dx)}{3\left(a^2-b^2\right)d} + \frac{a\tan^3(c+dx)}{3\left(a^2-b^2\right)d} + \frac{a^3\,\mathrm{Subst}\left(\int 1\,dx,x,-\tan(c+dx)\right)}{\left(a^2-b^2\right)^2 d}$$

$$= \frac{a^2b\sec(c+dx)}{\left(a^2-b^2\right)^2 d} + \frac{b\sec(c+dx)}{\left(a^2-b^2\right)d} - \frac{b\sec^3(c+dx)}{3\left(a^2-b^2\right)d} - \frac{a^3\tan(c+dx)}{\left(a^2-b^2\right)^2 d} + \frac{a\tan^3(c+dx)}{3\left(a^2-b^2\right)d} -$$

$$= \frac{2a^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2} d} + \frac{a^2b\sec(c+dx)}{\left(a^2-b^2\right)^2 d} + \frac{b\sec(c+dx)}{\left(a^2-b^2\right)d} - \frac{b\sec^3(c+dx)}{3\left(a^2-b^2\right)d} - \frac{a^3\tan}{\left(a^2\right.}$$

Mathematica [A] time = 1.40, size = 195, normalized size = 1.10

$$\frac{\frac{48a^4\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} - \frac{\sec^3(c+dx)\left(8a^3\sin(3(c+dx))+3b\left(11a^2-5b^2\right)\cos(c+dx)+12b\left(b^2-2a^2\right)\cos(2(c+dx))+11a^2b\cos(3(c+dx))-16a^2b+6ab\right.}{(a-b)^2(a+b)^2}}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((48*a^4*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2
) - (Sec[c + d*x]^3*(-16*a^2*b + 4*b^3 + 3*b*(11*a^2 - 5*b^2)*Cos[c + d*x]
+ 12*b*(-2*a^2 + b^2)*Cos[2*(c + d*x)] + 11*a^2*b*Cos[3*(c + d*x)] - 5*b^3*
Cos[3*(c + d*x)] + 6*a*b^2*Sin[c + d*x] + 8*a^3*Sin[3*(c + d*x)] - 2*a*b^2*
Sin[3*(c + d*x)]))/((a - b)^2*(a + b)^2))/(24*d)
```

fricas [A] time = 0.54, size = 476, normalized size = 2.69

$$\left[-\frac{3\sqrt{-a^2+b^2}\,a^4\cos(dx+c)^3\log\left(\frac{\left(2a^2-b^2\right)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2+2\left(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c)\right)\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)+}{6\left(a^6-3a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(-a^2 + b^2)*a^4*cos(d*x + c)^3*log(((2*a^2 - b^2)*cos(d*x + c
)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*c
os(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a
^2 - b^2)) + 2*a^4*b - 4*a^2*b^3 + 2*b^5 - 6*(2*a^4*b - 3*a^2*b^3 + b^5)*co
s(d*x + c)^2 - 2*(a^5 - 2*a^3*b^2 + a*b^4 - (4*a^5 - 5*a^3*b^2 + a*b^4)*cos
(d*x + c)^2)*sin(d*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x +
 c)^3), -1/3*(3*sqrt(a^2 - b^2)*a^4*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2
- b^2)*cos(d*x + c)))*cos(d*x + c)^3 + a^4*b - 2*a^2*b^3 + b^5 - 3*(2*a^4*b
 - 3*a^2*b^3 + b^5)*cos(d*x + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4 - (4*a^5 - 5*
a^3*b^2 + a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6)*d*cos(d*x + c)^3)]
```

giac [A] time = 3.70, size = 241, normalized size = 1.36

$$\frac{2\left(\frac{3\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a^4}{\left(a^4-2a^2b^2+b^4\right)\sqrt{a^2-b^2}}+\frac{3a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-3a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-10a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+4ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+1}{\left(a^4-2a^2b^2+b^4\right)\left(\tan\left(\frac{1}{2}dx\right.\right.}\right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 2/3*(3*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1
/2*c) + b)/sqrt(a^2 - b^2)))*a^4/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2))
+ (3*a^3*tan(1/2*d*x + 1/2*c)^5 - 3*a^2*b*tan(1/2*d*x + 1/2*c)^4 - 10*a^3*t
an(1/2*d*x + 1/2*c)^3 + 4*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 12*a^2*b*tan(1/2*d
*x + 1/2*c)^2 - 6*b^3*tan(1/2*d*x + 1/2*c)^2 + 3*a^3*tan(1/2*d*x + 1/2*c) -
 5*a^2*b + 2*b^3)/((a^4 - 2*a^2*b^2 + b^4)*(tan(1/2*d*x + 1/2*c)^2 - 1)^3))
/d
```

maple [A] time = 0.19, size = 269, normalized size = 1.52

$$-\frac{32}{3d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3(32a+32b)}-\frac{16}{d\,(32a+32b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{a}{d\,(a+b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{}{2d\,(a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+b*sin(d*x+c)),x)
```

```
[Out] -32/3/d/(tan(1/2*d*x+1/2*c)-1)^3/(32*a+32*b)-16/d/(32*a+32*b)/(tan(1/2*d*x+
1/2*c)-1)^2+1/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)*a+1/2/d/(a+b)^2/(tan(1/2*d*x
+1/2*c)-1)*b-32/3/d/(tan(1/2*d*x+1/2*c)+1)^3/(32*a-32*b)+16/d/(32*a-32*b)/(
tan(1/2*d*x+1/2*c)+1)^2+1/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)*a-1/2/d/(a-b)^2/
(tan(1/2*d*x+1/2*c)+1)*b+2/d*a^4/(a-b)^2/(a+b)^2/(a^2-b^2)^(1/2)*arctan(1/2
*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 9.55, size = 372, normalized size = 2.10

$$\frac{\frac{2a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^4-2a^2b^2+b^4}-\frac{2\left(5a^2b-2b^3\right)}{3\left(a^4-2a^2b^2+b^4\right)}+\frac{2a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{a^4-2a^2b^2+b^4}+\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(2ab^2-5a^3\right)}{3\left(a^4-2a^2b^2+b^4\right)}+\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(2a^2b-b^3\right)}{a^4-2a^2b^2+b^4}-\frac{2a^2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{a^4-2a^2b^2+b^4}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4/(a + b*sin(c + d*x)),x)
```

```
[Out] ((2*a^3*tan(c/2 + (d*x)/2))/(a^4 + b^4 - 2*a^2*b^2) - (2*(5*a^2*b - 2*b^3))
/(3*(a^4 + b^4 - 2*a^2*b^2)) + (2*a^3*tan(c/2 + (d*x)/2)^5)/(a^4 + b^4 - 2*
a^2*b^2) + (4*tan(c/2 + (d*x)/2)^3*(2*a*b^2 - 5*a^3))/(3*(a^4 + b^4 - 2*a^2
*b^2)) + (4*tan(c/2 + (d*x)/2)^2*(2*a^2*b - b^3))/(a^4 + b^4 - 2*a^2*b^2) -
 (2*a^2*b*tan(c/2 + (d*x)/2)^4)/(a^4 + b^4 - 2*a^2*b^2))/(d*(3*tan(c/2 + (d
*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1)) + (2*a^4*ata
n(((a^4*(2*a^4*b + 2*b^5 - 4*a^2*b^3))/((a + b)^(5/2)*(a - b)^(5/2)) + (2*a
^5*tan(c/2 + (d*x)/2)*(a^4 + b^4 - 2*a^2*b^2))/((a + b)^(5/2)*(a - b)^(5/2)
))/(2*a^4)))/(d*(a + b)^(5/2)*(a - b)^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**4/(a + b*sin(c + d*x)), x)
```

3.177 $\int \frac{\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=96

$$-\frac{2a^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{3/2}}+\frac{a\tan(c+dx)}{d\left(a^2-b^2\right)}-\frac{b\sec(c+dx)}{d\left(a^2-b^2\right)}$$

```
[Out] -2*a^2*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)/d-b
*sec(d*x+c)/(a^2-b^2)/d+a*tan(d*x+c)/(a^2-b^2)/d
```

Rubi [A] time = 0.10, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2727, 3767, 8, 2606, 2660, 618, 204}

$$-\frac{2a^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{3/2}}+\frac{a\tan(c+dx)}{d\left(a^2-b^2\right)}-\frac{b\sec(c+dx)}{d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*a^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(3/2
)*d) - (b*Sec[c + d*x])/((a^2 - b^2)*d) + (a*Tan[c + d*x])/((a^2 - b^2)*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2727

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[a/(a^2 - b^2), Int[(g*Tan[e + f*x])^p/Sin[e + f*x]^
2, x], x] + (-Dist[(b*g)/(a^2 - b^2), Int[(g*Tan[e + f*x])^(p - 1)/Cos[e +
f*x], x], x] - Dist[(a^2*g^2)/(a^2 - b^2), Int[(g*Tan[e + f*x])^(p - 2)/(a
+ b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0
] && IntegersQ[2*p] && GtQ[p, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\int \sec^2(c+dx)\,dx}{a^2-b^2} - \frac{a^2\int \frac{1}{a+b\sin(c+dx)}\,dx}{a^2-b^2} - \frac{b\int \sec(c+dx)\tan(c+dx)\,dx}{a^2-b^2}$$

$$= -\frac{a\,\text{Subst}(\int 1\,dx, x, -\tan(c+dx))}{(a^2-b^2)\,d} - \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{(a^2-b^2)\,d}$$

$$= -\frac{b\sec(c+dx)}{(a^2-b^2)\,d} + \frac{a\tan(c+dx)}{(a^2-b^2)\,d} + \frac{(4a^2)\,\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx, x, 2b+2a\tan\left(\frac{1}{2}(c+\right.\right.}{(a^2-b^2)\,d}$$

$$= -\frac{2a^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d} - \frac{b\sec(c+dx)}{(a^2-b^2)\,d} + \frac{a\tan(c+dx)}{(a^2-b^2)\,d}$$

Mathematica [A] time = 0.20, size = 152, normalized size = 1.58

$$\frac{\sqrt{a^2-b^2}\,(a\sin(c+dx)+b\cos(c+dx)-b) - 2a^2\cos(c+dx)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d(a-b)(a+b)\sqrt{a^2-b^2}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*a^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*Cos[c + d*x] + Sqr
t[a^2 - b^2]*(-b + b*Cos[c + d*x] + a*Sin[c + d*x]))/((a - b)*(a + b)*Sqrt[
a^2 - b^2]*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]))
```

fricas [A] time = 0.46, size = 305, normalized size = 3.18

$$\left[\frac{\sqrt{-a^2+b^2}\,a^2\cos(dx+c)\log\left(\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2+2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)-2a^2b}{2(a^4-2a^2b^2+b^4)d\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a^2 + b^2)*a^2*cos(d*x + c)*log(((2*a^2 - b^2)*cos(d*x + c)^2 -
 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*
x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
b^2)) - 2*a^2*b + 2*b^3 + 2*(a^3 - a*b^2)*sin(d*x + c))/((a^4 - 2*a^2*b^2 +
 b^4)*d*cos(d*x + c)), (sqrt(a^2 - b^2)*a^2*arctan(-(a*sin(d*x + c) + b)/(s
qrt(a^2 - b^2)*cos(d*x + c)))*cos(d*x + c) - a^2*b + b^3 + (a^3 - a*b^2)*si
n(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*d*cos(d*x + c))]
```

giac [A] time = 1.79, size = 107, normalized size = 1.11

$$-\frac{2\left(\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a^2}{\left(a^2-b^2\right)^{\frac{3}{2}}}+\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b}{\left(a^2-b^2\right)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -2*((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*
c) + b)/sqrt(a^2 - b^2)))*a^2/(a^2 - b^2)^(3/2) + (a*tan(1/2*d*x + 1/2*c) -
 b)/((a^2 - b^2)*(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

maple [A] time = 0.16, size = 117, normalized size = 1.22

$$-\frac{8}{d\left(8a+8b\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{8}{d\left(8a-8b\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{2a^2\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{d\left(a-b\right)\left(a+b\right)\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] -8/d/(8*a+8*b)/(tan(1/2*d*x+1/2*c)-1)-8/d/(8*a-8*b)/(tan(1/2*d*x+1/2*c)+1)-
2/d*a^2/(a-b)/(a+b)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)
/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 6.37, size = 148, normalized size = 1.54

$$\frac{\frac{2b}{a^2-b^2}-\frac{2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^2-b^2}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}-\frac{2a^2\,\mathrm{atan}\left(\frac{\frac{a^2\,(2a^2b-2b^3)}{(a+b)^{3/2}\,(a-b)^{3/2}}+\frac{2a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(a^2-b^2)}{(a+b)^{3/2}\,(a-b)^{3/2}}}{2a^2}\right)}{d\,(a+b)^{3/2}\,(a-b)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + b*sin(c + d*x)),x)
```

```
[Out] ((2*b)/(a^2 - b^2) - (2*a*tan(c/2 + (d*x)/2))/(a^2 - b^2))/(d*(tan(c/2 + (d
*x)/2)^2 - 1)) - (2*a^2*atan(((a^2*(2*a^2*b - 2*b^3))/((a + b)^(3/2)*(a - b
)^(3/2)) + (2*a^3*tan(c/2 + (d*x)/2)*(a^2 - b^2))/((a + b)^(3/2)*(a - b)^(3
/2)))/(2*a^2)))/(d*(a + b)^(3/2)*(a - b)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

3.178 $\int \frac{\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=80

$$-\frac{2\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{ad}$$

```
[Out] b*arctanh(cos(d*x+c))/a^2/d-cot(d*x+c)/a/d-2*arctan((b+a*tan(1/2*d*x+1/2*c)
)/(a^2-b^2)^(1/2))*(a^2-b^2)^(1/2)/a^2/d
```

Rubi [A] time = 0.24, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2723, 3056, 3001, 3770, 2660, 618, 204}

$$-\frac{2\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2*
d) + (b*ArcTanh[Cos[c + d*x]])/(a^2*d) - Cot[c + d*x]/(a*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2))/Sin[e + f*x]
^2, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\csc^2(c+dx)\left(1-\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx$$

$$= -\frac{\cot(c+dx)}{ad} + \frac{\int \frac{\csc(c+dx)(-b-a\sin(c+dx))}{a+b\sin(c+dx)}\,dx}{a}$$

$$= -\frac{\cot(c+dx)}{ad} - \frac{b\int\csc(c+dx)\,dx}{a^2} + \frac{\left(-a^2+b^2\right)\int\frac{1}{a+b\sin(c+dx)}\,dx}{a^2}$$

$$= \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cot(c+dx)}{ad} - \frac{\left(2\left(a^2-b^2\right)\right)\text{Subst}\left(\int\frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(c\right.\right.}{a^2d}$$

$$= \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cot(c+dx)}{ad} + \frac{\left(4\left(a^2-b^2\right)\right)\text{Subst}\left(\int\frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx, x, 2b+2\right.}{a^2d}$$

$$= -\frac{2\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2d} + \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cot(c+dx)}{ad}$$

Mathematica [A] time = 0.24, size = 108, normalized size = 1.35

$$\frac{-4\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right) + a\tan\left(\frac{1}{2}(c+dx)\right) - a\cot\left(\frac{1}{2}(c+dx)\right) - 2b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + 2b\log\left(\text{co}\right.}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-4*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] - a*Co
t[(c + d*x)/2] + 2*b*Log[Cos[(c + d*x)/2]] - 2*b*Log[Sin[(c + d*x)/2]] + a*
Tan[(c + d*x)/2])/(2*a^2*d)
```

fricas [A] time = 0.53, size = 314, normalized size = 3.92

$$\left[\frac{b\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c) - b\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c) + \sqrt{-a^2+b^2}\,\log\left(\frac{\left(2a^2-b^2\right)\cos(dx\right.}{}\right.}{2a^2d\sin(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(b*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - b*log(-1/2*cos(d*x + c)
+ 1/2)*sin(d*x + c) + sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 -
2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x
 + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b
^2))*sin(d*x + c) - 2*a*cos(d*x + c))/(a^2*d*sin(d*x + c)), 1/2*(b*log(1/2*
cos(d*x + c) + 1/2)*sin(d*x + c) - b*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x +
 c) + 2*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d
*x + c)))*sin(d*x + c) - 2*a*cos(d*x + c))/(a^2*d*sin(d*x + c))]
```

giac [A] time = 0.42, size = 129, normalized size = 1.61

$$-\frac{\frac{2\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2}-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}+\frac{4\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)\sqrt{a^2-b^2}}{a^2}-\frac{2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*b*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - tan(1/2*d*x + 1/2*c)/a + 4*(
pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) +
b)/sqrt(a^2 - b^2)))*sqrt(a^2 - b^2)/a^2 - (2*b*tan(1/2*d*x + 1/2*c) - a)/(
a^2*tan(1/2*d*x + 1/2*c)))/d
```

maple [B] time = 0.17, size = 155, normalized size = 1.94

$$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}-\frac{1}{2ad\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2}-\frac{2\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{d\sqrt{a^2-b^2}}+\frac{2b^2\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{d\,a^2\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/2/a/d*tan(1/2*d*x+1/2*c)-1/2/a/d/tan(1/2*d*x+1/2*c)-1/d/a^2*b*ln(tan(1/2*
d*x+1/2*c))-2/d/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^
2-b^2)^(1/2))+2/d*b^2/a^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c
)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 7.00, size = 204, normalized size = 2.55

$$-\frac{\cot (c+d\,x)}{a\,d}-\frac{b\,\ln \left(\tan \left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}+\frac{\mathrm{atan}\left(\frac{a^3\,\sqrt{b^2-a^2}\,1\mathrm{i}-a\,b^2\,\sqrt{b^2-a^2}\,2\mathrm{i}-b^3\,\tan \left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}\,4\mathrm{i}+a^2\,b\,\tan \left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}\,3\mathrm{i}}{\tan \left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^4-2\,a^3\,b-5\,\tan \left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b^2+2\,a\,b^3+4\,\tan \left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^4}\right)\,\sqrt{b}}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + b*sin(c + d*x)),x)
```

```
[Out] (atan((a^3*(b^2 - a^2)^(1/2)*1i - a*b^2*(b^2 - a^2)^(1/2)*2i - b^3*tan(c/2
+ (d*x)/2)*(b^2 - a^2)^(1/2)*4i + a^2*b*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2
)*3i)/(2*a*b^3 - 2*a^3*b + a^4*tan(c/2 + (d*x)/2) + 4*b^4*tan(c/2 + (d*x)/2
) - 5*a^2*b^2*tan(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2)*2i)/(a^2*d) - cot(c +
d*x)/(a*d) - (b*log(tan(c/2 + (d*x)/2)))/(a^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot ^2(c+dx)}{a+b\sin (c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

3.179 $\int \frac{\cot^4(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=154

$$\frac{b\cot(c+dx)\csc(c+dx)}{2a^2d}+\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4d}-\frac{b\left(3a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d}+\frac{\left(4a^2-3b^2\right.}{3a}$$

```
[Out] 2*(a^2-b^2)^(3/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^4/d-1/
2*b*(3*a^2-2*b^2)*arctanh(cos(d*x+c))/a^4/d+1/3*(4*a^2-3*b^2)*cot(d*x+c)/a^
3/d+1/2*b*cot(d*x+c)*csc(d*x+c)/a^2/d-1/3*cot(d*x+c)*csc(d*x+c)^2/a/d
```

Rubi [A] time = 0.43, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2725, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4d}+\frac{\left(4a^2-3b^2\right)\cot(c+dx)}{3a^3d}-\frac{b\left(3a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d}+\frac{b\cot(c+dx}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^4
*d) - (b*(3*a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*a^4*d) + ((4*a^2 - 3*b^2
)*Cot[c + d*x])/(3*a^3*d) + (b*Cot[c + d*x]*Csc[c + d*x])/(2*a^2*d) - (Cot[
c + d*x]*Csc[c + d*x]^2)/(3*a*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2725

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a*f*Sin[e
 + f*x]^3), x] + (-Dist[1/(6*a^2), Int[((a + b*Sin[e + f*x])^m*Simp[8*a^2 -
 b^2*(m - 1)*(m - 2) + a*b*m*Sin[e + f*x] - (6*a^2 - b^2*m*(m - 2))*Sin[e +
 f*x]^2, x])/Sin[e + f*x]^2, x], x] - Simp[(b*(m - 2)*Cos[e + f*x]*(a + b*S
in[e + f*x])^(m + 1))/(6*a^2*f*Sin[e + f*x]^2), x]) /; FreeQ[{a, b, e, f, m
}, x] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b\cot(c+dx)\csc(c+dx)}{2a^2d} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad} - \frac{\int \frac{\csc^2(c+dx)\left(2\left(4a^2-3b^2\right)-ab\sin(c+dx)\right.}{a+b\sin(c+dx)}}{6a^2}$$

$$= \frac{\left(4a^2-3b^2\right)\cot(c+dx)}{3a^3d} + \frac{b\cot(c+dx)\csc(c+dx)}{2a^2d} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad} - \int$$

$$= \frac{\left(4a^2-3b^2\right)\cot(c+dx)}{3a^3d} + \frac{b\cot(c+dx)\csc(c+dx)}{2a^2d} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad} + \frac{\left(b\left(3\right.\right.}{}$$

$$= -\frac{b\left(3a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d} + \frac{\left(4a^2-3b^2\right)\cot(c+dx)}{3a^3d} + \frac{b\cot(c+dx)\csc(c+}{2a^2d}$$

$$= -\frac{b\left(3a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d} + \frac{\left(4a^2-3b^2\right)\cot(c+dx)}{3a^3d} + \frac{b\cot(c+dx)\csc(c+}{2a^2d}$$

$$= \frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4d} - \frac{b\left(3a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d} + \frac{\left(4a^2-3b\right.}{}$$

Mathematica [B] time = 6.12, size = 350, normalized size = 2.27

$$\frac{b\csc^2\left(\frac{1}{2}(c+dx)\right)}{8a^2d} - \frac{b\sec^2\left(\frac{1}{2}(c+dx)\right)}{8a^2d} + \frac{\left(3a^2b-2b^3\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2a^4d} + \frac{\left(2b^3-3a^2b\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2a^4d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*(a^2 - b^2)^(3/2)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2] + a*Sin[(
c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^4*d) + ((4*a^2*Cos[(c + d*x)/2] - 3*b^2*
Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*a^3*d) + (b*Csc[(c + d*x)/2]^2)/(8*a
^2*d) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(24*a*d) + ((-3*a^2*b + 2*b^3
)*Log[Cos[(c + d*x)/2]])/(2*a^4*d) + ((3*a^2*b - 2*b^3)*Log[Sin[(c + d*x)/2
]])/(2*a^4*d) - (b*Sec[(c + d*x)/2]^2)/(8*a^2*d) + (Sec[(c + d*x)/2]*(-4*a^
2*Sin[(c + d*x)/2] + 3*b^2*Sin[(c + d*x)/2]))/(6*a^3*d) + (Sec[(c + d*x)/2]
^2*Tan[(c + d*x)/2])/(24*a*d)
```

fricas [A] time = 0.64, size = 633, normalized size = 4.11

$$\left[-\frac{6\,a^2b\cos\left(dx+c\right)\sin\left(dx+c\right)-4\left(4\,a^3-3\,ab^2\right)\cos\left(dx+c\right)^3+6\left(\left(a^2-b^2\right)\cos\left(dx+c\right)^2-a^2+b^2\right)\sqrt{-a^2+b^2}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/12*(6*a^2*b*cos(d*x + c)*sin(d*x + c) - 4*(4*a^3 - 3*a*b^2)*cos(d*x + c
)^3 + 6*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)*sqrt(-a^2 + b^2)*log(((2*a
^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x +
c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2
*a*b*sin(d*x + c) - a^2 - b^2))*sin(d*x + c) - 3*(3*a^2*b - 2*b^3 - (3*a^2*
b - 2*b^3)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*(3*
a^2*b - 2*b^3 - (3*a^2*b - 2*b^3)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1
/2)*sin(d*x + c) + 12*(a^3 - a*b^2)*cos(d*x + c))/((a^4*d*cos(d*x + c)^2 -
a^4*d)*sin(d*x + c)), -1/12*(6*a^2*b*cos(d*x + c)*sin(d*x + c) - 4*(4*a^3 -
 3*a*b^2)*cos(d*x + c)^3 + 12*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)*sqrt
(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*si
n(d*x + c) - 3*(3*a^2*b - 2*b^3 - (3*a^2*b - 2*b^3)*cos(d*x + c)^2)*log(1/2
*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*(3*a^2*b - 2*b^3 - (3*a^2*b - 2*b^3)*
cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 12*(a^3 - a*b^2
)*cos(d*x + c))/((a^4*d*cos(d*x + c)^2 - a^4*d)*sin(d*x + c))]
```

giac [A] time = 0.39, size = 273, normalized size = 1.77

$$\frac{\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+12\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3}+\frac{12\left(3\,a^2b-2\,b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^4}+\frac{48\left(a^4-2\,a^2b^2+b^4\right)\left(\pi\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*((a^2*tan(1/2*d*x + 1/2*c)^3 - 3*a*b*tan(1/2*d*x + 1/2*c)^2 - 15*a^2*t
an(1/2*d*x + 1/2*c) + 12*b^2*tan(1/2*d*x + 1/2*c))/a^3 + 12*(3*a^2*b - 2*b^
3)*log(abs(tan(1/2*d*x + 1/2*c)))/a^4 + 48*(a^4 - 2*a^2*b^2 + b^4)*(pi*floo
r(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt
(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^4) - (66*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 44
*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*a^3*tan(1/2*d*x + 1/2*c)^2 + 12*a*b^2*tan(
```

```
1/2*d*x + 1/2*c)^2 - 3*a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/(a^4*tan(1/2*d*x +
 1/2*c)^3))/d
```

maple [B] time = 0.20, size = 348, normalized size = 2.26

$$\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{24ad}-\frac{b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^2}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^3}-\frac{1}{24da\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{5}{8ad\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/24/d/a*tan(1/2*d*x+1/2*c)^3-1/8/d/a^2*b*tan(1/2*d*x+1/2*c)^2-5/8/a/d*tan(
1/2*d*x+1/2*c)+1/2/d/a^3*b^2*tan(1/2*d*x+1/2*c)-1/24/d/a/tan(1/2*d*x+1/2*c)
^3+5/8/a/d/tan(1/2*d*x+1/2*c)-1/2/d/a^3/tan(1/2*d*x+1/2*c)*b^2+1/8/d/a^2*b/
tan(1/2*d*x+1/2*c)^2+3/2/d/a^2*b*ln(tan(1/2*d*x+1/2*c))-1/d/a^4*b^3*ln(tan(
1/2*d*x+1/2*c))+2/d/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)
/(a^2-b^2)^(1/2))-4/d*b^2/a^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1
/2*c)+2*b)/(a^2-b^2)^(1/2))+2/d/a^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2
*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 7.14, size = 654, normalized size = 4.25

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a\,d}+\frac{5\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a\,d}-\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a\,d}+\frac{3\,b\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2\,a^2\,d}-\frac{b^3\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a^4\,d}+\frac{b\cot\left(\frac{c}{2}\right.}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4/(a + b*sin(c + d*x)),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(24*a*d) - cot(c/2 + (d*x)/2)^3/(24*a*d) + (5*cot(c/2
+ (d*x)/2))/(8*a*d) - (5*tan(c/2 + (d*x)/2))/(8*a*d) + (3*b*log(sin(c/2 + (
d*x)/2)/cos(c/2 + (d*x)/2)))/(2*a^2*d) - (b^3*log(sin(c/2 + (d*x)/2)/cos(c/
2 + (d*x)/2)))/(a^4*d) + (b*cot(c/2 + (d*x)/2)^2)/(8*a^2*d) - (b^2*cot(c/2
+ (d*x)/2))/(2*a^3*d) - (b*tan(c/2 + (d*x)/2)^2)/(8*a^2*d) + (b^2*tan(c/2 +
 (d*x)/2))/(2*a^3*d) + (atan((2*a^5*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b
^4 + 3*a^4*b^2)^(1/2) + 8*b^5*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3
*a^4*b^2)^(1/2) - 7*a^3*b^2*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a
^4*b^2)^(1/2) - 16*a^2*b^3*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^
4*b^2)^(1/2) + 4*a*b^4*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^
2)^(1/2) + 7*a^4*b*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(
1/2))/(a^8*sin(c/2 + (d*x)/2)*2i + b^8*sin(c/2 + (d*x)/2)*8i + a*b^7*cos(c/
2 + (d*x)/2)*4i - a^7*b*cos(c/2 + (d*x)/2)*5i - a^3*b^5*cos(c/2 + (d*x)/2)*
13i + a^5*b^3*cos(c/2 + (d*x)/2)*14i - a^2*b^6*sin(c/2 + (d*x)/2)*28i + a^4
*b^4*sin(c/2 + (d*x)/2)*34i - a^6*b^2*sin(c/2 + (d*x)/2)*16i))*(-(a + b)^3*
(a - b)^3)^(1/2)*2i)/(a^4*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**4/(a + b*sin(c + d*x)), x)
```

3.180 $\int \frac{\cot^6(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=307

$$\frac{b\cot(c+dx)\csc^3(c+dx)}{4a^2d}-\frac{2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^6d}+\frac{\left(8a^4-9a^2b^2+4b^4\right)\cot(c+dx)\csc(c+dx)}{8a^4bd}+\cdots$$

```
[Out] -2*(a^2-b^2)^(5/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^6/d+1
/8*b*(15*a^4-20*a^2*b^2+8*b^4)*arctanh(cos(d*x+c))/a^6/d-1/15*(23*a^4-35*a^
2*b^2+15*b^4)*cot(d*x+c)/a^5/d-cot(d*x+c)*csc(d*x+c)/b/d+1/8*(8*a^4-9*a^2*b
^2+4*b^4)*cot(d*x+c)*csc(d*x+c)/a^4/b/d+1/2*a*cot(d*x+c)*csc(d*x+c)^2/b^2/d
-1/30*(15*a^4-22*a^2*b^2+10*b^4)*cot(d*x+c)*csc(d*x+c)^2/a^3/b^2/d+1/4*b*co
t(d*x+c)*csc(d*x+c)^3/a^2/d-1/5*cot(d*x+c)*csc(d*x+c)^4/a/d
```

Rubi [A] time = 1.11, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2726, 3055, 3001, 3770, 2660, 618, 204}

$$-\frac{2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^6d}-\frac{\left(-35a^2b^2+23a^4+15b^4\right)\cot(c+dx)}{15a^5d}+\frac{b\left(-20a^2b^2+15a^4+8b^4\right)\tanh^{-1}\cdots}{8a^6d}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^
6*d) + (b*(15*a^4 - 20*a^2*b^2 + 8*b^4)*ArcTanh[Cos[c + d*x]])/(8*a^6*d) -
((23*a^4 - 35*a^2*b^2 + 15*b^4)*Cot[c + d*x])/(15*a^5*d) - (Cot[c + d*x]*Cs
c[c + d*x])/(b*d) + ((8*a^4 - 9*a^2*b^2 + 4*b^4)*Cot[c + d*x]*Csc[c + d*x])
/(8*a^4*b*d) + (a*Cot[c + d*x]*Csc[c + d*x]^2)/(2*b^2*d) - ((15*a^4 - 22*a^
2*b^2 + 10*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(30*a^3*b^2*d) + (b*Cot[c + d*
x]*Csc[c + d*x]^3)/(4*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^4)/(5*a*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^6,
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(5*a*f*Sin[e
 + f*x]^5), x] + (Dist[1/(20*a^2*b^2*m*(m - 1)), Int[((a + b*Sin[e + f*x])^
m*Simp[60*a^4 - 44*a^2*b^2*(m - 1)*m + b^4*m*(m - 1)*(m - 3)*(m - 4) + a*b*
m*(20*a^2 - b^2*m*(m - 1))*Sin[e + f*x] - (40*a^4 + b^4*m*(m - 1)*(m - 2)*(
m - 4) - 20*a^2*b^2*(m - 1)*(2*m + 1))*Sin[e + f*x]^2, x])/Sin[e + f*x]^4,
x], x] + Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*m*Sin[e + f*
x]^2), x] + Simp[(a*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*m*(m
- 1)*Sin[e + f*x]^3), x] - Simp[(b*(m - 4)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m + 1))/(20*a^2*f*Sin[e + f*x]^4), x]) /; FreeQ[{a, b, e, f, m}, x] && N
eQ[a^2 - b^2, 0] && NeQ[m, 1] && IntegerQ[2*m]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
```

```
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\cot(c+dx)\csc(c+dx)}{bd}+\frac{a\cot(c+dx)\csc^2(c+dx)}{2b^2d}+\frac{b\cot(c+dx)\csc^3(c+dx)}{4a^2d}-$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{bd}+\frac{a\cot(c+dx)\csc^2(c+dx)}{2b^2d}-\frac{\left(15a^4-22a^2b^2+10b^4\right)\cot(c}{30a^3b^2d}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{bd}+\frac{\left(8a^4-9a^2b^2+4b^4\right)\cot(c+dx)\csc(c+dx)}{8a^4bd}+\frac{a\cot(c+d}{2}$$

$$= -\frac{\left(23a^4-35a^2b^2+15b^4\right)\cot(c+dx)}{15a^5d}-\frac{\cot(c+dx)\csc(c+dx)}{bd}+\frac{\left(8a^4-9a^2b^2+4b^4\right)}{8}$$

$$= -\frac{\left(23a^4-35a^2b^2+15b^4\right)\cot(c+dx)}{15a^5d}-\frac{\cot(c+dx)\csc(c+dx)}{bd}+\frac{\left(8a^4-9a^2b^2+4b^4\right)}{8}$$

$$= \frac{b\left(15a^4-20a^2b^2+8b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^6d}-\frac{\left(23a^4-35a^2b^2+15b^4\right)\cot(c+dx)}{15a^5d}-$$

$$= \frac{b\left(15a^4-20a^2b^2+8b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^6d}-\frac{\left(23a^4-35a^2b^2+15b^4\right)\cot(c+dx)}{15a^5d}-$$

$$= -\frac{2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^6d}+\frac{b\left(15a^4-20a^2b^2+8b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^6d}$$

Mathematica [A] time = 1.41, size = 504, normalized size = 1.64

$$\frac{736a^5\tan\left(\frac{1}{2}(c+dx)\right)-3a^5\sin(c+dx)\csc^6\left(\frac{1}{2}(c+dx)\right)+41a^5\sin(c+dx)\csc^4\left(\frac{1}{2}(c+dx)\right)-656a^5\sin^4\left(\frac{1}{2}(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-1920*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] -
 32*(23*a^5 - 35*a^3*b^2 + 15*a*b^4)*Cot[(c + d*x)/2] - 270*a^4*b*Csc[(c +
d*x)/2]^2 + 120*a^2*b^3*Csc[(c + d*x)/2]^2 + 15*a^4*b*Csc[(c + d*x)/2]^4 +
1800*a^4*b*Log[Cos[(c + d*x)/2]] - 2400*a^2*b^3*Log[Cos[(c + d*x)/2]] + 960
*b^5*Log[Cos[(c + d*x)/2]] - 1800*a^4*b*Log[Sin[(c + d*x)/2]] + 2400*a^2*b^
3*Log[Sin[(c + d*x)/2]] - 960*b^5*Log[Sin[(c + d*x)/2]] + 270*a^4*b*Sec[(c
+ d*x)/2]^2 - 120*a^2*b^3*Sec[(c + d*x)/2]^2 - 15*a^4*b*Sec[(c + d*x)/2]^4
- 656*a^5*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 320*a^3*b^2*Csc[c + d*x]^3*Si
n[(c + d*x)/2]^4 + 41*a^5*Csc[(c + d*x)/2]^4*Sin[c + d*x] - 20*a^3*b^2*Csc[
(c + d*x)/2]^4*Sin[c + d*x] - 3*a^5*Csc[(c + d*x)/2]^6*Sin[c + d*x] + 736*a
^5*Tan[(c + d*x)/2] - 1120*a^3*b^2*Tan[(c + d*x)/2] + 480*a*b^4*Tan[(c + d*
x)/2] + 6*a^5*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/(960*a^6*d)
```

fricas [A] time = 1.04, size = 1079, normalized size = 3.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/240*(16*(23*a^5 - 35*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^5 - 80*(7*a^5 - 1
3*a^3*b^2 + 6*a*b^4)*cos(d*x + c)^3 - 120*((a^4 - 2*a^2*b^2 + b^4)*cos(d*x
+ c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*
sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a
^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2
))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2))*sin(d*x + c) - 15
*(15*a^4*b - 20*a^2*b^3 + 8*b^5 + (15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x +
 c)^4 - 2*(15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^2)*log(1/2*cos(d*x +
 c) + 1/2)*sin(d*x + c) + 15*(15*a^4*b - 20*a^2*b^3 + 8*b^5 + (15*a^4*b - 2
0*a^2*b^3 + 8*b^5)*cos(d*x + c)^4 - 2*(15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d
*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 240*(a^5 - 2*a^3*b^2
 + a*b^4)*cos(d*x + c) - 30*((9*a^4*b - 4*a^2*b^3)*cos(d*x + c)^3 - (7*a^4*
b - 4*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^6*d*cos(d*x + c)^4 - 2*a^6*d
*cos(d*x + c)^2 + a^6*d)*sin(d*x + c)), -1/240*(16*(23*a^5 - 35*a^3*b^2 + 1
5*a*b^4)*cos(d*x + c)^5 - 80*(7*a^5 - 13*a^3*b^2 + 6*a*b^4)*cos(d*x + c)^3
- 240*((a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(
a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x +
 c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*sin(d*x + c) - 15*(15*a^4*b - 20*a
^2*b^3 + 8*b^5 + (15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^4 - 2*(15*a^4
*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*
x + c) + 15*(15*a^4*b - 20*a^2*b^3 + 8*b^5 + (15*a^4*b - 20*a^2*b^3 + 8*b^5
)*cos(d*x + c)^4 - 2*(15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^2)*log(-1
/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 240*(a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x
```

```
 + c) - 30*((9*a^4*b - 4*a^2*b^3)*cos(d*x + c)^3 - (7*a^4*b - 4*a^2*b^3)*co
s(d*x + c))*sin(d*x + c))/((a^6*d*cos(d*x + c)^4 - 2*a^6*d*cos(d*x + c)^2 +
 a^6*d)*sin(d*x + c))]
```

giac [A] time = 0.60, size = 490, normalized size = 1.60

$$\frac{\frac{6\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-15\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-70\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+40\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+240\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-120\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+660}{a^5}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/960*((6*a^4*tan(1/2*d*x + 1/2*c)^5 - 15*a^3*b*tan(1/2*d*x + 1/2*c)^4 - 70
*a^4*tan(1/2*d*x + 1/2*c)^3 + 40*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 240*a^3*b
*tan(1/2*d*x + 1/2*c)^2 - 120*a*b^3*tan(1/2*d*x + 1/2*c)^2 + 660*a^4*tan(1/
2*d*x + 1/2*c) - 1080*a^2*b^2*tan(1/2*d*x + 1/2*c) + 480*b^4*tan(1/2*d*x +
1/2*c))/a^5 - 120*(15*a^4*b - 20*a^2*b^3 + 8*b^5)*log(abs(tan(1/2*d*x + 1/2
*c)))/a^6 - 1920*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(pi*floor(1/2*(d*x + c
)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/
(sqrt(a^2 - b^2)*a^6) + (4110*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 5480*a^2*b^3*t
an(1/2*d*x + 1/2*c)^5 + 2192*b^5*tan(1/2*d*x + 1/2*c)^5 - 660*a^5*tan(1/2*d
*x + 1/2*c)^4 + 1080*a^3*b^2*tan(1/2*d*x + 1/2*c)^4 - 480*a*b^4*tan(1/2*d*x
 + 1/2*c)^4 - 240*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 120*a^2*b^3*tan(1/2*d*x +
1/2*c)^3 + 70*a^5*tan(1/2*d*x + 1/2*c)^2 - 40*a^3*b^2*tan(1/2*d*x + 1/2*c)^
2 + 15*a^4*b*tan(1/2*d*x + 1/2*c) - 6*a^5)/(a^6*tan(1/2*d*x + 1/2*c)^5))/d
```

maple [B] time = 0.21, size = 629, normalized size = 2.05

$$-\frac{b\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d\,a^2}+\frac{b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d\,a^3}-\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^3}{8d\,a^4}+\frac{6b^2\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{d\,a^2\sqrt{a^2-b^2}}-\frac{15b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{8d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6/(a+b*sin(d*x+c)),x)
```

```
[Out] 6/d*b^2/a^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^
2)^(1/2))-15/8/d/a^2*b*ln(tan(1/2*d*x+1/2*c))-2/d/(a^2-b^2)^(1/2)*arctan(1/
2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+1/2/d/a^5*b^4*tan(1/2*d*x+1
/2*c)-1/24/d/a^3/tan(1/2*d*x+1/2*c)^3*b^2-1/2/d/a^5/tan(1/2*d*x+1/2*c)*b^4+
1/64/d/a^2*b/tan(1/2*d*x+1/2*c)^4+1/8/d/a^4*b^3/tan(1/2*d*x+1/2*c)^2-1/d/a^
```

```
6*b^5*ln(tan(1/2*d*x+1/2*c))-1/64/d/a^2*b*tan(1/2*d*x+1/2*c)^4+1/24/d/a^3*b
^2*tan(1/2*d*x+1/2*c)^3-1/8/d/a^4*tan(1/2*d*x+1/2*c)^2*b^3+1/4/d/a^2*b*tan(
1/2*d*x+1/2*c)^2-9/8/d/a^3*b^2*tan(1/2*d*x+1/2*c)+9/8/d/a^3/tan(1/2*d*x+1/2
*c)*b^2-1/4/d/a^2*b/tan(1/2*d*x+1/2*c)^2+5/2/d/a^4*b^3*ln(tan(1/2*d*x+1/2*c
))+11/16/a/d*tan(1/2*d*x+1/2*c)-11/16/a/d/tan(1/2*d*x+1/2*c)+1/160/d/a*tan(
1/2*d*x+1/2*c)^5-1/160/d/a/tan(1/2*d*x+1/2*c)^5-7/96/d/a*tan(1/2*d*x+1/2*c)
^3+7/96/d/a/tan(1/2*d*x+1/2*c)^3+2/d/a^6/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*ta
n(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^6-6/d/a^4/(a^2-b^2)^(1/2)*arctan(1
/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 7.13, size = 1099, normalized size = 3.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6/(a + b*sin(c + d*x)),x)
```

```
[Out] tan(c/2 + (d*x)/2)^5/(160*a*d) + (tan(c/2 + (d*x)/2)^2*(b/(32*a^2) + (b*(7/
(32*a) - b^2/(8*a^3)))/a))/d - (tan(c/2 + (d*x)/2)*(b^2/(8*a^3) - 11/(16*a)
 + (2*b*(b/(16*a^2) + (2*b*(7/(32*a) - b^2/(8*a^3)))/a))/a))/d - (tan(c/2 +
 (d*x)/2)^3*(7/(96*a) - b^2/(24*a^3)))/d - (b*tan(c/2 + (d*x)/2)^4)/(64*a^2
*d) - (log(tan(c/2 + (d*x)/2))*((15*a^4*b)/8 + b^5 - (5*a^2*b^3)/2))/(a^6*d
) + (tan(c/2 + (d*x)/2)^2*((7*a^4)/3 - (4*a^2*b^2)/3) - a^4/5 - tan(c/2 + (
d*x)/2)^4*(22*a^4 + 16*b^4 - 36*a^2*b^2) + tan(c/2 + (d*x)/2)^3*(4*a*b^3 -
8*a^3*b) + (a^3*b*tan(c/2 + (d*x)/2))/2)/(32*a^5*d*tan(c/2 + (d*x)/2)^5) -
(atan((((-(a + b)^5*(a - b)^5)^(1/2)*((8*a^12 - 16*a^6*b^6 + 44*a^8*b^4 - 3
9*a^10*b^2)/(4*a^10) + ((2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^12 - 32*a^10*b
^2))/(4*a^9))*(-(a + b)^5*(a - b)^5)^(1/2))/a^6 + (tan(c/2 + (d*x)/2)*(31*a
^10*b - 32*a^4*b^7 + 96*a^6*b^5 - 98*a^8*b^3))/(4*a^9))*1i)/a^6 + ((-(a + b
)^5*(a - b)^5)^(1/2)*((8*a^12 - 16*a^6*b^6 + 44*a^8*b^4 - 39*a^10*b^2)/(4*a
^10) - ((2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^12 - 32*a^10*b^2))/(4*a^9))*(-
(a + b)^5*(a - b)^5)^(1/2))/a^6 + (tan(c/2 + (d*x)/2)*(31*a^10*b - 32*a^4*b
^7 + 96*a^6*b^5 - 98*a^8*b^3))/(4*a^9))*1i)/a^6)/((15*a^10*b - 8*b^11 + 44*
```

```
a^2*b^9 - 99*a^4*b^7 + 113*a^6*b^5 - 65*a^8*b^3)/(2*a^10) + (tan(c/2 + (d*x
)/2)*(16*a^10 - 8*b^10 + 42*a^2*b^8 - 94*a^4*b^6 + 110*a^6*b^4 - 66*a^8*b^2
))/(2*a^9) - ((-(a + b)^5*(a - b)^5)^(1/2)*((8*a^12 - 16*a^6*b^6 + 44*a^8*b
^4 - 39*a^10*b^2)/(4*a^10) + ((2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^12 - 32*
a^10*b^2))/(4*a^9))*(-(a + b)^5*(a - b)^5)^(1/2))/a^6 + (tan(c/2 + (d*x)/2)
*(31*a^10*b - 32*a^4*b^7 + 96*a^6*b^5 - 98*a^8*b^3))/(4*a^9)))/a^6 + ((-(a
+ b)^5*(a - b)^5)^(1/2)*((8*a^12 - 16*a^6*b^6 + 44*a^8*b^4 - 39*a^10*b^2)/(
4*a^10) - ((2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^12 - 32*a^10*b^2))/(4*a^9))
*(-(a + b)^5*(a - b)^5)^(1/2))/a^6 + (tan(c/2 + (d*x)/2)*(31*a^10*b - 32*a^
4*b^7 + 96*a^6*b^5 - 98*a^8*b^3))/(4*a^9)))/a^6))*(-(a + b)^5*(a - b)^5)^(1
/2)*2i)/(a^6*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^6(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**6/(a + b*sin(c + d*x)), x)
```

3.181 $\int \frac{\tan^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=242

$$\frac{\sec^4(c+dx)\left(a^2-2ab\sin(c+dx)+b^2\right)}{4d\left(a^2-b^2\right)^2}-\frac{a^5}{d\left(a^2-b^2\right)^3(a+b\sin(c+dx))}+\frac{a^4\left(a^2+5b^2\right)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^4}-\cdots$$

```
[Out] -1/8*a*(4*a+b)*ln(1-sin(d*x+c))/(a+b)^4/d-1/8*a*(4*a-b)*ln(1+sin(d*x+c))/(a
-b)^4/d+a^4*(a^2+5*b^2)*ln(a+b*sin(d*x+c))/(a^2-b^2)^4/d-a^5/(a^2-b^2)^3/d/
(a+b*sin(d*x+c))+1/4*sec(d*x+c)^4*(a^2+b^2-2*a*b*sin(d*x+c))/(a^2-b^2)^2/d-
1/4*sec(d*x+c)^2*(4*a^4+6*a^2*b^2-2*b^4-a*b*(9*a^2-b^2)*sin(d*x+c))/(a^2-b^
2)^3/d
```

Rubi [A] time = 0.63, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2721, 1647, 1629}

$$-\frac{a^5}{d\left(a^2-b^2\right)^3(a+b\sin(c+dx))}+\frac{a^4\left(a^2+5b^2\right)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^4}+\frac{\sec^4(c+dx)\left(a^2-2ab\sin(c+dx)+b^2\right)}{4d\left(a^2-b^2\right)^2}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(a*(4*a + b)*Log[1 - Sin[c + d*x]])/(8*(a + b)^4*d) - (a*(4*a - b)*Log[1 +
 Sin[c + d*x]])/(8*(a - b)^4*d) + (a^4*(a^2 + 5*b^2)*Log[a + b*Sin[c + d*x]
])/((a^2 - b^2)^4*d) - a^5/((a^2 - b^2)^3*d*(a + b*Sin[c + d*x])) + (Sec[c
+ d*x]^4*(a^2 + b^2 - 2*a*b*Sin[c + d*x]))/(4*(a^2 - b^2)^2*d) - (Sec[c + d
*x]^2*(2*(2*a^4 + 3*a^2*b^2 - b^4) - a*b*(9*a^2 - b^2)*Sin[c + d*x]))/(4*(a
^2 - b^2)^3*d)
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
```

```
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(a+x)^2\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)\left(a^2+b^2-2ab\sin(c+dx)\right)}{4\left(a^2-b^2\right)^2 d} + \frac{\text{Subst}\left(\int \frac{\frac{2a^3b^6}{\left(a^2-b^2\right)^2}-\frac{4a^4b^4x}{\left(a^2-b^2\right)^2}-\frac{6ab^6x^2}{\left(a^2-b^2\right)^2}-4b^2x^3}{(a+x)^2\left(b^2-x^2\right)^2}\,dx\right.}{4b^2d}$$

$$= \frac{\sec^4(c+dx)\left(a^2+b^2-2ab\sin(c+dx)\right)}{4\left(a^2-b^2\right)^2 d} - \frac{\sec^2(c+dx)\left(2\left(2a^4+3a^2b^2-b^4\right)-ab\right.}{4\left(a^2-b^2\right)^3 d}$$

$$= \frac{\sec^4(c+dx)\left(a^2+b^2-2ab\sin(c+dx)\right)}{4\left(a^2-b^2\right)^2 d} - \frac{\sec^2(c+dx)\left(2\left(2a^4+3a^2b^2-b^4\right)-ab\right.}{4\left(a^2-b^2\right)^3 d}$$

$$= -\frac{a(4a+b)\log(1-\sin(c+dx))}{8(a+b)^4d} - \frac{a(4a-b)\log(1+\sin(c+dx))}{8(a-b)^4d} + \frac{a^4\left(a^2+5b^2\right)\log}{\left(a^2\right.}$$

Mathematica [A] time = 6.25, size = 240, normalized size = 0.99

$$-\frac{a^5}{d\left(a^2-b^2\right)^3(a+b\sin(c+dx))}+\frac{a^4\left(a^2+5b^2\right)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^4}-\frac{7a+3b}{16d(a+b)^3(1-\sin(c+dx))}-\frac{}{16d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/8*(a*(4*a + b)*Log[1 - Sin[c + d*x]])/((a + b)^4*d) - (a*(4*a - b)*Log[1
 + Sin[c + d*x]])/(8*(a - b)^4*d) + (a^4*(a^2 + 5*b^2)*Log[a + b*Sin[c + d*
x]])/((a^2 - b^2)^4*d) + 1/(16*(a + b)^2*d*(1 - Sin[c + d*x])^2) - (7*a + 3
*b)/(16*(a + b)^3*d*(1 - Sin[c + d*x])) + 1/(16*(a - b)^2*d*(1 + Sin[c + d*
x])^2) - (7*a - 3*b)/(16*(a - b)^3*d*(1 + Sin[c + d*x])) - a^5/((a^2 - b^2)
^3*d*(a + b*Sin[c + d*x]))
```

fricas [B] time = 0.97, size = 555, normalized size = 2.29

$$\frac{2\,a^{7}-6\,a^{5}b^{2}+6\,a^{3}b^{4}-2\,ab^{6}-2\left(4\,a^{7}+5\,a^{5}b^{2}-10\,a^{3}b^{4}+ab^{6}\right)\cos\left(dx+c\right)^{4}-2\left(4\,a^{7}-9\,a^{5}b^{2}+6\,a^{3}b^{4}-ab^{6}\right)\mathrm{c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*a^7 - 6*a^5*b^2 + 6*a^3*b^4 - 2*a*b^6 - 2*(4*a^7 + 5*a^5*b^2 - 10*a^
3*b^4 + a*b^6)*cos(d*x + c)^4 - 2*(4*a^7 - 9*a^5*b^2 + 6*a^3*b^4 - a*b^6)*c
os(d*x + c)^2 + 8*((a^6*b + 5*a^4*b^3)*cos(d*x + c)^4*sin(d*x + c) + (a^7 +
 5*a^5*b^2)*cos(d*x + c)^4)*log(b*sin(d*x + c) + a) - ((4*a^6*b + 15*a^5*b^
2 + 20*a^4*b^3 + 10*a^3*b^4 - a*b^6)*cos(d*x + c)^4*sin(d*x + c) + (4*a^7 +
 15*a^6*b + 20*a^5*b^2 + 10*a^4*b^3 - a^2*b^5)*cos(d*x + c)^4)*log(sin(d*x
+ c) + 1) - ((4*a^6*b - 15*a^5*b^2 + 20*a^4*b^3 - 10*a^3*b^4 + a*b^6)*cos(d
*x + c)^4*sin(d*x + c) + (4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*
b^5)*cos(d*x + c)^4)*log(-sin(d*x + c) + 1) - 2*(a^6*b - 3*a^4*b^3 + 3*a^2*
b^5 - b^7 - (5*a^6*b - 12*a^4*b^3 + 9*a^2*b^5 - 2*b^7)*cos(d*x + c)^2)*sin(
d*x + c))/((a^8*b - 4*a^6*b^3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9)*d*cos(d*x + c)
^4*sin(d*x + c) + (a^9 - 4*a^7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d
*x + c)^4)
```

giac [B] time = 9.56, size = 494, normalized size = 2.04

$$\frac{\frac{8\left(a^{6}b+5\,a^{4}b^{3}\right)\log\left(|b\sin(dx+c)+a|\right)}{a^{8}b-4\,a^{6}b^{3}+6\,a^{4}b^{5}-4\,a^{2}b^{7}+b^{9}}-\frac{\left(4\,a^{2}-ab\right)\log\left(|\sin(dx+c)+1|\right)}{a^{4}-4\,a^{3}b+6\,a^{2}b^{2}-4\,ab^{3}+b^{4}}-\frac{\left(4\,a^{2}+ab\right)\log\left(|\sin(dx+c)-1|\right)}{a^{4}+4\,a^{3}b+6\,a^{2}b^{2}+4\,ab^{3}+b^{4}}-\frac{8\left(a^{6}b\sin(dx+c)+5\,a^{4}b^{3}\sin(dx+c)+2\,a^{7}+4\,a^{5}}{\left(a^{8}-4\,a^{6}b^{2}+6\,a^{4}b^{4}-4\,a^{2}b^{6}+b^{8}\right)(b\sin(dx+c)+}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(8*(a^6*b + 5*a^4*b^3)*log(abs(b*sin(d*x + c) + a))/(a^8*b - 4*a^6*b^3
+ 6*a^4*b^5 - 4*a^2*b^7 + b^9) - (4*a^2 - a*b)*log(abs(sin(d*x + c) + 1))/(
```

```
a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) - (4*a^2 + a*b)*log(abs(sin(d*x
+ c) - 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 8*(a^6*b*sin(d*x +
 c) + 5*a^4*b^3*sin(d*x + c) + 2*a^7 + 4*a^5*b^2)/((a^8 - 4*a^6*b^2 + 6*a^4
*b^4 - 4*a^2*b^6 + b^8)*(b*sin(d*x + c) + a)) + 2*(3*a^6*sin(d*x + c)^4 + 1
5*a^4*b^2*sin(d*x + c)^4 - 9*a^5*b*sin(d*x + c)^3 + 10*a^3*b^3*sin(d*x + c)
^3 - a*b^5*sin(d*x + c)^3 - 2*a^6*sin(d*x + c)^2 - 28*a^4*b^2*sin(d*x + c)^
2 - 8*a^2*b^4*sin(d*x + c)^2 + 2*b^6*sin(d*x + c)^2 + 7*a^5*b*sin(d*x + c)
- 6*a^3*b^3*sin(d*x + c) - a*b^5*sin(d*x + c) + 12*a^4*b^2 + 7*a^2*b^4 - b^
6)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(sin(d*x + c)^2 - 1)^2)
)/d
```

maple [A] time = 0.27, size = 318, normalized size = 1.31

$$\frac{1}{16d\,(a+b)^{2}\,(\sin(dx+c)-1)^{2}}+\frac{3b}{16d\,(a+b)^{3}\,(\sin(dx+c)-1)}+\frac{7a}{16d\,(a+b)^{3}\,(\sin(dx+c)-1)}-\frac{a^{2}\ln(\sin(dx+}{2d\,(a+b)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/16/d/(a+b)^2/(sin(d*x+c)-1)^2+3/16/d/(a+b)^3/(sin(d*x+c)-1)*b+7/16/d/(a+b
)^3/(sin(d*x+c)-1)*a-1/2/d*a^2/(a+b)^4*ln(sin(d*x+c)-1)-1/8/d*a/(a+b)^4*ln(
sin(d*x+c)-1)*b-1/d*a^5/(a+b)^3/(a-b)^3/(a+b*sin(d*x+c))+1/d*a^6/(a+b)^4/(a
-b)^4*ln(a+b*sin(d*x+c))+5/d*a^4/(a+b)^4/(a-b)^4*ln(a+b*sin(d*x+c))*b^2+1/1
6/d/(a-b)^2/(1+sin(d*x+c))^2+3/16/d/(a-b)^3/(1+sin(d*x+c))*b-7/16/d/(a-b)^3
/(1+sin(d*x+c))*a-1/2/d*a^2/(a-b)^4*ln(1+sin(d*x+c))+1/8/d*a/(a-b)^4*ln(1+s
in(d*x+c))*b
```

maxima [B] time = 1.06, size = 505, normalized size = 2.09

$$\frac{\frac{8\left(a^{6}+5a^{4}b^{2}\right)\log(b\sin(dx+c)+a)}{a^{8}-4a^{6}b^{2}+6a^{4}b^{4}-4a^{2}b^{6}+b^{8}}-\frac{\left(4a^{2}-ab\right)\log(\sin(dx+c)+1)}{a^{4}-4a^{3}b+6a^{2}b^{2}-4ab^{3}+b^{4}}-\frac{\left(4a^{2}+ab\right)\log(\sin(dx+c)-1)}{a^{4}+4a^{3}b+6a^{2}b^{2}+4ab^{3}+b^{4}}-\frac{2\left(7a^{5}+6a^{3}b^{2}-\right.}{a^{7}-3a^{5}b^{2}+3a^{3}b^{4}-ab^{6}+\left(a^{6}b-3a^{4}b^{3}+3a^{2}b^{5}-b^{7}\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/8*(8*(a^6 + 5*a^4*b^2)*log(b*sin(d*x + c) + a)/(a^8 - 4*a^6*b^2 + 6*a^4*b
^4 - 4*a^2*b^6 + b^8) - (4*a^2 - a*b)*log(sin(d*x + c) + 1)/(a^4 - 4*a^3*b
+ 6*a^2*b^2 - 4*a*b^3 + b^4) - (4*a^2 + a*b)*log(sin(d*x + c) - 1)/(a^4 + 4
*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 2*(7*a^5 + 6*a^3*b^2 - a*b^4 + (4*a^5
 + 9*a^3*b^2 - a*b^4)*sin(d*x + c)^4 + (5*a^4*b - 7*a^2*b^3 + 2*b^5)*sin(d*
x + c)^3 - (12*a^5 + 13*a^3*b^2 - a*b^4)*sin(d*x + c)^2 - (4*a^4*b - 5*a^2*
b^3 + b^5)*sin(d*x + c))/(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6 + (a^6*b - 3*
a^4*b^3 + 3*a^2*b^5 - b^7)*sin(d*x + c)^5 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 -
```

```
a*b^6)*sin(d*x + c)^4 - 2*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*sin(d*x + c
)^3 - 2*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*sin(d*x + c)^2 + (a^6*b - 3*a
^4*b^3 + 3*a^2*b^5 - b^7)*sin(d*x + c)))/d
```

mupad [B] time = 7.78, size = 755, normalized size = 3.12

$$\frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)\left(a^6+5\,a^4\,b^2\right)}{d\,\left(a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)\left(\frac{1}{(a+b)^2}-\frac{7\,b}{4\,(a+b)^3}+\frac{3\,b^2}{4\,(a+b)^4}\right)}{d}-\frac{\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + b*sin(c + d*x))^2,x)
```

```
[Out] (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)*(a^6 + 5*a^4*b^2)
)/(d*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) - (log(tan(c/2 + (d*x
)/2) - 1)*(1/(a + b)^2 - (7*b)/(4*(a + b)^3) + (3*b^2)/(4*(a + b)^4)))/d -
(log(tan(c/2 + (d*x)/2) + 1)*((3*b^2)/(4*(a - b)^4) + (7*b)/(4*(a - b)^3) +
 1/(a - b)^2))/d - ((tan(c/2 + (d*x)/2)^2*(a*b^2 + 2*a^3))/(a^4 + b^4 - 2*a
^2*b^2) + (3*tan(c/2 + (d*x)/2)^4*(a*b^2 - 2*a^3))/(a^4 + b^4 - 2*a^2*b^2)
+ (3*tan(c/2 + (d*x)/2)^6*(a*b^2 - 2*a^3))/(a^4 + b^4 - 2*a^2*b^2) + (tan(c
/2 + (d*x)/2)^8*(a*b^2 + 2*a^3))/(a^4 + b^4 - 2*a^2*b^2) - (b*tan(c/2 + (d*
x)/2)^9*(11*a^4 + a^2*b^2))/(2*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (8*b*
tan(c/2 + (d*x)/2)^3*(2*a^4 + a^2*b^2))/((a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2
)) + (8*b*tan(c/2 + (d*x)/2)^7*(2*a^4 + a^2*b^2))/((a^2 - b^2)*(a^4 + b^4 -
 2*a^2*b^2)) - (b*tan(c/2 + (d*x)/2)^5*(13*a^4 - 8*b^4 + 31*a^2*b^2))/((a^2
 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) - (b*tan(c/2 + (d*x)/2)*(11*a^4 + a^2*b^2)
)/(2*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)))/(d*(a + 2*b*tan(c/2 + (d*x)/2) -
 3*a*tan(c/2 + (d*x)/2)^2 + 2*a*tan(c/2 + (d*x)/2)^4 + 2*a*tan(c/2 + (d*x)/
2)^6 - 3*a*tan(c/2 + (d*x)/2)^8 + a*tan(c/2 + (d*x)/2)^10 - 8*b*tan(c/2 + (
d*x)/2)^3 + 12*b*tan(c/2 + (d*x)/2)^5 - 8*b*tan(c/2 + (d*x)/2)^7 + 2*b*tan(
c/2 + (d*x)/2)^9))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**5/(a + b*sin(c + d*x))**2, x)
```

3.182 $\int \frac{\tan^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=161

$$-\frac{a^2\left(a^2+3b^2\right)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^3}+\frac{\sec^2(c+dx)\left(a^2-2ab\sin(c+dx)+b^2\right)}{2d\left(a^2-b^2\right)^2}+\frac{a^3}{d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}$$

```
[Out] 1/2*a*ln(1-sin(d*x+c))/(a+b)^3/d+1/2*a*ln(1+sin(d*x+c))/(a-b)^3/d-a^2*(a^2+
3*b^2)*ln(a+b*sin(d*x+c))/(a^2-b^2)^3/d+a^3/(a^2-b^2)^2/d/(a+b*sin(d*x+c))+
1/2*sec(d*x+c)^2*(a^2+b^2-2*a*b*sin(d*x+c))/(a^2-b^2)^2/d
```

Rubi [A] time = 0.31, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2721, 1647, 1629}

$$\frac{a^3}{d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}-\frac{a^2\left(a^2+3b^2\right)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^3}+\frac{\sec^2(c+dx)\left(a^2-2ab\sin(c+dx)+b^2\right)}{2d\left(a^2-b^2\right)^2}+$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a*Log[1 - Sin[c + d*x]])/(2*(a + b)^3*d) + (a*Log[1 + Sin[c + d*x]])/(2*(a
 - b)^3*d) - (a^2*(a^2 + 3*b^2)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^3*d)
+ a^3/((a^2 - b^2)^2*d*(a + b*Sin[c + d*x])) + (Sec[c + d*x]^2*(a^2 + b^2 -
 2*a*b*Sin[c + d*x]))/(2*(a^2 - b^2)^2*d)
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{(a+x)^2\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^2(c+dx)\left(a^2+b^2-2ab\sin(c+dx)\right)}{2\left(a^2-b^2\right)^2 d} + \frac{\text{Subst}\left(\int \frac{\frac{2a^3b^4}{\left(a^2-b^2\right)^2}-\frac{2a^2b^2x}{a^2-b^2}-\frac{2ab^4x^2}{\left(a^2-b^2\right)^2}}{(a+x)^2\left(b^2-x^2\right)}\,dx, x, b\sin \ldots}{2b^2d}$$

$$= \frac{\sec^2(c+dx)\left(a^2+b^2-2ab\sin(c+dx)\right)}{2\left(a^2-b^2\right)^2 d} + \frac{\text{Subst}\left(\int\left(-\frac{ab^2}{(a+b)^3(b-x)}-\frac{2a^3b^2}{(a-b)^2(a+b)^2(a+x)^2} \ldots\right.\right.}{\ldots}$$

$$= \frac{a\log(1-\sin(c+dx))}{2(a+b)^3d} + \frac{a\log(1+\sin(c+dx))}{2(a-b)^3d} - \frac{a^2\left(a^2+3b^2\right)\log(a+b\sin(c+dx \ldots}{\left(a^2-b^2\right)^3 d}$$

Mathematica [A] time = 0.79, size = 145, normalized size = 0.90

$$\frac{-\frac{4a^2\left(a^2+3b^2\right)\log(a+b\sin(c+dx))}{\left(a^2-b^2\right)^3}+\frac{4a^3}{\left(a^2-b^2\right)^2(a+b\sin(c+dx))}-\frac{1}{(a+b)^2(\sin(c+dx)-1)}+\frac{1}{(a-b)^2(\sin(c+dx)+1)}+\frac{2a\log(1-\sin(c+dx))}{(a+b)^3}+\frac{2a\log(\ldots}{\ldots}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((2*a*Log[1 - Sin[c + d*x]])/(a + b)^3 + (2*a*Log[1 + Sin[c + d*x]])/(a - b
)^3 - (4*a^2*(a^2 + 3*b^2)*Log[a + b*Sin[c + d*x]])/(a^2 - b^2)^3 - 1/((a +
 b)^2*(-1 + Sin[c + d*x])) + 1/((a - b)^2*(1 + Sin[c + d*x])) + (4*a^3)/((a
^2 - b^2)^2*(a + b*Sin[c + d*x])))/(4*d)
```

fricas [B] time = 0.68, size = 388, normalized size = 2.41

$$\frac{a^5 - 2\,a^3b^2 + ab^4 + 2\left(a^5 - ab^4\right)\cos\left(dx+c\right)^2 - 2\left(\left(a^4b + 3\,a^2b^3\right)\cos\left(dx+c\right)^2\sin\left(dx+c\right) + \left(a^5 + 3\,a^3b^2\right)\cos\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a^5 - 2*a^3*b^2 + a*b^4 + 2*(a^5 - a*b^4)*cos(d*x + c)^2 - 2*((a^4*b +
 3*a^2*b^3)*cos(d*x + c)^2*sin(d*x + c) + (a^5 + 3*a^3*b^2)*cos(d*x + c)^2)
*log(b*sin(d*x + c) + a) + ((a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(d*x
 + c)^2*sin(d*x + c) + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cos(d*x + c)^2
)*log(sin(d*x + c) + 1) + ((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*cos(d*x
+ c)^2*sin(d*x + c) + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*cos(d*x + c)^2)
*log(-sin(d*x + c) + 1) - (a^4*b - 2*a^2*b^3 + b^5)*sin(d*x + c))/((a^6*b -
 3*a^4*b^3 + 3*a^2*b^5 - b^7)*d*cos(d*x + c)^2*sin(d*x + c) + (a^7 - 3*a^5*
b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c)^2)
```

giac [A] time = 1.98, size = 248, normalized size = 1.54

$$-\frac{\frac{2\left(a^4b+3\,a^2b^3\right)\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{a^6b-3\,a^4b^3+3\,a^2b^5-b^7} - \frac{a\log\left(\left|\sin\left(dx+c\right)+1\right|\right)}{a^3-3\,a^2b+3\,ab^2-b^3} - \frac{a\log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{a^3+3\,a^2b+3\,ab^2+b^3} - \frac{2\,a^3\sin\left(dx+c\right)^2+2\,ab^2\sin\left(dx+c\right)^2+a^2b\sin\left(dx+c\right)-b^3\sin\left(d\right.}{\left(a^4-2\,a^2b^2+b^4\right)\left(b\sin\left(dx+c\right)^3+a\sin\left(dx+c\right)^2-b\sin\left(d\right.\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*(a^4*b + 3*a^2*b^3)*log(abs(b*sin(d*x + c) + a))/(a^6*b - 3*a^4*b^3
 + 3*a^2*b^5 - b^7) - a*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2*b + 3*a*b^2
 - b^3) - a*log(abs(sin(d*x + c) - 1))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (2
*a^3*sin(d*x + c)^2 + 2*a*b^2*sin(d*x + c)^2 + a^2*b*sin(d*x + c) - b^3*sin
(d*x + c) - 3*a^3 - a*b^2)/((a^4 - 2*a^2*b^2 + b^4)*(b*sin(d*x + c)^3 + a*s
in(d*x + c)^2 - b*sin(d*x + c) - a)))/d
```

maple [A] time = 0.26, size = 182, normalized size = 1.13

$$-\frac{1}{4d\left(a+b\right)^2\left(\sin\left(dx+c\right)-1\right)}+\frac{a\ln\left(\sin\left(dx+c\right)-1\right)}{2d\left(a+b\right)^3}+\frac{a^3}{d\left(a+b\right)^2\left(a-b\right)^2\left(a+b\sin\left(dx+c\right)\right)}-\frac{a^4\ln\left(a+b\sin\left(d\right.\right.}{d\left(a+b\right)^3\left(a-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/4/d/(a+b)^2/(sin(d*x+c)-1)+1/2/d*a/(a+b)^3*ln(sin(d*x+c)-1)+1/d*a^3/(a+b
)^2/(a-b)^2/(a+b*sin(d*x+c))-1/d*a^4/(a+b)^3/(a-b)^3*ln(a+b*sin(d*x+c))-3/d
```

```
*a^2/(a+b)^3/(a-b)^3*ln(a+b*sin(d*x+c))*b^2+1/4/d/(a-b)^2/(1+sin(d*x+c))+1/
2*a*ln(1+sin(d*x+c))/(a-b)^3/d
```

maxima [A] time = 1.45, size = 274, normalized size = 1.70

$$-\frac{\frac{2\left(a^4+3a^2b^2\right)\log(b\sin(dx+c)+a)}{a^6-3a^4b^2+3a^2b^4-b^6}-\frac{a\log(\sin(dx+c)+1)}{a^3-3a^2b+3ab^2-b^3}-\frac{a\log(\sin(dx+c)-1)}{a^3+3a^2b+3ab^2+b^3}-\frac{3a^3+ab^2-2\left(a^3+ab^2\right)\sin(dx+c)^2-\left(a^2b-\right.}{a^5-2a^3b^2+ab^4-\left(a^4b-2a^2b^3+b^5\right)\sin(dx+c)^3-\left(a^5-2a^3b^2+ab^4\right)\text{s}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(a^4 + 3*a^2*b^2)*log(b*sin(d*x + c) + a)/(a^6 - 3*a^4*b^2 + 3*a^2*
b^4 - b^6) - a*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - a*lo
g(sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (3*a^3 + a*b^2 - 2*(a
^3 + a*b^2)*sin(d*x + c)^2 - (a^2*b - b^3)*sin(d*x + c))/(a^5 - 2*a^3*b^2 +
 a*b^4 - (a^4*b - 2*a^2*b^3 + b^5)*sin(d*x + c)^3 - (a^5 - 2*a^3*b^2 + a*b^
4)*sin(d*x + c)^2 + (a^4*b - 2*a^2*b^3 + b^5)*sin(d*x + c)))/d
```

mupad [B] time = 7.25, size = 351, normalized size = 2.18

$$\frac{\frac{2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{a^2-b^2}+\frac{2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{a^2-b^2}+\frac{4b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(a^2+b^2\right)}{\left(a^2-b^2\right)^2}-\frac{4a^2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{a^4-2a^2b^2+b^4}-\frac{4a^2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{\left(a^2-b^2\right)^2}}{d\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-4b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + b*sin(c + d*x))^2,x)
```

```
[Out] ((2*a*tan(c/2 + (d*x)/2)^2)/(a^2 - b^2) + (2*a*tan(c/2 + (d*x)/2)^4)/(a^2 -
 b^2) + (4*b*tan(c/2 + (d*x)/2)^3*(a^2 + b^2))/(a^2 - b^2)^2 - (4*a^2*b*tan
(c/2 + (d*x)/2)^5)/(a^4 + b^4 - 2*a^2*b^2) - (4*a^2*b*tan(c/2 + (d*x)/2))/(
a^2 - b^2)^2)/(d*(a + 2*b*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^2 - a*t
an(c/2 + (d*x)/2)^4 + a*tan(c/2 + (d*x)/2)^6 - 4*b*tan(c/2 + (d*x)/2)^3 + 2
*b*tan(c/2 + (d*x)/2)^5)) + (a*log(tan(c/2 + (d*x)/2) + 1))/(d*(a - b)^3) -
 (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)*(a^4 + 3*a^2*b^2
))/(d*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (a*log(tan(c/2 + (d*x)/2) - 1)
)/(d*(a + b)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**3/(a + b*sin(c + d*x))**2, x)
```

3.183 $\int \frac{\tan(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=109

$$-\frac{a}{d\left(a^2-b^2\right)(a+b\sin(c+dx))}+\frac{\left(a^2+b^2\right)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^2}-\frac{\log(1-\sin(c+dx))}{2d(a+b)^2}-\frac{\log(\sin(c+dx)+1)}{2d(a-b)^2}$$

```
[Out] -1/2*ln(1-sin(d*x+c))/(a+b)^2/d-1/2*ln(1+sin(d*x+c))/(a-b)^2/d+(a^2+b^2)*ln
(a+b*sin(d*x+c))/(a^2-b^2)^2/d-a/(a^2-b^2)/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.10, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2721, 801}

$$-\frac{a}{d\left(a^2-b^2\right)(a+b\sin(c+dx))}+\frac{\left(a^2+b^2\right)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^2}-\frac{\log(1-\sin(c+dx))}{2d(a+b)^2}-\frac{\log(\sin(c+dx)+1)}{2d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -Log[1 - Sin[c + d*x]]/(2*(a + b)^2*d) - Log[1 + Sin[c + d*x]]/(2*(a - b)^2
*d) + ((a^2 + b^2)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^2*d) - a/((a^2 - b
^2)*d*(a + b*Sin[c + d*x]))
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tan(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(a+x)^2(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{2(a+b)^2(b-x)} + \frac{a}{(a-b)(a+b)(a+x)^2} + \frac{a^2+b^2}{(a-b)^2(a+b)^2(a+x)} - \frac{1}{2(a-b)^2(b+x)}\right) dx, x, b\sin(c$$
$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)^2d} - \frac{\log(1+\sin(c+dx))}{2(a-b)^2d} + \frac{(a^2+b^2)\log(a+b\sin(c+dx))}{(a^2-b^2)^2 d} -$$

Mathematica [A] time = 0.30, size = 162, normalized size = 1.49

$$-\frac{a\left(-2\left((a^2+b^2)\log(a+b\sin(c+dx))-a^2+b^2\right)+(a-b)^2\log(1-\sin(c+dx))+(a+b)^2\log(\sin(c+dx)+1}{2d(a-b)^2(a+}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/2*(a*((a - b)^2*Log[1 - Sin[c + d*x]] + (a + b)^2*Log[1 + Sin[c + d*x]]
- 2*(-a^2 + b^2 + (a^2 + b^2)*Log[a + b*Sin[c + d*x]])) + b*((a - b)^2*Log[
1 - Sin[c + d*x]] + (a + b)^2*Log[1 + Sin[c + d*x]] - 2*(a^2 + b^2)*Log[a +
 b*Sin[c + d*x]])*Sin[c + d*x])/((a - b)^2*(a + b)^2*d*(a + b*Sin[c + d*x])
)
```

fricas [A] time = 0.54, size = 195, normalized size = 1.79

$$-\frac{2a^3-2ab^2-2\left(a^3+ab^2+(a^2b+b^3)\sin(dx+c)\right)\log(b\sin(dx+c)+a)+\left(a^3+2a^2b+ab^2+\left(a^2b+2ab^2\right.\right.}{2\left((a^4b-2a^2b^3+b^5)d\sin\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^3 - 2*a*b^2 - 2*(a^3 + a*b^2 + (a^2*b + b^3)*sin(d*x + c))*log(b*
sin(d*x + c) + a) + (a^3 + 2*a^2*b + a*b^2 + (a^2*b + 2*a*b^2 + b^3)*sin(d*
x + c))*log(sin(d*x + c) + 1) + (a^3 - 2*a^2*b + a*b^2 + (a^2*b - 2*a*b^2 +
 b^3)*sin(d*x + c))*log(-sin(d*x + c) + 1))/((a^4*b - 2*a^2*b^3 + b^5)*d*si
n(d*x + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d)
```

giac [A] time = 0.46, size = 156, normalized size = 1.43

$$\frac{\frac{2(a^2b+b^3)\log(|b\sin(dx+c)+a|)}{a^4b-2a^2b^3+b^5} - \frac{\log(|\sin(dx+c)+1|)}{a^2-2ab+b^2} - \frac{\log(|\sin(dx+c)-1|)}{a^2+2ab+b^2} - \frac{2(a^2b\sin(dx+c)+b^3\sin(dx+c)+2a^3)}{(a^4-2a^2b^2+b^4)(b\sin(dx+c)+a)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*(a^2*b + b^3)*log(abs(b*sin(d*x + c) + a))/(a^4*b - 2*a^2*b^3 + b^5)
 - log(abs(sin(d*x + c) + 1))/(a^2 - 2*a*b + b^2) - log(abs(sin(d*x + c) -
1))/(a^2 + 2*a*b + b^2) - 2*(a^2*b*sin(d*x + c) + b^3*sin(d*x + c) + 2*a^3)
/((a^4 - 2*a^2*b^2 + b^4)*(b*sin(d*x + c) + a)))/d
```

maple [A] time = 0.24, size = 132, normalized size = 1.21

$$-\frac{\ln(\sin(dx+c)-1)}{2d(a+b)^2}-\frac{a}{d(a+b)(a-b)(a+b\sin(dx+c))}+\frac{\ln(a+b\sin(dx+c))a^2}{d(a+b)^2(a-b)^2}+\frac{\ln(a+b\sin(dx+c))b^2}{d(a+b)^2(a-b)^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/2/d/(a+b)^2*ln(sin(d*x+c)-1)-1/d*a/(a+b)/(a-b)/(a+b*sin(d*x+c))+1/d/(a+b
)^2/(a-b)^2*ln(a+b*sin(d*x+c))*a^2+1/d/(a+b)^2/(a-b)^2*ln(a+b*sin(d*x+c))*b
^2-1/2*ln(1+sin(d*x+c))/(a-b)^2/d
```

maxima [A] time = 0.92, size = 124, normalized size = 1.14

$$\frac{\frac{2\left(a^2+b^2\right)\log(b\sin(dx+c)+a)}{a^4-2a^2b^2+b^4}-\frac{2a}{a^3-ab^2+\left(a^2b-b^3\right)\sin(dx+c)}-\frac{\log(\sin(dx+c)+1)}{a^2-2ab+b^2}-\frac{\log(\sin(dx+c)-1)}{a^2+2ab+b^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a^2 + b^2)*log(b*sin(d*x + c) + a)/(a^4 - 2*a^2*b^2 + b^4) - 2*a/(a
^3 - a*b^2 + (a^2*b - b^3)*sin(d*x + c)) - log(sin(d*x + c) + 1)/(a^2 - 2*a
*b + b^2) - log(sin(d*x + c) - 1)/(a^2 + 2*a*b + b^2))/d
```

mupad [B] time = 6.76, size = 158, normalized size = 1.45

$$\frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)\left(a^2+b^2\right)}{d\left(a^4-2a^2b^2+b^4\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{d(a-b)^2}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)}{d(a+b)^2}+\frac{}{d\left(a^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + b*sin(c + d*x))^2,x)
```

```
[Out] (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)*(a^2 + b^2))/(d*(
a^4 + b^4 - 2*a^2*b^2)) - log(tan(c/2 + (d*x)/2) + 1)/(d*(a - b)^2) - log(t
an(c/2 + (d*x)/2) - 1)/(d*(a + b)^2) + (2*b*tan(c/2 + (d*x)/2))/(d*(a^2 - b
^2)*(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)/(a + b*sin(c + d*x))**2, x)
```

3.184 $\int \frac{\cot(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=53

$$-\frac{\log(a+b\sin(c+dx))}{a^2d}+\frac{\log(\sin(c+dx))}{a^2d}+\frac{1}{ad(a+b\sin(c+dx))}$$

```
[Out] ln(sin(d*x+c))/a^2/d-ln(a+b*sin(d*x+c))/a^2/d+1/a/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2721, 44}

$$-\frac{\log(a+b\sin(c+dx))}{a^2d}+\frac{\log(\sin(c+dx))}{a^2d}+\frac{1}{ad(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] Log[Sin[c + d*x]]/(a^2*d) - Log[a + b*Sin[c + d*x]]/(a^2*d) + 1/(a*d*(a + b
*Sin[c + d*x]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2x} - \frac{1}{a(a+x)^2} - \frac{1}{a^2(a+x)}\right)dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\log(\sin(c+dx))}{a^2d} - \frac{\log(a+b\sin(c+dx))}{a^2d} + \frac{1}{ad(a+b\sin(c+dx))}$$

Mathematica [A] time = 0.08, size = 42, normalized size = 0.79

$$\frac{\frac{a}{a+b\sin(c+dx)} - \log(a+b\sin(c+dx)) + \log(\sin(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (Log[Sin[c + d*x]] - Log[a + b*Sin[c + d*x]] + a/(a + b*Sin[c + d*x]))/(a^2
*d)
```

fricas [A] time = 0.51, size = 69, normalized size = 1.30

$$-\frac{(b\sin(dx+c)+a)\log(b\sin(dx+c)+a) - (b\sin(dx+c)+a)\log\left(-\frac{1}{2}\sin(dx+c)\right) - a}{a^2bd\sin(dx+c)+a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -((b*sin(d*x + c) + a)*log(b*sin(d*x + c) + a) - (b*sin(d*x + c) + a)*log(-
1/2*sin(d*x + c)) - a)/(a^2*b*d*sin(d*x + c) + a^3*d)
```

giac [A] time = 0.42, size = 51, normalized size = 0.96

$$\frac{b\left(\frac{\log\left(\left|-\frac{a}{b\sin(dx+c)+a}+1\right|\right)}{a^2b} + \frac{1}{(b\sin(dx+c)+a)ab}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] b*(log(abs(-a/(b*sin(d*x + c) + a) + 1))/(a^2*b) + 1/((b*sin(d*x + c) + a)*
a*b))/d
```

maple [A] time = 0.15, size = 54, normalized size = 1.02

$$\frac{\ln\left(\sin\left(dx+c\right)\right)}{a^{2}d}-\frac{\ln\left(a+b\sin\left(dx+c\right)\right)}{a^{2}d}+\frac{1}{ad\left(a+b\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] ln(sin(d*x+c))/a^2/d-ln(a+b*sin(d*x+c))/a^2/d+1/a/d/(a+b*sin(d*x+c))
```

maxima [A] time = 0.60, size = 47, normalized size = 0.89

$$\frac{\frac{1}{ab\sin(dx+c)+a^{2}}-\frac{\log(b\sin(dx+c)+a)}{a^{2}}+\frac{\log(\sin(dx+c))}{a^{2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (1/(a*b*sin(d*x + c) + a^2) - log(b*sin(d*x + c) + a)/a^2 + log(sin(d*x + c
))/a^2)/d
```

mupad [B] time = 6.38, size = 105, normalized size = 1.98

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}-\frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}{a^2\,d}-\frac{2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^3+2\,b\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + b*sin(c + d*x))^2,x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a^2*d) - log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/
2 + (d*x)/2)^2)/(a^2*d) - (2*b*tan(c/2 + (d*x)/2))/(d*(a^3*tan(c/2 + (d*x)/
2)^2 + a^3 + 2*a^2*b*tan(c/2 + (d*x)/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cot\left(c+dx\right)}{\left(a+b\sin\left(c+dx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c))**2,x)
[Out] Integral(cot(c + d*x)/(a + b*sin(c + d*x))**2, x)
```

3.185 $\int \frac{\cot^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=114

$$\frac{2b\csc(c+dx)}{a^3d}-\frac{\csc^2(c+dx)}{2a^2d}-\frac{\left(a^2-3b^2\right)\log(\sin(c+dx))}{a^4d}+\frac{\left(a^2-3b^2\right)\log(a+b\sin(c+dx))}{a^4d}-\frac{a^2-b^2}{a^3d(a+b\sin(c+d$$

```
[Out] 2*b*csc(d*x+c)/a^3/d-1/2*csc(d*x+c)^2/a^2/d-(a^2-3*b^2)*ln(sin(d*x+c))/a^4/
d+(a^2-3*b^2)*ln(a+b*sin(d*x+c))/a^4/d+(-a^2+b^2)/a^3/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.11, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2721, 894}

$$-\frac{a^2-b^2}{a^3d(a+b\sin(c+dx))}-\frac{\left(a^2-3b^2\right)\log(\sin(c+dx))}{a^4d}+\frac{\left(a^2-3b^2\right)\log(a+b\sin(c+dx))}{a^4d}+\frac{2b\csc(c+dx)}{a^3d}-\frac{\csc^2(c-}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*b*Csc[c + d*x])/(a^3*d) - Csc[c + d*x]^2/(2*a^2*d) - ((a^2 - 3*b^2)*Log[
Sin[c + d*x]])/(a^4*d) + ((a^2 - 3*b^2)*Log[a + b*Sin[c + d*x]])/(a^4*d) -
(a^2 - b^2)/(a^3*d*(a + b*Sin[c + d*x]))
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{b^2-x^2}{x^3(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{b^2}{a^2x^3}-\frac{2b^2}{a^3x^2}+\frac{-a^2+3b^2}{a^4x}+\frac{a^2-b^2}{a^3(a+x)^2}+\frac{a^2-3b^2}{a^4(a+x)}\right)dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{2b\csc(c+dx)}{a^3d}-\frac{\csc^2(c+dx)}{2a^2d}-\frac{\left(a^2-3b^2\right)\log(\sin(c+dx))}{a^4d}+\frac{\left(a^2-3b^2\right)\log(a+b}{a^4d}$$

Mathematica [A] time = 0.63, size = 96, normalized size = 0.84

$$-\frac{2\left(a^2-3b^2\right)\log(\sin(c+dx))-2\left(a^2-3b^2\right)\log(a+b\sin(c+dx))+a^2\csc^2(c+dx)+\frac{2a(a-b)(a+b)}{a+b\sin(c+dx)}-4ab\csc(c}{2a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/2*(-4*a*b*Csc[c + d*x] + a^2*Csc[c + d*x]^2 + 2*(a^2 - 3*b^2)*Log[Sin[c
+ d*x]] - 2*(a^2 - 3*b^2)*Log[a + b*Sin[c + d*x]] + (2*a*(a - b)*(a + b))/(
a + b*Sin[c + d*x]))/(a^4*d)
```

fricas [B] time = 0.53, size = 259, normalized size = 2.27

$$-\frac{3\,a^2b\sin(dx+c)-3\,a^3+6\,ab^2+2\left(a^3-3\,ab^2\right)\cos(dx+c)^2+2\left(a^3-3\,ab^2-\left(a^3-3\,ab^2\right)\cos(dx+c)^2+\left(a^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(3*a^2*b*sin(d*x + c) - 3*a^3 + 6*a*b^2 + 2*(a^3 - 3*a*b^2)*cos(d*x +
c)^2 + 2*(a^3 - 3*a*b^2 - (a^3 - 3*a*b^2)*cos(d*x + c)^2 + (a^2*b - 3*b^3 -
 (a^2*b - 3*b^3)*cos(d*x + c)^2)*sin(d*x + c))*log(b*sin(d*x + c) + a) - 2*
(a^3 - 3*a*b^2 - (a^3 - 3*a*b^2)*cos(d*x + c)^2 + (a^2*b - 3*b^3 - (a^2*b -
 3*b^3)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*sin(d*x + c)))/(a^5*d*cos(d*
x + c)^2 - a^5*d + (a^4*b*d*cos(d*x + c)^2 - a^4*b*d)*sin(d*x + c))
```

giac [A] time = 0.43, size = 165, normalized size = 1.45

$$-\frac{\frac{2\left(a^2-3\,b^2\right)\log(|\sin(dx+c)|)}{a^4}-\frac{2\left(a^2b-3\,b^3\right)\log(|b\sin(dx+c)+a|)}{a^4b}+\frac{2\left(a^2b\sin(dx+c)-3\,b^3\sin(dx+c)+2\,a^3-4\,ab^2\right)}{(b\sin(dx+c)+a)a^4}-\frac{3\,a^2\sin(dx+c)^2-9\,b^2\sin(dx+}{a^4\sin(dx}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*(a^2 - 3*b^2)*log(abs(sin(d*x + c)))/a^4 - 2*(a^2*b - 3*b^3)*log(ab
s(b*sin(d*x + c) + a))/(a^4*b) + 2*(a^2*b*sin(d*x + c) - 3*b^3*sin(d*x + c)
 + 2*a^3 - 4*a*b^2)/((b*sin(d*x + c) + a)*a^4) - (3*a^2*sin(d*x + c)^2 - 9*
b^2*sin(d*x + c)^2 + 4*a*b*sin(d*x + c) - a^2)/(a^4*sin(d*x + c)^2))/d
```

maple [A] time = 0.29, size = 150, normalized size = 1.32

$$\frac{\ln\left(a+b\sin\left(dx+c\right)\right)}{a^{2}d}-\frac{3\ln\left(a+b\sin\left(dx+c\right)\right)b^{2}}{d\,a^{4}}-\frac{1}{ad\left(a+b\sin\left(dx+c\right)\right)}+\frac{b^{2}}{d\,a^{3}\left(a+b\sin\left(dx+c\right)\right)}-\frac{1}{2d\,a^{2}\sin\left(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+b*sin(d*x+c))^2,x)
```

```
[Out] ln(a+b*sin(d*x+c))/a^2/d-3/d/a^4*ln(a+b*sin(d*x+c))*b^2-1/a/d/(a+b*sin(d*x+
c))+1/d/a^3/(a+b*sin(d*x+c))*b^2-1/2/d/a^2/sin(d*x+c)^2-ln(sin(d*x+c))/a^2/
d+3/d/a^4*ln(sin(d*x+c))*b^2+2/d/a^3*b/sin(d*x+c)
```

maxima [A] time = 0.30, size = 116, normalized size = 1.02

$$\frac{\frac{3\,ab\sin(dx+c)-2\left(a^{2}-3\,b^{2}\right)\sin(dx+c)^{2}-a^{2}}{a^{3}b\sin(dx+c)^{3}+a^{4}\sin(dx+c)^{2}}+\frac{2\left(a^{2}-3\,b^{2}\right)\log(b\sin(dx+c)+a)}{a^{4}}-\frac{2\left(a^{2}-3\,b^{2}\right)\log(\sin(dx+c))}{a^{4}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*((3*a*b*sin(d*x + c) - 2*(a^2 - 3*b^2)*sin(d*x + c)^2 - a^2)/(a^3*b*sin
(d*x + c)^3 + a^4*sin(d*x + c)^2) + 2*(a^2 - 3*b^2)*log(b*sin(d*x + c) + a)
/a^4 - 2*(a^2 - 3*b^2)*log(sin(d*x + c))/a^4)/d
```

mupad [B] time = 6.38, size = 235, normalized size = 2.06

$$\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^{3}\,d}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{2}\,\left(\frac{a^{2}}{2}-8\,b^{2}\right)+\frac{a^{2}}{2}-\frac{4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{3}\,\left(3\,a^{2}\,b-2\,b^{3}\right)}{a}-3\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(4\,a^{4}\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{4}+4\,a^{4}\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{2}+8\,b\,a^{3}\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{3}\right)}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{2}}{8\,a^{2}\,d}-\frac{\ln\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3/(a + b*sin(c + d*x))^2,x)
```

```
[Out] (b*tan(c/2 + (d*x)/2))/(a^3*d) - (tan(c/2 + (d*x)/2)^2*(a^2/2 - 8*b^2) + a^
2/2 - (4*tan(c/2 + (d*x)/2)^3*(3*a^2*b - 2*b^3))/a - 3*a*b*tan(c/2 + (d*x)/
2))/(d*(4*a^4*tan(c/2 + (d*x)/2)^2 + 4*a^4*tan(c/2 + (d*x)/2)^4 + 8*a^3*b*t
an(c/2 + (d*x)/2)^3)) - tan(c/2 + (d*x)/2)^2/(8*a^2*d) - (log(tan(c/2 + (d*
x)/2))*(a^2 - 3*b^2))/(a^4*d) + (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2
 + (d*x)/2)^2)*(a^2 - 3*b^2))/(a^4*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**3/(a + b*sin(c + d*x))**2, x)
```

3.186 $\int \frac{\cot^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=188

$$\frac{2b\csc^3(c+dx)}{3a^3d}-\frac{\csc^4(c+dx)}{4a^2d}+\frac{\left(a^2-b^2\right)^2}{a^5d(a+b\sin(c+dx))}-\frac{4b\left(a^2-b^2\right)\csc(c+dx)}{a^5d}+\frac{\left(2a^2-3b^2\right)\csc^2(c+dx)}{2a^4d}+\frac{\left(a^4-\right.}{}$$

```
[Out] -4*b*(a^2-b^2)*csc(d*x+c)/a^5/d+1/2*(2*a^2-3*b^2)*csc(d*x+c)^2/a^4/d+2/3*b*
csc(d*x+c)^3/a^3/d-1/4*csc(d*x+c)^4/a^2/d+(a^4-6*a^2*b^2+5*b^4)*ln(sin(d*x+
c))/a^6/d-(a^4-6*a^2*b^2+5*b^4)*ln(a+b*sin(d*x+c))/a^6/d+(a^2-b^2)^2/a^5/d/
(a+b*sin(d*x+c))
```

Rubi [A] time = 0.18, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2721, 894}

$$\frac{\left(a^2-b^2\right)^2}{a^5d(a+b\sin(c+dx))}+\frac{\left(2a^2-3b^2\right)\csc^2(c+dx)}{2a^4d}-\frac{4b\left(a^2-b^2\right)\csc(c+dx)}{a^5d}+\frac{\left(-6a^2b^2+a^4+5b^4\right)\log(\sin(c+dx))}{a^6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-4*b*(a^2 - b^2)*Csc[c + d*x])/(a^5*d) + ((2*a^2 - 3*b^2)*Csc[c + d*x]^2)/
(2*a^4*d) + (2*b*Csc[c + d*x]^3)/(3*a^3*d) - Csc[c + d*x]^4/(4*a^2*d) + ((a
^4 - 6*a^2*b^2 + 5*b^4)*Log[Sin[c + d*x]])/(a^6*d) - ((a^4 - 6*a^2*b^2 + 5*
b^4)*Log[a + b*Sin[c + d*x]])/(a^6*d) + (a^2 - b^2)^2/(a^5*d*(a + b*Sin[c +
 d*x]))
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^2}{x^5(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{b^4}{a^2x^5} - \frac{2b^4}{a^3x^4} + \frac{-2a^2b^2+3b^4}{a^4x^3} + \frac{4b^2\left(a^2-b^2\right)}{a^5x^2} + \frac{a^4-6a^2b^2+5b^4}{a^6x} - \frac{\left(a^2-b^2\right)^2}{a^5(a+x)^2} + \frac{-a^4+6a^2b^2-5}{a^6(a+x)}\right.\right.}{d}$$

$$= -\frac{4b\left(a^2-b^2\right)\csc(c+dx)}{a^5d} + \frac{\left(2a^2-3b^2\right)\csc^2(c+dx)}{2a^4d} + \frac{2b\csc^3(c+dx)}{3a^3d} - \frac{\csc^4(c+}{4a^2d}$$

Mathematica [A] time = 6.14, size = 187, normalized size = 0.99

$$-\frac{4b(a-b)(a+b)\csc(c+dx)}{a^5d} + \frac{2b\csc^3(c+dx)}{3a^3d} - \frac{\csc^4(c+dx)}{4a^2d} + \frac{\left(a^2-b^2\right)^2}{a^5d(a+b\sin(c+dx))} + \frac{\left(2a^2-3b^2\right)\csc^2(c+dx)}{2a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-4*(a - b)*b*(a + b)*Csc[c + d*x])/(a^5*d) + ((2*a^2 - 3*b^2)*Csc[c + d*x]
^2)/(2*a^4*d) + (2*b*Csc[c + d*x]^3)/(3*a^3*d) - Csc[c + d*x]^4/(4*a^2*d) +
 ((a^4 - 6*a^2*b^2 + 5*b^4)*Log[Sin[c + d*x]])/(a^6*d) - ((a^4 - 6*a^2*b^2
+ 5*b^4)*Log[a + b*Sin[c + d*x]])/(a^6*d) + (a^2 - b^2)^2/(a^5*d*(a + b*Sin
[c + d*x]))
```

fricas [B] time = 0.55, size = 542, normalized size = 2.88

$$\frac{21\,a^5 - 82\,a^3b^2 + 60\,ab^4 + 12\left(a^5 - 6\,a^3b^2 + 5\,ab^4\right)\cos(dx+c)^4 - 2\left(18\,a^5 - 77\,a^3b^2 + 60\,ab^4\right)\cos(dx+c)^2 - 12}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(21*a^5 - 82*a^3*b^2 + 60*a*b^4 + 12*(a^5 - 6*a^3*b^2 + 5*a*b^4)*cos(d
*x + c)^4 - 2*(18*a^5 - 77*a^3*b^2 + 60*a*b^4)*cos(d*x + c)^2 - 12*(a^5 - 6
*a^3*b^2 + 5*a*b^4 + (a^5 - 6*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^4 - 2*(a^5 -
6*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^2 + (a^4*b - 6*a^2*b^3 + 5*b^5 + (a^4*b -
 6*a^2*b^3 + 5*b^5)*cos(d*x + c)^4 - 2*(a^4*b - 6*a^2*b^3 + 5*b^5)*cos(d*x
```

```
+ c)^2)*sin(d*x + c))*log(b*sin(d*x + c) + a) + 12*(a^5 - 6*a^3*b^2 + 5*a*b
^4 + (a^5 - 6*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^4 - 2*(a^5 - 6*a^3*b^2 + 5*a*
b^4)*cos(d*x + c)^2 + (a^4*b - 6*a^2*b^3 + 5*b^5 + (a^4*b - 6*a^2*b^3 + 5*b
^5)*cos(d*x + c)^4 - 2*(a^4*b - 6*a^2*b^3 + 5*b^5)*cos(d*x + c)^2)*sin(d*x
+ c))*log(-1/2*sin(d*x + c)) - (31*a^4*b - 30*a^2*b^3 - 6*(6*a^4*b - 5*a^2*
b^3)*cos(d*x + c)^2)*sin(d*x + c))/(a^7*d*cos(d*x + c)^4 - 2*a^7*d*cos(d*x
+ c)^2 + a^7*d + (a^6*b*d*cos(d*x + c)^4 - 2*a^6*b*d*cos(d*x + c)^2 + a^6*b
*d)*sin(d*x + c))
```

giac [A] time = 0.42, size = 278, normalized size = 1.48

$$\frac{\frac{12\left(a^{4}-6a^{2}b^{2}+5b^{4}\right)\log(|\sin(dx+c)|)}{a^{6}}-\frac{12\left(a^{4}b-6a^{2}b^{3}+5b^{5}\right)\log(|b\sin(dx+c)+a|)}{a^{6}b}+\frac{12\left(a^{4}b\sin(dx+c)-6a^{2}b^{3}\sin(dx+c)+5b^{5}\sin(dx+c)+2a^{5}-8a^{3}b^{2}\cdots}{(b\sin(dx+c)+a)a^{6}}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/12*(12*(a^4 - 6*a^2*b^2 + 5*b^4)*log(abs(sin(d*x + c)))/a^6 - 12*(a^4*b -
 6*a^2*b^3 + 5*b^5)*log(abs(b*sin(d*x + c) + a))/(a^6*b) + 12*(a^4*b*sin(d*
x + c) - 6*a^2*b^3*sin(d*x + c) + 5*b^5*sin(d*x + c) + 2*a^5 - 8*a^3*b^2 +
6*a*b^4)/((b*sin(d*x + c) + a)*a^6) - (25*a^4*sin(d*x + c)^4 - 150*a^2*b^2*
sin(d*x + c)^4 + 125*b^4*sin(d*x + c)^4 + 48*a^3*b*sin(d*x + c)^3 - 48*a*b^
3*sin(d*x + c)^3 - 12*a^4*sin(d*x + c)^2 + 18*a^2*b^2*sin(d*x + c)^2 - 8*a^
3*b*sin(d*x + c) + 3*a^4)/(a^6*sin(d*x + c)^4))/d
```

maple [A] time = 0.27, size = 282, normalized size = 1.50

$$-\frac{\ln\left(a+b\sin\left(dx+c\right)\right)}{a^{2}d}+\frac{6\ln\left(a+b\sin\left(dx+c\right)\right)b^{2}}{d\,a^{4}}-\frac{5\ln\left(a+b\sin\left(dx+c\right)\right)b^{4}}{d\,a^{6}}+\frac{1}{ad\left(a+b\sin\left(dx+c\right)\right)}-\frac{\cdots}{d\,a^{3}\left(a+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -ln(a+b*sin(d*x+c))/a^2/d+6/d/a^4*ln(a+b*sin(d*x+c))*b^2-5/d/a^6*ln(a+b*sin
(d*x+c))*b^4+1/a/d/(a+b*sin(d*x+c))-2/d/a^3/(a+b*sin(d*x+c))*b^2+1/d/a^5/(a
+b*sin(d*x+c))*b^4-1/4/d/a^2/sin(d*x+c)^4+1/d/a^2/sin(d*x+c)^2-3/2/d/a^4/si
n(d*x+c)^2*b^2+ln(sin(d*x+c))/a^2/d-6/d/a^4*ln(sin(d*x+c))*b^2+5/d/a^6*ln(s
in(d*x+c))*b^4+2/3/d/a^3*b/sin(d*x+c)^3-4/d/a^3*b/sin(d*x+c)+4/d*b^3/a^5/si
n(d*x+c)
```

maxima [A] time = 0.81, size = 189, normalized size = 1.01

$$\frac{\frac{5a^{3}b\sin(dx+c)+12\left(a^{4}-6a^{2}b^{2}+5b^{4}\right)\sin(dx+c)^{4}-3a^{4}-6\left(6a^{3}b-5ab^{3}\right)\sin(dx+c)^{3}+2\left(6a^{4}-5a^{2}b^{2}\right)\sin(dx+c)^{2}}{a^{5}b\sin(dx+c)^{5}+a^{6}\sin(dx+c)^{4}}-\frac{12\left(a^{4}-6a^{2}b^{2}+5b^{4}\right)\log(b\sin(dx+c)\cdots}{a^{6}}\cdots}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/12*((5*a^3*b*sin(d*x + c) + 12*(a^4 - 6*a^2*b^2 + 5*b^4)*sin(d*x + c)^4 -
 3*a^4 - 6*(6*a^3*b - 5*a*b^3)*sin(d*x + c)^3 + 2*(6*a^4 - 5*a^2*b^2)*sin(d
*x + c)^2)/(a^5*b*sin(d*x + c)^5 + a^6*sin(d*x + c)^4) - 12*(a^4 - 6*a^2*b^
2 + 5*b^4)*log(b*sin(d*x + c) + a)/a^6 + 12*(a^4 - 6*a^2*b^2 + 5*b^4)*log(s
in(d*x + c))/a^6)/d
```

mupad [B] time = 6.90, size = 439, normalized size = 2.34

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(3\,a^4-62\,a^2\,b^2+64\,b^4\right)-\frac{a^4}{4}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{11\,a^4}{4}-\frac{10\,a^2\,b^2}{3}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(20\,a\,b^3-\frac{62\,a^3\,b}{3}\right)}{d\,\left(16\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+16\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+32\,b\,a^5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5/(a + b*sin(c + d*x))^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^4*(3*a^4 + 64*b^4 - 62*a^2*b^2) - a^4/4 + tan(c/2 + (d*
x)/2)^2*((11*a^4)/4 - (10*a^2*b^2)/3) + tan(c/2 + (d*x)/2)^3*(20*a*b^3 - (6
2*a^3*b)/3) - (tan(c/2 + (d*x)/2)^5*(60*a^4*b + 32*b^5 - 96*a^2*b^3))/a + (
5*a^3*b*tan(c/2 + (d*x)/2))/6)/(d*(16*a^6*tan(c/2 + (d*x)/2)^4 + 16*a^6*tan
(c/2 + (d*x)/2)^6 + 32*a^5*b*tan(c/2 + (d*x)/2)^5)) - tan(c/2 + (d*x)/2)^4/
(64*a^2*d) + (tan(c/2 + (d*x)/2)^2*((a^2/16 + b^2/8)/a^4 + 1/(8*a^2) - b^2/
(2*a^4)))/d - (tan(c/2 + (d*x)/2)*((b*(32*a^2 + 64*b^2))/(64*a^5) - b/(4*a^
3) + (4*b*((a^2/8 + b^2/4)/a^4 + 1/(4*a^2) - b^2/a^4))/a))/d + (log(tan(c/2
 + (d*x)/2))*(a^4 + 5*b^4 - 6*a^2*b^2))/(a^6*d) + (b*tan(c/2 + (d*x)/2)^3)/
(12*a^3*d) - (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)*(a^4
 + 5*b^4 - 6*a^2*b^2))/(a^6*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**5/(a + b*sin(c + d*x))**2, x)
```

3.187 $\int \frac{\tan^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=333

$$\frac{2a^5\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}+\frac{a^4b\cos(c+dx)}{d\left(a^2-b^2\right)^3(a+b\sin(c+dx))}+\frac{8a^3b^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}-\frac{(3a+b)\cos(c+d}{4d(a+b)^3(1-\sin(c+}\cdots$$

```
[Out] 2*a^5*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(7/2)/d+8*
a^3*b^2*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(7/2)/d+
1/12*cos(d*x+c)/(a+b)^2/d/(1-sin(d*x+c))^2+1/12*cos(d*x+c)/(a+b)^2/d/(1-sin
(d*x+c))-1/4*(3*a+b)*cos(d*x+c)/(a+b)^3/d/(1-sin(d*x+c))-1/12*cos(d*x+c)/(a
-b)^2/d/(1+sin(d*x+c))^2-1/12*cos(d*x+c)/(a-b)^2/d/(1+sin(d*x+c))+1/4*(3*a-
b)*cos(d*x+c)/(a-b)^3/d/(1+sin(d*x+c))+a^4*b*cos(d*x+c)/(a^2-b^2)^3/d/(a+b*
sin(d*x+c))
```

Rubi [A] time = 0.63, antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2731, 2650, 2648, 2664, 12, 2660, 618, 204}

$$\frac{2a^5\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}+\frac{8a^3b^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}+\frac{a^4b\cos(c+dx)}{d\left(a^2-b^2\right)^3(a+b\sin(c+dx))}-\frac{(3a+b)\cos(c+d}{4d(a+b)^3(1-\sin(c+}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*a^5*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(7/2)
*d) + (8*a^3*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 -
b^2)^(7/2)*d) + Cos[c + d*x]/(12*(a + b)^2*d*(1 - Sin[c + d*x])^2) + Cos[c
+ d*x]/(12*(a + b)^2*d*(1 - Sin[c + d*x])) - ((3*a + b)*Cos[c + d*x])/(4*(a
 + b)^3*d*(1 - Sin[c + d*x])) - Cos[c + d*x]/(12*(a - b)^2*d*(1 + Sin[c + d
*x])^2) - Cos[c + d*x]/(12*(a - b)^2*d*(1 + Sin[c + d*x])) + ((3*a - b)*Cos
[c + d*x])/(4*(a - b)^3*d*(1 + Sin[c + d*x])) + (a^4*b*Cos[c + d*x])/((a^2
- b^2)^3*d*(a + b*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e + f*x])^m)
/(1 - Sin[e + f*x]^2)^(p/2), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 -
```

```
b^2, 0] && IntegersQ[m, p/2]
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \left(\frac{1}{4(a+b)^2(-1+\sin(c+dx))^2} + \frac{3a+b}{4(a+b)^3(-1+\sin(c+dx))} + \frac{1}{4(a-b)^2(1+\sin(\right.$$

$$= \frac{\int \frac{1}{(1+\sin(c+dx))^2}\,dx}{4(a-b)^2} - \frac{(3a-b)\int \frac{1}{1+\sin(c+dx)}\,dx}{4(a-b)^3} + \frac{\int \frac{1}{(-1+\sin(c+dx))^2}\,dx}{4(a+b)^2} + \frac{(3a+b)\int \frac{}{-1+\sin}}{4(a+b)}$$

$$= \frac{\cos(c+dx)}{12(a+b)^2d(1-\sin(c+dx))^2} - \frac{(3a+b)\cos(c+dx)}{4(a+b)^3d(1-\sin(c+dx))} - \frac{\cos(c+dx)}{12(a-b)^2d(1+\sin(c}$$

$$= \frac{\cos(c+dx)}{12(a+b)^2d(1-\sin(c+dx))^2} + \frac{\cos(c+dx)}{12(a+b)^2d(1-\sin(c+dx))} - \frac{(3a+b)\cos(c+}{4(a+b)^3d(1-\sin(c}$$

$$= \frac{8a^3b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d} + \frac{\cos(c+dx)}{12(a+b)^2d(1-\sin(c+dx))^2} + \frac{\cos(c+dx)}{12(a+b)^2d(1-\sin(}$$

$$= \frac{8a^3b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d} + \frac{\cos(c+dx)}{12(a+b)^2d(1-\sin(c+dx))^2} + \frac{\cos(c+dx)}{12(a+b)^2d(1-\sin(}$$

$$= \frac{2a^5\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d} + \frac{8a^3b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d} + \frac{\cos(c+dx)}{12(a+b)^2d(1-\sin(c+}$$

Mathematica [A] time = 2.03, size = 341, normalized size = 1.02

$$\frac{\frac{12a^4b\cos(c+dx)}{(a-b)^3(a+b)^3(a+b\sin(c+dx))} + \frac{24a^3\left(a^2+4b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}} - \frac{4(4a+b)\sin\left(\frac{1}{2}(c+dx)\right)}{(a+b)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{2\sin\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+\right.\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((24*a^3*(a^2 + 4*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a
^2 - b^2)^(7/2) + 1/((a + b)^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (
2*Sin[(c + d*x)/2])/((a + b)^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3) - (
4*(4*a + b)*Sin[(c + d*x)/2])/((a + b)^3*(Cos[(c + d*x)/2] - Sin[(c + d*x)/
2])) + (2*Sin[(c + d*x)/2])/((a - b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
)^3) - 1/((a - b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (4*(-4*a + b
)*Sin[(c + d*x)/2])/((a - b)^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (12
*a^4*b*Cos[c + d*x])/((a - b)^3*(a + b)^3*(a + b*Sin[c + d*x]))))/(12*d)
```

fricas [A] time = 0.57, size = 815, normalized size = 2.45

$$\left[-\frac{2\,a^6b - 6\,a^4b^3 + 6\,a^2b^5 - 2\,b^7 - 2\left(7\,a^6b + 2\,a^4b^3 - 10\,a^2b^5 + b^7\right)\cos\left(dx+c\right)^4 - 2\left(7\,a^6b - 16\,a^4b^3 + 11\,a^2b^5 - \cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(2*a^6*b - 6*a^4*b^3 + 6*a^2*b^5 - 2*b^7 - 2*(7*a^6*b + 2*a^4*b^3 - 1
0*a^2*b^5 + b^7)*cos(d*x + c)^4 - 2*(7*a^6*b - 16*a^4*b^3 + 11*a^2*b^5 - 2*
b^7)*cos(d*x + c)^2 - 3*((a^5*b + 4*a^3*b^3)*cos(d*x + c)^3*sin(d*x + c) +
(a^6 + 4*a^4*b^2)*cos(d*x + c)^3)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(
d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c
) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x +
 c) - a^2 - b^2)) - 2*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6 - (4*a^7 - 7*a^5
*b^2 + 2*a^3*b^4 + a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^8*b - 4*a^6*b^3
 + 6*a^4*b^5 - 4*a^2*b^7 + b^9)*d*cos(d*x + c)^3*sin(d*x + c) + (a^9 - 4*a^
7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d*x + c)^3), -1/3*(a^6*b - 3*a
^4*b^3 + 3*a^2*b^5 - b^7 - (7*a^6*b + 2*a^4*b^3 - 10*a^2*b^5 + b^7)*cos(d*x
 + c)^4 - (7*a^6*b - 16*a^4*b^3 + 11*a^2*b^5 - 2*b^7)*cos(d*x + c)^2 + 3*((
a^5*b + 4*a^3*b^3)*cos(d*x + c)^3*sin(d*x + c) + (a^6 + 4*a^4*b^2)*cos(d*x
+ c)^3)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d
*x + c))) - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6 - (4*a^7 - 7*a^5*b^2 + 2*a
^3*b^4 + a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^8*b - 4*a^6*b^3 + 6*a^4*b
^5 - 4*a^2*b^7 + b^9)*d*cos(d*x + c)^3*sin(d*x + c) + (a^9 - 4*a^7*b^2 + 6*
a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d*x + c)^3)]
```

giac [A] time = 2.98, size = 406, normalized size = 1.22

$$2\left(\frac{3\left(a^5+4a^3b^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^6-3a^4b^2+3a^2b^4-b^6\right)\sqrt{a^2-b^2}}+\frac{3\left(a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a^4b\right)}{\left(a^6-3a^4b^2+3a^2b^4-b^6\right)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a\right)}+\frac{3a^4\tan\left(\frac{1}{2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2/3*(3*(a^5 + 4*a^3*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan(
(a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6)*sqrt(a^2 - b^2)) + 3*(a^3*b^2*tan(1/2*d*x + 1/2*c) + a^4*b)/((a^6
- 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x
+ 1/2*c) + a)) + (3*a^4*tan(1/2*d*x + 1/2*c)^5 + 9*a^2*b^2*tan(1/2*d*x + 1/
2*c)^5 - 6*a^3*b*tan(1/2*d*x + 1/2*c)^4 - 6*a*b^3*tan(1/2*d*x + 1/2*c)^4 -
10*a^4*tan(1/2*d*x + 1/2*c)^3 - 18*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 4*b^4*t
an(1/2*d*x + 1/2*c)^3 + 24*a^3*b*tan(1/2*d*x + 1/2*c)^2 + 3*a^4*tan(1/2*d*x
 + 1/2*c) + 9*a^2*b^2*tan(1/2*d*x + 1/2*c) - 10*a^3*b - 2*a*b^3)/((a^6 - 3*
a^4*b^2 + 3*a^2*b^4 - b^6)*(tan(1/2*d*x + 1/2*c)^2 - 1)^3))/d
```

maple [A] time = 0.26, size = 382, normalized size = 1.15

$$-\frac{1}{3d\left(a+b\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{2d\left(a+b\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{a}{d\left(a+b\right)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{1}{3d\left(a-b\right)^2\left(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/3/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)^3-1/2/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1
)^2+1/d*a/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)-1/3/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+
1)^3+1/2/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)^2+1/d*a/(a-b)^3/(tan(1/2*d*x+1/2*
c)+1)+2/d*a^3/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*
b+a)*b^2*tan(1/2*d*x+1/2*c)+2/d*a^4/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a
+2*tan(1/2*d*x+1/2*c)*b+a)*b+2/d*a^5/(a-b)^3/(a+b)^3/(a^2-b^2)^(1/2)*arctan
(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+8/d*a^3/(a-b)^3/(a+b)^3/
(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^
2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 9.90, size = 722, normalized size = 2.17

$$\frac{\frac{2\left(13\,a^4\,b+2\,a^2\,b^3\right)}{3\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(-3\,a^5+14\,a^3\,b^2+4\,a\,b^4\right)}{3\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(a^4\,b+4\,a^2\,b^3\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(29\,a^4\,b+16\,a^2\,b^3\right)}{3\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}+\frac{2\tan\left(\right.}{3}}{d\left(-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\,b\tan\left(\frac{c}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4/(a + b*sin(c + d*x))^2,x)
```

```
[Out] ((2*(13*a^4*b + 2*a^2*b^3))/(3*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (2*ta
n(c/2 + (d*x)/2)*(4*a*b^4 - 3*a^5 + 14*a^3*b^2))/(3*(a^6 - b^6 + 3*a^2*b^4
- 3*a^4*b^2)) - (2*tan(c/2 + (d*x)/2)^6*(a^4*b + 4*a^2*b^3))/(a^6 - b^6 + 3
*a^2*b^4 - 3*a^4*b^2) - (2*tan(c/2 + (d*x)/2)^2*(29*a^4*b + 16*a^2*b^3))/(3
*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (2*tan(c/2 + (d*x)/2)^5*(8*a*b^4 +
7*a^5 + 30*a^3*b^2))/(3*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (2*tan(c/2 +
 (d*x)/2)^3*(4*a*b^4 - 7*a^5 + 48*a^3*b^2))/(3*(a^6 - b^6 + 3*a^2*b^4 - 3*a
^4*b^2)) + (2*tan(c/2 + (d*x)/2)^4*(11*a^4*b - 8*b^5 + 42*a^2*b^3))/(3*(a^6
 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (2*a^3*tan(c/2 + (d*x)/2)^7*(a^2 + 4*b^2
))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))/(d*(a + 2*b*tan(c/2 + (d*x)/2) - 2*
a*tan(c/2 + (d*x)/2)^2 + 2*a*tan(c/2 + (d*x)/2)^6 - a*tan(c/2 + (d*x)/2)^8
- 6*b*tan(c/2 + (d*x)/2)^3 + 6*b*tan(c/2 + (d*x)/2)^5 - 2*b*tan(c/2 + (d*x)
/2)^7)) + (2*a^3*atan(((a^3*(a^2 + 4*b^2)*(2*a^6*b - 2*b^7 + 6*a^2*b^5 - 6*
a^4*b^3))/((a + b)^(7/2)*(a - b)^(7/2)) + (2*a^4*tan(c/2 + (d*x)/2)*(a^2 +
4*b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))/((a + b)^(7/2)*(a - b)^(7/2)))/
(2*a^5 + 8*a^3*b^2))*(a^2 + 4*b^2))/(d*(a + b)^(7/2)*(a - b)^(7/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+b*sin(d*x+c))**2,x)
[Out] Integral(tan(c + d*x)**4/(a + b*sin(c + d*x))**2, x)
```

3.188 $\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=200

$$-\frac{4ab^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}-\frac{a^2b\cos(c+dx)}{d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}-\frac{2a^3\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}+\frac{\cos(c+dx)}{2d(a+b)^2(1-\sin(c}$$

```
[Out] -2*a^3*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)/d-4
*a*b^2*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)/d+1
/2*cos(d*x+c)/(a+b)^2/d/(1-sin(d*x+c))-1/2*cos(d*x+c)/(a-b)^2/d/(1+sin(d*x+
c))-a^2*b*cos(d*x+c)/(a^2-b^2)^2/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.30, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2731, 2648, 2664, 12, 2660, 618, 204}

$$-\frac{2a^3\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}-\frac{4ab^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}-\frac{a^2b\cos(c+dx)}{d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{\cos(c+dx)}{2d(a+b)^2(1-\sin(c}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a^3*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5/2
)*d) - (4*a*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b
^2)^(5/2)*d) + Cos[c + d*x]/(2*(a + b)^2*d*(1 - Sin[c + d*x])) - Cos[c + d*
x]/(2*(a - b)^2*d*(1 + Sin[c + d*x])) - (a^2*b*Cos[c + d*x])/((a^2 - b^2)^2
*d*(a + b*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e + f*x])^m)
/(1 - Sin[e + f*x]^2)^(p/2), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 -
b^2, 0] && IntegersQ[m, p/2]
```

Rubi steps

$$\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \left(-\frac{1}{2(a+b)^2(-1+\sin(c+dx))} + \frac{1}{2(a-b)^2(1+\sin(c+dx))} - \frac{a^2}{\left(a^2-b^2\right)(a+b\,\mathrm{si}\right.$$

$$= \frac{\int \frac{1}{1+\sin(c+dx)}\,dx}{2(a-b)^2} - \frac{\int \frac{1}{-1+\sin(c+dx)}\,dx}{2(a+b)^2} - \frac{\left(2ab^2\right)\int \frac{1}{a+b\sin(c+dx)}\,dx}{\left(a^2-b^2\right)^2} - \frac{a^2\int \frac{1}{(a+b\sin(c+dx))^2}\,\mathrm{c}}{a^2-b^2}$$

$$= \frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))} - \frac{\cos(c+dx)}{2(a-b)^2d(1+\sin(c+dx))} - \frac{a^2b\cos(c+dx)}{\left(a^2-b^2\right)^2d(a+b\sin(\mathrm{c}}$$

$$= \frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))} - \frac{\cos(c+dx)}{2(a-b)^2d(1+\sin(c+dx))} - \frac{a^2b\cos(c+dx)}{\left(a^2-b^2\right)^2d(a+b\sin(\mathrm{c}}$$

$$= -\frac{4ab^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}d} + \frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))} - \frac{\cos(c+dx)}{2(a-b)^2d(1+\sin(c}$$

$$= -\frac{4ab^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}d} + \frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))} - \frac{\cos(c+dx)}{2(a-b)^2d(1+\sin(c}$$

$$= -\frac{2a^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}d} - \frac{4ab^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}d} + \frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c\,-}$$

Mathematica [A] time = 1.06, size = 169, normalized size = 0.84

$$\frac{-\frac{2a\left(a^2+2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} - \frac{a^2b\cos(c+dx)}{(a-b)^2(a+b)^2(a+b\sin(c+dx))} + \sin\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{(a-b)^2\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{}{(a+b)^2\left(\mathrm{c}\right.}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((-2*a*(a^2 + 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2
 - b^2)^(5/2) + Sin[(c + d*x)/2]*(1/((a + b)^2*(Cos[(c + d*x)/2] - Sin[(c +
```

```
d*x)/2])) + 1/((a - b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) - (a^2*b*
Cos[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Sin[c + d*x])))/d
```

fricas [A] time = 0.55, size = 569, normalized size = 2.84

$$\left[-\frac{2\,a^4b - 4\,a^2b^3 + 2\,b^5 + 2\left(2\,a^4b - a^2b^3 - b^5\right)\cos\left(dx+c\right)^2 + \left(\left(a^3b + 2\,ab^3\right)\cos\left(dx+c\right)\sin\left(dx+c\right) + \left(a^4 + 2\,a^2\right.\right.}{2\left(\left(a^6b - 3\,a^4b^3 + 3\,a^2b^5 - b^7\right)d\cos\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a^4*b - 4*a^2*b^3 + 2*b^5 + 2*(2*a^4*b - a^2*b^3 - b^5)*cos(d*x +
c)^2 + ((a^3*b + 2*a*b^3)*cos(d*x + c)*sin(d*x + c) + (a^4 + 2*a^2*b^2)*cos
(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(
d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqr
t(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 2*(
a^5 - 2*a^3*b^2 + a*b^4)*sin(d*x + c))/((a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^
7)*d*cos(d*x + c)*sin(d*x + c) + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*co
s(d*x + c)), -(a^4*b - 2*a^2*b^3 + b^5 + (2*a^4*b - a^2*b^3 - b^5)*cos(d*x
+ c)^2 - ((a^3*b + 2*a*b^3)*cos(d*x + c)*sin(d*x + c) + (a^4 + 2*a^2*b^2)*c
os(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*
cos(d*x + c))) - (a^5 - 2*a^3*b^2 + a*b^4)*sin(d*x + c))/((a^6*b - 3*a^4*b^
3 + 3*a^2*b^5 - b^7)*d*cos(d*x + c)*sin(d*x + c) + (a^7 - 3*a^5*b^2 + 3*a^3
*b^4 - a*b^6)*d*cos(d*x + c))]
```

giac [A] time = 0.87, size = 251, normalized size = 1.26

$$-\frac{2\left(\frac{\left(a^3+2\,ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(a\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{a^2-b^2}}+\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,b\tan\left(\frac{1}{2}\,dx+\right.\right.}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2*((a^3 + 2*a*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*ta
n(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2
 - b^2)) + (a^3*tan(1/2*d*x + 1/2*c)^3 + 2*a*b^2*tan(1/2*d*x + 1/2*c)^3 + a
^2*b*tan(1/2*d*x + 1/2*c)^2 + 2*b^3*tan(1/2*d*x + 1/2*c)^2 + a^3*tan(1/2*d*
x + 1/2*c) - 4*a*b^2*tan(1/2*d*x + 1/2*c) - 3*a^2*b)/((a*tan(1/2*d*x + 1/2*
```

```
c)^4 + 2*b*tan(1/2*d*x + 1/2*c)^3 - 2*b*tan(1/2*d*x + 1/2*c) - a)*(a^4 - 2*
a^2*b^2 + b^4)))/d
```

maple [A] time = 0.31, size = 282, normalized size = 1.41

$$-\frac{1}{d\left(a+b\right)^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{1}{d\left(a-b\right)^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{2a\,b^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(a-b\right)^{2}\left(a+b\right)^{2}\left(\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+2\tan\left(\frac{dx}{2}\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)-1/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)-2/d*
a/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*b^2*tan
(1/2*d*x+1/2*c)-2/d*a^2/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d
*x+1/2*c)*b+a)*b-2/d*a^3/(a-b)^2/(a+b)^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*ta
n(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-4/d*a/(a-b)^2/(a+b)^2/(a^2-b^2)^(1/2
)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 8.68, size = 313, normalized size = 1.56

$$-\frac{\frac{6\,a^2\,b}{{\left(a^2-b^2\right)}^2}+\frac{2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(4\,a\,b^2-a^3\right)}{{\left(a^2-b^2\right)}^2}-\frac{2\,b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(a^2+2\,b^2\right)}{a^4-2\,a^2\,b^2+b^4}-\frac{2\,a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(a^2+2\,b^2\right)}{{\left(a^2-b^2\right)}^2}}{d\,\left(-a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-2\,b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+2\,b\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}-\frac{2\,a\,\mathrm{atan}\left(\frac{\frac{a\,\left(a^2+2\,b^2\right)\,\left(2\,a^4\,b-4\,a^2\,b^3+2\,b^5\right)}{{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{5/2}}+\cdots}{2\,a\cdots}\right)\cdots}{d\,\left(a+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + b*sin(c + d*x))^2,x)
```

```
[Out] - ((6*a^2*b)/(a^2 - b^2)^2 + (2*tan(c/2 + (d*x)/2)*(4*a*b^2 - a^3))/(a^2 -
b^2)^2 - (2*b*tan(c/2 + (d*x)/2)^2*(a^2 + 2*b^2))/(a^4 + b^4 - 2*a^2*b^2) -
 (2*a*tan(c/2 + (d*x)/2)^3*(a^2 + 2*b^2))/(a^2 - b^2)^2)/(d*(a + 2*b*tan(c/
2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^4 - 2*b*tan(c/2 + (d*x)/2)^3)) - (2*a*a
tan(((a*(a^2 + 2*b^2)*(2*a^4*b + 2*b^5 - 4*a^2*b^3))/((a + b)^(5/2)*(a - b)
^(5/2)) + (2*a^2*tan(c/2 + (d*x)/2)*(a^2 + 2*b^2)*(a^4 + b^4 - 2*a^2*b^2))/
((a + b)^(5/2)*(a - b)^(5/2)))/(4*a*b^2 + 2*a^3))*(a^2 + 2*b^2))/(d*(a + b)
^(5/2)*(a - b)^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**2/(a + b*sin(c + d*x))**2, x)
```

3.189 $\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=115

$$\frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{2\cot(c+dx)}{a^2d}-\frac{2\left(a^2-2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3d\sqrt{a^2-b^2}}+\frac{\cot(c+dx)}{ad(a+b\sin(c+dx))}$$

```
[Out] 2*b*arctanh(cos(d*x+c))/a^3/d-2*cot(d*x+c)/a^2/d+cot(d*x+c)/a/d/(a+b*sin(d*
x+c))-2*(a^2-2*b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^3/d/
(a^2-b^2)^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2723, 3056, 3001, 3770, 2660, 618, 204}

$$-\frac{2\left(a^2-2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3d\sqrt{a^2-b^2}}+\frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{2\cot(c+dx)}{a^2d}+\frac{\cot(c+dx)}{ad(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*(a^2 - 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3*Sq
rt[a^2 - b^2]*d) + (2*b*ArcTanh[Cos[c + d*x]])/(a^3*d) - (2*Cot[c + d*x])/(
a^2*d) + Cot[c + d*x]/(a*d*(a + b*Sin[c + d*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2))/Sin[e + f*x]
^2, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \frac{\csc^2(c+dx)\left(1-\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^2}\,dx$$

$$= \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))} + \frac{\int \frac{\csc^2(c+dx)\left(2\left(a^2-b^2\right)-\left(a^2-b^2\right)\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{a\left(a^2-b^2\right)}$$

$$= -\frac{2\cot(c+dx)}{a^2d} + \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))} + \frac{\int \frac{\csc(c+dx)\left(-2b\left(a^2-b^2\right)-a\left(a^2-b^2\right)\sin(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{a^2\left(a^2-b^2\right)}$$

$$= -\frac{2\cot(c+dx)}{a^2d} + \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))} - \frac{(2b)\int \csc(c+dx)\,dx}{a^3} - \frac{\left(a^2-2b^2\right)\int \frac{}{a+b\,\mathrm{s}}}{a^3}$$

$$= \frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{2\cot(c+dx)}{a^2d} + \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))} - \frac{\left(2\left(a^2-2b^2\right)\right)\mathrm{Su}}{}$$

$$= \frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{2\cot(c+dx)}{a^2d} + \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))} + \frac{\left(4\left(a^2-2b^2\right)\right)\mathrm{Su}}{}$$

$$= -\frac{2\left(a^2-2b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\sqrt{a^2-b^2}\,d} + \frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{2\cot(c+dx)}{a^2d} + \frac{}{a}$$

Mathematica [A] time = 0.76, size = 139, normalized size = 1.21

$$-\frac{\frac{4\left(a^2-2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{2ab\cos(c+dx)}{a+b\sin(c+dx)} - a\tan\left(\frac{1}{2}(c+dx)\right) + a\cot\left(\frac{1}{2}(c+dx)\right) + 4b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - 4}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/2*((4*(a^2 - 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sq
rt[a^2 - b^2] + a*Cot[(c + d*x)/2] - 4*b*Log[Cos[(c + d*x)/2]] + 4*b*Log[Si
n[(c + d*x)/2]] + (2*a*b*Cos[c + d*x])/(a + b*Sin[c + d*x]) - a*Tan[(c + d*
x)/2])/(a^3*d)
```

fricas [B] time = 0.66, size = 768, normalized size = 6.68

$$\left[\frac{4\left(a^3b-ab^3\right)\cos\left(dx+c\right)\sin\left(dx+c\right)-\left(a^2b-2\,b^3-\left(a^2b-2\,b^3\right)\cos\left(dx+c\right)^2+\left(a^3-2\,ab^2\right)\sin\left(dx+c\right)\right)\sqrt{-a^2}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(4*(a^3*b - a*b^3)*cos(d*x + c)*sin(d*x + c) - (a^2*b - 2*b^3 - (a^2*b
 - 2*b^3)*cos(d*x + c)^2 + (a^3 - 2*a*b^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*l
og(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*co
s(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x +
c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 2*(a^4 - a^2*b^2)*cos(d*x + c) -
2*(a^2*b^2 - b^4 - (a^2*b^2 - b^4)*cos(d*x + c)^2 + (a^3*b - a*b^3)*sin(d*x
 + c))*log(1/2*cos(d*x + c) + 1/2) + 2*(a^2*b^2 - b^4 - (a^2*b^2 - b^4)*cos
(d*x + c)^2 + (a^3*b - a*b^3)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/(
(a^5*b - a^3*b^3)*d*cos(d*x + c)^2 - (a^6 - a^4*b^2)*d*sin(d*x + c) - (a^5*
b - a^3*b^3)*d), (2*(a^3*b - a*b^3)*cos(d*x + c)*sin(d*x + c) - (a^2*b - 2*
b^3 - (a^2*b - 2*b^3)*cos(d*x + c)^2 + (a^3 - 2*a*b^2)*sin(d*x + c))*sqrt(a
^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + (a
^4 - a^2*b^2)*cos(d*x + c) - (a^2*b^2 - b^4 - (a^2*b^2 - b^4)*cos(d*x + c)^
2 + (a^3*b - a*b^3)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + (a^2*b^2 -
b^4 - (a^2*b^2 - b^4)*cos(d*x + c)^2 + (a^3*b - a*b^3)*sin(d*x + c))*log(-1
/2*cos(d*x + c) + 1/2))/((a^5*b - a^3*b^3)*d*cos(d*x + c)^2 - (a^6 - a^4*b^
2)*d*sin(d*x + c) - (a^5*b - a^3*b^3)*d)]
```

giac [A] time = 0.40, size = 218, normalized size = 1.90

$$-\frac{\frac{12\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}-\frac{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2}+\frac{12\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)\left(a^2-2\,b^2\right)}{\sqrt{a^2-b^2}\,a^3}-\frac{4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,a^2\tan\left(\frac{1}{2}\cdots\right.}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3\cdots\right.}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/6*(12*b*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 3*tan(1/2*d*x + 1/2*c)/a^2
+ 12*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2
*c) + b)/sqrt(a^2 - b^2)))*(a^2 - 2*b^2)/(sqrt(a^2 - b^2)*a^3) - (4*a*b*tan
(1/2*d*x + 1/2*c)^3 - 3*a^2*tan(1/2*d*x + 1/2*c)^2 - 4*b^2*tan(1/2*d*x + 1/
```

```
2*c)^2 - 14*a*b*tan(1/2*d*x + 1/2*c) - 3*a^2)/((a*tan(1/2*d*x + 1/2*c)^3 +
2*b*tan(1/2*d*x + 1/2*c)^2 + a*tan(1/2*d*x + 1/2*c))*a^3))/d
```

maple [B] time = 0.28, size = 245, normalized size = 2.13

$$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}-\frac{1}{2d\,a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3}-\frac{2b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^3\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b+a\right)}-\frac{}{d\,a^2\left(\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/2/d/a^2*tan(1/2*d*x+1/2*c)-1/2/d/a^2/tan(1/2*d*x+1/2*c)-2/d/a^3*b*ln(tan(
1/2*d*x+1/2*c))-2/d/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*b
^2*tan(1/2*d*x+1/2*c)-2/d/a^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*
b+a)*b-2/d/a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b
^2)^(1/2))+4/d/a^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/
(a^2-b^2)^(1/2))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 8.32, size = 1616, normalized size = 14.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + b*sin(c + d*x))^2,x)
```

```
[Out] -(a^4*cos(c + d*x) - b^4/2 - b^4*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))
 + (a^2*b^2)/2 + (b^4*cos(2*c + 2*d*x))/2 - a^2*b^2*cos(c + d*x) - a*b^3*si
n(2*c + 2*d*x) + a^3*b*sin(2*c + 2*d*x) + a^2*b^2*log(sin(c/2 + (d*x)/2)/co
s(c/2 + (d*x)/2)) + b^4*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c
+ 2*d*x) + b^3*atan((a^3*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*1i - b^3*sin(
```

```
c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*8i - a*b^2*cos(c/2 + (d*x)/2)*(b^2 - a^2)^
(1/2)*4i + a^2*b*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i)/(a^4*sin(c/2 + (d
*x)/2) + 8*b^4*sin(c/2 + (d*x)/2) + 4*a*b^3*cos(c/2 + (d*x)/2) - 3*a^3*b*co
s(c/2 + (d*x)/2) - 8*a^2*b^2*sin(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2)*2i - (a
^2*b^2*cos(2*c + 2*d*x))/2 - a*b^3*sin(c + d*x) + a^3*b*sin(c + d*x) - a^2*
b*atan((a^3*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*1i - b^3*sin(c/2 + (d*x)/2
)*(b^2 - a^2)^(1/2)*8i - a*b^2*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i + a^
2*b*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i)/(a^4*sin(c/2 + (d*x)/2) + 8*b^
4*sin(c/2 + (d*x)/2) + 4*a*b^3*cos(c/2 + (d*x)/2) - 3*a^3*b*cos(c/2 + (d*x)
/2) - 8*a^2*b^2*sin(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2)*1i - a^3*sin(c + d*x
)*atan((a^3*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*1i - b^3*sin(c/2 + (d*x)/2
)*(b^2 - a^2)^(1/2)*8i - a*b^2*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i + a^
2*b*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i)/(a^4*sin(c/2 + (d*x)/2) + 8*b^
4*sin(c/2 + (d*x)/2) + 4*a*b^3*cos(c/2 + (d*x)/2) - 3*a^3*b*cos(c/2 + (d*x)
/2) - 8*a^2*b^2*sin(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2)*2i - a^2*b^2*log(sin
(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x) - b^3*cos(2*c + 2*d*x)
*atan((a^3*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*1i - b^3*sin(c/2 + (d*x)/2)
*(b^2 - a^2)^(1/2)*8i - a*b^2*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i + a^2
*b*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i)/(a^4*sin(c/2 + (d*x)/2) + 8*b^4
*sin(c/2 + (d*x)/2) + 4*a*b^3*cos(c/2 + (d*x)/2) - 3*a^3*b*cos(c/2 + (d*x)/
2) - 8*a^2*b^2*sin(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2)*2i - 2*a*b^3*sin(c +
d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + 2*a^3*b*sin(c + d*x)*log(
sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + a*b^2*sin(c + d*x)*atan((a^3*cos(c
/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*1i - b^3*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/
2)*8i - a*b^2*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i + a^2*b*sin(c/2 + (d*
x)/2)*(b^2 - a^2)^(1/2)*4i)/(a^4*sin(c/2 + (d*x)/2) + 8*b^4*sin(c/2 + (d*x)
/2) + 4*a*b^3*cos(c/2 + (d*x)/2) - 3*a^3*b*cos(c/2 + (d*x)/2) - 8*a^2*b^2*s
in(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2)*4i + a^2*b*cos(2*c + 2*d*x)*atan((a^3
*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*1i - b^3*sin(c/2 + (d*x)/2)*(b^2 - a^
2)^(1/2)*8i - a*b^2*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i + a^2*b*sin(c/2
 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i)/(a^4*sin(c/2 + (d*x)/2) + 8*b^4*sin(c/2 +
 (d*x)/2) + 4*a*b^3*cos(c/2 + (d*x)/2) - 3*a^3*b*cos(c/2 + (d*x)/2) - 8*a^2
*b^2*sin(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2)*1i)/(2*a^3*d*(a^2 - b^2)*(b/4 +
 (a*sin(c + d*x))/2 - (b*cos(2*c + 2*d*x))/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**2/(a + b*sin(c + d*x))**2, x)
```

3.190 $\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=238

$$\frac{(3a^2-4b^2)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))}-\frac{b(3a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{a^5d}+\frac{(7a^2-12b^2)\cot(c+dx)}{3a^4d}-\frac{(a^2-2b^2)\mathrm{co}}{}$$

```
[Out] -b*(3*a^2-4*b^2)*arctanh(cos(d*x+c))/a^5/d+1/3*(7*a^2-12*b^2)*cot(d*x+c)/a^
4/d-(a^2-2*b^2)*cot(d*x+c)*csc(d*x+c)/a^3/b/d+1/3*(3*a^2-4*b^2)*cot(d*x+c)*
csc(d*x+c)/a^2/b/d/(a+b*sin(d*x+c))-1/3*cot(d*x+c)*csc(d*x+c)^2/a/d/(a+b*si
n(d*x+c))+2*(a^4-5*a^2*b^2+4*b^4)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)
^(1/2))/a^5/d/(a^2-b^2)^(1/2)
```

Rubi [A] time = 0.70, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2724, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{2(-5a^2b^2+a^4+4b^4)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5d\sqrt{a^2-b^2}}+\frac{(7a^2-12b^2)\cot(c+dx)}{3a^4d}-\frac{b(3a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{a^5d}-\frac{(a^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*(a^4 - 5*a^2*b^2 + 4*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2
]])/(a^5*Sqrt[a^2 - b^2]*d) - (b*(3*a^2 - 4*b^2)*ArcTanh[Cos[c + d*x]])/(a^
5*d) + ((7*a^2 - 12*b^2)*Cot[c + d*x])/(3*a^4*d) - ((a^2 - 2*b^2)*Cot[c + d
*x]*Csc[c + d*x])/(a^3*b*d) + ((3*a^2 - 4*b^2)*Cot[c + d*x]*Csc[c + d*x])/(
3*a^2*b*d*(a + b*Sin[c + d*x])) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*a*d*(a +
 b*Sin[c + d*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2724

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a*f*Sin[e
 + f*x]^3), x] + (-Dist[1/(3*a^2*b*(m + 1)), Int[((a + b*Sin[e + f*x])^(m +
 1)*Simp[6*a^2 - b^2*(m - 1)*(m - 2) + a*b*(m + 1)*Sin[e + f*x] - (3*a^2 -
b^2*m*(m - 2))*Sin[e + f*x]^2, x])/Sin[e + f*x]^3, x], x] - Simp[((3*a^2 +
b^2*(m - 2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a^2*b*f*(m + 1)*
Sin[e + f*x]^2), x]) /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[
m, -1] && IntegerQ[2*m]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\left(3a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))} + \frac{\int \csc^3(c+dx)\left(6\left(a^2-2b\right.\right.}{}$$

$$= -\frac{\left(a^2-2b^2\right)\cot(c+dx)\csc(c+dx)}{a^3bd} + \frac{\left(3a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))} - \frac{\cot(c}{3ad(a}$$

$$= \frac{\left(7a^2-12b^2\right)\cot(c+dx)}{3a^4d} - \frac{\left(a^2-2b^2\right)\cot(c+dx)\csc(c+dx)}{a^3bd} + \frac{\left(3a^2-4b^2\right)\cot(c}{3a^2bd(a+b\,s}$$

$$= \frac{\left(7a^2-12b^2\right)\cot(c+dx)}{3a^4d} - \frac{\left(a^2-2b^2\right)\cot(c+dx)\csc(c+dx)}{a^3bd} + \frac{\left(3a^2-4b^2\right)\cot(c}{3a^2bd(a+b\,s}$$

$$= -\frac{b\left(3a^2-4b^2\right)\tanh^{-1}(\cos(c+dx))}{a^5d} + \frac{\left(7a^2-12b^2\right)\cot(c+dx)}{3a^4d} - \frac{\left(a^2-2b^2\right)\cot(c}{a^3}$$

$$= -\frac{b\left(3a^2-4b^2\right)\tanh^{-1}(\cos(c+dx))}{a^5d} + \frac{\left(7a^2-12b^2\right)\cot(c+dx)}{3a^4d} - \frac{\left(a^2-2b^2\right)\cot(c}{a^3}$$

$$= \frac{2\left(a^4-5a^2b^2+4b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5\sqrt{a^2-b^2}\,d} - \frac{b\left(3a^2-4b^2\right)\tanh^{-1}(\cos(c+dx))}{a^5d} +$$

Mathematica [A] time = 6.20, size = 403, normalized size = 1.69

$$\frac{b\csc^2\left(\frac{1}{2}(c+dx)\right)}{4a^3d} - \frac{b\sec^2\left(\frac{1}{2}(c+dx)\right)}{4a^3d} - \frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{24a^2d} + \frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)}{24a^2d} + \frac{(3a}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*(a^4 - 5*a^2*b^2 + 4*b^4)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2] +
 a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^5*Sqrt[a^2 - b^2]*d) + ((4*a^2*C
```

```
os[(c + d*x)/2] - 9*b^2*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*a^4*d) + (b*
Csc[(c + d*x)/2]^2)/(4*a^3*d) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(24*a
^2*d) + ((-3*a^2*b + 4*b^3)*Log[Cos[(c + d*x)/2]])/(a^5*d) + ((3*a^2*b - 4*
b^3)*Log[Sin[(c + d*x)/2]])/(a^5*d) - (b*Sec[(c + d*x)/2]^2)/(4*a^3*d) + (S
ec[(c + d*x)/2]*(-4*a^2*Sin[(c + d*x)/2] + 9*b^2*Sin[(c + d*x)/2]))/(6*a^4*
d) + (a^2*b*Cos[c + d*x] - b^3*Cos[c + d*x])/(a^4*d*(a + b*Sin[c + d*x])) +
 (Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24*a^2*d)
```

fricas [B] time = 0.72, size = 1149, normalized size = 4.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(4*(2*a^4 - 3*a^2*b^2)*cos(d*x + c)^3 + 3*((a^2*b - 4*b^3)*cos(d*x +
c)^4 + a^2*b - 4*b^3 - 2*(a^2*b - 4*b^3)*cos(d*x + c)^2 + (a^3 - 4*a*b^2 -
(a^3 - 4*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2
- b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*
sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*
b*sin(d*x + c) - a^2 - b^2)) - 6*(a^4 - 2*a^2*b^2)*cos(d*x + c) + 3*((3*a^2
*b^2 - 4*b^4)*cos(d*x + c)^4 + 3*a^2*b^2 - 4*b^4 - 2*(3*a^2*b^2 - 4*b^4)*co
s(d*x + c)^2 + (3*a^3*b - 4*a*b^3 - (3*a^3*b - 4*a*b^3)*cos(d*x + c)^2)*sin
(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*((3*a^2*b^2 - 4*b^4)*cos(d*x + c
)^4 + 3*a^2*b^2 - 4*b^4 - 2*(3*a^2*b^2 - 4*b^4)*cos(d*x + c)^2 + (3*a^3*b -
 4*a*b^3 - (3*a^3*b - 4*a*b^3)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d
*x + c) + 1/2) + 2*((7*a^3*b - 12*a*b^3)*cos(d*x + c)^3 - 3*(3*a^3*b - 4*a*
b^3)*cos(d*x + c))*sin(d*x + c))/(a^5*b*d*cos(d*x + c)^4 - 2*a^5*b*d*cos(d*
x + c)^2 + a^5*b*d - (a^6*d*cos(d*x + c)^2 - a^6*d)*sin(d*x + c)), -1/6*(4*
(2*a^4 - 3*a^2*b^2)*cos(d*x + c)^3 + 6*((a^2*b - 4*b^3)*cos(d*x + c)^4 + a^
2*b - 4*b^3 - 2*(a^2*b - 4*b^3)*cos(d*x + c)^2 + (a^3 - 4*a*b^2 - (a^3 - 4*
a*b^2)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c
) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 6*(a^4 - 2*a^2*b^2)*cos(d*x + c) +
 3*((3*a^2*b^2 - 4*b^4)*cos(d*x + c)^4 + 3*a^2*b^2 - 4*b^4 - 2*(3*a^2*b^2 -
 4*b^4)*cos(d*x + c)^2 + (3*a^3*b - 4*a*b^3 - (3*a^3*b - 4*a*b^3)*cos(d*x +
 c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*((3*a^2*b^2 - 4*b^4)*c
os(d*x + c)^4 + 3*a^2*b^2 - 4*b^4 - 2*(3*a^2*b^2 - 4*b^4)*cos(d*x + c)^2 +
(3*a^3*b - 4*a*b^3 - (3*a^3*b - 4*a*b^3)*cos(d*x + c)^2)*sin(d*x + c))*log(
-1/2*cos(d*x + c) + 1/2) + 2*((7*a^3*b - 12*a*b^3)*cos(d*x + c)^3 - 3*(3*a^
3*b - 4*a*b^3)*cos(d*x + c))*sin(d*x + c))/(a^5*b*d*cos(d*x + c)^4 - 2*a^5*
b*d*cos(d*x + c)^2 + a^5*b*d - (a^6*d*cos(d*x + c)^2 - a^6*d)*sin(d*x + c))
]
```

giac [A] time = 0.54, size = 356, normalized size = 1.50

$$\frac{24\left(3\,a^2b-4\,b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^5}+\frac{48\left(a^4-5\,a^2b^2+4\,b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^5}+\frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,a^3b\tan\left(\frac{1}{2}\,dx\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(24*(3*a^2*b - 4*b^3)*log(abs(tan(1/2*d*x + 1/2*c)))/a^5 + 48*(a^4 - 5
*a^2*b^2 + 4*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(
1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^5) + (a^4*tan(1/
2*d*x + 1/2*c)^3 - 6*a^3*b*tan(1/2*d*x + 1/2*c)^2 - 15*a^4*tan(1/2*d*x + 1/
2*c) + 36*a^2*b^2*tan(1/2*d*x + 1/2*c))/a^6 + 48*(a^2*b^2*tan(1/2*d*x + 1/2
*c) - b^4*tan(1/2*d*x + 1/2*c) + a^3*b - a*b^3)/((a*tan(1/2*d*x + 1/2*c)^2
+ 2*b*tan(1/2*d*x + 1/2*c) + a)*a^5) - (132*a^2*b*tan(1/2*d*x + 1/2*c)^3 -
176*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*a^3*tan(1/2*d*x + 1/2*c)^2 + 36*a*b^2*t
an(1/2*d*x + 1/2*c)^2 - 6*a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/(a^5*tan(1/2*d*
x + 1/2*c)^3))/d
```

maple [B] time = 0.29, size = 527, normalized size = 2.21

$$\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{24d\,a^2}-\frac{b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d\,a^3}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^2}+\frac{3b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^4}-\frac{1}{24d\,a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{5}{8d\,a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/24/d/a^2*tan(1/2*d*x+1/2*c)^3-1/4/d/a^3*b*tan(1/2*d*x+1/2*c)^2-5/8/d/a^2*
tan(1/2*d*x+1/2*c)+3/2/d/a^4*b^2*tan(1/2*d*x+1/2*c)-1/24/d/a^2/tan(1/2*d*x+
1/2*c)^3+5/8/d/a^2/tan(1/2*d*x+1/2*c)-3/2/d/a^4/tan(1/2*d*x+1/2*c)*b^2+1/4/
d/a^3*b/tan(1/2*d*x+1/2*c)^2+3/d/a^3*b*ln(tan(1/2*d*x+1/2*c))-4/d/a^5*b^3*l
n(tan(1/2*d*x+1/2*c))+2/d/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*
b+a)*b^2*tan(1/2*d*x+1/2*c)-2/d/a^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1
/2*c)*b+a)*tan(1/2*d*x+1/2*c)*b^4+2/d/a^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2
*d*x+1/2*c)*b+a)*b-2/d/a^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a
)*b^3+2/d/a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^
2)^(1/2))-10/d/a^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/
(a^2-b^2)^(1/2))*b^2+8/d/a^5/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/
2*c)+2*b)/(a^2-b^2)^(1/2))*b^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 6.83, size = 973, normalized size = 4.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4/(a + b*sin(c + d*x))^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*(28*a^2*b - 40*b^3) - tan(c/2 + (d*x)/2)^2*(8*a*b^2 -
 (14*a^3)/3) - a^3/3 + (4*a^2*b*tan(c/2 + (d*x)/2))/3 + (tan(c/2 + (d*x)/2)
^4*(5*a^4 - 16*b^4 + 4*a^2*b^2))/a)/(d*(8*a^5*tan(c/2 + (d*x)/2)^3 + 8*a^5*
tan(c/2 + (d*x)/2)^5 + 16*a^4*b*tan(c/2 + (d*x)/2)^4)) + tan(c/2 + (d*x)/2)
^3/(24*a^2*d) - (tan(c/2 + (d*x)/2)*((16*a^2 + 32*b^2)/(64*a^4) + 3/(8*a^2)
 - (2*b^2)/a^4))/d - (b*tan(c/2 + (d*x)/2)^2)/(4*a^3*d) + (b*log(tan(c/2 +
(d*x)/2))*(3*a^2 - 4*b^2))/(a^5*d) + (atan((((b^2 - a^2)^(1/2)*(a^2 - 4*b^2
)*((2*(a^9 + 8*a^5*b^4 - 8*a^7*b^2))/a^8 + (2*tan(c/2 + (d*x)/2)*(5*a^7*b +
 16*a^3*b^5 - 20*a^5*b^3))/a^7 + ((2*a^2*b - (2*tan(c/2 + (d*x)/2)*(3*a^10
- 4*a^8*b^2))/a^7)*(b^2 - a^2)^(1/2)*(a^2 - 4*b^2))/a^5)*1i)/a^5 + ((b^2 -
a^2)^(1/2)*(a^2 - 4*b^2)*((2*(a^9 + 8*a^5*b^4 - 8*a^7*b^2))/a^8 + (2*tan(c/
2 + (d*x)/2)*(5*a^7*b + 16*a^3*b^5 - 20*a^5*b^3))/a^7 - ((2*a^2*b - (2*tan(
c/2 + (d*x)/2)*(3*a^10 - 4*a^8*b^2))/a^7)*(b^2 - a^2)^(1/2)*(a^2 - 4*b^2))/
a^5)*1i)/a^5)/((4*(3*a^6*b - 16*b^7 + 32*a^2*b^5 - 19*a^4*b^3))/a^8 + (4*ta
n(c/2 + (d*x)/2)*(2*a^6 - 16*b^6 + 28*a^2*b^4 - 14*a^4*b^2))/a^7 - ((b^2 -
a^2)^(1/2)*(a^2 - 4*b^2)*((2*(a^9 + 8*a^5*b^4 - 8*a^7*b^2))/a^8 + (2*tan(c/
2 + (d*x)/2)*(5*a^7*b + 16*a^3*b^5 - 20*a^5*b^3))/a^7 + ((2*a^2*b - (2*tan(
c/2 + (d*x)/2)*(3*a^10 - 4*a^8*b^2))/a^7)*(b^2 - a^2)^(1/2)*(a^2 - 4*b^2))/
a^5))/a^5 + ((b^2 - a^2)^(1/2)*(a^2 - 4*b^2)*((2*(a^9 + 8*a^5*b^4 - 8*a^7*b
^2))/a^8 + (2*tan(c/2 + (d*x)/2)*(5*a^7*b + 16*a^3*b^5 - 20*a^5*b^3))/a^7 -
 ((2*a^2*b - (2*tan(c/2 + (d*x)/2)*(3*a^10 - 4*a^8*b^2))/a^7)*(b^2 - a^2)^(
1/2)*(a^2 - 4*b^2))/a^5))/a^5))*(b^2 - a^2)^(1/2)*(a^2 - 4*b^2)*2i)/(a^5*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**4/(a + b*sin(c + d*x))**2, x)
```

3.191 $\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=424

$$\frac{3b\cot(c+dx)\csc^3(c+dx)}{10a^2d(a+b\sin(c+dx))}-\frac{2\left(a^2-6b^2\right)\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^7d}-\frac{\left(15a^4-82a^2b^2+60b^4\right)\cot(c+dx)}{30a^4b^2d}$$

```
[Out] -2*(a^2-6*b^2)*(a^2-b^2)^(3/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1
/2))/a^7/d+1/4*b*(15*a^4-40*a^2*b^2+24*b^4)*arctanh(cos(d*x+c))/a^7/d-1/15*
(38*a^4-135*a^2*b^2+90*b^4)*cot(d*x+c)/a^6/d+1/4*(4*a^4-17*a^2*b^2+12*b^4)*
cot(d*x+c)*csc(d*x+c)/a^5/b/d-1/30*(15*a^4-82*a^2*b^2+60*b^4)*cot(d*x+c)*cs
c(d*x+c)^2/a^4/b^2/d-1/2*cot(d*x+c)*csc(d*x+c)/b/d/(a+b*sin(d*x+c))+1/6*a*c
ot(d*x+c)*csc(d*x+c)^2/b^2/d/(a+b*sin(d*x+c))+1/6*(2*a^4-12*a^2*b^2+9*b^4)*
cot(d*x+c)*csc(d*x+c)^2/a^3/b^2/d/(a+b*sin(d*x+c))+3/10*b*cot(d*x+c)*csc(d*
x+c)^3/a^2/d/(a+b*sin(d*x+c))-1/5*cot(d*x+c)*csc(d*x+c)^4/a/d/(a+b*sin(d*x+
c))
```

Rubi [A] time = 1.49, antiderivative size = 424, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2726, 3055, 3001, 3770, 2660, 618, 204}

$$-\frac{2\left(a^2-6b^2\right)\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^7d}-\frac{\left(-135a^2b^2+38a^4+90b^4\right)\cot(c+dx)}{15a^6d}+\frac{b\left(-40a^2b^2+15a^4+2\right.}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*(a^2 - 6*b^2)*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^
2 - b^2]])/(a^7*d) + (b*(15*a^4 - 40*a^2*b^2 + 24*b^4)*ArcTanh[Cos[c + d*x]
])/(4*a^7*d) - ((38*a^4 - 135*a^2*b^2 + 90*b^4)*Cot[c + d*x])/(15*a^6*d) +
((4*a^4 - 17*a^2*b^2 + 12*b^4)*Cot[c + d*x]*Csc[c + d*x])/(4*a^5*b*d) - ((1
5*a^4 - 82*a^2*b^2 + 60*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(30*a^4*b^2*d) -
(Cot[c + d*x]*Csc[c + d*x])/(2*b*d*(a + b*Sin[c + d*x])) + (a*Cot[c + d*x]*
Csc[c + d*x]^2)/(6*b^2*d*(a + b*Sin[c + d*x])) + ((2*a^4 - 12*a^2*b^2 + 9*b
^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(6*a^3*b^2*d*(a + b*Sin[c + d*x])) + (3*b*
Cot[c + d*x]*Csc[c + d*x]^3)/(10*a^2*d*(a + b*Sin[c + d*x])) - (Cot[c + d*x
]*Csc[c + d*x]^4)/(5*a*d*(a + b*Sin[c + d*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^6,
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(5*a*f*Sin[e
 + f*x]^5), x] + (Dist[1/(20*a^2*b^2*m*(m - 1)), Int[((a + b*Sin[e + f*x])^
m*Simp[60*a^4 - 44*a^2*b^2*(m - 1)*m + b^4*m*(m - 1)*(m - 3)*(m - 4) + a*b*
m*(20*a^2 - b^2*m*(m - 1))*Sin[e + f*x] - (40*a^4 + b^4*m*(m - 1)*(m - 2)*(
m - 4) - 20*a^2*b^2*(m - 1)*(2*m + 1))*Sin[e + f*x]^2, x])/Sin[e + f*x]^4,
x], x] + Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*m*Sin[e + f*
x]^2), x] + Simp[(a*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*m*(m
- 1)*Sin[e + f*x]^3), x] - Simp[(b*(m - 4)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m + 1))/(20*a^2*f*Sin[e + f*x]^4), x]) /; FreeQ[{a, b, e, f, m}, x] && N
eQ[a^2 - b^2, 0] && NeQ[m, 1] && IntegerQ[2*m]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
```

```
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]
) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{\cot(c+dx)\csc(c+dx)}{2bd(a+b\sin(c+dx))} + \frac{a\cot(c+dx)\csc^2(c+dx)}{6b^2d(a+b\sin(c+dx))} + \frac{3b\cot(c+dx)\csc^3(c+dx}{10a^2d(a+b\sin(c+dx))}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{2bd(a+b\sin(c+dx))} + \frac{a\cot(c+dx)\csc^2(c+dx)}{6b^2d(a+b\sin(c+dx))} + \frac{\left(2a^4-12a^2b^2+9b^4\right)\cot(c}{6a^3b^2d(a+b\sin}$$

$$= -\frac{\left(15a^4-82a^2b^2+60b^4\right)\cot(c+dx)\csc^2(c+dx)}{30a^4b^2d} - \frac{\cot(c+dx)\csc(c+dx)}{2bd(a+b\sin(c+dx))} + \frac{a\,\mathrm{co}}{6b}$$

$$= \frac{\left(4a^4-17a^2b^2+12b^4\right)\cot(c+dx)\csc(c+dx)}{4a^5bd} - \frac{\left(15a^4-82a^2b^2+60b^4\right)\cot(c+dx}{30a^4b^2d}$$

$$= -\frac{\left(38a^4-135a^2b^2+90b^4\right)\cot(c+dx)}{15a^6d} + \frac{\left(4a^4-17a^2b^2+12b^4\right)\cot(c+dx)\csc(c+}{4a^5bd}$$

$$= -\frac{\left(38a^4-135a^2b^2+90b^4\right)\cot(c+dx)}{15a^6d} + \frac{\left(4a^4-17a^2b^2+12b^4\right)\cot(c+dx)\csc(c+}{4a^5bd}$$

$$= \frac{b\left(15a^4-40a^2b^2+24b^4\right)\tanh^{-1}(\cos(c+dx))}{4a^7d} - \frac{\left(38a^4-135a^2b^2+90b^4\right)\cot(c+}{15a^6d}$$

$$= \frac{b\left(15a^4-40a^2b^2+24b^4\right)\tanh^{-1}(\cos(c+dx))}{4a^7d} - \frac{\left(38a^4-135a^2b^2+90b^4\right)\cot(c+}{15a^6d}$$

$$= -\frac{2\left(a^2-6b^2\right)\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^7d} + \frac{b\left(15a^4-40a^2b^2+24b^4\right)\tanh}{4a^7d}$$

Mathematica [A] time = 1.57, size = 361, normalized size = 0.85

$$-\frac{1920\left(a^2-6b^2\right)\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)+240b\left(15a^4-40a^2b^2+24b^4\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-240}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/960*(1920*(a^2 - 6*b^2)*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2]
)/Sqrt[a^2 - b^2]] - 240*b*(15*a^4 - 40*a^2*b^2 + 24*b^4)*Log[Cos[(c + d*x)
/2]] + 240*b*(15*a^4 - 40*a^2*b^2 + 24*b^4)*Log[Sin[(c + d*x)/2]] + (2*a*Co
t[c + d*x]*Csc[c + d*x]^5*(196*a^5 - 735*a^3*b^2 + 540*a*b^4 - 12*(16*a^5 -
 85*a^3*b^2 + 60*a*b^4)*Cos[2*(c + d*x)] + (92*a^5 - 285*a^3*b^2 + 180*a*b^
4)*Cos[4*(c + d*x)] + 1162*a^4*b*Sin[c + d*x] - 3060*a^2*b^3*Sin[c + d*x] +
 1800*b^5*Sin[c + d*x] - 562*a^4*b*Sin[3*(c + d*x)] + 1470*a^2*b^3*Sin[3*(c
 + d*x)] - 900*b^5*Sin[3*(c + d*x)] + 76*a^4*b*Sin[5*(c + d*x)] - 270*a^2*b
^3*Sin[5*(c + d*x)] + 180*b^5*Sin[5*(c + d*x)]))/(b + a*Csc[c + d*x]))/(a^7
*d)
```

fricas [B] time = 1.06, size = 2011, normalized size = 4.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/120*(2*(92*a^6 - 285*a^4*b^2 + 180*a^2*b^4)*cos(d*x + c)^5 - 40*(7*a^6 -
 27*a^4*b^2 + 18*a^2*b^4)*cos(d*x + c)^3 + 60*((a^4*b - 7*a^2*b^3 + 6*b^5)*
cos(d*x + c)^6 - a^4*b + 7*a^2*b^3 - 6*b^5 - 3*(a^4*b - 7*a^2*b^3 + 6*b^5)*
cos(d*x + c)^4 + 3*(a^4*b - 7*a^2*b^3 + 6*b^5)*cos(d*x + c)^2 - (a^5 - 7*a^
3*b^2 + 6*a*b^4 + (a^5 - 7*a^3*b^2 + 6*a*b^4)*cos(d*x + c)^4 - 2*(a^5 - 7*a
^3*b^2 + 6*a*b^4)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^
2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c
)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*
a*b*sin(d*x + c) - a^2 - b^2)) + 30*(4*a^6 - 17*a^4*b^2 + 12*a^2*b^4)*cos(d
*x + c) + 15*((15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x + c)^6 - 15*a^4*b^
2 + 40*a^2*b^4 - 24*b^6 - 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x + c)
^4 + 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x + c)^2 - (15*a^5*b - 40*a
^3*b^3 + 24*a*b^5 + (15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c)^4 - 2*(
15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos
(d*x + c) + 1/2) - 15*((15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x + c)^6 -
15*a^4*b^2 + 40*a^2*b^4 - 24*b^6 - 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos
(d*x + c)^4 + 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x + c)^2 - (15*a^5
*b - 40*a^3*b^3 + 24*a*b^5 + (15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c
)^4 - 2*(15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*lo
g(-1/2*cos(d*x + c) + 1/2) + 2*(4*(38*a^5*b - 135*a^3*b^3 + 90*a*b^5)*cos(d
*x + c)^5 - 5*(79*a^5*b - 228*a^3*b^3 + 144*a*b^5)*cos(d*x + c)^3 + 15*(15*
a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c))*sin(d*x + c))/(a^7*b*d*cos(d*x
 + c)^6 - 3*a^7*b*d*cos(d*x + c)^4 + 3*a^7*b*d*cos(d*x + c)^2 - a^7*b*d - (
a^8*d*cos(d*x + c)^4 - 2*a^8*d*cos(d*x + c)^2 + a^8*d)*sin(d*x + c)), 1/120
*(2*(92*a^6 - 285*a^4*b^2 + 180*a^2*b^4)*cos(d*x + c)^5 - 40*(7*a^6 - 27*a^
4*b^2 + 18*a^2*b^4)*cos(d*x + c)^3 + 120*((a^4*b - 7*a^2*b^3 + 6*b^5)*cos(d
*x + c)^6 - a^4*b + 7*a^2*b^3 - 6*b^5 - 3*(a^4*b - 7*a^2*b^3 + 6*b^5)*cos(d
```

```
*x + c)^4 + 3*(a^4*b - 7*a^2*b^3 + 6*b^5)*cos(d*x + c)^2 - (a^5 - 7*a^3*b^2
 + 6*a*b^4 + (a^5 - 7*a^3*b^2 + 6*a*b^4)*cos(d*x + c)^4 - 2*(a^5 - 7*a^3*b^
2 + 6*a*b^4)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d
*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 30*(4*a^6 - 17*a^4*b^2 + 12*
a^2*b^4)*cos(d*x + c) + 15*((15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x + c)
^6 - 15*a^4*b^2 + 40*a^2*b^4 - 24*b^6 - 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6
)*cos(d*x + c)^4 + 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x + c)^2 - (1
5*a^5*b - 40*a^3*b^3 + 24*a*b^5 + (15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*
x + c)^4 - 2*(15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c)^2)*sin(d*x + c
))*log(1/2*cos(d*x + c) + 1/2) - 15*((15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos
(d*x + c)^6 - 15*a^4*b^2 + 40*a^2*b^4 - 24*b^6 - 3*(15*a^4*b^2 - 40*a^2*b^4
 + 24*b^6)*cos(d*x + c)^4 + 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x +
c)^2 - (15*a^5*b - 40*a^3*b^3 + 24*a*b^5 + (15*a^5*b - 40*a^3*b^3 + 24*a*b^
5)*cos(d*x + c)^4 - 2*(15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c)^2)*si
n(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 2*(4*(38*a^5*b - 135*a^3*b^3 + 9
0*a*b^5)*cos(d*x + c)^5 - 5*(79*a^5*b - 228*a^3*b^3 + 144*a*b^5)*cos(d*x +
c)^3 + 15*(15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c))*sin(d*x + c))/(a
^7*b*d*cos(d*x + c)^6 - 3*a^7*b*d*cos(d*x + c)^4 + 3*a^7*b*d*cos(d*x + c)^2
 - a^7*b*d - (a^8*d*cos(d*x + c)^4 - 2*a^8*d*cos(d*x + c)^2 + a^8*d)*sin(d*
x + c))]
```

giac [A] time = 0.90, size = 596, normalized size = 1.41

$$-\frac{\frac{120\left(15\,a^4b-40\,a^2b^3+24\,b^5\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^7}+\frac{960\left(a^6-8\,a^4b^2+13\,a^2b^4-6\,b^6\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^7}+\frac{960\left(a^4b^2\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/480*(120*(15*a^4*b - 40*a^2*b^3 + 24*b^5)*log(abs(tan(1/2*d*x + 1/2*c)))
/a^7 + 960*(a^6 - 8*a^4*b^2 + 13*a^2*b^4 - 6*b^6)*(pi*floor(1/2*(d*x + c)/p
i + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sq
rt(a^2 - b^2)*a^7) + 960*(a^4*b^2*tan(1/2*d*x + 1/2*c) - 2*a^2*b^4*tan(1/2*
d*x + 1/2*c) + b^6*tan(1/2*d*x + 1/2*c) + a^5*b - 2*a^3*b^3 + a*b^5)/((a*ta
n(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*a^7) - (3*a^8*tan(1/2*
d*x + 1/2*c)^5 - 15*a^7*b*tan(1/2*d*x + 1/2*c)^4 - 35*a^8*tan(1/2*d*x + 1/2
*c)^3 + 60*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 + 240*a^7*b*tan(1/2*d*x + 1/2*c)^
2 - 240*a^5*b^3*tan(1/2*d*x + 1/2*c)^2 + 330*a^8*tan(1/2*d*x + 1/2*c) - 162
0*a^6*b^2*tan(1/2*d*x + 1/2*c) + 1200*a^4*b^4*tan(1/2*d*x + 1/2*c))/a^10 -
(4110*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 10960*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 +
 6576*b^5*tan(1/2*d*x + 1/2*c)^5 - 330*a^5*tan(1/2*d*x + 1/2*c)^4 + 1620*a^
3*b^2*tan(1/2*d*x + 1/2*c)^4 - 1200*a*b^4*tan(1/2*d*x + 1/2*c)^4 - 240*a^4*
```

```
b*tan(1/2*d*x + 1/2*c)^3 + 240*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 35*a^5*tan(
1/2*d*x + 1/2*c)^2 - 60*a^3*b^2*tan(1/2*d*x + 1/2*c)^2 + 15*a^4*b*tan(1/2*d
*x + 1/2*c) - 3*a^5)/(a^7*tan(1/2*d*x + 1/2*c)^5))/d
```

maple [B] time = 0.32, size = 897, normalized size = 2.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/2/d/a^5*b^3/tan(1/2*d*x+1/2*c)^2-6/d/a^7*b^5*ln(tan(1/2*d*x+1/2*c))-1/32/
d/a^3*b*tan(1/2*d*x+1/2*c)^4+1/8/d/a^4*b^2*tan(1/2*d*x+1/2*c)^3-1/2/d/a^5*t
an(1/2*d*x+1/2*c)^2*b^3+5/2/d/a^6*b^4*tan(1/2*d*x+1/2*c)-1/8/d/a^4/tan(1/2*
d*x+1/2*c)^3*b^2-2/d/a^6/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*
b^5-5/2/d/a^6/tan(1/2*d*x+1/2*c)*b^4+1/32/d/a^3*b/tan(1/2*d*x+1/2*c)^4-15/4
/d/a^3*b*ln(tan(1/2*d*x+1/2*c))-2/d/a^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d
*x+1/2*c)*b+a)*b-2/d/a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2
*b)/(a^2-b^2)^(1/2))-2/d/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b
+a)*b^2*tan(1/2*d*x+1/2*c)+16/d/a^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2
*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^2+11/16/d/a^2*tan(1/2*d*x+1/2*c)-11/16/
d/a^2/tan(1/2*d*x+1/2*c)+1/2/d/a^3*b*tan(1/2*d*x+1/2*c)^2-27/8/d/a^4*b^2*ta
n(1/2*d*x+1/2*c)+27/8/d/a^4/tan(1/2*d*x+1/2*c)*b^2-1/2/d/a^3*b/tan(1/2*d*x+
1/2*c)^2+10/d/a^5*b^3*ln(tan(1/2*d*x+1/2*c))+4/d/a^4/(tan(1/2*d*x+1/2*c)^2*
a+2*tan(1/2*d*x+1/2*c)*b+a)*b^3+4/d/a^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d
*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)*b^4+1/160/d/a^2*tan(1/2*d*x+1/2*c)^5-1/16
0/d/a^2/tan(1/2*d*x+1/2*c)^5-7/96/d/a^2*tan(1/2*d*x+1/2*c)^3+7/96/d/a^2/tan
(1/2*d*x+1/2*c)^3-26/d/a^5/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*
c)+2*b)/(a^2-b^2)^(1/2))*b^4-2/d/a^7/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+
1/2*c)*b+a)*tan(1/2*d*x+1/2*c)*b^6+12/d/a^7/(a^2-b^2)^(1/2)*arctan(1/2*(2*a
*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^6
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 6.91, size = 1424, normalized size = 3.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6/(a + b*sin(c + d*x))^2,x)
```

```
[Out] tan(c/2 + (d*x)/2)^5/(160*a^2*d) + (tan(c/2 + (d*x)/2)*(1/(4*a^2) + b^2/(2*
a^4) - (4*b*((b*(64*a^2 + 128*b^2))/(256*a^5) - b/(8*a^3) + (4*b*((64*a^2 +
 128*b^2)/(1024*a^4) + 5/(32*a^2) - b^2/(2*a^4)))/a))/a + ((64*a^2 + 128*b^
2)*((64*a^2 + 128*b^2)/(1024*a^4) + 5/(32*a^2) - b^2/(2*a^4)))/(32*a^2)))/d
 - (tan(c/2 + (d*x)/2)^3*((64*a^2 + 128*b^2)/(3072*a^4) + 5/(96*a^2) - b^2/
(6*a^4)))/d - (tan(c/2 + (d*x)/2)^3*((31*a^4*b)/3 - 8*a^2*b^3) + tan(c/2 +
(d*x)/2)^4*(48*a*b^4 + (59*a^5)/3 - 72*a^3*b^2) + tan(c/2 + (d*x)/2)^5*(124
*a^4*b + 224*b^5 - 360*a^2*b^3) + a^5/5 - tan(c/2 + (d*x)/2)^2*((32*a^5)/15
 - 2*a^3*b^2) - (3*a^4*b*tan(c/2 + (d*x)/2))/5 + (2*tan(c/2 + (d*x)/2)^6*(1
1*a^6 + 32*b^6 - 24*a^2*b^4 - 22*a^4*b^2))/a)/(d*(32*a^7*tan(c/2 + (d*x)/2)
^5 + 32*a^7*tan(c/2 + (d*x)/2)^7 + 64*a^6*b*tan(c/2 + (d*x)/2)^6)) + (tan(c
/2 + (d*x)/2)^2*((b*(64*a^2 + 128*b^2))/(512*a^5) - b/(16*a^3) + (2*b*((64*
a^2 + 128*b^2)/(1024*a^4) + 5/(32*a^2) - b^2/(2*a^4)))/a))/d - (log(tan(c/2
 + (d*x)/2))*(15*a^4*b + 24*b^5 - 40*a^2*b^3))/(4*a^7*d) - (b*tan(c/2 + (d*
x)/2)^4)/(32*a^3*d) - (atan((((a^2 - 6*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((
4*a^13 - 48*a^7*b^6 + 92*a^9*b^4 - 47*a^11*b^2)/(2*a^12) + (tan(c/2 + (d*x)
/2)*(23*a^11*b - 96*a^5*b^7 + 208*a^7*b^5 - 134*a^9*b^3))/(2*a^11) + ((2*a^
2*b - (tan(c/2 + (d*x)/2)*(12*a^14 - 16*a^12*b^2))/(2*a^11))*(a^2 - 6*b^2)*
(-(a + b)^3*(a - b)^3)^(1/2))/a^7)*1i)/a^7 + ((a^2 - 6*b^2)*(-(a + b)^3*(a
- b)^3)^(1/2)*((4*a^13 - 48*a^7*b^6 + 92*a^9*b^4 - 47*a^11*b^2)/(2*a^12) +
(tan(c/2 + (d*x)/2)*(23*a^11*b - 96*a^5*b^7 + 208*a^7*b^5 - 134*a^9*b^3))/(
2*a^11) - ((2*a^2*b - (tan(c/2 + (d*x)/2)*(12*a^14 - 16*a^12*b^2))/(2*a^11)
)*(a^2 - 6*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/a^7)*1i)/a^7)/((15*a^10*b - 1
44*b^11 + 552*a^2*b^9 - 802*a^4*b^7 + 539*a^6*b^5 - 160*a^8*b^3)/a^12 + (ta
n(c/2 + (d*x)/2)*(8*a^10 - 144*b^10 + 516*a^2*b^8 - 682*a^4*b^6 + 400*a^6*b
^4 - 98*a^8*b^2))/a^11 - ((a^2 - 6*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((4*a^
13 - 48*a^7*b^6 + 92*a^9*b^4 - 47*a^11*b^2)/(2*a^12) + (tan(c/2 + (d*x)/2)*
(23*a^11*b - 96*a^5*b^7 + 208*a^7*b^5 - 134*a^9*b^3))/(2*a^11) + ((2*a^2*b
- (tan(c/2 + (d*x)/2)*(12*a^14 - 16*a^12*b^2))/(2*a^11))*(a^2 - 6*b^2)*(-(a
 + b)^3*(a - b)^3)^(1/2))/a^7))/a^7 + ((a^2 - 6*b^2)*(-(a + b)^3*(a - b)^3)
^(1/2)*((4*a^13 - 48*a^7*b^6 + 92*a^9*b^4 - 47*a^11*b^2)/(2*a^12) + (tan(c/
2 + (d*x)/2)*(23*a^11*b - 96*a^5*b^7 + 208*a^7*b^5 - 134*a^9*b^3))/(2*a^11)
 - ((2*a^2*b - (tan(c/2 + (d*x)/2)*(12*a^14 - 16*a^12*b^2))/(2*a^11))*(a^2
- 6*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/a^7))/a^7))*(a^2 - 6*b^2)*(-(a + b)^
3*(a - b)^3)^(1/2)*2i)/(a^7*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6/(a+b*sin(d*x+c))**2,x)
[Out] Integral(cot(c + d*x)**6/(a + b*sin(c + d*x))**2, x)
```

3.192 $\int \frac{\tan^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=321

$$-\frac{(8a^2-5ab-b^2)\log(1-\sin(c+dx))}{16d(a+b)^5}-\frac{(8a^2+5ab-b^2)\log(\sin(c+dx)+1)}{16d(a-b)^5}+\frac{\sec^4(c+dx)\left(a\left(a^2+3b^2\right)-b\left(3\right.\right.}{4d\left(a^2-b^2\right.}$$

```
[Out] -1/16*(8*a^2-5*a*b-b^2)*ln(1-sin(d*x+c))/(a+b)^5/d-1/16*(8*a^2+5*a*b-b^2)*l
n(1+sin(d*x+c))/(a-b)^5/d+a^3*(a^4+13*a^2*b^2+10*b^4)*ln(a+b*sin(d*x+c))/(a
^2-b^2)^5/d-1/2*a^5/(a^2-b^2)^3/d/(a+b*sin(d*x+c))^2-a^4*(a^2+5*b^2)/(a^2-b
^2)^4/d/(a+b*sin(d*x+c))+1/4*sec(d*x+c)^4*(a*(a^2+3*b^2)-b*(3*a^2+b^2)*sin(
d*x+c))/(a^2-b^2)^3/d-1/8*sec(d*x+c)^2*(8*a^3*(a^2+5*b^2)-b*(27*a^4+22*a^2*
b^2-b^4)*sin(d*x+c))/(a^2-b^2)^4/d
```

Rubi [A] time = 0.88, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2721, 1647, 1629}

$$-\frac{a^5}{2d\left(a^2-b^2\right)^3(a+b\sin(c+dx))^2}-\frac{a^4\left(a^2+5b^2\right)}{d\left(a^2-b^2\right)^4(a+b\sin(c+dx))}+\frac{a^3\left(13a^2b^2+a^4+10b^4\right)\log(a+b\sin(c+}{d\left(a^2-b^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -((8*a^2 - 5*a*b - b^2)*Log[1 - Sin[c + d*x]])/(16*(a + b)^5*d) - ((8*a^2 +
 5*a*b - b^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^5*d) + (a^3*(a^4 + 13*a^2*
b^2 + 10*b^4)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^5*d) - a^5/(2*(a^2 - b^
2)^3*d*(a + b*Sin[c + d*x])^2) - (a^4*(a^2 + 5*b^2))/((a^2 - b^2)^4*d*(a +
b*Sin[c + d*x])) + (Sec[c + d*x]^4*(a*(a^2 + 3*b^2) - b*(3*a^2 + b^2)*Sin[c
 + d*x]))/(4*(a^2 - b^2)^3*d) - (Sec[c + d*x]^2*(8*a^3*(a^2 + 5*b^2) - b*(2
7*a^4 + 22*a^2*b^2 - b^4)*Sin[c + d*x]))/(8*(a^2 - b^2)^4*d)
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
```

```
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(a+x)^3\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)\left(a\left(a^2+3b^2\right)-b\left(3a^2+b^2\right)\sin(c+dx)\right)}{4\left(a^2-b^2\right)^3 d} + \frac{\text{Subst}\left(\int \frac{\frac{a^3b^6\left(3a^2+b^2\right)}{\left(a^2-b^2\right)^3} - \frac{a^2b^4\left(4a^4+\cdots\right.}{\left(a^2\cdots\right.}\cdots}{\cdots}\right.}{\cdots}$$

$$= \frac{\sec^4(c+dx)\left(a\left(a^2+3b^2\right)-b\left(3a^2+b^2\right)\sin(c+dx)\right)}{4\left(a^2-b^2\right)^3 d} - \frac{\sec^2(c+dx)\left(8a^3\left(a^2+5b^2\right)\cdots\right.}{8\cdots}$$

$$= \frac{\sec^4(c+dx)\left(a\left(a^2+3b^2\right)-b\left(3a^2+b^2\right)\sin(c+dx)\right)}{4\left(a^2-b^2\right)^3 d} - \frac{\sec^2(c+dx)\left(8a^3\left(a^2+5b^2\right)\cdots\right.}{8\cdots}$$

$$= -\frac{\left(8a^2-5ab-b^2\right)\log(1-\sin(c+dx))}{16(a+b)^5 d} - \frac{\left(8a^2+5ab-b^2\right)\log(1+\sin(c+dx))}{16(a-b)^5 d} + \frac{a^3\cdots}{\cdots}$$

Mathematica [A] time = 6.34, size = 304, normalized size = 0.95

$$-\frac{(8a^2-5ab-b^2)\log(1-\sin(c+dx))}{16d(a+b)^5}-\frac{(8a^2+5ab-b^2)\log(\sin(c+dx)+1)}{16d(a-b)^5}-\frac{a^5}{2d(a^2-b^2)^3(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/16*((8*a^2 - 5*a*b - b^2)*Log[1 - Sin[c + d*x]])/((a + b)^5*d) - ((8*a^2
 + 5*a*b - b^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^5*d) + (a^3*(a^4 + 13*a^
2*b^2 + 10*b^4)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^5*d) + 1/(16*(a + b)^
3*d*(1 - Sin[c + d*x])^2) - (7*a + b)/(16*(a + b)^4*d*(1 - Sin[c + d*x])) +
 1/(16*(a - b)^3*d*(1 + Sin[c + d*x])^2) - (7*a - b)/(16*(a - b)^4*d*(1 + S
in[c + d*x])) - a^5/(2*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])^2) - (a^4*(a^2
+ 5*b^2))/((a^2 - b^2)^4*d*(a + b*Sin[c + d*x]))
```

fricas [B] time = 1.35, size = 981, normalized size = 3.06

$$-\frac{4a^9-16a^7b^2+24a^5b^4-16a^3b^6+4ab^8-4(6a^9+35a^7b^2-39a^5b^4-3a^3b^6+ab^8)\cos(dx+c)^4-16(a^9-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/16*(4*a^9 - 16*a^7*b^2 + 24*a^5*b^4 - 16*a^3*b^6 + 4*a*b^8 - 4*(6*a^9 +
35*a^7*b^2 - 39*a^5*b^4 - 3*a^3*b^6 + a*b^8)*cos(d*x + c)^4 - 16*(a^9 - 3*a
^7*b^2 + 3*a^5*b^4 - a^3*b^6)*cos(d*x + c)^2 - 16*((a^7*b^2 + 13*a^5*b^4 +
10*a^3*b^6)*cos(d*x + c)^6 - 2*(a^8*b + 13*a^6*b^3 + 10*a^4*b^5)*cos(d*x +
c)^4*sin(d*x + c) - (a^9 + 14*a^7*b^2 + 23*a^5*b^4 + 10*a^3*b^6)*cos(d*x +
c)^4)*log(b*sin(d*x + c) + a) + ((8*a^7*b^2 + 45*a^6*b^3 + 104*a^5*b^4 + 12
5*a^4*b^5 + 80*a^3*b^6 + 23*a^2*b^7 - b^9)*cos(d*x + c)^6 - 2*(8*a^8*b + 45
*a^7*b^2 + 104*a^6*b^3 + 125*a^5*b^4 + 80*a^4*b^5 + 23*a^3*b^6 - a*b^8)*cos
(d*x + c)^4*sin(d*x + c) - (8*a^9 + 45*a^8*b + 112*a^7*b^2 + 170*a^6*b^3 +
184*a^5*b^4 + 148*a^4*b^5 + 80*a^3*b^6 + 22*a^2*b^7 - b^9)*cos(d*x + c)^4)*
log(sin(d*x + c) + 1) + ((8*a^7*b^2 - 45*a^6*b^3 + 104*a^5*b^4 - 125*a^4*b^
5 + 80*a^3*b^6 - 23*a^2*b^7 + b^9)*cos(d*x + c)^6 - 2*(8*a^8*b - 45*a^7*b^2
 + 104*a^6*b^3 - 125*a^5*b^4 + 80*a^4*b^5 - 23*a^3*b^6 + a*b^8)*cos(d*x + c
)^4*sin(d*x + c) - (8*a^9 - 45*a^8*b + 112*a^7*b^2 - 170*a^6*b^3 + 184*a^5*
b^4 - 148*a^4*b^5 + 80*a^3*b^6 - 22*a^2*b^7 + b^9)*cos(d*x + c)^4)*log(-sin
(d*x + c) + 1) - 2*(2*a^8*b - 8*a^6*b^3 + 12*a^4*b^5 - 8*a^2*b^7 + 2*b^9 +
(8*a^8*b + 59*a^6*b^3 - 45*a^4*b^5 - 23*a^2*b^7 + b^9)*cos(d*x + c)^4 - (11
*a^8*b - 36*a^6*b^3 + 42*a^4*b^5 - 20*a^2*b^7 + 3*b^9)*cos(d*x + c)^2)*sin(
d*x + c))/((a^10*b^2 - 5*a^8*b^4 + 10*a^6*b^6 - 10*a^4*b^8 + 5*a^2*b^10 - b
```

```
^12)*d*cos(d*x + c)^6 - 2*(a^11*b - 5*a^9*b^3 + 10*a^7*b^5 - 10*a^5*b^7 + 5
*a^3*b^9 - a*b^11)*d*cos(d*x + c)^4*sin(d*x + c) - (a^12 - 4*a^10*b^2 + 5*a
^8*b^4 - 5*a^4*b^8 + 4*a^2*b^10 - b^12)*d*cos(d*x + c)^4)
```

giac [A] time = 6.81, size = 585, normalized size = 1.82

$$\frac{16\left(a^7b+13\,a^5b^3+10\,a^3b^5\right)\log(|b\sin(dx+c)+a|)}{a^{10}b-5\,a^8b^3+10\,a^6b^5-10\,a^4b^7+5\,a^2b^9-b^{11}}-\frac{\left(8\,a^2+5\,ab-b^2\right)\log(|\sin(dx+c)+1|)}{a^5-5\,a^4b+10\,a^3b^2-10\,a^2b^3+5\,ab^4-b^5}-\frac{\left(8\,a^2-5\,ab-b^2\right)\log(|\sin(dx+c)-1|)}{a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5}-\frac{2\left(8\,a^6b\sin(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/16*(16*(a^7*b + 13*a^5*b^3 + 10*a^3*b^5)*log(abs(b*sin(d*x + c) + a))/(a^
10*b - 5*a^8*b^3 + 10*a^6*b^5 - 10*a^4*b^7 + 5*a^2*b^9 - b^11) - (8*a^2 + 5
*a*b - b^2)*log(abs(sin(d*x + c) + 1))/(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2
*b^3 + 5*a*b^4 - b^5) - (8*a^2 - 5*a*b - b^2)*log(abs(sin(d*x + c) - 1))/(a
^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5) - 2*(8*a^6*b*sin(d*
x + c)^5 + 67*a^4*b^3*sin(d*x + c)^5 + 22*a^2*b^5*sin(d*x + c)^5 - b^7*sin(
d*x + c)^5 + 12*a^7*sin(d*x + c)^4 + 82*a^5*b^2*sin(d*x + c)^4 + 4*a^3*b^4*
sin(d*x + c)^4 - 2*a*b^6*sin(d*x + c)^4 - 5*a^6*b*sin(d*x + c)^3 - 159*a^4*
b^3*sin(d*x + c)^3 - 27*a^2*b^5*sin(d*x + c)^3 - b^7*sin(d*x + c)^3 - 32*a^
7*sin(d*x + c)^2 - 148*a^5*b^2*sin(d*x + c)^2 - 16*a^3*b^4*sin(d*x + c)^2 +
 4*a*b^6*sin(d*x + c)^2 - a^6*b*sin(d*x + c) + 86*a^4*b^3*sin(d*x + c) + 11
*a^2*b^5*sin(d*x + c) + 18*a^7 + 72*a^5*b^2 + 6*a^3*b^4)/((a^8 - 4*a^6*b^2
+ 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(b*sin(d*x + c)^3 + a*sin(d*x + c)^2 - b*sin
(d*x + c) - a)^2))/d
```

maple [A] time = 0.33, size = 465, normalized size = 1.45

$$\frac{1}{16d\left(a+b\right)^{3}\left(\sin\left(dx+c\right)-1\right)^{2}}+\frac{b}{16d\left(a+b\right)^{4}\left(\sin\left(dx+c\right)-1\right)}+\frac{7a}{16d\left(a+b\right)^{4}\left(\sin\left(dx+c\right)-1\right)}-\frac{\ln\left(\sin\left(dx+c\right)-\right.}{2d\left(a+b\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/16/d/(a+b)^3/(sin(d*x+c)-1)^2+1/16/d/(a+b)^4/(sin(d*x+c)-1)*b+7/16/d/(a+b
)^4/(sin(d*x+c)-1)*a-1/2/d/(a+b)^5*ln(sin(d*x+c)-1)*a^2+5/16/d/(a+b)^5*ln(s
in(d*x+c)-1)*a*b+1/16/d/(a+b)^5*ln(sin(d*x+c)-1)*b^2-1/2/d*a^5/(a+b)^3/(a-b
)^3/(a+b*sin(d*x+c))^2+1/d*a^7/(a+b)^5/(a-b)^5*ln(a+b*sin(d*x+c))+13/d*a^5/
(a+b)^5/(a-b)^5*ln(a+b*sin(d*x+c))*b^2+10/d*a^3/(a+b)^5/(a-b)^5*ln(a+b*sin(
d*x+c))*b^4-1/d*a^6/(a+b)^4/(a-b)^4/(a+b*sin(d*x+c))-5/d*a^4/(a+b)^4/(a-b)^
4/(a+b*sin(d*x+c))*b^2+1/16/d/(a-b)^3/(1+sin(d*x+c))^2+1/16/d/(a-b)^4/(1+si
```

```
n(d*x+c))*b-7/16/d/(a-b)^4/(1+sin(d*x+c))*a-1/2/d/(a-b)^5*ln(1+sin(d*x+c))*
a^2-5/16/d/(a-b)^5*ln(1+sin(d*x+c))*a*b+1/16/d/(a-b)^5*ln(1+sin(d*x+c))*b^2
```

maxima [B] time = 1.44, size = 730, normalized size = 2.27

$$\frac{16\left(a^{7}+13\,a^{5}b^{2}+10\,a^{3}b^{4}\right)\log(b\sin(dx+c)+a)}{a^{10}-5\,a^{8}b^{2}+10\,a^{6}b^{4}-10\,a^{4}b^{6}+5\,a^{2}b^{8}-b^{10}}-\frac{\left(8\,a^{2}+5\,ab-b^{2}\right)\log(\sin(dx+c)+1)}{a^{5}-5\,a^{4}b+10\,a^{3}b^{2}-10\,a^{2}b^{3}+5\,ab^{4}-b^{5}}-\frac{\left(8\,a^{2}-5\,ab-b^{2}\right)\log(\sin(dx+c)-1)}{a^{5}+5\,a^{4}b+10\,a^{3}b^{2}+10\,a^{2}b^{3}+5\,ab^{4}+b^{5}}-\frac{}{a^{10}-4\,a^{8}b^{2}+6\,a^{6}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/16*(16*(a^7 + 13*a^5*b^2 + 10*a^3*b^4)*log(b*sin(d*x + c) + a)/(a^10 - 5*
a^8*b^2 + 10*a^6*b^4 - 10*a^4*b^6 + 5*a^2*b^8 - b^10) - (8*a^2 + 5*a*b - b^
2)*log(sin(d*x + c) + 1)/(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4
 - b^5) - (8*a^2 - 5*a*b - b^2)*log(sin(d*x + c) - 1)/(a^5 + 5*a^4*b + 10*a
^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5) - 2*(18*a^7 + 72*a^5*b^2 + 6*a^3*b^4 +
 (8*a^6*b + 67*a^4*b^3 + 22*a^2*b^5 - b^7)*sin(d*x + c)^5 + 2*(6*a^7 + 41*a
^5*b^2 + 2*a^3*b^4 - a*b^6)*sin(d*x + c)^4 - (5*a^6*b + 159*a^4*b^3 + 27*a^
2*b^5 + b^7)*sin(d*x + c)^3 - 4*(8*a^7 + 37*a^5*b^2 + 4*a^3*b^4 - a*b^6)*si
n(d*x + c)^2 - (a^6*b - 86*a^4*b^3 - 11*a^2*b^5)*sin(d*x + c))/(a^10 - 4*a^
8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8 + (a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6
- 4*a^2*b^8 + b^10)*sin(d*x + c)^6 + 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a
^3*b^7 + a*b^9)*sin(d*x + c)^5 + (a^10 - 6*a^8*b^2 + 14*a^6*b^4 - 16*a^4*b^
6 + 9*a^2*b^8 - 2*b^10)*sin(d*x + c)^4 - 4*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 -
 4*a^3*b^7 + a*b^9)*sin(d*x + c)^3 - (2*a^10 - 9*a^8*b^2 + 16*a^6*b^4 - 14*
a^4*b^6 + 6*a^2*b^8 - b^10)*sin(d*x + c)^2 + 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b
^5 - 4*a^3*b^7 + a*b^9)*sin(d*x + c)))/d
```

mupad [B] time = 10.74, size = 1229, normalized size = 3.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + b*sin(c + d*x))^3,x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^2*(a*b^6 - 2*a^7 + 38*a^3*b^4 + 11*a^5*b^2))/(a^8 + b^
8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2) - (4*tan(c/2 + (d*x)/2)^4*(4*a*b^6 -
 a^7 + 33*a^3*b^4 + 12*a^5*b^2))/(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6
*b^2) - (4*tan(c/2 + (d*x)/2)^8*(4*a*b^6 - a^7 + 33*a^3*b^4 + 12*a^5*b^2))/
(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2) + (tan(c/2 + (d*x)/2)^10*(a
*b^6 - 2*a^7 + 38*a^3*b^4 + 11*a^5*b^2))/(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4
 - 4*a^6*b^2) + (2*tan(c/2 + (d*x)/2)^6*(7*a*b^6 + 6*a^7 + 118*a^3*b^4 + 13
*a^5*b^2))/(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2) + (b*tan(c/2 + (
d*x)/2)^11*(37*a^6 + a^2*b^4 + 58*a^4*b^2))/(4*(a^8 + b^8 - 4*a^2*b^6 + 6*a
```

```
^4*b^4 - 4*a^6*b^2)) + (b*tan(c/2 + (d*x)/2)^5*(7*a^6 + 14*b^6 - 57*a^2*b^4
 + 132*a^4*b^2))/(2*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) + (b*t
an(c/2 + (d*x)/2)^7*(7*a^6 + 14*b^6 - 57*a^2*b^4 + 132*a^4*b^2))/(2*(a^8 +
b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) - (b*tan(c/2 + (d*x)/2)^3*(83*a^6
 - 4*b^6 - 17*a^2*b^4 + 226*a^4*b^2))/(4*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4
 - 4*a^6*b^2)) - (b*tan(c/2 + (d*x)/2)^9*(83*a^6 - 4*b^6 - 17*a^2*b^4 + 226
*a^4*b^2))/(4*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) + (b*tan(c/2
 + (d*x)/2)*(37*a^6 + a^2*b^4 + 58*a^4*b^2))/(4*(a^8 + b^8 - 4*a^2*b^6 + 6*
a^4*b^4 - 4*a^6*b^2)))/(d*(tan(c/2 + (d*x)/2)^6*(4*a^2 + 24*b^2) - tan(c/2
+ (d*x)/2)^10*(2*a^2 - 4*b^2) - tan(c/2 + (d*x)/2)^2*(2*a^2 - 4*b^2) + a^2*
tan(c/2 + (d*x)/2)^12 + a^2 - tan(c/2 + (d*x)/2)^4*(a^2 + 16*b^2) - tan(c/2
 + (d*x)/2)^8*(a^2 + 16*b^2) - 12*a*b*tan(c/2 + (d*x)/2)^3 + 8*a*b*tan(c/2
+ (d*x)/2)^5 + 8*a*b*tan(c/2 + (d*x)/2)^7 - 12*a*b*tan(c/2 + (d*x)/2)^9 + 4
*a*b*tan(c/2 + (d*x)/2)^11 + 4*a*b*tan(c/2 + (d*x)/2))) - (log(tan(c/2 + (d
*x)/2) + 1)*((3*b^2)/(2*(a - b)^5) + (21*b)/(8*(a - b)^4) + 1/(a - b)^3))/d
 - (log(tan(c/2 + (d*x)/2) - 1)*(1/(a + b)^3 - (21*b)/(8*(a + b)^4) + (3*b^
2)/(2*(a + b)^5)))/d + (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/
2)^2)*(a^7 + 10*a^3*b^4 + 13*a^5*b^2))/(d*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4
*b^6 + 10*a^6*b^4 - 5*a^8*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**5/(a + b*sin(c + d*x))**3, x)
```

3.193 $\int \frac{\tan^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=232

$$\frac{a^2\left(a^2+3b^2\right)}{d\left(a^2-b^2\right)^3(a+b\sin(c+dx))}+\frac{\sec^2(c+dx)\left(a\left(a^2+3b^2\right)-b\left(3a^2+b^2\right)\sin(c+dx)\right)}{2d\left(a^2-b^2\right)^3}-\frac{a\left(a^4+8a^2b^2+3b^4\right)\log}{d\left(a^2-b\right.}$$

```
[Out] 1/4*(2*a-b)*ln(1-sin(d*x+c))/(a+b)^4/d+1/4*(2*a+b)*ln(1+sin(d*x+c))/(a-b)^4
/d-a*(a^4+8*a^2*b^2+3*b^4)*ln(a+b*sin(d*x+c))/(a^2-b^2)^4/d+1/2*a^3/(a^2-b^
2)^2/d/(a+b*sin(d*x+c))^2+a^2*(a^2+3*b^2)/(a^2-b^2)^3/d/(a+b*sin(d*x+c))+1/
2*sec(d*x+c)^2*(a*(a^2+3*b^2)-b*(3*a^2+b^2)*sin(d*x+c))/(a^2-b^2)^3/d
```

Rubi [A] time = 0.48, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2721, 1647, 1629}

$$\frac{a^3}{2d\left(a^2-b^2\right)^2(a+b\sin(c+dx))^2}+\frac{a^2\left(a^2+3b^2\right)}{d\left(a^2-b^2\right)^3(a+b\sin(c+dx))}-\frac{a\left(8a^2b^2+a^4+3b^4\right)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^4}+$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((2*a - b)*Log[1 - Sin[c + d*x]])/(4*(a + b)^4*d) + ((2*a + b)*Log[1 + Sin[
c + d*x]])/(4*(a - b)^4*d) - (a*(a^4 + 8*a^2*b^2 + 3*b^4)*Log[a + b*Sin[c +
 d*x]])/((a^2 - b^2)^4*d) + a^3/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^2)
+ (a^2*(a^2 + 3*b^2))/((a^2 - b^2)^3*d*(a + b*Sin[c + d*x])) + (Sec[c + d*x
]^2*(a*(a^2 + 3*b^2) - b*(3*a^2 + b^2)*Sin[c + d*x]))/(2*(a^2 - b^2)^3*d)
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
```

```
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{(a+x)^3\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^2(c+dx)\left(a\left(a^2+3b^2\right)-b\left(3a^2+b^2\right)\sin(c+dx)\right)}{2\left(a^2-b^2\right)^3 d} + \frac{\text{Subst}\left(\int \frac{\frac{a^3b^4\left(3a^2+b^2\right)}{\left(a^2-b^2\right)^3}-\frac{a^2b^2\left(2a^4-\right.}{\left(a^2\right.}}{}\right.}{}$$

$$= \frac{\sec^2(c+dx)\left(a\left(a^2+3b^2\right)-b\left(3a^2+b^2\right)\sin(c+dx)\right)}{2\left(a^2-b^2\right)^3 d} + \frac{\text{Subst}\left(\int\left(\frac{b^2(-2a+b)}{2(a+b)^4(b-x)}-\frac{}{\left(a^2-\right.}\right.\right.}{}$$

$$= \frac{(2a-b)\log(1-\sin(c+dx))}{4(a+b)^4 d} + \frac{(2a+b)\log(1+\sin(c+dx))}{4(a-b)^4 d} - \frac{a\left(a^4+8a^2b^2+3b^4\right)}{\left(a^2-\right.}$$

Mathematica [A] time = 2.28, size = 196, normalized size = 0.84

$$\frac{\frac{4a^2\left(a^2+3b^2\right)}{\left(a^2-b^2\right)^3(a+b\sin(c+dx))}-\frac{4a\left(a^4+8a^2b^2+3b^4\right)\log(a+b\sin(c+dx))}{\left(a^2-b^2\right)^4}+\frac{2a^3}{\left(a^2-b^2\right)^2(a+b\sin(c+dx))^2}-\frac{1}{(a+b)^3(\sin(c+dx)-1)}+\frac{1}{(a-b)^3(\sin(c+dx)+1)}+}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (((2*a - b)*Log[1 - Sin[c + d*x]])/(a + b)^4 + ((2*a + b)*Log[1 + Sin[c + d
*x]])/(a - b)^4 - (4*a*(a^4 + 8*a^2*b^2 + 3*b^4)*Log[a + b*Sin[c + d*x]])/(
```

```
a^2 - b^2)^4 - 1/((a + b)^3*(-1 + Sin[c + d*x])) + 1/((a - b)^3*(1 + Sin[c
+ d*x])) + (2*a^3)/((a^2 - b^2)^2*(a + b*Sin[c + d*x])^2) + (4*a^2*(a^2 + 3
*b^2))/((a^2 - b^2)^3*(a + b*Sin[c + d*x])))/(4*d)
```

fricas [B] time = 0.86, size = 788, normalized size = 3.40

$$-\frac{2\,a^7-6\,a^5b^2+6\,a^3b^4-2\,ab^6+2\left(3\,a^7+7\,a^5b^2-11\,a^3b^4+ab^6\right)\cos\left(dx+c\right)^2+4\left(\left(a^5b^2+8\,a^3b^4+3\,ab^6\right)\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^7 - 6*a^5*b^2 + 6*a^3*b^4 - 2*a*b^6 + 2*(3*a^7 + 7*a^5*b^2 - 11*a
^3*b^4 + a*b^6)*cos(d*x + c)^2 + 4*((a^5*b^2 + 8*a^3*b^4 + 3*a*b^6)*cos(d*x
 + c)^4 - 2*(a^6*b + 8*a^4*b^3 + 3*a^2*b^5)*cos(d*x + c)^2*sin(d*x + c) - (
a^7 + 9*a^5*b^2 + 11*a^3*b^4 + 3*a*b^6)*cos(d*x + c)^2)*log(b*sin(d*x + c)
+ a) - ((2*a^5*b^2 + 9*a^4*b^3 + 16*a^3*b^4 + 14*a^2*b^5 + 6*a*b^6 + b^7)*c
os(d*x + c)^4 - 2*(2*a^6*b + 9*a^5*b^2 + 16*a^4*b^3 + 14*a^3*b^4 + 6*a^2*b^
5 + a*b^6)*cos(d*x + c)^2*sin(d*x + c) - (2*a^7 + 9*a^6*b + 18*a^5*b^2 + 23
*a^4*b^3 + 22*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)*cos(d*x + c)^2)*log(sin
(d*x + c) + 1) - ((2*a^5*b^2 - 9*a^4*b^3 + 16*a^3*b^4 - 14*a^2*b^5 + 6*a*b^
6 - b^7)*cos(d*x + c)^4 - 2*(2*a^6*b - 9*a^5*b^2 + 16*a^4*b^3 - 14*a^3*b^4
+ 6*a^2*b^5 - a*b^6)*cos(d*x + c)^2*sin(d*x + c) - (2*a^7 - 9*a^6*b + 18*a^
5*b^2 - 23*a^4*b^3 + 22*a^3*b^4 - 15*a^2*b^5 + 6*a*b^6 - b^7)*cos(d*x + c)^
2)*log(-sin(d*x + c) + 1) - 2*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7 - (2*a^6
*b + 7*a^4*b^3 - 8*a^2*b^5 - b^7)*cos(d*x + c)^2)*sin(d*x + c))/((a^8*b^2 -
 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10)*d*cos(d*x + c)^4 - 2*(a^9*b - 4*
a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*d*cos(d*x + c)^2*sin(d*x + c) - (a
^10 - 3*a^8*b^2 + 2*a^6*b^4 + 2*a^4*b^6 - 3*a^2*b^8 + b^10)*d*cos(d*x + c)^
2)
```

giac [B] time = 2.39, size = 464, normalized size = 2.00

$$-\frac{\frac{4\left(a^5b+8\,a^3b^3+3\,ab^5\right)\log(|b\sin(dx+c)+a|)}{a^8b-4\,a^6b^3+6\,a^4b^5-4\,a^2b^7+b^9}-\frac{(2\,a+b)\log(|\sin(dx+c)+1|)}{a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4}-\frac{(2\,a-b)\log(|\sin(dx+c)-1|)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}+\frac{2\left(a^5\sin(dx+c)^2+8\,a^3b^2\sin(dx+c)^2+\right.}{(a}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/4*(4*(a^5*b + 8*a^3*b^3 + 3*a*b^5)*log(abs(b*sin(d*x + c) + a))/(a^8*b -
 4*a^6*b^3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9) - (2*a + b)*log(abs(sin(d*x + c)
+ 1))/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) - (2*a - b)*log(abs(sin(d
*x + c) - 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 2*(a^5*sin(d*x
```

```
+ c)^2 + 8*a^3*b^2*sin(d*x + c)^2 + 3*a*b^4*sin(d*x + c)^2 - 3*a^4*b*sin(d*
x + c) + 2*a^2*b^3*sin(d*x + c) + b^5*sin(d*x + c) - 6*a^3*b^2 - 6*a*b^4)/(
(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(sin(d*x + c)^2 - 1)) - 2*(
3*a^5*b^2*sin(d*x + c)^2 + 24*a^3*b^4*sin(d*x + c)^2 + 9*a*b^6*sin(d*x + c)
^2 + 8*a^6*b*sin(d*x + c) + 52*a^4*b^3*sin(d*x + c) + 12*a^2*b^5*sin(d*x +
c) + 6*a^7 + 26*a^5*b^2 + 4*a^3*b^4)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*
b^6 + b^8)*(b*sin(d*x + c) + a)^2))/d
```

maple [A] time = 0.30, size = 323, normalized size = 1.39

$$-\frac{1}{4d\left(a+b\right)^{3}\left(\sin\left(dx+c\right)-1\right)}+\frac{\ln\left(\sin\left(dx+c\right)-1\right)a}{2d\left(a+b\right)^{4}}-\frac{\ln\left(\sin\left(dx+c\right)-1\right)b}{4d\left(a+b\right)^{4}}+\frac{a^{3}}{2d\left(a+b\right)^{2}\left(a-b\right)^{2}\left(a+b\sin\left(dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/4/d/(a+b)^3/(sin(d*x+c)-1)+1/2/d/(a+b)^4*ln(sin(d*x+c)-1)*a-1/4/d/(a+b)^
4*ln(sin(d*x+c)-1)*b+1/2/d*a^3/(a+b)^2/(a-b)^2/(a+b*sin(d*x+c))^2-1/d*a^5/(
a+b)^4/(a-b)^4*ln(a+b*sin(d*x+c))-8/d*a^3/(a+b)^4/(a-b)^4*ln(a+b*sin(d*x+c)
)*b^2-3/d*a/(a+b)^4/(a-b)^4*ln(a+b*sin(d*x+c))*b^4+1/d*a^4/(a+b)^3/(a-b)^3/
(a+b*sin(d*x+c))+3/d*a^2/(a+b)^3/(a-b)^3/(a+b*sin(d*x+c))*b^2+1/4/d/(a-b)^3
/(1+sin(d*x+c))+1/2/d/(a-b)^4*ln(1+sin(d*x+c))*a+1/4/d/(a-b)^4*ln(1+sin(d*x
+c))*b
```

maxima [A] time = 0.74, size = 441, normalized size = 1.90

$$-\frac{\frac{4\left(a^{5}+8a^{3}b^{2}+3ab^{4}\right)\log(b\sin(dx+c)+a)}{a^{8}-4a^{6}b^{2}+6a^{4}b^{4}-4a^{2}b^{6}+b^{8}}-\frac{(2a+b)\log(\sin(dx+c)+1)}{a^{4}-4a^{3}b+6a^{2}b^{2}-4ab^{3}+b^{4}}-\frac{(2a-b)\log(\sin(dx+c)-1)}{a^{4}+4a^{3}b+6a^{2}b^{2}+4ab^{3}+b^{4}}-\frac{2(}{a^{8}-3a^{6}b^{2}+3a^{4}b^{4}-a^{2}b^{6}-\left(a^{6}b^{2}-3a^{4}b^{4}+3\right.}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*(4*(a^5 + 8*a^3*b^2 + 3*a*b^4)*log(b*sin(d*x + c) + a)/(a^8 - 4*a^6*b^
2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) - (2*a + b)*log(sin(d*x + c) + 1)/(a^4 - 4
*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) - (2*a - b)*log(sin(d*x + c) - 1)/(a^4
+ 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 2*(4*a^5 + 8*a^3*b^2 - (2*a^4*b +
9*a^2*b^3 + b^5)*sin(d*x + c)^3 - (3*a^5 + 10*a^3*b^2 - a*b^4)*sin(d*x + c)
^2 + (a^4*b + 11*a^2*b^3)*sin(d*x + c))/(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*
b^6 - (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*sin(d*x + c)^4 - 2*(a^7*b - 3
*a^5*b^3 + 3*a^3*b^5 - a*b^7)*sin(d*x + c)^3 - (a^8 - 4*a^6*b^2 + 6*a^4*b^4
 - 4*a^2*b^6 + b^8)*sin(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b
^7)*sin(d*x + c)))/d
```

mupad [B] time = 7.46, size = 690, normalized size = 2.97

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,(2\,a-b)}{2\,d\,(a+b)^4}-\frac{\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(-a^5+6\,a^3\,b^2+7\,a\,b^4\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}-\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(a^5+2\,a^3\,b^2+9\,a\,b^4\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\left(3\,a^4\,b+13\,a^2\right.}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-}}{d\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(2\,a^2+8\,b^2\right)+4\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + b*sin(c + d*x))^3,x)
```

```
[Out] (log(tan(c/2 + (d*x)/2) - 1)*(2*a - b))/(2*d*(a + b)^4) - ((2*tan(c/2 + (d*
x)/2)^6*(7*a*b^4 - a^5 + 6*a^3*b^2))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) -
(4*tan(c/2 + (d*x)/2)^4*(9*a*b^4 + a^5 + 2*a^3*b^2))/(a^6 - b^6 + 3*a^2*b^4
 - 3*a^4*b^2) - (tan(c/2 + (d*x)/2)^5*(3*a^4*b - 4*b^5 + 13*a^2*b^3))/(a^6
- b^6 + 3*a^2*b^4 - 3*a^4*b^2) + (2*tan(c/2 + (d*x)/2)^2*(7*a*b^4 - a^5 + 6
*a^3*b^2))/((a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) + (b*tan(c/2 + (d*x)/2)^7*
(7*a^4 + 5*a^2*b^2))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) - (b*tan(c/2 + (d*
x)/2)^3*(3*a^4 - 4*b^4 + 13*a^2*b^2))/((a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2))
 + (a*tan(c/2 + (d*x)/2)*(5*a*b^3 + 7*a^3*b))/((a^2 - b^2)*(a^4 + b^4 - 2*a
^2*b^2)))/(d*(a^2*tan(c/2 + (d*x)/2)^8 - tan(c/2 + (d*x)/2)^4*(2*a^2 + 8*b^
2) + 4*b^2*tan(c/2 + (d*x)/2)^2 + 4*b^2*tan(c/2 + (d*x)/2)^6 + a^2 - 4*a*b*
tan(c/2 + (d*x)/2)^3 - 4*a*b*tan(c/2 + (d*x)/2)^5 + 4*a*b*tan(c/2 + (d*x)/2
)^7 + 4*a*b*tan(c/2 + (d*x)/2))) + (log(tan(c/2 + (d*x)/2) + 1)*(2*a + b))/
(2*d*(a - b)^4) - (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)
*(3*a*b^4 + a^5 + 8*a^3*b^2))/(d*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6
*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**3/(a + b*sin(c + d*x))**3, x)
```

3.194 $\int \frac{\tan(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=149

$$-\frac{a}{2d\left(a^2-b^2\right)(a+b\sin(c+dx))^2}-\frac{a^2+b^2}{d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{a\left(a^2+3b^2\right)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^3}-\frac{\log(1-\mathrm{s}}{2d(}$$

```
[Out] -1/2*ln(1-sin(d*x+c))/(a+b)^3/d-1/2*ln(1+sin(d*x+c))/(a-b)^3/d+a*(a^2+3*b^2
)*ln(a+b*sin(d*x+c))/(a^2-b^2)^3/d-1/2*a/(a^2-b^2)/d/(a+b*sin(d*x+c))^2+(-a
^2-b^2)/(a^2-b^2)^2/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.13, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2721, 801}

$$-\frac{a}{2d\left(a^2-b^2\right)(a+b\sin(c+dx))^2}-\frac{a^2+b^2}{d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{a\left(a^2+3b^2\right)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^3}-\frac{\log(1-\mathrm{s}}{2d(}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -Log[1 - Sin[c + d*x]]/(2*(a + b)^3*d) - Log[1 + Sin[c + d*x]]/(2*(a - b)^3
*d) + (a*(a^2 + 3*b^2)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^3*d) - a/(2*(a
^2 - b^2)*d*(a + b*Sin[c + d*x])^2) - (a^2 + b^2)/((a^2 - b^2)^2*d*(a + b*S
in[c + d*x]))
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tan(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(a+x)^3(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{2(a+b)^3(b-x)} + \frac{a}{(a-b)(a+b)(a+x)^3} + \frac{a^2+b^2}{(a-b)^2(a+b)^2(a+x)^2} + \frac{a^3+3ab^2}{(a-b)^3(a+b)^3(a+x)} - \frac{1}{2(a-b)^3(}\right.\right.}{d}$$
$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)^3d} - \frac{\log(1+\sin(c+dx))}{2(a-b)^3d} + \frac{a\left(a^2+3b^2\right)\log(a+b\sin(c+dx))}{\left(a^2-b^2\right)^3 d}$$

Mathematica [A] time = 2.16, size = 213, normalized size = 1.43

$$\frac{\frac{2b}{\left(a^2-b^2\right)(a+b\sin(c+dx))} - \frac{4ab\log(a+b\sin(c+dx))}{\left(a^2-b^2\right)^2} + a\left(\frac{b\left(\frac{\left(a^2-b^2\right)\left(-5a^2-4ab\sin(c+dx)+b^2\right)}{(a+b\sin(c+dx))^2}+2\left(3a^2+b^2\right)\log(a+b\sin(c+dx))\right)}{\left(a^2-b^2\right)^3} + \frac{\log(1-\sin(c+dx))}{(a+b)^3} - \right.}{2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-(Log[1 - Sin[c + d*x]]/(a + b)^2) + Log[1 + Sin[c + d*x]]/(a - b)^2 - (4*
a*b*Log[a + b*Sin[c + d*x]])/(a^2 - b^2)^2 + (2*b)/((a^2 - b^2)*(a + b*Sin[
c + d*x])) + a*(Log[1 - Sin[c + d*x]]/(a + b)^3 - Log[1 + Sin[c + d*x]]/(a
- b)^3 + (b*(2*(3*a^2 + b^2)*Log[a + b*Sin[c + d*x]] + ((a^2 - b^2)*(-5*a^2
 + b^2 - 4*a*b*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2))/(a^2 - b^2)^3))/(2*b
*d)
```

fricas [B] time = 0.66, size = 462, normalized size = 3.10

$$\frac{3\,a^5 - 2\,a^3b^2 - ab^4 - 2\left(a^5 + 4\,a^3b^2 + 3\,ab^4 - \left(a^3b^2 + 3\,ab^4\right)\cos\left(dx+c\right)^2 + 2\left(a^4b + 3\,a^2b^3\right)\sin\left(dx+c\right)\right)\log\left(b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(3*a^5 - 2*a^3*b^2 - a*b^4 - 2*(a^5 + 4*a^3*b^2 + 3*a*b^4 - (a^3*b^2 +
3*a*b^4)*cos(d*x + c)^2 + 2*(a^4*b + 3*a^2*b^3)*sin(d*x + c))*log(b*sin(d*x
 + c) + a) + (a^5 + 3*a^4*b + 4*a^3*b^2 + 4*a^2*b^3 + 3*a*b^4 + b^5 - (a^3*
b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cos(d*x + c)^2 + 2*(a^4*b + 3*a^3*b^2 + 3*
a^2*b^3 + a*b^4)*sin(d*x + c))*log(sin(d*x + c) + 1) + (a^5 - 3*a^4*b + 4*a
```

```
^3*b^2 - 4*a^2*b^3 + 3*a*b^4 - b^5 - (a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*
cos(d*x + c)^2 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*sin(d*x + c))*lo
g(-sin(d*x + c) + 1) + 2*(a^4*b - b^5)*sin(d*x + c))/((a^6*b^2 - 3*a^4*b^4
+ 3*a^2*b^6 - b^8)*d*cos(d*x + c)^2 - 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*
b^7)*d*sin(d*x + c) - (a^8 - 2*a^6*b^2 + 2*a^2*b^6 - b^8)*d)
```

giac [A] time = 0.53, size = 257, normalized size = 1.72

$$\frac{\frac{2\left(a^3b+3\,ab^3\right)\log(|b\sin(dx+c)+a|)}{a^6b-3\,a^4b^3+3\,a^2b^5-b^7}-\frac{\log(|\sin(dx+c)+1|)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{\log(|\sin(dx+c)-1|)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{3\,a^3b^2\sin(dx+c)^2+9\,ab^4\sin(dx+c)^2+8\,a^4b\sin(dx+c)+18\,a^2b^3\sin\cdots}{\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)(b\sin(dx\cdots}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(a^3*b + 3*a*b^3)*log(abs(b*sin(d*x + c) + a))/(a^6*b - 3*a^4*b^3 +
3*a^2*b^5 - b^7) - log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2*b + 3*a*b^2 - b^
3) - log(abs(sin(d*x + c) - 1))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (3*a^3*b^
2*sin(d*x + c)^2 + 9*a*b^4*sin(d*x + c)^2 + 8*a^4*b*sin(d*x + c) + 18*a^2*b
^3*sin(d*x + c) - 2*b^5*sin(d*x + c) + 6*a^5 + 7*a^3*b^2 - a*b^4)/((a^6 - 3
*a^4*b^2 + 3*a^2*b^4 - b^6)*(b*sin(d*x + c) + a)^2))/d
```

maple [A] time = 0.29, size = 198, normalized size = 1.33

$$-\frac{\ln\left(\sin\left(dx+c\right)-1\right)}{2d\left(a+b\right)^3}-\frac{a}{2d\left(a+b\right)\left(a-b\right)\left(a+b\sin\left(dx+c\right)\right)^2}-\frac{a^2}{d\left(a+b\right)^2\left(a-b\right)^2\left(a+b\sin\left(dx+c\right)\right)}-\frac{\cdots}{d\left(a+b\right)^2\left(a\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/2/d/(a+b)^3*ln(sin(d*x+c)-1)-1/2/d*a/(a+b)/(a-b)/(a+b*sin(d*x+c))^2-1/d/
(a+b)^2/(a-b)^2/(a+b*sin(d*x+c))*a^2-1/d/(a+b)^2/(a-b)^2/(a+b*sin(d*x+c))*b
^2+1/d*a^3/(a+b)^3/(a-b)^3*ln(a+b*sin(d*x+c))+3/d*a/(a+b)^3/(a-b)^3*ln(a+b*
sin(d*x+c))*b^2-1/2*ln(1+sin(d*x+c))/(a-b)^3/d
```

maxima [A] time = 0.66, size = 228, normalized size = 1.53

$$\frac{\frac{2\left(a^3+3\,ab^2\right)\log(b\sin(dx+c)+a)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}-\frac{3\,a^3+ab^2+2\left(a^2b+b^3\right)\sin(dx+c)}{a^6-2\,a^4b^2+a^2b^4+\left(a^4b^2-2\,a^2b^4+b^6\right)\sin(dx+c)^2+2\left(a^5b-2\,a^3b^3+ab^5\right)\sin(dx+c)}-\frac{\log(\sin(dx+c)+1)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{\log(\sin\cdots}{a^3+3\,a^2\cdots}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a^3 + 3*a*b^2)*log(b*sin(d*x + c) + a)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4
 - b^6) - (3*a^3 + a*b^2 + 2*(a^2*b + b^3)*sin(d*x + c))/(a^6 - 2*a^4*b^2 +
 a^2*b^4 + (a^4*b^2 - 2*a^2*b^4 + b^6)*sin(d*x + c)^2 + 2*(a^5*b - 2*a^3*b^
3 + a*b^5)*sin(d*x + c)) - log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 -
 b^3) - log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3))/d
```

mupad [B] time = 6.85, size = 304, normalized size = 2.04

$$\frac{\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(3\,a^2\,b^2+b^4\right)}{a\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{4\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^4-2\,a^2\,b^2+b^4}+\frac{4\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{a^4-2\,a^2\,b^2+b^4}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(2\,a^2+4\,b^2\right)+a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+a^2+4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}\right.\right.}{d\,(a+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + b*sin(c + d*x))^3,x)
```

```
[Out] ((2*tan(c/2 + (d*x)/2)^2*(b^4 + 3*a^2*b^2))/(a*(a^4 + b^4 - 2*a^2*b^2)) + (
4*a^2*b*tan(c/2 + (d*x)/2))/(a^4 + b^4 - 2*a^2*b^2) + (4*a^2*b*tan(c/2 + (d
*x)/2)^3)/(a^4 + b^4 - 2*a^2*b^2))/(d*(tan(c/2 + (d*x)/2)^2*(2*a^2 + 4*b^2)
 + a^2*tan(c/2 + (d*x)/2)^4 + a^2 + 4*a*b*tan(c/2 + (d*x)/2)^3 + 4*a*b*tan(
c/2 + (d*x)/2))) - log(tan(c/2 + (d*x)/2) - 1)/(d*(a + b)^3) - log(tan(c/2
+ (d*x)/2) + 1)/(d*(a - b)^3) + (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2
 + (d*x)/2)^2)*(3*a*b^2 + a^3))/(d*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan{\left(c + dx \right)}}{\left(a + b \sin{\left(c + dx \right)}\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)/(a + b*sin(c + d*x))**3, x)
```

3.195 $\int \frac{\cot(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=75

$$-\frac{\log(a+b\sin(c+dx))}{a^3d}+\frac{\log(\sin(c+dx))}{a^3d}+\frac{1}{a^2d(a+b\sin(c+dx))}+\frac{1}{2ad(a+b\sin(c+dx))^2}$$

```
[Out] ln(sin(d*x+c))/a^3/d-ln(a+b*sin(d*x+c))/a^3/d+1/2/a/d/(a+b*sin(d*x+c))^2+1/
a^2/d/(a+b*sin(d*x+c))
```

Rubi **[A]** time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2721, 44}

$$\frac{1}{a^2d(a+b\sin(c+dx))}-\frac{\log(a+b\sin(c+dx))}{a^3d}+\frac{\log(\sin(c+dx))}{a^3d}+\frac{1}{2ad(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] Log[Sin[c + d*x]]/(a^3*d) - Log[a + b*Sin[c + d*x]]/(a^3*d) + 1/(2*a*d*(a +
 b*Sin[c + d*x])^2) + 1/(a^2*d*(a + b*Sin[c + d*x]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+x)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^3x} - \frac{1}{a(a+x)^3} - \frac{1}{a^2(a+x)^2} - \frac{1}{a^3(a+x)}\right)dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\log(\sin(c+dx))}{a^3d} - \frac{\log(a+b\sin(c+dx))}{a^3d} + \frac{1}{2ad(a+b\sin(c+dx))^2} + \frac{1}{a^2d(a+b\sin$$

Mathematica [A] time = 0.26, size = 60, normalized size = 0.80

$$\frac{\frac{a(3a+2b\sin(c+dx))}{(a+b\sin(c+dx))^2} - 2\log(a+b\sin(c+dx)) + 2\log(\sin(c+dx))}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (2*Log[Sin[c + d*x]] - 2*Log[a + b*Sin[c + d*x]] + (a*(3*a + 2*b*Sin[c + d*
x]))/(a + b*Sin[c + d*x])^2)/(2*a^3*d)
```

fricas [B] time = 0.52, size = 154, normalized size = 2.05

$$-\frac{2\,ab\sin(dx+c) + 3\,a^2 + 2\left(b^2\cos(dx+c)^2 - 2\,ab\sin(dx+c) - a^2 - b^2\right)\log(b\sin(dx+c)+a) - 2\left(b^2\cos(d}{2\left(a^3b^2d\cos(dx+c)^2 - 2\,a^4bd\sin(dx+c) - \left(a^5 + a^3b^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*sin(d*x + c) + 3*a^2 + 2*(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x +
c) - a^2 - b^2)*log(b*sin(d*x + c) + a) - 2*(b^2*cos(d*x + c)^2 - 2*a*b*sin
(d*x + c) - a^2 - b^2)*log(-1/2*sin(d*x + c)))/(a^3*b^2*d*cos(d*x + c)^2 -
2*a^4*b*d*sin(d*x + c) - (a^5 + a^3*b^2)*d)
```

giac [A] time = 1.08, size = 69, normalized size = 0.92

$$-\frac{\frac{2\,\log(|b\sin(dx+c)+a|)}{a^3} - \frac{2\,\log(|\sin(dx+c)|)}{a^3} - \frac{2\,ab\sin(dx+c)+3\,a^2}{(b\sin(dx+c)+a)^2a^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*log(abs(b*sin(d*x + c) + a))/a^3 - 2*log(abs(sin(d*x + c)))/a^3 - (
2*a*b*sin(d*x + c) + 3*a^2)/((b*sin(d*x + c) + a)^2*a^3))/d
```

maple [A] time = 0.18, size = 74, normalized size = 0.99

$$\frac{\ln\left(\sin\left(dx+c\right)\right)}{a^3d}-\frac{\ln\left(a+b\sin\left(dx+c\right)\right)}{a^3d}+\frac{1}{2ad\left(a+b\sin\left(dx+c\right)\right)^2}+\frac{1}{a^2d\left(a+b\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+b*sin(d*x+c))^3,x)
```

```
[Out] ln(sin(d*x+c))/a^3/d-ln(a+b*sin(d*x+c))/a^3/d+1/2/a/d/(a+b*sin(d*x+c))^2+1/
a^2/d/(a+b*sin(d*x+c))
```

maxima [A] time = 0.69, size = 81, normalized size = 1.08

$$\frac{\frac{2\,b\sin(dx+c)+3\,a}{a^2b^2\sin(dx+c)^2+2\,a^3b\sin(dx+c)+a^4}-\frac{2\,\log(b\sin(dx+c)+a)}{a^3}+\frac{2\,\log(\sin(dx+c))}{a^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((2*b*sin(d*x + c) + 3*a)/(a^2*b^2*sin(d*x + c)^2 + 2*a^3*b*sin(d*x + c
) + a^4) - 2*log(b*sin(d*x + c) + a)/a^3 + 2*log(sin(d*x + c))/a^3)/d
```

mupad [B] time = 6.54, size = 369, normalized size = 4.92

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}-\frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}{a^3\,d}-\frac{}{d\left(a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+2\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^5+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + b*sin(c + d*x))^3,x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a^3*d) - log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/
2 + (d*x)/2)^2)/(a^3*d) - (6*b^2*tan(c/2 + (d*x)/2)^2)/(d*(2*a^5*tan(c/2 +
(d*x)/2)^2 + a^5*tan(c/2 + (d*x)/2)^4 + a^5 + 4*a^3*b^2*tan(c/2 + (d*x)/2)^
2 + 4*a^4*b*tan(c/2 + (d*x)/2) + 4*a^4*b*tan(c/2 + (d*x)/2)^3)) - (4*b*tan(
c/2 + (d*x)/2))/(d*(2*a^4*tan(c/2 + (d*x)/2)^2 + a^4*tan(c/2 + (d*x)/2)^4 +
 a^4 + 4*a^2*b^2*tan(c/2 + (d*x)/2)^2 + 4*a^3*b*tan(c/2 + (d*x)/2) + 4*a^3*
```

```
b*tan(c/2 + (d*x)/2)^3)) - (4*b*tan(c/2 + (d*x)/2)^3)/(d*(2*a^4*tan(c/2 + (
d*x)/2)^2 + a^4*tan(c/2 + (d*x)/2)^4 + a^4 + 4*a^2*b^2*tan(c/2 + (d*x)/2)^2
 + 4*a^3*b*tan(c/2 + (d*x)/2) + 4*a^3*b*tan(c/2 + (d*x)/2)^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)/(a + b*sin(c + d*x))**3, x)
```

3.196 $\int \frac{\cot^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=145

$$\frac{3b\csc(c+dx)}{a^4d}-\frac{\csc^2(c+dx)}{2a^3d}-\frac{(a^2-6b^2)\log(\sin(c+dx))}{a^5d}+\frac{(a^2-6b^2)\log(a+b\sin(c+dx))}{a^5d}-\frac{a^2-3b^2}{a^4d(a+b\sin(c+d$$

```
[Out] 3*b*csc(d*x+c)/a^4/d-1/2*csc(d*x+c)^2/a^3/d-(a^2-6*b^2)*ln(sin(d*x+c))/a^5/
d+(a^2-6*b^2)*ln(a+b*sin(d*x+c))/a^5/d+1/2*(-a^2+b^2)/a^3/d/(a+b*sin(d*x+c)
)^2+(-a^2+3*b^2)/a^4/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.13, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2721, 894}

$$-\frac{a^2-3b^2}{a^4d(a+b\sin(c+dx))}-\frac{a^2-b^2}{2a^3d(a+b\sin(c+dx))^2}-\frac{(a^2-6b^2)\log(\sin(c+dx))}{a^5d}+\frac{(a^2-6b^2)\log(a+b\sin(c+dx)}{a^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (3*b*Csc[c + d*x])/(a^4*d) - Csc[c + d*x]^2/(2*a^3*d) - ((a^2 - 6*b^2)*Log[
Sin[c + d*x]])/(a^5*d) + ((a^2 - 6*b^2)*Log[a + b*Sin[c + d*x]])/(a^5*d) -
(a^2 - b^2)/(2*a^3*d*(a + b*Sin[c + d*x])^2) - (a^2 - 3*b^2)/(a^4*d*(a + b*
Sin[c + d*x]))
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{b^2-x^2}{x^3(a+x)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{b^2}{a^3x^3} - \frac{3b^2}{a^4x^2} + \frac{-a^2+6b^2}{a^5x} + \frac{a^2-b^2}{a^3(a+x)^3} + \frac{a^2-3b^2}{a^4(a+x)^2} + \frac{a^2-6b^2}{a^5(a+x)}\right) dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{3b\csc(c+dx)}{a^4d} - \frac{\csc^2(c+dx)}{2a^3d} - \frac{\left(a^2-6b^2\right)\log(\sin(c+dx))}{a^5d} + \frac{\left(a^2-6b^2\right)\log(a+b}{a^5d}$$

Mathematica [A] time = 0.94, size = 121, normalized size = 0.83

$$-\frac{\frac{2a\left(a^2-3b^2\right)}{a+b\sin(c+dx)} + 2\left(a^2-6b^2\right)\log(\sin(c+dx)) - 2\left(a^2-6b^2\right)\log(a+b\sin(c+dx)) + \frac{a^2(a-b)(a+b)}{(a+b\sin(c+dx))^2} + a^2\csc^2(c+}{2a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/2*(-6*a*b*Csc[c + d*x] + a^2*Csc[c + d*x]^2 + 2*(a^2 - 6*b^2)*Log[Sin[c
+ d*x]] - 2*(a^2 - 6*b^2)*Log[a + b*Sin[c + d*x]] + (a^2*(a - b)*(a + b))/(
a + b*Sin[c + d*x])^2 + (2*a*(a^2 - 3*b^2))/(a + b*Sin[c + d*x]))/(a^5*d)
```

fricas [B] time = 0.58, size = 404, normalized size = 2.79

$$-\frac{4\,a^4 - 18\,a^2b^2 - 3\left(a^4 - 6\,a^2b^2\right)\cos\left(dx+c\right)^2 - 2\left(\left(a^2b^2 - 6\,b^4\right)\cos\left(dx+c\right)^4 + a^4 - 5\,a^2b^2 - 6\,b^4 - \left(a^4 - 4\,a^2b^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(4*a^4 - 18*a^2*b^2 - 3*(a^4 - 6*a^2*b^2)*cos(d*x + c)^2 - 2*((a^2*b^2
 - 6*b^4)*cos(d*x + c)^4 + a^4 - 5*a^2*b^2 - 6*b^4 - (a^4 - 4*a^2*b^2 - 12*
b^4)*cos(d*x + c)^2 + 2*(a^3*b - 6*a*b^3 - (a^3*b - 6*a*b^3)*cos(d*x + c)^2
)*sin(d*x + c))*log(b*sin(d*x + c) + a) + 2*((a^2*b^2 - 6*b^4)*cos(d*x + c)
^4 + a^4 - 5*a^2*b^2 - 6*b^4 - (a^4 - 4*a^2*b^2 - 12*b^4)*cos(d*x + c)^2 +
2*(a^3*b - 6*a*b^3 - (a^3*b - 6*a*b^3)*cos(d*x + c)^2)*sin(d*x + c))*log(-1
/2*sin(d*x + c)) - 2*(a^3*b + 6*a*b^3 + (a^3*b - 6*a*b^3)*cos(d*x + c)^2)*s
in(d*x + c))/(a^5*b^2*d*cos(d*x + c)^4 - (a^7 + 2*a^5*b^2)*d*cos(d*x + c)^2
 + (a^7 + a^5*b^2)*d - 2*(a^6*b*d*cos(d*x + c)^2 - a^6*b*d)*sin(d*x + c))
```

giac [A] time = 0.54, size = 154, normalized size = 1.06

$$-\frac{\frac{2\left(a^2-6\,b^2\right)\log(|\sin(dx+c)|)}{a^5}-\frac{2\left(a^2b-6\,b^3\right)\log(|b\sin(dx+c)+a|)}{a^5b}+\frac{2\,a^2b\sin(dx+c)^3-12\,b^3\sin(dx+c)^3+3\,a^3\sin(dx+c)^2-18\,ab^2\sin(dx+c)^2-4\,a^2b\sin}{\left(b\sin(dx+c)^2+a\sin(dx+c)\right)^2a^4}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*(a^2 - 6*b^2)*log(abs(sin(d*x + c)))/a^5 - 2*(a^2*b - 6*b^3)*log(ab
s(b*sin(d*x + c) + a))/(a^5*b) + (2*a^2*b*sin(d*x + c)^3 - 12*b^3*sin(d*x +
 c)^3 + 3*a^3*sin(d*x + c)^2 - 18*a*b^2*sin(d*x + c)^2 - 4*a^2*b*sin(d*x +
c) + a^3)/((b*sin(d*x + c)^2 + a*sin(d*x + c))^2*a^4))/d
```

maple [A] time = 0.33, size = 194, normalized size = 1.34

$$\frac{\ln(a+b\sin(dx+c))}{a^3d}-\frac{6\ln(a+b\sin(dx+c))\,b^2}{d\,a^5}-\frac{1}{a^2d\,(a+b\sin(dx+c))}+\frac{3b^2}{d\,a^4\,(a+b\sin(dx+c))}-\frac{1}{2ad\,(a+b\,\mathrm{s}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+b*sin(d*x+c))^3,x)
```

```
[Out] ln(a+b*sin(d*x+c))/a^3/d-6/d/a^5*ln(a+b*sin(d*x+c))*b^2-1/a^2/d/(a+b*sin(d*
x+c))+3/d/a^4/(a+b*sin(d*x+c))*b^2-1/2/a/d/(a+b*sin(d*x+c))^2+1/2/d/a^3/(a+
b*sin(d*x+c))^2*b^2-1/2/a^3/d/sin(d*x+c)^2-ln(sin(d*x+c))/a^3/d+6/d/a^5*ln(
sin(d*x+c))*b^2+3/d/a^4*b/sin(d*x+c)
```

maxima [A] time = 0.73, size = 156, normalized size = 1.08

$$\frac{\frac{4\,a^2b\sin(dx+c)-2\left(a^2b-6\,b^3\right)\sin(dx+c)^3-a^3-3\left(a^3-6\,ab^2\right)\sin(dx+c)^2}{a^4b^2\sin(dx+c)^4+2\,a^5b\sin(dx+c)^3+a^6\sin(dx+c)^2}+\frac{2\left(a^2-6\,b^2\right)\log(b\sin(dx+c)+a)}{a^5}-\frac{2\left(a^2-6\,b^2\right)\log(\sin(dx+c))}{a^5}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((4*a^2*b*sin(d*x + c) - 2*(a^2*b - 6*b^3)*sin(d*x + c)^3 - a^3 - 3*(a^
3 - 6*a*b^2)*sin(d*x + c)^2)/(a^4*b^2*sin(d*x + c)^4 + 2*a^5*b*sin(d*x + c)
^3 + a^6*sin(d*x + c)^2) + 2*(a^2 - 6*b^2)*log(b*sin(d*x + c) + a)/a^5 - 2*
(a^2 - 6*b^2)*log(sin(d*x + c))/a^5)/d
```

mupad [B] time = 6.79, size = 334, normalized size = 2.30

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(22\,a\,b^2-a^3\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(26\,a^2\,b-8\,b^3\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(22\,a^2\,b-32\,b^3\right)-\frac{a^3}{2}+4\,a^2\,b\tan}{d\,\left(4\,a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+4\,a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(8\,a^6+16\,a^4\,b^2\right)+16\,a^5\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3/(a + b*sin(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^2*(22*a*b^2 - a^3) + tan(c/2 + (d*x)/2)^3*(26*a^2*b - 8
*b^3) + tan(c/2 + (d*x)/2)^5*(22*a^2*b - 32*b^3) - a^3/2 + 4*a^2*b*tan(c/2
+ (d*x)/2) - (tan(c/2 + (d*x)/2)^4*(a^4 + 112*b^4 - 96*a^2*b^2))/(2*a))/(d*
(4*a^6*tan(c/2 + (d*x)/2)^2 + 4*a^6*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/
2)^4*(8*a^6 + 16*a^4*b^2) + 16*a^5*b*tan(c/2 + (d*x)/2)^3 + 16*a^5*b*tan(c/
2 + (d*x)/2)^5)) - tan(c/2 + (d*x)/2)^2/(8*a^3*d) + (3*b*tan(c/2 + (d*x)/2)
)/(2*a^4*d) - (log(tan(c/2 + (d*x)/2))*(a^2 - 6*b^2))/(a^5*d) + (log(a + 2*
b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)*(a^2 - 6*b^2))/(a^5*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)**3/(a + b*sin(c + d*x))**3, x)
```

3.197 $\int \frac{\cot^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=221

$$\frac{b\csc^3(c+dx)}{a^4d}-\frac{\csc^4(c+dx)}{4a^3d}-\frac{2b\left(3a^2-5b^2\right)\csc(c+dx)}{a^6d}+\frac{\left(a^2-b^2\right)^2}{2a^5d(a+b\sin(c+dx))^2}+\frac{\left(a^2-3b^2\right)\csc^2(c+dx)}{a^5d}+\frac{\left(a^4\right.}{}$$

```
[Out] -2*b*(3*a^2-5*b^2)*csc(d*x+c)/a^6/d+(a^2-3*b^2)*csc(d*x+c)^2/a^5/d+b*csc(d*
x+c)^3/a^4/d-1/4*csc(d*x+c)^4/a^3/d+(a^4-12*a^2*b^2+15*b^4)*ln(sin(d*x+c))/
a^7/d-(a^4-12*a^2*b^2+15*b^4)*ln(a+b*sin(d*x+c))/a^7/d+1/2*(a^2-b^2)^2/a^5/
d/(a+b*sin(d*x+c))^2+(a^4-6*a^2*b^2+5*b^4)/a^6/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.21, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2721, 894}

$$\frac{-6a^2b^2+a^4+5b^4}{a^6d(a+b\sin(c+dx))}+\frac{\left(a^2-b^2\right)^2}{2a^5d(a+b\sin(c+dx))^2}+\frac{\left(a^2-3b^2\right)\csc^2(c+dx)}{a^5d}-\frac{2b\left(3a^2-5b^2\right)\csc(c+dx)}{a^6d}+\frac{\left(-12a^2b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-2*b*(3*a^2 - 5*b^2)*Csc[c + d*x])/(a^6*d) + ((a^2 - 3*b^2)*Csc[c + d*x]^2
)/(a^5*d) + (b*Csc[c + d*x]^3)/(a^4*d) - Csc[c + d*x]^4/(4*a^3*d) + ((a^4 -
 12*a^2*b^2 + 15*b^4)*Log[Sin[c + d*x]])/(a^7*d) - ((a^4 - 12*a^2*b^2 + 15*
b^4)*Log[a + b*Sin[c + d*x]])/(a^7*d) + (a^2 - b^2)^2/(2*a^5*d*(a + b*Sin[c
 + d*x])^2) + (a^4 - 6*a^2*b^2 + 5*b^4)/(a^6*d*(a + b*Sin[c + d*x]))
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^2}{x^5(a+x)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{b^4}{a^3x^5} - \frac{3b^4}{a^4x^4} + \frac{2b^2\left(-a^2+3b^2\right)}{a^5x^3} + \frac{2\left(3a^2b^2-5b^4\right)}{a^6x^2} + \frac{a^4-12a^2b^2+15b^4}{a^7x} - \frac{\left(a^2-b^2\right)^2}{a^5(a+x)^3} + \frac{-a^4+6}{a^6(}\right.\right.}{d}$$

$$= -\frac{2b\left(3a^2-5b^2\right)\csc(c+dx)}{a^6d} + \frac{\left(a^2-3b^2\right)\csc^2(c+dx)}{a^5d} + \frac{b\csc^3(c+dx)}{a^4d} - \frac{\csc^4(c+}{4a^3d}$$

Mathematica [A] time = 5.31, size = 195, normalized size = 0.88

$$\frac{-a^4\csc^4(c+dx) + \frac{2\left(a^3-ab^2\right)^2}{(a+b\sin(c+dx))^2} + 4a^3b\csc^3(c+dx) + 4a^2\left(a^2-3b^2\right)\csc^2(c+dx) - 8ab\left(3a^2-5b^2\right)\csc(c+dx}{4a^7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-8*a*b*(3*a^2 - 5*b^2)*Csc[c + d*x] + 4*a^2*(a^2 - 3*b^2)*Csc[c + d*x]^2 +
 4*a^3*b*Csc[c + d*x]^3 - a^4*Csc[c + d*x]^4 + 4*(a^4 - 12*a^2*b^2 + 15*b^4
)*Log[Sin[c + d*x]] - 4*(a^4 - 12*a^2*b^2 + 15*b^4)*Log[a + b*Sin[c + d*x]]
 + (2*(a^3 - a*b^2)^2)/(a + b*Sin[c + d*x])^2 + (4*a*(a^4 - 6*a^2*b^2 + 5*b
^4))/(a + b*Sin[c + d*x]))/(4*a^7*d)
```

fricas [B] time = 0.63, size = 754, normalized size = 3.41

$$-\frac{9\,a^6 - 77\,a^4b^2 + 90\,a^2b^4 + 6\left(a^6 - 12\,a^4b^2 + 15\,a^2b^4\right)\cos\left(dx+c\right)^4 - \left(16\,a^6 - 149\,a^4b^2 + 180\,a^2b^4\right)\cos\left(dx+c\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(9*a^6 - 77*a^4*b^2 + 90*a^2*b^4 + 6*(a^6 - 12*a^4*b^2 + 15*a^2*b^4)*c
os(d*x + c)^4 - (16*a^6 - 149*a^4*b^2 + 180*a^2*b^4)*cos(d*x + c)^2 + 4*((a
^4*b^2 - 12*a^2*b^4 + 15*b^6)*cos(d*x + c)^6 - a^6 + 11*a^4*b^2 - 3*a^2*b^4
 - 15*b^6 - (a^6 - 9*a^4*b^2 - 21*a^2*b^4 + 45*b^6)*cos(d*x + c)^4 + (2*a^6
 - 21*a^4*b^2 - 6*a^2*b^4 + 45*b^6)*cos(d*x + c)^2 - 2*(a^5*b - 12*a^3*b^3
```

```
+ 15*a*b^5 + (a^5*b - 12*a^3*b^3 + 15*a*b^5)*cos(d*x + c)^4 - 2*(a^5*b - 12
*a^3*b^3 + 15*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(b*sin(d*x + c) + a)
- 4*((a^4*b^2 - 12*a^2*b^4 + 15*b^6)*cos(d*x + c)^6 - a^6 + 11*a^4*b^2 - 3*
a^2*b^4 - 15*b^6 - (a^6 - 9*a^4*b^2 - 21*a^2*b^4 + 45*b^6)*cos(d*x + c)^4 +
 (2*a^6 - 21*a^4*b^2 - 6*a^2*b^4 + 45*b^6)*cos(d*x + c)^2 - 2*(a^5*b - 12*a
^3*b^3 + 15*a*b^5 + (a^5*b - 12*a^3*b^3 + 15*a*b^5)*cos(d*x + c)^4 - 2*(a^5
*b - 12*a^3*b^3 + 15*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*sin(d*x
+ c)) - 2*(5*a^5*b + 14*a^3*b^3 - 30*a*b^5 - 2*(a^5*b - 12*a^3*b^3 + 15*a*b
^5)*cos(d*x + c)^4 - 2*(2*a^5*b + 19*a^3*b^3 - 30*a*b^5)*cos(d*x + c)^2)*si
n(d*x + c))/(a^7*b^2*d*cos(d*x + c)^6 - (a^9 + 3*a^7*b^2)*d*cos(d*x + c)^4
+ (2*a^9 + 3*a^7*b^2)*d*cos(d*x + c)^2 - (a^9 + a^7*b^2)*d - 2*(a^8*b*d*cos
(d*x + c)^4 - 2*a^8*b*d*cos(d*x + c)^2 + a^8*b*d)*sin(d*x + c))
```

giac [A] time = 0.84, size = 327, normalized size = 1.48

$$\frac{\frac{12\left(a^4-12a^2b^2+15b^4\right)\log(|\sin(dx+c)|)}{a^7}-\frac{12\left(a^4b-12a^2b^3+15b^5\right)\log(|b\sin(dx+c)+a|)}{a^7b}+\frac{6\left(3a^4b^2\sin(dx+c)^2-36a^2b^4\sin(dx+c)^2+45b^6\sin(dx+c)^2\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/12*(12*(a^4 - 12*a^2*b^2 + 15*b^4)*log(abs(sin(d*x + c)))/a^7 - 12*(a^4*b
 - 12*a^2*b^3 + 15*b^5)*log(abs(b*sin(d*x + c) + a))/(a^7*b) + 6*(3*a^4*b^2
*sin(d*x + c)^2 - 36*a^2*b^4*sin(d*x + c)^2 + 45*b^6*sin(d*x + c)^2 + 8*a^5
*b*sin(d*x + c) - 84*a^3*b^3*sin(d*x + c) + 100*a*b^5*sin(d*x + c) + 6*a^6
- 50*a^4*b^2 + 56*a^2*b^4)/((b*sin(d*x + c) + a)^2*a^7) - (25*a^4*sin(d*x +
 c)^4 - 300*a^2*b^2*sin(d*x + c)^4 + 375*b^4*sin(d*x + c)^4 + 72*a^3*b*sin(
d*x + c)^3 - 120*a*b^3*sin(d*x + c)^3 - 12*a^4*sin(d*x + c)^2 + 36*a^2*b^2*
sin(d*x + c)^2 - 12*a^3*b*sin(d*x + c) + 3*a^4)/(a^7*sin(d*x + c)^4))/d
```

maple [A] time = 0.35, size = 348, normalized size = 1.57

$$-\frac{\ln(a+b\sin(dx+c))}{a^3d}+\frac{12\ln(a+b\sin(dx+c))\,b^2}{d\,a^5}-\frac{15\ln(a+b\sin(dx+c))\,b^4}{d\,a^7}+\frac{1}{a^2d\,(a+b\sin(dx+c))}-\frac{\cdots}{d\,a^4\,(\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -ln(a+b*sin(d*x+c))/a^3/d+12/d/a^5*ln(a+b*sin(d*x+c))*b^2-15/d/a^7*ln(a+b*s
in(d*x+c))*b^4+1/a^2/d/(a+b*sin(d*x+c))-6/d/a^4/(a+b*sin(d*x+c))*b^2+5/d/a^
6/(a+b*sin(d*x+c))*b^4+1/2/a/d/(a+b*sin(d*x+c))^2-1/d/a^3/(a+b*sin(d*x+c))^
2*b^2+1/2/d/a^5/(a+b*sin(d*x+c))^2*b^4-1/4/d/a^3/sin(d*x+c)^4+1/a^3/d/sin(d
*x+c)^2-3/d/a^5/sin(d*x+c)^2*b^2+ln(sin(d*x+c))/a^3/d-12/d/a^5*ln(sin(d*x+c
```

```
))*b^2+15/d/a^7*ln(sin(d*x+c))*b^4+1/d/a^4*b/sin(d*x+c)^3-6/d/a^4*b/sin(d*x
+c)+10/d*b^3/a^6/sin(d*x+c)
```

maxima [A] time = 0.70, size = 236, normalized size = 1.07

$$\frac{\frac{2\,a^4b\sin(dx+c)+4\left(a^4b-12\,a^2b^3+15\,b^5\right)\sin(dx+c)^5-a^5+6\left(a^5-12\,a^3b^2+15\,ab^4\right)\sin(dx+c)^4-4\left(4\,a^4b-5\,a^2b^3\right)\sin(dx+c)^3+\left(4\,a^5-5\,a^3b^2\right)\sin(dx+c)^2}{a^6b^2\sin(dx+c)^6+2\,a^7b\sin(dx+c)^5+a^8\sin(dx+c)^4} - \cdots}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*((2*a^4*b*sin(d*x + c) + 4*(a^4*b - 12*a^2*b^3 + 15*b^5)*sin(d*x + c)^5
 - a^5 + 6*(a^5 - 12*a^3*b^2 + 15*a*b^4)*sin(d*x + c)^4 - 4*(4*a^4*b - 5*a^
2*b^3)*sin(d*x + c)^3 + (4*a^5 - 5*a^3*b^2)*sin(d*x + c)^2)/(a^6*b^2*sin(d*
x + c)^6 + 2*a^7*b*sin(d*x + c)^5 + a^8*sin(d*x + c)^4) - 4*(a^4 - 12*a^2*b
^2 + 15*b^4)*log(b*sin(d*x + c) + a)/a^7 + 4*(a^4 - 12*a^2*b^2 + 15*b^4)*lo
g(sin(d*x + c))/a^7)/d
```

mupad [B] time = 7.26, size = 563, normalized size = 2.55

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(\frac{23\,a^5}{4}-172\,a^3\,b^2+272\,a\,b^4\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(27\,a^4\,b-40\,a^2\,b^3\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(-134\,a^4\,b+2\cdots\right.}{d\,\left(16\,a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+16\,a^8\,\tan\left(\frac{c}{2}+\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5/(a + b*sin(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^4*(272*a*b^4 + (23*a^5)/4 - 172*a^3*b^2) - tan(c/2 + (d
*x)/2)^3*(27*a^4*b - 40*a^2*b^3) + tan(c/2 + (d*x)/2)^5*(128*b^5 - 134*a^4*
b + 200*a^2*b^3) - tan(c/2 + (d*x)/2)^7*(106*a^4*b + 192*b^5 - 336*a^2*b^3)
 - a^5/4 + tan(c/2 + (d*x)/2)^2*((5*a^5)/2 - 5*a^3*b^2) + a^4*b*tan(c/2 + (
d*x)/2) + (tan(c/2 + (d*x)/2)^6*(3*a^6 - 352*b^6 + 768*a^2*b^4 - 276*a^4*b^
2))/a)/(d*(16*a^8*tan(c/2 + (d*x)/2)^4 + 16*a^8*tan(c/2 + (d*x)/2)^8 + tan(
c/2 + (d*x)/2)^6*(32*a^8 + 64*a^6*b^2) + 64*a^7*b*tan(c/2 + (d*x)/2)^5 + 64
*a^7*b*tan(c/2 + (d*x)/2)^7)) - tan(c/2 + (d*x)/2)^4/(64*a^3*d) + (tan(c/2
+ (d*x)/2)^2*((3*(a^2 + 4*b^2))/(32*a^5) + 3/(32*a^3) - (9*b^2)/(8*a^5)))/d
 - (tan(c/2 + (d*x)/2)*((6*b*((3*(a^2 + 4*b^2))/(16*a^5) + 3/(16*a^3) - (9*
b^2)/(4*a^5)))/a - (192*a^2*b + 128*b^3)/(256*a^6) + (9*b*(a^2 + 4*b^2))/(8
*a^6)))/d + (log(tan(c/2 + (d*x)/2))*(a^4 + 15*b^4 - 12*a^2*b^2))/(a^7*d) +
 (b*tan(c/2 + (d*x)/2)^3)/(8*a^4*d) - (log(a + 2*b*tan(c/2 + (d*x)/2) + a*t
an(c/2 + (d*x)/2)^2)*(a^4 + 15*b^4 - 12*a^2*b^2))/(a^7*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)**5/(a + b*sin(c + d*x))**3, x)
```

3.198 $\int \frac{\tan^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=474

$$\frac{12a^2b^2\left(a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{9/2}}+\frac{3a^5b\cos(c+dx)}{2d\left(a^2-b^2\right)^4(a+b\sin(c+dx))}+\frac{a^4\left(2a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{9/2}}$$

```
[Out] 8*a^4*b^2*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(9/2)/
d+12*a^2*b^2*(a^2+b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^
2-b^2)^(9/2)/d+a^4*(2*a^2+b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1
/2))/(a^2-b^2)^(9/2)/d+1/12*cos(d*x+c)/(a+b)^3/d/(1-sin(d*x+c))^2-3/4*a*cos
(d*x+c)/(a+b)^4/d/(1-sin(d*x+c))+1/12*cos(d*x+c)/(a+b)^3/d/(1-sin(d*x+c))-1
/12*cos(d*x+c)/(a-b)^3/d/(1+sin(d*x+c))^2+3/4*a*cos(d*x+c)/(a-b)^4/d/(1+sin
(d*x+c))-1/12*cos(d*x+c)/(a-b)^3/d/(1+sin(d*x+c))+1/2*a^4*b*cos(d*x+c)/(a^2
-b^2)^3/d/(a+b*sin(d*x+c))^2+3/2*a^5*b*cos(d*x+c)/(a^2-b^2)^4/d/(a+b*sin(d*
x+c))+4*a^3*b^3*cos(d*x+c)/(a^2-b^2)^4/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.87, antiderivative size = 474, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2731, 2650, 2648, 2664, 2754, 12, 2660, 618, 204}

$$\frac{a^4\left(2a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{9/2}}+\frac{8a^4b^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{9/2}}+\frac{12a^2b^2\left(a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{9/2}}+\frac{}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (8*a^4*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(
9/2)*d) + (12*a^2*b^2*(a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2
- b^2]])/((a^2 - b^2)^(9/2)*d) + (a^4*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c +
d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(9/2)*d) + Cos[c + d*x]/(12*(a + b)
^3*d*(1 - Sin[c + d*x])^2) - (3*a*Cos[c + d*x])/(4*(a + b)^4*d*(1 - Sin[c +
 d*x])) + Cos[c + d*x]/(12*(a + b)^3*d*(1 - Sin[c + d*x])) - Cos[c + d*x]/(
12*(a - b)^3*d*(1 + Sin[c + d*x])^2) + (3*a*Cos[c + d*x])/(4*(a - b)^4*d*(1
 + Sin[c + d*x])) - Cos[c + d*x]/(12*(a - b)^3*d*(1 + Sin[c + d*x])) + (a^4
*b*Cos[c + d*x])/(2*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])^2) + (3*a^5*b*Cos[
c + d*x])/(2*(a^2 - b^2)^4*d*(a + b*Sin[c + d*x])) + (4*a^3*b^3*Cos[c + d*x
])/((a^2 - b^2)^4*d*(a + b*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e + f*x])^m)
/(1 - Sin[e + f*x]^2)^(p/2), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 -
b^2, 0] && IntegersQ[m, p/2]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int\left(\frac{1}{4(a+b)^3(-1+\sin(c+dx))^2}+\frac{3a}{4(a+b)^4(-1+\sin(c+dx))}+\frac{1}{4(a-b)^3(1+\sin(\right.$$

$$= -\frac{(3a)\int\frac{1}{1+\sin(c+dx)}\,dx}{4(a-b)^4}+\frac{\int\frac{1}{(1+\sin(c+dx))^2}\,dx}{4(a-b)^3}+\frac{(3a)\int\frac{1}{-1+\sin(c+dx)}\,dx}{4(a+b)^4}+\frac{\int\frac{1}{(-1+\sin(c+dx))^2}}{4(a+b)^3}$$

$$= \frac{\cos(c+dx)}{12(a+b)^3d(1-\sin(c+dx))^2}-\frac{3a\cos(c+dx)}{4(a+b)^4d(1-\sin(c+dx))}-\frac{\cos(c+dx)}{12(a-b)^3d(1+\sin(c}$$

$$= \frac{\cos(c+dx)}{12(a+b)^3d(1-\sin(c+dx))^2}-\frac{3a\cos(c+dx)}{4(a+b)^4d(1-\sin(c+dx))}+\frac{\cos(c+dx)}{12(a+b)^3d(1-\sin(c}$$

$$= \frac{12a^2b^2\left(a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{9/2}d}+\frac{\cos(c+dx)}{12(a+b)^3d(1-\sin(c+dx))^2}-\frac{3a\,c}{4(a+b)^4c}$$

$$= \frac{12a^2b^2\left(a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{9/2}d}+\frac{\cos(c+dx)}{12(a+b)^3d(1-\sin(c+dx))^2}-\frac{3a\,c}{4(a+b)^4c}$$

$$= \frac{8a^4b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{9/2}d}+\frac{12a^2b^2\left(a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{9/2}d}+\frac{c}{12(a+b)^3}$$

$$= \frac{8a^4b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{9/2}d}+\frac{12a^2b^2\left(a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{9/2}d}+\frac{a^4\left(2a^2+\right.}{}$$

Mathematica [A] time = 1.04, size = 351, normalized size = 0.74

$$\frac{\frac{96a^2\left(2a^4+21a^2b^2+12b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{9/2}}-\frac{\sec^3(c+dx)\left(32a^7\sin(3(c+dx))-264a^6b+22a^5b^2\sin(c+dx)-91a^5b^2\sin(3(c+dx))-17a^5b^2\sin(5(c+d\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((96*a^2*(2*a^4 + 21*a^2*b^2 + 12*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt
[a^2 - b^2]])/(a^2 - b^2)^(9/2) - (Sec[c + d*x]^3*(-264*a^6*b - 358*a^4*b^3
 + 8*a^2*b^5 - 16*b^7 - 8*(44*a^6*b + 55*a^4*b^3 + 8*a^2*b^5 - 2*b^7)*Cos[2
*(c + d*x)] - 2*(28*a^6*b + 89*a^4*b^3 - 12*a^2*b^5)*Cos[4*(c + d*x)] + 22*
a^5*b^2*Sin[c + d*x] - 264*a^3*b^4*Sin[c + d*x] + 32*a*b^6*Sin[c + d*x] + 3
2*a^7*Sin[3*(c + d*x)] - 91*a^5*b^2*Sin[3*(c + d*x)] - 244*a^3*b^4*Sin[3*(c
 + d*x)] - 12*a*b^6*Sin[3*(c + d*x)] - 17*a^5*b^2*Sin[5*(c + d*x)] - 76*a^3
*b^4*Sin[5*(c + d*x)] - 12*a*b^6*Sin[5*(c + d*x)]))/((a^2 - b^2)^4*(a + b*S
in[c + d*x])^2))/(96*d)
```

fricas [A] time = 0.64, size = 1249, normalized size = 2.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/12*(4*a^8*b - 16*a^6*b^3 + 24*a^4*b^5 - 16*a^2*b^7 + 4*b^9 - 2*(28*a^8*b
 + 61*a^6*b^3 - 101*a^4*b^5 + 12*a^2*b^7)*cos(d*x + c)^4 - 4*(8*a^8*b - 25*
a^6*b^3 + 27*a^4*b^5 - 11*a^2*b^7 + b^9)*cos(d*x + c)^2 - 3*((2*a^6*b^2 + 2
1*a^4*b^4 + 12*a^2*b^6)*cos(d*x + c)^5 - 2*(2*a^7*b + 21*a^5*b^3 + 12*a^3*b
^5)*cos(d*x + c)^3*sin(d*x + c) - (2*a^8 + 23*a^6*b^2 + 33*a^4*b^4 + 12*a^2
*b^6)*cos(d*x + c)^3)*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 -
2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x
 + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b
^2)) - 2*(2*a^9 - 8*a^7*b^2 + 12*a^5*b^4 - 8*a^3*b^6 + 2*a*b^8 + (17*a^7*b^
2 + 59*a^5*b^4 - 64*a^3*b^6 - 12*a*b^8)*cos(d*x + c)^4 - 2*(4*a^9 - 9*a^7*b
^2 + 3*a^5*b^4 + 5*a^3*b^6 - 3*a*b^8)*cos(d*x + c)^2)*sin(d*x + c))/((a^10*
b^2 - 5*a^8*b^4 + 10*a^6*b^6 - 10*a^4*b^8 + 5*a^2*b^10 - b^12)*d*cos(d*x +
c)^5 - 2*(a^11*b - 5*a^9*b^3 + 10*a^7*b^5 - 10*a^5*b^7 + 5*a^3*b^9 - a*b^11
)*d*cos(d*x + c)^3*sin(d*x + c) - (a^12 - 4*a^10*b^2 + 5*a^8*b^4 - 5*a^4*b^
8 + 4*a^2*b^10 - b^12)*d*cos(d*x + c)^3), 1/6*(2*a^8*b - 8*a^6*b^3 + 12*a^4
*b^5 - 8*a^2*b^7 + 2*b^9 - (28*a^8*b + 61*a^6*b^3 - 101*a^4*b^5 + 12*a^2*b^
7)*cos(d*x + c)^4 - 2*(8*a^8*b - 25*a^6*b^3 + 27*a^4*b^5 - 11*a^2*b^7 + b^9
)*cos(d*x + c)^2 - 3*((2*a^6*b^2 + 21*a^4*b^4 + 12*a^2*b^6)*cos(d*x + c)^5
- 2*(2*a^7*b + 21*a^5*b^3 + 12*a^3*b^5)*cos(d*x + c)^3*sin(d*x + c) - (2*a^
8 + 23*a^6*b^2 + 33*a^4*b^4 + 12*a^2*b^6)*cos(d*x + c)^3)*sqrt(a^2 - b^2)*a
rctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (2*a^9 - 8*a^
7*b^2 + 12*a^5*b^4 - 8*a^3*b^6 + 2*a*b^8 + (17*a^7*b^2 + 59*a^5*b^4 - 64*a^
3*b^6 - 12*a*b^8)*cos(d*x + c)^4 - 2*(4*a^9 - 9*a^7*b^2 + 3*a^5*b^4 + 5*a^3
*b^6 - 3*a*b^8)*cos(d*x + c)^2)*sin(d*x + c))/((a^10*b^2 - 5*a^8*b^4 + 10*a
^6*b^6 - 10*a^4*b^8 + 5*a^2*b^10 - b^12)*d*cos(d*x + c)^5 - 2*(a^11*b - 5*a
^9*b^3 + 10*a^7*b^5 - 10*a^5*b^7 + 5*a^3*b^9 - a*b^11)*d*cos(d*x + c)^3*sin
```

```
(d*x + c) - (a^12 - 4*a^10*b^2 + 5*a^8*b^4 - 5*a^4*b^8 + 4*a^2*b^10 - b^12)
*d*cos(d*x + c)^3)]
```

giac [A] time = 2.42, size = 632, normalized size = 1.33

$$\frac{\frac{3\left(2a^6+21a^4b^2+12a^2b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8\right)\sqrt{a^2-b^2}}+\frac{3\left(5a^5b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+6a^3b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+4a^6b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\cdots\right.}{\left(a^8-4a^6b^2+6a^4b^4\cdots\right.}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/3*(3*(2*a^6 + 21*a^4*b^2 + 12*a^2*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*
sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^8 - 4*a^
6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*sqrt(a^2 - b^2)) + 3*(5*a^5*b^2*tan(1/
2*d*x + 1/2*c)^3 + 6*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 + 4*a^6*b*tan(1/2*d*x +
 1/2*c)^2 + 15*a^4*b^3*tan(1/2*d*x + 1/2*c)^2 + 14*a^2*b^5*tan(1/2*d*x + 1/
2*c)^2 + 11*a^5*b^2*tan(1/2*d*x + 1/2*c) + 22*a^3*b^4*tan(1/2*d*x + 1/2*c)
+ 4*a^6*b + 7*a^4*b^3)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(a*
tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2) + 2*(3*a^5*tan(1/
2*d*x + 1/2*c)^5 + 24*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 + 9*a*b^4*tan(1/2*d*x
+ 1/2*c)^5 - 9*a^4*b*tan(1/2*d*x + 1/2*c)^4 - 24*a^2*b^3*tan(1/2*d*x + 1/2*
c)^4 - 3*b^5*tan(1/2*d*x + 1/2*c)^4 - 10*a^5*tan(1/2*d*x + 1/2*c)^3 - 56*a^
3*b^2*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 36*a^4*b*ta
n(1/2*d*x + 1/2*c)^2 + 36*a^2*b^3*tan(1/2*d*x + 1/2*c)^2 + 3*a^5*tan(1/2*d*
x + 1/2*c) + 24*a^3*b^2*tan(1/2*d*x + 1/2*c) + 9*a*b^4*tan(1/2*d*x + 1/2*c)
 - 15*a^4*b - 20*a^2*b^3 - b^5)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 +
 b^8)*(tan(1/2*d*x + 1/2*c)^2 - 1)^3))/d
```

maple [B] time = 0.32, size = 922, normalized size = 1.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/3/d/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)^3-1/2/d/(a+b)^3/(tan(1/2*d*x+1/2*c)-1
)^2+1/d/(a+b)^4/(tan(1/2*d*x+1/2*c)-1)*a-1/2/d/(a+b)^4/(tan(1/2*d*x+1/2*c)-
1)*b-1/3/d/(a-b)^3/(tan(1/2*d*x+1/2*c)+1)^3+1/2/d/(a-b)^3/(tan(1/2*d*x+1/2*
c)+1)^2+1/d/(a-b)^4/(tan(1/2*d*x+1/2*c)+1)*a+1/2/d/(a-b)^4/(tan(1/2*d*x+1/2
*c)+1)*b+5/d*a^5/(a-b)^4/(a+b)^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*
c)*b+a)^2*tan(1/2*d*x+1/2*c)^3*b^2+6/d*a^3/(a-b)^4/(a+b)^4/(tan(1/2*d*x+1/2
```

```
*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3*b^4+4/d*a^6/(a-b)^
4/(a+b)^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1
/2*c)^2*b+15/d*a^4/(a-b)^4/(a+b)^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/
2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*b^3+14/d*a^2/(a-b)^4/(a+b)^4/(tan(1/2*d*x+
1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*b^5+11/d*a^5/(a
-b)^4/(a+b)^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d
*x+1/2*c)*b^2+22/d*a^3/(a-b)^4/(a+b)^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*
x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)*b^4+4/d*a^6/(a-b)^4/(a+b)^4/(tan(1/2*d*x
+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b+7/d*a^4/(a-b)^4/(a+b)^4/(tan(1/2*
d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b^3+2/d*a^6/(a-b)^4/(a+b)^4/(a^2
-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+21/d*a
^4/(a-b)^4/(a+b)^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/
(a^2-b^2)^(1/2))*b^2+12/d*a^2/(a-b)^4/(a+b)^4/(a^2-b^2)^(1/2)*arctan(1/2*(2
*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 11.22, size = 1099, normalized size = 2.32

$$\frac{\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\left(2\,a^6\,b+21\,a^4\,b^3+12\,a^2\,b^5\right)}{a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8}-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\left(14\,a^7+23\,a^5\,b^2+242\,a^3\,b^4+36\,a\,b^6\right)}{3\left(a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8\right)}-\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\left(2\,a^5+21\,a^3\,b^2+12\,a\,b^4\right)}{3\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}+\frac{2\tan(}{}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(2\,a^2+12\,b^2\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4/(a + b*sin(c + d*x))^3,x)
```

```
[Out] ((3*tan(c/2 + (d*x)/2)^8*(2*a^6*b + 12*a^2*b^5 + 21*a^4*b^3))/(a^8 + b^8 -
4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2) - (2*tan(c/2 + (d*x)/2)^5*(36*a*b^6 + 14
*a^7 + 242*a^3*b^4 + 23*a^5*b^2))/(3*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4
*a^6*b^2)) - (4*tan(c/2 + (d*x)/2)^7*(12*a*b^4 + 2*a^5 + 21*a^3*b^2))/(3*(a
^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (2*tan(c/2 + (d*x)/2)^2*(30*a^4*b + 4*
```

```
b^5 + 71*a^2*b^3))/(3*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (a^2*(42*a^4*b
 + 2*b^5 + 61*a^2*b^3))/(3*(a^2 - b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))
 + (10*tan(c/2 + (d*x)/2)^4*(4*a^6*b + 43*a^2*b^5 + 16*a^4*b^3))/(3*(a^2 -
b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (4*tan(c/2 + (d*x)/2)^3*(16*a*b
^6 - 2*a^7 + 131*a^3*b^4 + 65*a^5*b^2))/(3*(a^2 - b^2)*(a^6 - b^6 + 3*a^2*b
^4 - 3*a^4*b^2)) - (2*tan(c/2 + (d*x)/2)^6*(22*a^6*b + 12*b^7 + 153*a^2*b^5
 + 233*a^4*b^3))/(3*(a^2 - b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (a^3
*tan(c/2 + (d*x)/2)^9*(2*a^4 + 12*b^4 + 21*a^2*b^2))/(a^8 + b^8 - 4*a^2*b^6
 + 6*a^4*b^4 - 4*a^6*b^2) - (a*tan(c/2 + (d*x)/2)*(8*b^6 - 6*a^6 + 208*a^2*
b^4 + 105*a^4*b^2))/(3*(a^2 - b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)))/(d
*(tan(c/2 + (d*x)/2)^4*(2*a^2 + 12*b^2) - tan(c/2 + (d*x)/2)^6*(2*a^2 + 12*
b^2) + a^2*tan(c/2 + (d*x)/2)^10 - a^2 + tan(c/2 + (d*x)/2)^2*(a^2 - 4*b^2)
 - tan(c/2 + (d*x)/2)^8*(a^2 - 4*b^2) + 8*a*b*tan(c/2 + (d*x)/2)^3 - 8*a*b*
tan(c/2 + (d*x)/2)^7 + 4*a*b*tan(c/2 + (d*x)/2)^9 - 4*a*b*tan(c/2 + (d*x)/2
))) + (a^2*atan(((a^2*(2*a^4 + 12*b^4 + 21*a^2*b^2)*(2*a^8*b + 2*b^9 - 8*a^
2*b^7 + 12*a^4*b^5 - 8*a^6*b^3))/(2*(a + b)^(9/2)*(a - b)^(9/2)) + (a^3*tan
(c/2 + (d*x)/2)*(2*a^4 + 12*b^4 + 21*a^2*b^2)*(a^8 + b^8 - 4*a^2*b^6 + 6*a^
4*b^4 - 4*a^6*b^2))/((a + b)^(9/2)*(a - b)^(9/2)))/(2*a^6 + 12*a^2*b^4 + 21
*a^4*b^2))*(2*a^4 + 12*b^4 + 21*a^2*b^2))/(d*(a + b)^(9/2)*(a - b)^(9/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**4/(a + b*sin(c + d*x))**3, x)
```

3.199 $\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=350

$$-\frac{a^2\left(2a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}-\frac{4a^2b^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}-\frac{2b^2\left(3a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}-\frac{}{2d}$$

```
[Out] -4*a^2*b^2*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(7/2)
/d-a^2*(2*a^2+b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^
2)^(7/2)/d-2*b^2*(3*a^2+b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2
))/(a^2-b^2)^(7/2)/d+1/2*cos(d*x+c)/(a+b)^3/d/(1-sin(d*x+c))-1/2*cos(d*x+c)
/(a-b)^3/d/(1+sin(d*x+c))-1/2*a^2*b*cos(d*x+c)/(a^2-b^2)^2/d/(a+b*sin(d*x+c
))^2-3/2*a^3*b*cos(d*x+c)/(a^2-b^2)^3/d/(a+b*sin(d*x+c))-2*a*b^3*cos(d*x+c)
/(a^2-b^2)^3/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.54, antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {2731, 2648, 2664, 2754, 12, 2660, 618, 204}

$$-\frac{a^2\left(2a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}-\frac{4a^2b^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}-\frac{2b^2\left(3a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}-\frac{}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-4*a^2*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^
(7/2)*d) - (a^2*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^
2]])/((a^2 - b^2)^(7/2)*d) - (2*b^2*(3*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*
x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(7/2)*d) + Cos[c + d*x]/(2*(a + b)^3*
d*(1 - Sin[c + d*x])) - Cos[c + d*x]/(2*(a - b)^3*d*(1 + Sin[c + d*x])) - (
a^2*b*Cos[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^2) - (3*a^3*b*C
os[c + d*x])/(2*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])) - (2*a*b^3*Cos[c + d*
x])/((a^2 - b^2)^3*d*(a + b*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e + f*x])^m)
/(1 - Sin[e + f*x]^2)^(p/2), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 -
b^2, 0] && IntegersQ[m, p/2]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
```

```
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int\left(-\frac{1}{2(a+b)^3(-1+\sin(c+dx))} + \frac{1}{2(a-b)^3(1+\sin(c+dx))} - \frac{a^2}{\left(a^2-b^2\right)(a+b\,\mathrm{si}\right.$$

$$= \frac{\int \frac{1}{1+\sin(c+dx)}\,dx}{2(a-b)^3} - \frac{\int \frac{1}{-1+\sin(c+dx)}\,dx}{2(a+b)^3} - \frac{\left(2ab^2\right)\int \frac{1}{(a+b\sin(c+dx))^2}\,dx}{\left(a^2-b^2\right)^2} - \frac{a^2\int \frac{1}{(a+b\sin(c+dx))}}{a^2-b^2}$$

$$= \frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))} - \frac{\cos(c+dx)}{2(a-b)^3d(1+\sin(c+dx))} - \frac{a^2b\cos(c+dx}{2\left(a^2-b^2\right)^2d(a+b\sin}$$

$$= \frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))} - \frac{\cos(c+dx)}{2(a-b)^3d(1+\sin(c+dx))} - \frac{a^2b\cos(c+dx}{2\left(a^2-b^2\right)^2d(a+b\sin}$$

$$= -\frac{2b^2\left(3a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d} + \frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))} - \frac{\cos}{2(a-b)^3d(}$$

$$= -\frac{2b^2\left(3a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d} + \frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))} - \frac{\cos}{2(a-b)^3d(}$$

$$= -\frac{4a^2b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d} - \frac{2b^2\left(3a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d} + \frac{\mathrm{co}}{2(a+b)^3}$$

$$= -\frac{4a^2b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d} - \frac{a^2\left(2a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d} - \frac{2b^2\left(3a^2+\right.}{}$$

Mathematica [A] time = 3.22, size = 212, normalized size = 0.61

$$\frac{-\frac{2\left(2a^4+11a^2b^2+2b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}}-\frac{ab\cos(c+dx)\left(4a^3+b\left(3a^2+4b^2\right)\sin(c+dx)+3ab^2\right)}{(a-b)^3(a+b)^3(a+b\sin(c+dx))^2}+\sin\left(\frac{1}{2}(c+dx)\right)\left(\frac{2}{(a-b)^3\left(\sin\left(\frac{1}{2}(c+dx)\right)+\text{co}\right.}\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-2*(2*a^4 + 11*a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2
- b^2]])/(a^2 - b^2)^(7/2) + Sin[(c + d*x)/2]*(2/((a + b)^3*(Cos[(c + d*x)/
2] - Sin[(c + d*x)/2])) + 2/((a - b)^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
))) - (a*b*Cos[c + d*x]*(4*a^3 + 3*a*b^2 + b*(3*a^2 + 4*b^2)*Sin[c + d*x]))
/((a - b)^3*(a + b)^3*(a + b*Sin[c + d*x])^2))/(2*d)
```

fricas [A] time = 0.59, size = 934, normalized size = 2.67

$$\left[\frac{4\,a^6b-12\,a^4b^3+12\,a^2b^5-4\,b^7+2\left(8\,a^6b+a^4b^3-11\,a^2b^5+2\,b^7\right)\cos\left(dx+c\right)^2+\left(\left(2\,a^4b^2+11\,a^2b^4+2\,b^6\right)\cos\right.}{4\left(\left(a^8b\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(4*a^6*b - 12*a^4*b^3 + 12*a^2*b^5 - 4*b^7 + 2*(8*a^6*b + a^4*b^3 - 11
*a^2*b^5 + 2*b^7)*cos(d*x + c)^2 + ((2*a^4*b^2 + 11*a^2*b^4 + 2*b^6)*cos(d*
x + c)^3 - 2*(2*a^5*b + 11*a^3*b^3 + 2*a*b^5)*cos(d*x + c)*sin(d*x + c) - (
2*a^6 + 13*a^4*b^2 + 13*a^2*b^4 + 2*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*log
(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(
d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)
^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 2*(2*a^7 - 6*a^5*b^2 + 6*a^3*b^4 -
2*a*b^6 - 5*(a^5*b^2 + a^3*b^4 - 2*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a
^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10)*d*cos(d*x + c)^3 - 2*(a^
9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*d*cos(d*x + c)*sin(d*x + c
) - (a^10 - 3*a^8*b^2 + 2*a^6*b^4 + 2*a^4*b^6 - 3*a^2*b^8 + b^10)*d*cos(d*x
 + c)), 1/2*(2*a^6*b - 6*a^4*b^3 + 6*a^2*b^5 - 2*b^7 + (8*a^6*b + a^4*b^3 -
 11*a^2*b^5 + 2*b^7)*cos(d*x + c)^2 + ((2*a^4*b^2 + 11*a^2*b^4 + 2*b^6)*cos
(d*x + c)^3 - 2*(2*a^5*b + 11*a^3*b^3 + 2*a*b^5)*cos(d*x + c)*sin(d*x + c)
- (2*a^6 + 13*a^4*b^2 + 13*a^2*b^4 + 2*b^6)*cos(d*x + c))*sqrt(a^2 - b^2)*a
rctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (2*a^7 - 6*a^
5*b^2 + 6*a^3*b^4 - 2*a*b^6 - 5*(a^5*b^2 + a^3*b^4 - 2*a*b^6)*cos(d*x + c)^
```

```
2)*sin(d*x + c))/((a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10)*d*co
s(d*x + c)^3 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*d*cos(
d*x + c)*sin(d*x + c) - (a^10 - 3*a^8*b^2 + 2*a^6*b^4 + 2*a^4*b^6 - 3*a^2*b
^8 + b^10)*d*cos(d*x + c))]
```

giac [A] time = 1.82, size = 384, normalized size = 1.10

$$-\frac{\frac{\left(2\,a^4+11\,a^2b^2+2\,b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\sqrt{a^2-b^2}}+\frac{2\left(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,a^2b-b^3\right)}{\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}+\frac{5\,a^3b^2\tan\left(\frac{1}{2}\,dx+\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -((2*a^4 + 11*a^2*b^2 + 2*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + a
rctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^6 - 3*a^4*b^2 + 3*
a^2*b^4 - b^6)*sqrt(a^2 - b^2)) + 2*(a^3*tan(1/2*d*x + 1/2*c) + 3*a*b^2*tan
(1/2*d*x + 1/2*c) - 3*a^2*b - b^3)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(ta
n(1/2*d*x + 1/2*c)^2 - 1)) + (5*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 2*a*b^4*ta
n(1/2*d*x + 1/2*c)^3 + 4*a^4*b*tan(1/2*d*x + 1/2*c)^2 + 11*a^2*b^3*tan(1/2*
d*x + 1/2*c)^2 + 6*b^5*tan(1/2*d*x + 1/2*c)^2 + 11*a^3*b^2*tan(1/2*d*x + 1/
2*c) + 10*a*b^4*tan(1/2*d*x + 1/2*c) + 4*a^4*b + 3*a^2*b^3)/((a^6 - 3*a^4*b
^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c)
+ a)^2))/d
```

maple [B] time = 0.29, size = 766, normalized size = 2.19

$$-\frac{1}{d\left(a+b\right)^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{1}{d\left(a-b\right)^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{5\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^{3}b^{2}}{d\left(a-b\right)^{3}\left(a+b\right)^{3}\left(\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+2\tan\left(\frac{dx}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/d/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)-1/d/(a-b)^3/(tan(1/2*d*x+1/2*c)+1)-5/d/
(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2
*d*x+1/2*c)^3*a^3*b^2-2/d/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2
*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3*a*b^4-4/d/(a-b)^3/(a+b)^3/(tan(1/2*
d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*a^4*b-11/d/
(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2
```

```
*d*x+1/2*c)^2*a^2*b^3-6/d/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2
*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*b^5-11/d/(a-b)^3/(a+b)^3/(tan(1/2*d
*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)*a^3*b^2-10/d/(
a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*
d*x+1/2*c)*a*b^4-4/d/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+
1/2*c)*b+a)^2*a^4*b-3/d/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d
*x+1/2*c)*b+a)^2*a^2*b^3-2/d/(a-b)^3/(a+b)^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*
a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*a^4-11/d/(a-b)^3/(a+b)^3/(a^2-b^
2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*a^2*b^2-2
/d/(a-b)^3/(a+b)^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/
(a^2-b^2)^(1/2))*b^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 10.36, size = 627, normalized size = 1.79

$$\frac{\frac{5\left(2\,a^4\,b+a^2\,b^3\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(2\,a^5+a^3\,b^2+12\,a\,b^4\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(2\,a^4\,b+6\,a^2\,b^3+7\,b^5\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(2\,a^4\,b+11\,a^2\,b^3+2\,b^5\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}+\frac{a\,\mathrm{t}}{}}{d\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-a^2-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(a^2+4\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(a^2+4\,b^2\right)+4\,a\,b\,\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + b*sin(c + d*x))^3,x)
```

```
[Out] ((5*(2*a^4*b + a^2*b^3))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) - (2*tan(c/2 +
 (d*x)/2)^3*(12*a*b^4 + 2*a^5 + a^3*b^2))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^
2) + (2*tan(c/2 + (d*x)/2)^2*(2*a^4*b + 7*b^5 + 6*a^2*b^3))/(a^6 - b^6 + 3*
a^2*b^4 - 3*a^4*b^2) - (3*tan(c/2 + (d*x)/2)^4*(2*a^4*b + 2*b^5 + 11*a^2*b^
3))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) + (a*tan(c/2 + (d*x)/2)*(18*b^4 - 2
*a^4 + 29*a^2*b^2))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) - (a*tan(c/2 + (d*x
)/2)^5*(2*a^4 + 2*b^4 + 11*a^2*b^2))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))/(
d*(a^2*tan(c/2 + (d*x)/2)^6 - a^2 - tan(c/2 + (d*x)/2)^2*(a^2 + 4*b^2) + ta
```

```
n(c/2 + (d*x)/2)^4*(a^2 + 4*b^2) + 4*a*b*tan(c/2 + (d*x)/2)^5 - 4*a*b*tan(c
/2 + (d*x)/2))) - (atan((((2*a^4 + 2*b^4 + 11*a^2*b^2)*(2*a^6*b - 2*b^7 + 6
*a^2*b^5 - 6*a^4*b^3))/(2*(a + b)^(7/2)*(a - b)^(7/2)) + (a*tan(c/2 + (d*x)
/2)*(2*a^4 + 2*b^4 + 11*a^2*b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))/((a +
 b)^(7/2)*(a - b)^(7/2)))/(2*a^4 + 2*b^4 + 11*a^2*b^2))*(2*a^4 + 2*b^4 + 11
*a^2*b^2))/(d*(a + b)^(7/2)*(a - b)^(7/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**2/(a + b*sin(c + d*x))**3, x)
```

3.200 $\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=202

$$\frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d}+\frac{(2a^2-3b^2)\cot(c+dx)}{2a^2d(a^2-b^2)(a+b\sin(c+dx))}-\frac{(2a^4-9a^2b^2+6b^4)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4d(a^2-b^2)^{3/2}}-\frac{(5a^2-6}{2a^3}$$

```
[Out] -(2*a^4-9*a^2*b^2+6*b^4)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a
^4/(a^2-b^2)^(3/2)/d+3*b*arctanh(cos(d*x+c))/a^4/d-1/2*(5*a^2-6*b^2)*cot(d*
x+c)/a^3/(a^2-b^2)/d+1/2*cot(d*x+c)/a/d/(a+b*sin(d*x+c))^2+1/2*(2*a^2-3*b^2
)*cot(d*x+c)/a^2/(a^2-b^2)/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.79, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2723, 3056, 3055, 3001, 3770, 2660, 618, 204}

$$-\frac{(-9a^2b^2+2a^4+6b^4)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4d(a^2-b^2)^{3/2}}-\frac{(5a^2-6b^2)\cot(c+dx)}{2a^3d(a^2-b^2)}+\frac{(2a^2-3b^2)\cot(c+dx)}{2a^2d(a^2-b^2)(a+b\sin(c+dx))}+\frac{3b\,\mathrm{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -(((2*a^4 - 9*a^2*b^2 + 6*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b
^2]])/(a^4*(a^2 - b^2)^(3/2)*d)) + (3*b*ArcTanh[Cos[c + d*x]])/(a^4*d) - ((
5*a^2 - 6*b^2)*Cot[c + d*x])/(2*a^3*(a^2 - b^2)*d) + Cot[c + d*x]/(2*a*d*(a
 + b*Sin[c + d*x])^2) + ((2*a^2 - 3*b^2)*Cot[c + d*x])/(2*a^2*(a^2 - b^2)*d
*(a + b*Sin[c + d*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2))/Sin[e + f*x]
^2, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
```

```
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \frac{\csc^2(c+dx)\left(1-\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^3}\,dx$$

$$= \frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\int \frac{\csc^2(c+dx)\left(3\left(a^2-b^2\right)-2\left(a^2-b^2\right)\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^2}\,dx}{2a\left(a^2-b^2\right)}$$

$$= \frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\left(2a^2-3b^2\right)\cot(c+dx)}{2a^2\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\int \frac{\csc^2(c+dx)\left(5a^4-11a^2b^2\right.}{}}{}$$

$$= -\frac{\left(5a^2-6b^2\right)\cot(c+dx)}{2a^3\left(a^2-b^2\right)d} + \frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\left(2a^2-3b^2\right)\cot(c+dx)}{2a^2\left(a^2-b^2\right)d(a+b\sin(c+}$$

$$= -\frac{\left(5a^2-6b^2\right)\cot(c+dx)}{2a^3\left(a^2-b^2\right)d} + \frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\left(2a^2-3b^2\right)\cot(c+dx)}{2a^2\left(a^2-b^2\right)d(a+b\sin(c+}$$

$$= \frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d} - \frac{\left(5a^2-6b^2\right)\cot(c+dx)}{2a^3\left(a^2-b^2\right)d} + \frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{}{2a^2}$$

$$= \frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d} - \frac{\left(5a^2-6b^2\right)\cot(c+dx)}{2a^3\left(a^2-b^2\right)d} + \frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{}{2a^2}$$

$$= -\frac{\left(2a^4-9a^2b^2+6b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4\left(a^2-b^2\right)^{3/2}d} + \frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d} - \frac{\left(5a^2-6b\right.}{2a^3\left(\right.}$$

Mathematica [A] time = 5.78, size = 195, normalized size = 0.97

$$\frac{\frac{ab\left(4b^2-3a^2\right)\cos(c+dx)}{(a-b)(a+b)(a+b\sin(c+dx))} - \frac{a^2b\cos(c+dx)}{(a+b\sin(c+dx))^2} - \frac{2\left(2a^4-9a^2b^2+6b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + a\tan\left(\frac{1}{2}(c+dx)\right) - a\cot\left(\frac{1}{2}(c+dx)\right)}{2a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-2*(2*a^4 - 9*a^2*b^2 + 6*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 -
 b^2]])/(a^2 - b^2)^(3/2) - a*Cot[(c + d*x)/2] + 6*b*Log[Cos[(c + d*x)/2]]
```

```
- 6*b*Log[Sin[(c + d*x)/2]] - (a^2*b*Cos[c + d*x])/(a + b*Sin[c + d*x])^2 +
 (a*b*(-3*a^2 + 4*b^2)*Cos[c + d*x])/((a - b)*(a + b)*(a + b*Sin[c + d*x]))
 + a*Tan[(c + d*x)/2])/(2*a^4*d)
```

fricas [B] time = 1.04, size = 1394, normalized size = 6.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(5*a^5*b^2 - 11*a^3*b^4 + 6*a*b^6)*cos(d*x + c)^3 - 2*(8*a^6*b - 1
7*a^4*b^3 + 9*a^2*b^5)*cos(d*x + c)*sin(d*x + c) + (4*a^5*b - 18*a^3*b^3 +
12*a*b^5 - 2*(2*a^5*b - 9*a^3*b^3 + 6*a*b^5)*cos(d*x + c)^2 + (2*a^6 - 7*a^
4*b^2 - 3*a^2*b^4 + 6*b^6 - (2*a^4*b^2 - 9*a^2*b^4 + 6*b^6)*cos(d*x + c)^2)
*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*s
in(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*
sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) -
2*(2*a^7 + a^5*b^2 - 9*a^3*b^4 + 6*a*b^6)*cos(d*x + c) + 6*(2*a^5*b^2 - 4*a
^3*b^4 + 2*a*b^6 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x + c)^2 + (a^6*b
- a^4*b^3 - a^2*b^5 + b^7 - (a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^2)*sin
(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 6*(2*a^5*b^2 - 4*a^3*b^4 + 2*a*b^6
 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x + c)^2 + (a^6*b - a^4*b^3 - a^2*
b^5 + b^7 - (a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(-
1/2*cos(d*x + c) + 1/2))/(2*(a^9*b - 2*a^7*b^3 + a^5*b^5)*d*cos(d*x + c)^2
- 2*(a^9*b - 2*a^7*b^3 + a^5*b^5)*d + ((a^8*b^2 - 2*a^6*b^4 + a^4*b^6)*d*co
s(d*x + c)^2 - (a^10 - a^8*b^2 - a^6*b^4 + a^4*b^6)*d)*sin(d*x + c)), -1/2*
((5*a^5*b^2 - 11*a^3*b^4 + 6*a*b^6)*cos(d*x + c)^3 - (8*a^6*b - 17*a^4*b^3
+ 9*a^2*b^5)*cos(d*x + c)*sin(d*x + c) + (4*a^5*b - 18*a^3*b^3 + 12*a*b^5 -
 2*(2*a^5*b - 9*a^3*b^3 + 6*a*b^5)*cos(d*x + c)^2 + (2*a^6 - 7*a^4*b^2 - 3*
a^2*b^4 + 6*b^6 - (2*a^4*b^2 - 9*a^2*b^4 + 6*b^6)*cos(d*x + c)^2)*sin(d*x +
 c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x
+ c))) - (2*a^7 + a^5*b^2 - 9*a^3*b^4 + 6*a*b^6)*cos(d*x + c) + 3*(2*a^5*b^
2 - 4*a^3*b^4 + 2*a*b^6 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x + c)^2 +
(a^6*b - a^4*b^3 - a^2*b^5 + b^7 - (a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)
^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*(2*a^5*b^2 - 4*a^3*b^4 +
2*a*b^6 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x + c)^2 + (a^6*b - a^4*b^3
 - a^2*b^5 + b^7 - (a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^2)*sin(d*x + c)
)*log(-1/2*cos(d*x + c) + 1/2))/(2*(a^9*b - 2*a^7*b^3 + a^5*b^5)*d*cos(d*x
+ c)^2 - 2*(a^9*b - 2*a^7*b^3 + a^5*b^5)*d + ((a^8*b^2 - 2*a^6*b^4 + a^4*b^
6)*d*cos(d*x + c)^2 - (a^10 - a^8*b^2 - a^6*b^4 + a^4*b^6)*d)*sin(d*x + c))
]
```

giac [A] time = 0.50, size = 339, normalized size = 1.68

$$-\frac{\frac{2\left(2a^4-9a^2b^2+6b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^6-a^4b^2\right)\sqrt{a^2-b^2}}+\frac{2\left(5a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-6ab^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+4a^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3\cdots}{\left(a^6-a^4b^2\right)\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*(2*a^4 - 9*a^2*b^2 + 6*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a
) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^6 - a^4*b^2)*
sqrt(a^2 - b^2)) + 2*(5*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^4*tan(1/2*d*
x + 1/2*c)^3 + 4*a^4*b*tan(1/2*d*x + 1/2*c)^2 + 3*a^2*b^3*tan(1/2*d*x + 1/2
*c)^2 - 10*b^5*tan(1/2*d*x + 1/2*c)^2 + 11*a^3*b^2*tan(1/2*d*x + 1/2*c) - 1
4*a*b^4*tan(1/2*d*x + 1/2*c) + 4*a^4*b - 5*a^2*b^3)/((a^6 - a^4*b^2)*(a*tan
(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2) + 6*b*log(abs(tan(1/
2*d*x + 1/2*c)))/a^4 - tan(1/2*d*x + 1/2*c)/a^3 - (6*b*tan(1/2*d*x + 1/2*c)
 - a)/(a^4*tan(1/2*d*x + 1/2*c)))/d
```

maple [B] time = 0.30, size = 729, normalized size = 3.61

$$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^3}-\frac{1}{2d\,a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{3b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^4}-\frac{5b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b+a\right)^2\left(a^2-b^2\cdots\right.}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/2/d/a^3*tan(1/2*d*x+1/2*c)-1/2/d/a^3/tan(1/2*d*x+1/2*c)-3/d/a^4*b*ln(tan(
1/2*d*x+1/2*c))-5/d/a/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b
^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)^3+6/d/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2
*d*x+1/2*c)*b+a)^2*b^4/(a^2-b^2)*tan(1/2*d*x+1/2*c)^3-4/d/(tan(1/2*d*x+1/2*
c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)^2-3/d/a^2
/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b^3/(a^2-b^2)*tan(1/2*
d*x+1/2*c)^2+10/d/a^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b
^5/(a^2-b^2)*tan(1/2*d*x+1/2*c)^2-11/d/a/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*
d*x+1/2*c)*b+a)^2*b^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)+14/d/a^3/(tan(1/2*d*x+1/
2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b^4/(a^2-b^2)*tan(1/2*d*x+1/2*c)-4/d/(
tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b/(a^2-b^2)+5/d/a^2/(tan
(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b^3/(a^2-b^2)-2/d/(a^2-b^2)
```

```
^(3/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+9/d/a^2/(a^
2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^2-6
/d/a^4/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1
/2))*b^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 7.85, size = 1762, normalized size = 8.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + b*sin(c + d*x))^3,x)
```

```
[Out] tan(c/2 + (d*x)/2)/(2*a^3*d) - (a^2 - (2*tan(c/2 + (d*x)/2)*(7*a*b^3 - 6*a^
3*b))/(a^2 - b^2) + (tan(c/2 + (d*x)/2)^4*(a^4 - 12*b^4 + 9*a^2*b^2))/(a^2
- b^2) + (2*tan(c/2 + (d*x)/2)^2*(a^4 - 16*b^4 + 12*a^2*b^2))/(a^2 - b^2) +
 (2*tan(c/2 + (d*x)/2)^3*(6*a^4*b - 10*b^5 + a^2*b^3))/(a*(a^2 - b^2)))/(d*
(2*a^5*tan(c/2 + (d*x)/2)^5 + tan(c/2 + (d*x)/2)^3*(4*a^5 + 8*a^3*b^2) + 2*
a^5*tan(c/2 + (d*x)/2) + 8*a^4*b*tan(c/2 + (d*x)/2)^2 + 8*a^4*b*tan(c/2 + (
d*x)/2)^4)) - (3*b*log(tan(c/2 + (d*x)/2)))/(a^4*d) - (atan((((-(a + b)^3*(
a - b)^3)^(1/2)*(a^4 + 3*b^4 - (9*a^2*b^2)/2)*((2*a^8 + 12*a^4*b^4 - 15*a^6
*b^2)/(a^8 - a^6*b^2) + (tan(c/2 + (d*x)/2)*(10*a^8*b - 24*a^2*b^7 + 60*a^4
*b^5 - 46*a^6*b^3))/(a^9 + a^5*b^4 - 2*a^7*b^2) + (((2*a^10*b - 2*a^8*b^3)/
(a^8 - a^6*b^2) - (tan(c/2 + (d*x)/2)*(6*a^12 - 8*a^6*b^6 + 22*a^8*b^4 - 20
*a^10*b^2))/(a^9 + a^5*b^4 - 2*a^7*b^2))*(-(a + b)^3*(a - b)^3)^(1/2)*(a^4
+ 3*b^4 - (9*a^2*b^2)/2))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2))*1i)/(a^
10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2) + ((-(a + b)^3*(a - b)^3)^(1/2)*(a^4
+ 3*b^4 - (9*a^2*b^2)/2)*((2*a^8 + 12*a^4*b^4 - 15*a^6*b^2)/(a^8 - a^6*b^2)
 + (tan(c/2 + (d*x)/2)*(10*a^8*b - 24*a^2*b^7 + 60*a^4*b^5 - 46*a^6*b^3))/(
a^9 + a^5*b^4 - 2*a^7*b^2) - (((2*a^10*b - 2*a^8*b^3)/(a^8 - a^6*b^2) - (ta
n(c/2 + (d*x)/2)*(6*a^12 - 8*a^6*b^6 + 22*a^8*b^4 - 20*a^10*b^2))/(a^9 + a^
5*b^4 - 2*a^7*b^2))*(-(a + b)^3*(a - b)^3)^(1/2)*(a^4 + 3*b^4 - (9*a^2*b^2)
/2))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2))*1i)/(a^10 - a^4*b^6 + 3*a^6*
b^4 - 3*a^8*b^2))/((2*(6*a^4*b + 18*b^5 - 27*a^2*b^3))/(a^8 - a^6*b^2) + (2
```

```
*tan(c/2 + (d*x)/2)*(4*a^6 - 18*b^6 + 39*a^2*b^4 - 24*a^4*b^2))/(a^9 + a^5*
b^4 - 2*a^7*b^2) - ((-(a + b)^3*(a - b)^3)^(1/2)*(a^4 + 3*b^4 - (9*a^2*b^2)
/2)*((2*a^8 + 12*a^4*b^4 - 15*a^6*b^2)/(a^8 - a^6*b^2) + (tan(c/2 + (d*x)/2
)*(10*a^8*b - 24*a^2*b^7 + 60*a^4*b^5 - 46*a^6*b^3))/(a^9 + a^5*b^4 - 2*a^7
*b^2) + (((2*a^10*b - 2*a^8*b^3)/(a^8 - a^6*b^2) - (tan(c/2 + (d*x)/2)*(6*a
^12 - 8*a^6*b^6 + 22*a^8*b^4 - 20*a^10*b^2))/(a^9 + a^5*b^4 - 2*a^7*b^2))*(
-(a + b)^3*(a - b)^3)^(1/2)*(a^4 + 3*b^4 - (9*a^2*b^2)/2))/(a^10 - a^4*b^6
+ 3*a^6*b^4 - 3*a^8*b^2)))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2) + ((-(a
 + b)^3*(a - b)^3)^(1/2)*(a^4 + 3*b^4 - (9*a^2*b^2)/2)*((2*a^8 + 12*a^4*b^4
 - 15*a^6*b^2)/(a^8 - a^6*b^2) + (tan(c/2 + (d*x)/2)*(10*a^8*b - 24*a^2*b^7
 + 60*a^4*b^5 - 46*a^6*b^3))/(a^9 + a^5*b^4 - 2*a^7*b^2) - (((2*a^10*b - 2*
a^8*b^3)/(a^8 - a^6*b^2) - (tan(c/2 + (d*x)/2)*(6*a^12 - 8*a^6*b^6 + 22*a^8
*b^4 - 20*a^10*b^2))/(a^9 + a^5*b^4 - 2*a^7*b^2))*(-(a + b)^3*(a - b)^3)^(1
/2)*(a^4 + 3*b^4 - (9*a^2*b^2)/2))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)
))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)))*(-(a + b)^3*(a - b)^3)^(1/2)*
(a^4 + 3*b^4 - (9*a^2*b^2)/2)*2i)/(d*(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^
2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)**2/(a + b*sin(c + d*x))**3, x)
```

3.201 $\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=289

$$\frac{(3a^2-5b^2)\cot(c+dx)\csc(c+dx)}{6a^2bd(a+b\sin(c+dx))^2}-\frac{b(9a^2-20b^2)\tanh^{-1}(\cos(c+dx))}{2a^6d}+\frac{(17a^2-60b^2)\cot(c+dx)}{6a^5d}-\frac{(a^2-5b^2)\,c}{}$$

```
[Out] -1/2*b*(9*a^2-20*b^2)*arctanh(cos(d*x+c))/a^6/d+1/6*(17*a^2-60*b^2)*cot(d*x
+c)/a^5/d-(a^2-5*b^2)*cot(d*x+c)*csc(d*x+c)/a^4/b/d+1/6*(3*a^2-5*b^2)*cot(d
*x+c)*csc(d*x+c)/a^2/b/d/(a+b*sin(d*x+c))^2-1/3*cot(d*x+c)*csc(d*x+c)^2/a/d
/(a+b*sin(d*x+c))^2+1/6*(3*a^2-20*b^2)*cot(d*x+c)*csc(d*x+c)/a^3/b/d/(a+b*s
in(d*x+c))+(2*a^4-19*a^2*b^2+20*b^4)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b
^2)^(1/2))/a^6/d/(a^2-b^2)^(1/2)
```

Rubi [A] time = 1.07, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2724, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{(-19a^2b^2+2a^4+20b^4)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^6d\sqrt{a^2-b^2}}+\frac{(17a^2-60b^2)\cot(c+dx)}{6a^5d}-\frac{b(9a^2-20b^2)\tanh^{-1}(\cos(c+dx))}{2a^6d}-$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((2*a^4 - 19*a^2*b^2 + 20*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b
^2]])/(a^6*Sqrt[a^2 - b^2]*d) - (b*(9*a^2 - 20*b^2)*ArcTanh[Cos[c + d*x]])/
(2*a^6*d) + ((17*a^2 - 60*b^2)*Cot[c + d*x])/(6*a^5*d) - ((a^2 - 5*b^2)*Cot
[c + d*x]*Csc[c + d*x])/(a^4*b*d) + ((3*a^2 - 5*b^2)*Cot[c + d*x]*Csc[c + d
*x])/(6*a^2*b*d*(a + b*Sin[c + d*x])^2) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*
a*d*(a + b*Sin[c + d*x])^2) + ((3*a^2 - 20*b^2)*Cot[c + d*x]*Csc[c + d*x])/
(6*a^3*b*d*(a + b*Sin[c + d*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2724

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a*f*Sin[e
 + f*x]^3), x] + (-Dist[1/(3*a^2*b*(m + 1)), Int[((a + b*Sin[e + f*x])^(m +
 1)*Simp[6*a^2 - b^2*(m - 1)*(m - 2) + a*b*(m + 1)*Sin[e + f*x] - (3*a^2 -
b^2*m*(m - 2))*Sin[e + f*x]^2, x])/Sin[e + f*x]^3, x], x] - Simp[((3*a^2 +
b^2*(m - 2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a^2*b*f*(m + 1)*
Sin[e + f*x]^2), x]) /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[
m, -1] && IntegerQ[2*m]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
```

```
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{(3a^2-5b^2)\cot(c+dx)\csc(c+dx)}{6a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^2} + \frac{\int \frac{\csc^3(c+dx)(2(3a^2-10b$$

$$= \frac{(3a^2-5b^2)\cot(c+dx)\csc(c+dx)}{6a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^2} + \frac{(3a^2-20b^2)\cot(c}{6a^3bd(a+b\,s}$$

$$= -\frac{(a^2-5b^2)\cot(c+dx)\csc(c+dx)}{a^4bd} + \frac{(3a^2-5b^2)\cot(c+dx)\csc(c+dx)}{6a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+}{3ad(a+}$$

$$= \frac{(17a^2-60b^2)\cot(c+dx)}{6a^5d} - \frac{(a^2-5b^2)\cot(c+dx)\csc(c+dx)}{a^4bd} + \frac{(3a^2-5b^2)\cot(c+}{6a^2bd(a+b\,si}$$

$$= \frac{(17a^2-60b^2)\cot(c+dx)}{6a^5d} - \frac{(a^2-5b^2)\cot(c+dx)\csc(c+dx)}{a^4bd} + \frac{(3a^2-5b^2)\cot(c+}{6a^2bd(a+b\,si}$$

$$= -\frac{b(9a^2-20b^2)\tanh^{-1}(\cos(c+dx))}{2a^6d} + \frac{(17a^2-60b^2)\cot(c+dx)}{6a^5d} - \frac{(a^2-5b^2)\cot(c}{a}$$

$$= -\frac{b(9a^2-20b^2)\tanh^{-1}(\cos(c+dx))}{2a^6d} + \frac{(17a^2-60b^2)\cot(c+dx)}{6a^5d} - \frac{(a^2-5b^2)\cot(c}{a}$$

$$= \frac{(2a^4-19a^2b^2+20b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^6\sqrt{a^2-b^2}\,d} - \frac{b(9a^2-20b^2)\tanh^{-1}(\cos(c+dx))}{2a^6d}$$

Mathematica [A] time = 6.21, size = 459, normalized size = 1.59

$$\frac{3b\csc^2\left(\frac{1}{2}(c+dx)\right)}{8a^4d} - \frac{3b\sec^2\left(\frac{1}{2}(c+dx)\right)}{8a^4d} - \frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{24a^3d} + \frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)}{24a^3d} + \frac{(9}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((2*a^4 - 19*a^2*b^2 + 20*b^4)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2]
 + a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^6*Sqrt[a^2 - b^2]*d) + ((2*a^2
*Cos[(c + d*x)/2] - 9*b^2*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(3*a^5*d) + (
3*b*Csc[(c + d*x)/2]^2)/(8*a^4*d) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(
24*a^3*d) + ((-9*a^2*b + 20*b^3)*Log[Cos[(c + d*x)/2]])/(2*a^6*d) + ((9*a^2
*b - 20*b^3)*Log[Sin[(c + d*x)/2]])/(2*a^6*d) - (3*b*Sec[(c + d*x)/2]^2)/(8
*a^4*d) + (Sec[(c + d*x)/2]*(-2*a^2*Sin[(c + d*x)/2] + 9*b^2*Sin[(c + d*x)/
2]))/(3*a^5*d) + (a^2*b*Cos[c + d*x] - b^3*Cos[c + d*x])/(2*a^4*d*(a + b*Si
n[c + d*x])^2) + (3*a^2*b*Cos[c + d*x] - 8*b^3*Cos[c + d*x])/(2*a^5*d*(a +
b*Sin[c + d*x])) + (Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24*a^3*d)
```

fricas [B] time = 1.05, size = 2027, normalized size = 7.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/12*(2*(17*a^5*b^2 - 77*a^3*b^4 + 60*a*b^6)*cos(d*x + c)^5 - 4*(4*a^7 + 3
*a^5*b^2 - 67*a^3*b^4 + 60*a*b^6)*cos(d*x + c)^3 - 3*(4*a^5*b - 38*a^3*b^3
+ 40*a*b^5 + 2*(2*a^5*b - 19*a^3*b^3 + 20*a*b^5)*cos(d*x + c)^4 - 4*(2*a^5*
b - 19*a^3*b^3 + 20*a*b^5)*cos(d*x + c)^2 + (2*a^6 - 17*a^4*b^2 + a^2*b^4 +
 20*b^6 + (2*a^4*b^2 - 19*a^2*b^4 + 20*b^6)*cos(d*x + c)^4 - (2*a^6 - 15*a^
4*b^2 - 18*a^2*b^4 + 40*b^6)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)
*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*
cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x
+ c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 6*(2*a^7 - 3*a^5*b^2 - 19*a^3*b
^4 + 20*a*b^6)*cos(d*x + c) - 3*(18*a^5*b^2 - 58*a^3*b^4 + 40*a*b^6 + 2*(9*
a^5*b^2 - 29*a^3*b^4 + 20*a*b^6)*cos(d*x + c)^4 - 4*(9*a^5*b^2 - 29*a^3*b^4
 + 20*a*b^6)*cos(d*x + c)^2 + (9*a^6*b - 20*a^4*b^3 - 9*a^2*b^5 + 20*b^7 +
(9*a^4*b^3 - 29*a^2*b^5 + 20*b^7)*cos(d*x + c)^4 - (9*a^6*b - 11*a^4*b^3 -
38*a^2*b^5 + 40*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1
/2) + 3*(18*a^5*b^2 - 58*a^3*b^4 + 40*a*b^6 + 2*(9*a^5*b^2 - 29*a^3*b^4 + 2
0*a*b^6)*cos(d*x + c)^4 - 4*(9*a^5*b^2 - 29*a^3*b^4 + 20*a*b^6)*cos(d*x + c
)^2 + (9*a^6*b - 20*a^4*b^3 - 9*a^2*b^5 + 20*b^7 + (9*a^4*b^3 - 29*a^2*b^5
+ 20*b^7)*cos(d*x + c)^4 - (9*a^6*b - 11*a^4*b^3 - 38*a^2*b^5 + 40*b^7)*cos
(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 2*(2*(14*a^6*b -
59*a^4*b^3 + 45*a^2*b^5)*cos(d*x + c)^3 - 3*(11*a^6*b - 41*a^4*b^3 + 30*a^2
*b^5)*cos(d*x + c))*sin(d*x + c))/(2*(a^9*b - a^7*b^3)*d*cos(d*x + c)^4 - 4
*(a^9*b - a^7*b^3)*d*cos(d*x + c)^2 + 2*(a^9*b - a^7*b^3)*d + ((a^8*b^2 - a
^6*b^4)*d*cos(d*x + c)^4 - (a^10 + a^8*b^2 - 2*a^6*b^4)*d*cos(d*x + c)^2 +
```

```
(a^10 - a^6*b^4)*d)*sin(d*x + c)), 1/12*(2*(17*a^5*b^2 - 77*a^3*b^4 + 60*a*
b^6)*cos(d*x + c)^5 - 4*(4*a^7 + 3*a^5*b^2 - 67*a^3*b^4 + 60*a*b^6)*cos(d*x
 + c)^3 - 6*(4*a^5*b - 38*a^3*b^3 + 40*a*b^5 + 2*(2*a^5*b - 19*a^3*b^3 + 20
*a*b^5)*cos(d*x + c)^4 - 4*(2*a^5*b - 19*a^3*b^3 + 20*a*b^5)*cos(d*x + c)^2
 + (2*a^6 - 17*a^4*b^2 + a^2*b^4 + 20*b^6 + (2*a^4*b^2 - 19*a^2*b^4 + 20*b^
6)*cos(d*x + c)^4 - (2*a^6 - 15*a^4*b^2 - 18*a^2*b^4 + 40*b^6)*cos(d*x + c)
^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 -
b^2)*cos(d*x + c))) + 6*(2*a^7 - 3*a^5*b^2 - 19*a^3*b^4 + 20*a*b^6)*cos(d*x
 + c) - 3*(18*a^5*b^2 - 58*a^3*b^4 + 40*a*b^6 + 2*(9*a^5*b^2 - 29*a^3*b^4 +
 20*a*b^6)*cos(d*x + c)^4 - 4*(9*a^5*b^2 - 29*a^3*b^4 + 20*a*b^6)*cos(d*x +
 c)^2 + (9*a^6*b - 20*a^4*b^3 - 9*a^2*b^5 + 20*b^7 + (9*a^4*b^3 - 29*a^2*b^
5 + 20*b^7)*cos(d*x + c)^4 - (9*a^6*b - 11*a^4*b^3 - 38*a^2*b^5 + 40*b^7)*c
os(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 3*(18*a^5*b^2 -
58*a^3*b^4 + 40*a*b^6 + 2*(9*a^5*b^2 - 29*a^3*b^4 + 20*a*b^6)*cos(d*x + c)^
4 - 4*(9*a^5*b^2 - 29*a^3*b^4 + 20*a*b^6)*cos(d*x + c)^2 + (9*a^6*b - 20*a^
4*b^3 - 9*a^2*b^5 + 20*b^7 + (9*a^4*b^3 - 29*a^2*b^5 + 20*b^7)*cos(d*x + c)
^4 - (9*a^6*b - 11*a^4*b^3 - 38*a^2*b^5 + 40*b^7)*cos(d*x + c)^2)*sin(d*x +
 c))*log(-1/2*cos(d*x + c) + 1/2) - 2*(2*(14*a^6*b - 59*a^4*b^3 + 45*a^2*b^
5)*cos(d*x + c)^3 - 3*(11*a^6*b - 41*a^4*b^3 + 30*a^2*b^5)*cos(d*x + c))*si
n(d*x + c))/(2*(a^9*b - a^7*b^3)*d*cos(d*x + c)^4 - 4*(a^9*b - a^7*b^3)*d*c
os(d*x + c)^2 + 2*(a^9*b - a^7*b^3)*d + ((a^8*b^2 - a^6*b^4)*d*cos(d*x + c)
^4 - (a^10 + a^8*b^2 - 2*a^6*b^4)*d*cos(d*x + c)^2 + (a^10 - a^6*b^4)*d)*si
n(d*x + c))]
```

giac [A] time = 0.92, size = 451, normalized size = 1.56

$$\frac{12\left(9a^2b-20b^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^6}+\frac{24\left(2a^4-19a^2b^2+20b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^6}+\frac{24\left(5a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-10a\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/24*(12*(9*a^2*b - 20*b^3)*log(abs(tan(1/2*d*x + 1/2*c)))/a^6 + 24*(2*a^4
- 19*a^2*b^2 + 20*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a
*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^6) + 24*(5*
a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 10*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 4*a^4*b*
tan(1/2*d*x + 1/2*c)^2 - a^2*b^3*tan(1/2*d*x + 1/2*c)^2 - 18*b^5*tan(1/2*d*
x + 1/2*c)^2 + 11*a^3*b^2*tan(1/2*d*x + 1/2*c) - 26*a*b^4*tan(1/2*d*x + 1/2
*c) + 4*a^4*b - 9*a^2*b^3)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1
/2*c) + a)^2*a^6) + (a^6*tan(1/2*d*x + 1/2*c)^3 - 9*a^5*b*tan(1/2*d*x + 1/2
*c)^2 - 15*a^6*tan(1/2*d*x + 1/2*c) + 72*a^4*b^2*tan(1/2*d*x + 1/2*c))/a^9
- (198*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 440*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*a
```

```
^3*tan(1/2*d*x + 1/2*c)^2 + 72*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 9*a^2*b*tan(1
/2*d*x + 1/2*c) + a^3)/(a^6*tan(1/2*d*x + 1/2*c)^3))/d
```

maple [B] time = 0.33, size = 780, normalized size = 2.70

$$\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{24d\,a^3}-\frac{3b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^4}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^3}+\frac{3b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^5}-\frac{1}{24d\,a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{5}{8d\,a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/24/d/a^3*tan(1/2*d*x+1/2*c)^3-3/8/d/a^4*b*tan(1/2*d*x+1/2*c)^2-5/8/d/a^3*
tan(1/2*d*x+1/2*c)+3/d/a^5*b^2*tan(1/2*d*x+1/2*c)-1/24/d/a^3/tan(1/2*d*x+1/
2*c)^3+5/8/d/a^3/tan(1/2*d*x+1/2*c)-3/d/a^5/tan(1/2*d*x+1/2*c)*b^2+3/8/d/a^
4*b/tan(1/2*d*x+1/2*c)^2+9/2/d/a^4*b*ln(tan(1/2*d*x+1/2*c))-10/d/a^6*b^3*ln
(tan(1/2*d*x+1/2*c))+5/d/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b
+a)^2*tan(1/2*d*x+1/2*c)^3*b^2-10/d/a^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d
*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3*b^4+4/d/a^2/(tan(1/2*d*x+1/2*c)^2*a+2
*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*b-1/d/a^4/(tan(1/2*d*x+1/2*
c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*b^3-18/d/a^6/(tan(1
/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*b^5+11/d
/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)
*b^2-26/d/a^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d
*x+1/2*c)*b^4+4/d/a^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b
-9/d/a^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b^3+2/d/a^2/(a
^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-19/d
/a^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2
))*b^2+20/d/a^6/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^
2-b^2)^(1/2))*b^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 7.31, size = 1261, normalized size = 4.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4/(a + b*sin(c + d*x))^3,x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(24*a^3*d) - (tan(c/2 + (d*x)/2)*((3*(a^2 + 4*b^2))/(8
*a^5) + 1/(4*a^3) - (9*b^2)/(2*a^5)))/d + (tan(c/2 + (d*x)/2)^6*(5*a^4 - 80
*b^4 + 16*a^2*b^2) + tan(c/2 + (d*x)/2)^4*((29*a^4)/3 - 304*b^4 + 72*a^2*b^
2) - a^4/3 + tan(c/2 + (d*x)/2)^2*((13*a^4)/3 - (40*a^2*b^2)/3) - tan(c/2 +
 (d*x)/2)^3*(156*a*b^3 - (170*a^3*b)/3) - (tan(c/2 + (d*x)/2)^5*(144*b^5 -
55*a^4*b + 104*a^2*b^3))/a + (5*a^3*b*tan(c/2 + (d*x)/2))/3)/(d*(8*a^7*tan(
c/2 + (d*x)/2)^3 + 8*a^7*tan(c/2 + (d*x)/2)^7 + tan(c/2 + (d*x)/2)^5*(16*a^
7 + 32*a^5*b^2) + 32*a^6*b*tan(c/2 + (d*x)/2)^4 + 32*a^6*b*tan(c/2 + (d*x)/
2)^6)) + (log(tan(c/2 + (d*x)/2))*(9*a^2*b - 20*b^3))/(2*a^6*d) - (3*b*tan(
c/2 + (d*x)/2)^2)/(8*a^4*d) + (atan((((-(a + b)*(a - b))^(1/2)*(a^4 + 10*b^
4 - (19*a^2*b^2)/2)*((2*a^10 + 40*a^6*b^4 - 28*a^8*b^2)/a^10 + (tan(c/2 + (
d*x)/2)*(13*a^8*b + 80*a^4*b^5 - 76*a^6*b^3))/a^9 + ((-(a + b)*(a - b))^(1/
2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(6*a^12 - 8*a^10*b^2))/a^9)*(a^4 + 10*b^4
 - (19*a^2*b^2)/2))/(a^8 - a^6*b^2))*1i)/(a^8 - a^6*b^2) + ((-(a + b)*(a -
b))^(1/2)*(a^4 + 10*b^4 - (19*a^2*b^2)/2)*((2*a^10 + 40*a^6*b^4 - 28*a^8*b^
2)/a^10 + (tan(c/2 + (d*x)/2)*(13*a^8*b + 80*a^4*b^5 - 76*a^6*b^3))/a^9 - (
(-(a + b)*(a - b))^(1/2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(6*a^12 - 8*a^10*b^
2))/a^9)*(a^4 + 10*b^4 - (19*a^2*b^2)/2))/(a^8 - a^6*b^2))*1i)/(a^8 - a^6*b
^2))/((18*a^6*b - 400*b^7 + 560*a^2*b^5 - 211*a^4*b^3)/a^10 + (2*tan(c/2 +
(d*x)/2)*(4*a^6 - 200*b^6 + 230*a^2*b^4 - 58*a^4*b^2))/a^9 - ((-(a + b)*(a
- b))^(1/2)*(a^4 + 10*b^4 - (19*a^2*b^2)/2)*((2*a^10 + 40*a^6*b^4 - 28*a^8*
b^2)/a^10 + (tan(c/2 + (d*x)/2)*(13*a^8*b + 80*a^4*b^5 - 76*a^6*b^3))/a^9 +
 ((-(a + b)*(a - b))^(1/2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(6*a^12 - 8*a^10*
b^2))/a^9)*(a^4 + 10*b^4 - (19*a^2*b^2)/2))/(a^8 - a^6*b^2)))/(a^8 - a^6*b^
2) + ((-(a + b)*(a - b))^(1/2)*(a^4 + 10*b^4 - (19*a^2*b^2)/2)*((2*a^10 + 4
0*a^6*b^4 - 28*a^8*b^2)/a^10 + (tan(c/2 + (d*x)/2)*(13*a^8*b + 80*a^4*b^5 -
 76*a^6*b^3))/a^9 - ((-(a + b)*(a - b))^(1/2)*(2*a^2*b - (tan(c/2 + (d*x)/2
)*(6*a^12 - 8*a^10*b^2))/a^9)*(a^4 + 10*b^4 - (19*a^2*b^2)/2))/(a^8 - a^6*b
^2)))/(a^8 - a^6*b^2)))*(-(a + b)*(a - b))^(1/2)*(a^4 + 10*b^4 - (19*a^2*b^
2)/2)*2i)/(d*(a^8 - a^6*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4(c + dx)}{(a + b \sin(c + dx))^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+b*sin(d*x+c))**3,x)
[Out] Integral(cot(c + d*x)**4/(a + b*sin(c + d*x))**3, x)
```

3.202 $\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=492

$$\frac{7b\cot(c+dx)\csc^3(c+dx)}{20a^2d(a+b\sin(c+dx))^2}+\frac{\left(4a^4-54a^2b^2+63b^4\right)\cot(c+dx)\csc^2(c+dx)}{12a^4b^2d(a+b\sin(c+dx))}-\frac{\sqrt{a^2-b^2}\left(2a^4-29a^2b^2+42b^4\right)\tan}{a^8d}$$

```
[Out] 1/8*b*(45*a^4-200*a^2*b^2+168*b^4)*arctanh(cos(d*x+c))/a^8/d-1/30*(91*a^4-6
45*a^2*b^2+630*b^4)*cot(d*x+c)/a^7/d+1/8*(8*a^4-79*a^2*b^2+84*b^4)*cot(d*x+
c)*csc(d*x+c)/a^6/b/d-1/30*(15*a^4-187*a^2*b^2+210*b^4)*cot(d*x+c)*csc(d*x+
c)^2/a^5/b^2/d-1/3*cot(d*x+c)*csc(d*x+c)/b/d/(a+b*sin(d*x+c))^2+1/12*a*cot(
d*x+c)*csc(d*x+c)^2/b^2/d/(a+b*sin(d*x+c))^2+1/60*(5*a^4-60*a^2*b^2+63*b^4)
*cot(d*x+c)*csc(d*x+c)^2/a^3/b^2/d/(a+b*sin(d*x+c))^2+7/20*b*cot(d*x+c)*csc
(d*x+c)^3/a^2/d/(a+b*sin(d*x+c))^2-1/5*cot(d*x+c)*csc(d*x+c)^4/a/d/(a+b*sin
(d*x+c))^2+1/12*(4*a^4-54*a^2*b^2+63*b^4)*cot(d*x+c)*csc(d*x+c)^2/a^4/b^2/d
/(a+b*sin(d*x+c))-(2*a^4-29*a^2*b^2+42*b^4)*arctan((b+a*tan(1/2*d*x+1/2*c))
/(a^2-b^2)^(1/2))*(a^2-b^2)^(1/2)/a^8/d
```

Rubi [A] time = 2.13, antiderivative size = 492, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2726, 3055, 3001, 3770, 2660, 618, 204}

$$-\frac{\sqrt{a^2-b^2}\left(-29a^2b^2+2a^4+42b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^8d}-\frac{\left(-645a^2b^2+91a^4+630b^4\right)\cot(c+dx)}{30a^7d}+\frac{b\left(-200a^2b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -((Sqrt[a^2 - b^2]*(2*a^4 - 29*a^2*b^2 + 42*b^4)*ArcTan[(b + a*Tan[(c + d*x
)/2])/Sqrt[a^2 - b^2]])/(a^8*d)) + (b*(45*a^4 - 200*a^2*b^2 + 168*b^4)*ArcT
anh[Cos[c + d*x]])/(8*a^8*d) - ((91*a^4 - 645*a^2*b^2 + 630*b^4)*Cot[c + d*
x])/(30*a^7*d) + ((8*a^4 - 79*a^2*b^2 + 84*b^4)*Cot[c + d*x]*Csc[c + d*x])/
(8*a^6*b*d) - ((15*a^4 - 187*a^2*b^2 + 210*b^4)*Cot[c + d*x]*Csc[c + d*x]^2
)/(30*a^5*b^2*d) - (Cot[c + d*x]*Csc[c + d*x])/(3*b*d*(a + b*Sin[c + d*x])^
2) + (a*Cot[c + d*x]*Csc[c + d*x]^2)/(12*b^2*d*(a + b*Sin[c + d*x])^2) + ((
5*a^4 - 60*a^2*b^2 + 63*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(60*a^3*b^2*d*(a
+ b*Sin[c + d*x])^2) + (7*b*Cot[c + d*x]*Csc[c + d*x]^3)/(20*a^2*d*(a + b*S
in[c + d*x])^2) - (Cot[c + d*x]*Csc[c + d*x]^4)/(5*a*d*(a + b*Sin[c + d*x])
^2) + ((4*a^4 - 54*a^2*b^2 + 63*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(12*a^4*b
^2*d*(a + b*Sin[c + d*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^6,
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(5*a*f*Sin[e
 + f*x]^5), x] + (Dist[1/(20*a^2*b^2*m*(m - 1)), Int[((a + b*Sin[e + f*x])^
m*Simp[60*a^4 - 44*a^2*b^2*(m - 1)*m + b^4*m*(m - 1)*(m - 3)*(m - 4) + a*b*
m*(20*a^2 - b^2*m*(m - 1))*Sin[e + f*x] - (40*a^4 + b^4*m*(m - 1)*(m - 2)*(
m - 4) - 20*a^2*b^2*(m - 1)*(2*m + 1))*Sin[e + f*x]^2, x])/Sin[e + f*x]^4,
x], x] + Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*m*Sin[e + f*
x]^2), x] + Simp[(a*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*m*(m
- 1)*Sin[e + f*x]^3), x] - Simp[(b*(m - 4)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m + 1))/(20*a^2*f*Sin[e + f*x]^4), x]) /; FreeQ[{a, b, e, f, m}, x] && N
eQ[a^2 - b^2, 0] && NeQ[m, 1] && IntegerQ[2*m]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2} + \frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2} + \frac{7b\cot(c+dx)\csc^3(c+dx}{20a^2d(a+b\sin(c+dx))^2}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2} + \frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2} + \frac{\left(5a^4-60a^2b^2+63b^4\right)\cot}{60a^3b^2d(a+b\sin}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2} + \frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2} + \frac{\left(5a^4-60a^2b^2+63b^4\right)\cot}{60a^3b^2d(a+b\sin}$$

$$= -\frac{\left(15a^4-187a^2b^2+210b^4\right)\cot(c+dx)\csc^2(c+dx)}{30a^5b^2d} - \frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2} + \frac{a}{1}$$

$$= \frac{\left(8a^4-79a^2b^2+84b^4\right)\cot(c+dx)\csc(c+dx)}{8a^6bd} - \frac{\left(15a^4-187a^2b^2+210b^4\right)\cot(c+}{30a^5b^2d}$$

$$= -\frac{\left(91a^4-645a^2b^2+630b^4\right)\cot(c+dx)}{30a^7d} + \frac{\left(8a^4-79a^2b^2+84b^4\right)\cot(c+dx)\csc(c}{8a^6bd}$$

$$= -\frac{\left(91a^4-645a^2b^2+630b^4\right)\cot(c+dx)}{30a^7d} + \frac{\left(8a^4-79a^2b^2+84b^4\right)\cot(c+dx)\csc(c}{8a^6bd}$$

$$= \frac{b\left(45a^4-200a^2b^2+168b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^8d} - \frac{\left(91a^4-645a^2b^2+630b^4\right)\cot(c}{30a^7d}$$

$$= \frac{b\left(45a^4-200a^2b^2+168b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^8d} - \frac{\left(91a^4-645a^2b^2+630b^4\right)\cot(c}{30a^7d}$$

$$= -\frac{\sqrt{a^2-b^2}\left(2a^4-29a^2b^2+42b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^8d} + \frac{b\left(45a^4-200a^2b^2+16\right.}{8}$$

Mathematica [A] time = 1.76, size = 448, normalized size = 0.91

$$\frac{-480b\left(45a^4-200a^2b^2+168b^4\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+480b\left(45a^4-200a^2b^2+168b^4\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-3840*(2*a^6 - 31*a^4*b^2 + 71*a^2*b^4 - 42*b^6)*ArcTan[(b + a*Tan[(c + d
*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + 480*b*(45*a^4 - 200*a^2*b^2 + 1
68*b^4)*Log[Cos[(c + d*x)/2]] - 480*b*(45*a^4 - 200*a^2*b^2 + 168*b^4)*Log[
Sin[(c + d*x)/2]] + (2*a*Cot[c + d*x]*Csc[c + d*x]^6*(-784*a^6 + 3256*a^4*b
^2 + 7860*a^2*b^4 - 12600*b^6 + 2*(384*a^6 - 2131*a^4*b^2 - 6315*a^2*b^4 +
9450*b^6)*Cos[2*(c + d*x)] + (-368*a^6 + 824*a^4*b^2 + 6060*a^2*b^4 - 7560*
b^6)*Cos[4*(c + d*x)] + 182*a^4*b^2*Cos[6*(c + d*x)] - 1290*a^2*b^4*Cos[6*(
c + d*x)] + 1260*b^6*Cos[6*(c + d*x)] - 8156*a^5*b*Sin[c + d*x] + 42270*a^3
*b^3*Sin[c + d*x] - 37800*a*b^5*Sin[c + d*x] + 3956*a^5*b*Sin[3*(c + d*x)]
- 20715*a^3*b^3*Sin[3*(c + d*x)] + 18900*a*b^5*Sin[3*(c + d*x)] - 608*a^5*b
*Sin[5*(c + d*x)] + 3975*a^3*b^3*Sin[5*(c + d*x)] - 3780*a*b^5*Sin[5*(c + d
*x)]))/(b + a*Csc[c + d*x])^2)/(3840*a^8*d)
```

fricas [B] time = 1.18, size = 2571, normalized size = 5.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/240*(8*(91*a^5*b^2 - 645*a^3*b^4 + 630*a*b^6)*cos(d*x + c)^7 - 4*(92*a^
7 + 67*a^5*b^2 - 3450*a^3*b^4 + 3780*a*b^6)*cos(d*x + c)^5 + 40*(14*a^7 - 3
7*a^5*b^2 - 303*a^3*b^4 + 378*a*b^6)*cos(d*x + c)^3 - 60*(2*(2*a^5*b - 29*a
^3*b^3 + 42*a*b^5)*cos(d*x + c)^6 - 4*a^5*b + 58*a^3*b^3 - 84*a*b^5 - 6*(2*
a^5*b - 29*a^3*b^3 + 42*a*b^5)*cos(d*x + c)^4 + 6*(2*a^5*b - 29*a^3*b^3 + 4
2*a*b^5)*cos(d*x + c)^2 + ((2*a^4*b^2 - 29*a^2*b^4 + 42*b^6)*cos(d*x + c)^6
 - 2*a^6 + 27*a^4*b^2 - 13*a^2*b^4 - 42*b^6 - (2*a^6 - 23*a^4*b^2 - 45*a^2*
b^4 + 126*b^6)*cos(d*x + c)^4 + (4*a^6 - 52*a^4*b^2 - 3*a^2*b^4 + 126*b^6)*
cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x +
 c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b
*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) -
 a^2 - b^2)) - 60*(4*a^7 - 17*a^5*b^2 - 58*a^3*b^4 + 84*a*b^6)*cos(d*x + c)
 + 15*(90*a^5*b^2 - 400*a^3*b^4 + 336*a*b^6 - 2*(45*a^5*b^2 - 200*a^3*b^4 +
 168*a*b^6)*cos(d*x + c)^6 + 6*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d
*x + c)^4 - 6*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^2 + (45*a
^6*b - 155*a^4*b^3 - 32*a^2*b^5 + 168*b^7 - (45*a^4*b^3 - 200*a^2*b^5 + 168
*b^7)*cos(d*x + c)^6 + (45*a^6*b - 65*a^4*b^3 - 432*a^2*b^5 + 504*b^7)*cos(
d*x + c)^4 - (90*a^6*b - 265*a^4*b^3 - 264*a^2*b^5 + 504*b^7)*cos(d*x + c)^
2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 15*(90*a^5*b^2 - 400*a^3*b^4
 + 336*a*b^6 - 2*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^6 + 6*
(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^4 - 6*(45*a^5*b^2 - 200
```

```
*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^2 + (45*a^6*b - 155*a^4*b^3 - 32*a^2*b^5
 + 168*b^7 - (45*a^4*b^3 - 200*a^2*b^5 + 168*b^7)*cos(d*x + c)^6 + (45*a^6*
b - 65*a^4*b^3 - 432*a^2*b^5 + 504*b^7)*cos(d*x + c)^4 - (90*a^6*b - 265*a^
4*b^3 - 264*a^2*b^5 + 504*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d
*x + c) + 1/2) - 2*((608*a^6*b - 3975*a^4*b^3 + 3780*a^2*b^5)*cos(d*x + c)^
5 - 5*(289*a^6*b - 1632*a^4*b^3 + 1512*a^2*b^5)*cos(d*x + c)^3 + 15*(53*a^6
*b - 279*a^4*b^3 + 252*a^2*b^5)*cos(d*x + c))*sin(d*x + c))/(2*a^9*b*d*cos(
d*x + c)^6 - 6*a^9*b*d*cos(d*x + c)^4 + 6*a^9*b*d*cos(d*x + c)^2 - 2*a^9*b*
d + (a^8*b^2*d*cos(d*x + c)^6 - (a^10 + 3*a^8*b^2)*d*cos(d*x + c)^4 + (2*a^
10 + 3*a^8*b^2)*d*cos(d*x + c)^2 - (a^10 + a^8*b^2)*d)*sin(d*x + c)), -1/24
0*(8*(91*a^5*b^2 - 645*a^3*b^4 + 630*a*b^6)*cos(d*x + c)^7 - 4*(92*a^7 + 67
*a^5*b^2 - 3450*a^3*b^4 + 3780*a*b^6)*cos(d*x + c)^5 + 40*(14*a^7 - 37*a^5*
b^2 - 303*a^3*b^4 + 378*a*b^6)*cos(d*x + c)^3 - 120*(2*(2*a^5*b - 29*a^3*b^
3 + 42*a*b^5)*cos(d*x + c)^6 - 4*a^5*b + 58*a^3*b^3 - 84*a*b^5 - 6*(2*a^5*b
 - 29*a^3*b^3 + 42*a*b^5)*cos(d*x + c)^4 + 6*(2*a^5*b - 29*a^3*b^3 + 42*a*b
^5)*cos(d*x + c)^2 + ((2*a^4*b^2 - 29*a^2*b^4 + 42*b^6)*cos(d*x + c)^6 - 2*
a^6 + 27*a^4*b^2 - 13*a^2*b^4 - 42*b^6 - (2*a^6 - 23*a^4*b^2 - 45*a^2*b^4 +
 126*b^6)*cos(d*x + c)^4 + (4*a^6 - 52*a^4*b^2 - 3*a^2*b^4 + 126*b^6)*cos(d
*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt
(a^2 - b^2)*cos(d*x + c))) - 60*(4*a^7 - 17*a^5*b^2 - 58*a^3*b^4 + 84*a*b^6
)*cos(d*x + c) + 15*(90*a^5*b^2 - 400*a^3*b^4 + 336*a*b^6 - 2*(45*a^5*b^2 -
 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^6 + 6*(45*a^5*b^2 - 200*a^3*b^4 + 16
8*a*b^6)*cos(d*x + c)^4 - 6*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x
+ c)^2 + (45*a^6*b - 155*a^4*b^3 - 32*a^2*b^5 + 168*b^7 - (45*a^4*b^3 - 200
*a^2*b^5 + 168*b^7)*cos(d*x + c)^6 + (45*a^6*b - 65*a^4*b^3 - 432*a^2*b^5 +
 504*b^7)*cos(d*x + c)^4 - (90*a^6*b - 265*a^4*b^3 - 264*a^2*b^5 + 504*b^7)
*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 15*(90*a^5*b^2
 - 400*a^3*b^4 + 336*a*b^6 - 2*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d
*x + c)^6 + 6*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^4 - 6*(45
*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^2 + (45*a^6*b - 155*a^4*b^
3 - 32*a^2*b^5 + 168*b^7 - (45*a^4*b^3 - 200*a^2*b^5 + 168*b^7)*cos(d*x + c
)^6 + (45*a^6*b - 65*a^4*b^3 - 432*a^2*b^5 + 504*b^7)*cos(d*x + c)^4 - (90*
a^6*b - 265*a^4*b^3 - 264*a^2*b^5 + 504*b^7)*cos(d*x + c)^2)*sin(d*x + c))*
log(-1/2*cos(d*x + c) + 1/2) - 2*((608*a^6*b - 3975*a^4*b^3 + 3780*a^2*b^5)
*cos(d*x + c)^5 - 5*(289*a^6*b - 1632*a^4*b^3 + 1512*a^2*b^5)*cos(d*x + c)^
3 + 15*(53*a^6*b - 279*a^4*b^3 + 252*a^2*b^5)*cos(d*x + c))*sin(d*x + c))/(
2*a^9*b*d*cos(d*x + c)^6 - 6*a^9*b*d*cos(d*x + c)^4 + 6*a^9*b*d*cos(d*x + c
)^2 - 2*a^9*b*d + (a^8*b^2*d*cos(d*x + c)^6 - (a^10 + 3*a^8*b^2)*d*cos(d*x
+ c)^4 + (2*a^10 + 3*a^8*b^2)*d*cos(d*x + c)^2 - (a^10 + a^8*b^2)*d)*sin(d*
x + c))]
```

giac [A] time = 1.52, size = 731, normalized size = 1.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/960*(120*(45*a^4*b - 200*a^2*b^3 + 168*b^5)*log(abs(tan(1/2*d*x + 1/2*c)
))/a^8 + 960*(2*a^6 - 31*a^4*b^2 + 71*a^2*b^4 - 42*b^6)*(pi*floor(1/2*(d*x
+ c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)
))/(sqrt(a^2 - b^2)*a^8) + 960*(5*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 - 19*a^3*b
^4*tan(1/2*d*x + 1/2*c)^3 + 14*a*b^6*tan(1/2*d*x + 1/2*c)^3 + 4*a^6*b*tan(1
/2*d*x + 1/2*c)^2 - 9*a^4*b^3*tan(1/2*d*x + 1/2*c)^2 - 21*a^2*b^5*tan(1/2*d
*x + 1/2*c)^2 + 26*b^7*tan(1/2*d*x + 1/2*c)^2 + 11*a^5*b^2*tan(1/2*d*x + 1/
2*c) - 49*a^3*b^4*tan(1/2*d*x + 1/2*c) + 38*a*b^6*tan(1/2*d*x + 1/2*c) + 4*
a^6*b - 17*a^4*b^3 + 13*a^2*b^5)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d
*x + 1/2*c) + a)^2*a^8) - (12330*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 54800*a^2*b
^3*tan(1/2*d*x + 1/2*c)^5 + 46032*b^5*tan(1/2*d*x + 1/2*c)^5 - 660*a^5*tan(
1/2*d*x + 1/2*c)^4 + 6480*a^3*b^2*tan(1/2*d*x + 1/2*c)^4 - 7200*a*b^4*tan(1
/2*d*x + 1/2*c)^4 - 720*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 1200*a^2*b^3*tan(1/2
*d*x + 1/2*c)^3 + 70*a^5*tan(1/2*d*x + 1/2*c)^2 - 240*a^3*b^2*tan(1/2*d*x +
 1/2*c)^2 + 45*a^4*b*tan(1/2*d*x + 1/2*c) - 6*a^5)/(a^8*tan(1/2*d*x + 1/2*c
)^5) - (6*a^12*tan(1/2*d*x + 1/2*c)^5 - 45*a^11*b*tan(1/2*d*x + 1/2*c)^4 -
70*a^12*tan(1/2*d*x + 1/2*c)^3 + 240*a^10*b^2*tan(1/2*d*x + 1/2*c)^3 + 720*
a^11*b*tan(1/2*d*x + 1/2*c)^2 - 1200*a^9*b^3*tan(1/2*d*x + 1/2*c)^2 + 660*a
^12*tan(1/2*d*x + 1/2*c) - 6480*a^10*b^2*tan(1/2*d*x + 1/2*c) + 7200*a^8*b^
4*tan(1/2*d*x + 1/2*c))/a^15)/d
```

maple [B] time = 0.41, size = 1252, normalized size = 2.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6/(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/160/d/a^3*tan(1/2*d*x+1/2*c)^5-1/160/d/a^3/tan(1/2*d*x+1/2*c)^5-45/8/d/a^
4*b*ln(tan(1/2*d*x+1/2*c))+11/16/d/a^3*tan(1/2*d*x+1/2*c)-11/16/d/a^3/tan(1
/2*d*x+1/2*c)+3/4/d/a^4*b*tan(1/2*d*x+1/2*c)^2-27/4/d/a^5*b^2*tan(1/2*d*x+1
/2*c)+27/4/d/a^5/tan(1/2*d*x+1/2*c)*b^2-3/4/d/a^4*b/tan(1/2*d*x+1/2*c)^2+25
/d/a^6*b^3*ln(tan(1/2*d*x+1/2*c))-4/d/a^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2
*d*x+1/2*c)*b+a)^2*b+17/d/a^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*
b+a)^2*b^3-2/d/a^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/
(a^2-b^2)^(1/2))-13/d/a^6/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)
^2*b^5+3/64/d/a^4*b/tan(1/2*d*x+1/2*c)^4+5/4/d/a^6*b^3/tan(1/2*d*x+1/2*c)^2
-21/d/a^8*b^5*ln(tan(1/2*d*x+1/2*c))-3/64/d/a^4*b*tan(1/2*d*x+1/2*c)^4+1/4/
d/a^5*b^2*tan(1/2*d*x+1/2*c)^3-5/4/d/a^6*tan(1/2*d*x+1/2*c)^2*b^3+15/2/d/a^
7*b^4*tan(1/2*d*x+1/2*c)-1/4/d/a^5/tan(1/2*d*x+1/2*c)^3*b^2-15/2/d/a^7/tan(
1/2*d*x+1/2*c)*b^4-26/d/a^8/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+
a)^2*tan(1/2*d*x+1/2*c)^2*b^7-38/d/a^7/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*
x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)*b^6+42/d/a^8/(a^2-b^2)^(1/2)*arctan(1/2*
```

```
(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^6-14/d/a^7/(tan(1/2*d*x+1/2
*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3*b^6-7/96/d/a^3*tan
(1/2*d*x+1/2*c)^3+7/96/d/a^3/tan(1/2*d*x+1/2*c)^3+31/d/a^4/(a^2-b^2)^(1/2)*
arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^2-71/d/a^6/(a^2-
b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^4+19/
d/a^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c
)^3*b^4-4/d/a^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2
*d*x+1/2*c)^2*b+9/d/a^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2
*tan(1/2*d*x+1/2*c)^2*b^3+21/d/a^6/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/
2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*b^5-11/d/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan
(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)*b^2+49/d/a^5/(tan(1/2*d*x+1/2*c)^
2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)*b^4-5/d/a^3/(tan(1/2*d*x
+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 7.30, size = 1614, normalized size = 3.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6/(a + b*sin(c + d*x))^3,x)
```

```
[Out] tan(c/2 + (d*x)/2)^5/(160*a^3*d) - (tan(c/2 + (d*x)/2)^3*((a^2 + 4*b^2)/(32
*a^5) + 1/(24*a^3) - (3*b^2)/(8*a^5)))/d + (tan(c/2 + (d*x)/2)*(1/(8*a^3) -
 (3*(a^2 + 4*b^2))/(32*a^5) - (6*b*((6*b*((3*(a^2 + 4*b^2))/(32*a^5) + 1/(8
*a^3) - (9*b^2)/(8*a^5)))/a - (384*a^2*b + 256*b^3)/(1024*a^6) + (9*b*(a^2
+ 4*b^2))/(16*a^6)))/a + (3*(a^2 + 4*b^2)*((3*(a^2 + 4*b^2))/(32*a^5) + 1/(
8*a^3) - (9*b^2)/(8*a^5)))/a^2 + (3*b*(384*a^2*b + 256*b^3))/(512*a^7)))/d
- (tan(c/2 + (d*x)/2)^3*((187*a^5*b)/15 - 14*a^3*b^3) + a^6/5 + tan(c/2 + (
d*x)/2)^4*((263*a^6)/15 + 112*a^2*b^4 - (358*a^4*b^2)/3) + tan(c/2 + (d*x)/
2)^5*(1216*a*b^5 + (1519*a^5*b)/6 - 1360*a^3*b^3) - tan(c/2 + (d*x)/2)^2*((
29*a^6)/15 - (14*a^4*b^2)/5) + tan(c/2 + (d*x)/2)^8*(22*a^6 + 448*b^6 - 368
*a^2*b^4 - 56*a^4*b^2) + tan(c/2 + (d*x)/2)^6*((125*a^6)/3 + 2176*b^6 - 211
2*a^2*b^4 + 112*a^4*b^2) + (8*tan(c/2 + (d*x)/2)^7*(30*a^6*b + 104*b^7 + 36
```

```
*a^2*b^5 - 149*a^4*b^3))/a - (7*a^5*b*tan(c/2 + (d*x)/2))/10)/(d*(32*a^9*ta
n(c/2 + (d*x)/2)^5 + 32*a^9*tan(c/2 + (d*x)/2)^9 + tan(c/2 + (d*x)/2)^7*(64
*a^9 + 128*a^7*b^2) + 128*a^8*b*tan(c/2 + (d*x)/2)^6 + 128*a^8*b*tan(c/2 +
(d*x)/2)^8)) + (tan(c/2 + (d*x)/2)^2*((3*b*((3*(a^2 + 4*b^2))/(32*a^5) + 1/
(8*a^3) - (9*b^2)/(8*a^5)))/a - (384*a^2*b + 256*b^3)/(2048*a^6) + (9*b*(a^
2 + 4*b^2))/(32*a^6)))/d - (log(tan(c/2 + (d*x)/2))*(45*a^4*b + 168*b^5 - 2
00*a^2*b^3))/(8*a^8*d) - (3*b*tan(c/2 + (d*x)/2)^4)/(64*a^4*d) - (atan((((-
(a + b)*(a - b))^(1/2)*(a^4 + 21*b^4 - (29*a^2*b^2)/2)*((2*a^14 - 84*a^8*b^
6 + 121*a^10*b^4 - (169*a^12*b^2)/4)/a^14 + (tan(c/2 + (d*x)/2)*(61*a^12*b
- 672*a^6*b^7 + 1136*a^8*b^5 - 538*a^10*b^3))/(4*a^13) + ((-(a + b)*(a - b)
)^(1/2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^16 - 32*a^14*b^2))/(4*a^13))*(
a^4 + 21*b^4 - (29*a^2*b^2)/2))/a^8)*1i)/a^8 + ((-(a + b)*(a - b))^(1/2)*(a
^4 + 21*b^4 - (29*a^2*b^2)/2)*((2*a^14 - 84*a^8*b^6 + 121*a^10*b^4 - (169*a
^12*b^2)/4)/a^14 + (tan(c/2 + (d*x)/2)*(61*a^12*b - 672*a^6*b^7 + 1136*a^8*
b^5 - 538*a^10*b^3))/(4*a^13) - ((-(a + b)*(a - b))^(1/2)*(2*a^2*b - (tan(c
/2 + (d*x)/2)*(24*a^16 - 32*a^14*b^2))/(4*a^13))*(a^4 + 21*b^4 - (29*a^2*b^
2)/2))/a^8)*1i)/a^8)/(((45*a^10*b)/2 - 1764*b^11 + 5082*a^2*b^9 - (10649*a^
4*b^7)/2 + (9731*a^6*b^5)/4 - (1795*a^8*b^3)/4)/a^14 + (tan(c/2 + (d*x)/2)*
(16*a^10 - 3528*b^10 + 9282*a^2*b^8 - 8549*a^4*b^6 + 3185*a^6*b^4 - 406*a^8
*b^2))/(2*a^13) - ((-(a + b)*(a - b))^(1/2)*(a^4 + 21*b^4 - (29*a^2*b^2)/2)
*((2*a^14 - 84*a^8*b^6 + 121*a^10*b^4 - (169*a^12*b^2)/4)/a^14 + (tan(c/2 +
 (d*x)/2)*(61*a^12*b - 672*a^6*b^7 + 1136*a^8*b^5 - 538*a^10*b^3))/(4*a^13)
 + ((-(a + b)*(a - b))^(1/2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^16 - 32*a
^14*b^2))/(4*a^13))*(a^4 + 21*b^4 - (29*a^2*b^2)/2))/a^8))/a^8 + ((-(a + b)
*(a - b))^(1/2)*(a^4 + 21*b^4 - (29*a^2*b^2)/2)*((2*a^14 - 84*a^8*b^6 + 121
*a^10*b^4 - (169*a^12*b^2)/4)/a^14 + (tan(c/2 + (d*x)/2)*(61*a^12*b - 672*a
^6*b^7 + 1136*a^8*b^5 - 538*a^10*b^3))/(4*a^13) - ((-(a + b)*(a - b))^(1/2)
*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^16 - 32*a^14*b^2))/(4*a^13))*(a^4 + 2
1*b^4 - (29*a^2*b^2)/2))/a^8))/a^8))*(-(a + b)*(a - b))^(1/2)*(a^4 + 21*b^4
 - (29*a^2*b^2)/2)*2i)/(a^8*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)**6/(a + b*sin(c + d*x))**3, x)
```

3.203 $\int (a+b\sin(e+fx))^3 (g\tan(e+fx))^p \, dx$

Optimal. Leaf size=271

$$\frac{a^3(g\tan(e+fx))^{p+1}\,{}_2F_1\left(1,\frac{p+1}{2};\frac{p+3}{2};-\tan^2(e+fx)\right)}{fg(p+1)}+\frac{3a^2b\sin(e+fx)\cos^2(e+fx)^{\frac{p+1}{2}}(g\tan(e+fx))^{p+1}\,{}_2F_1\left(\right.}{fg(p+2)}$$

```
[Out] a^3*hypergeom([1, 1/2+1/2*p],[3/2+1/2*p],-tan(f*x+e)^2)*(g*tan(f*x+e))^(1+p
)/f/g/(1+p)+3*a^2*b*(cos(f*x+e)^2)^(1/2+1/2*p)*hypergeom([1+1/2*p, 1/2+1/2*
p],[2+1/2*p],sin(f*x+e)^2)*sin(f*x+e)*(g*tan(f*x+e))^(1+p)/f/g/(2+p)+b^3*(c
os(f*x+e)^2)^(1/2+1/2*p)*hypergeom([2+1/2*p, 1/2+1/2*p],[3+1/2*p],sin(f*x+e
)^2)*sin(f*x+e)^3*(g*tan(f*x+e))^(1+p)/f/g/(4+p)+3*a*b^2*hypergeom([2, 3/2+
1/2*p],[5/2+1/2*p],-tan(f*x+e)^2)*(g*tan(f*x+e))^(3+p)/f/g^3/(3+p)
```

Rubi [A] time = 0.38, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2722, 3476, 364, 2602, 2577, 2591}

$$\frac{3a^2b\sin(e+fx)\cos^2(e+fx)^{\frac{p+1}{2}}(g\tan(e+fx))^{p+1}\,{}_2F_1\left(\frac{p+1}{2},\frac{p+2}{2};\frac{p+4}{2};\sin^2(e+fx)\right)}{fg(p+2)}+\frac{a^3(g\tan(e+fx))^{p+1}\,{}_2F_1\left(\right.}{fg(p}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3*(g*Tan[e + f*x])^p,x]
```

```
[Out] (a^3*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e +
 f*x])^(1 + p))/(f*g*(1 + p)) + (3*a^2*b*(Cos[e + f*x]^2)^((1 + p)/2)*Hyper
geometric2F1[(1 + p)/2, (2 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*
(g*Tan[e + f*x])^(1 + p))/(f*g*(2 + p)) + (b^3*(Cos[e + f*x]^2)^((1 + p)/2)
*Hypergeometric2F1[(1 + p)/2, (4 + p)/2, (6 + p)/2, Sin[e + f*x]^2]*Sin[e +
 f*x]^3*(g*Tan[e + f*x])^(1 + p))/(f*g*(4 + p)) + (3*a*b^2*Hypergeometric2F
1[2, (3 + p)/2, (5 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e + f*x])^(3 + p))/(f*g^
3*(3 + p))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
```

```
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int (a+b\sin(e+fx))^3(g\tan(e+fx))^p\,dx = \int \left(a^3(g\tan(e+fx))^p + 3a^2b\sin(e+fx)(g\tan(e+fx))^p + 3ab^2\sin\right.$$

$$= a^3\int (g\tan(e+fx))^p\,dx + \left(3a^2b\right)\int \sin(e+fx)(g\tan(e+fx))^p\,dx$$

$$= \frac{\left(a^3g\right)\text{Subst}\left(\int \frac{x^p}{g^2+x^2}\,dx, x, g\tan(e+fx)\right)}{f} + \frac{\left(3ab^2g\right)\text{Subst}\left(\int \frac{x^{2+}}{(g^2+x}\right.}{}$$

$$= \frac{a^3\,{}_2F_1\left(1,\frac{1+p}{2};\frac{3+p}{2};-\tan^2(e+fx)\right)(g\tan(e+fx))^{1+p}}{fg(1+p)} + \frac{3a^2b\cos^2(e}{}$$

Mathematica [C] time = 16.10, size = 4791, normalized size = 17.68

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])^3*(g*Tan[e + f*x])^p,x]
```

```
[Out] (2*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^p*Tan[(e + f*x)/2]*(a^3*(2 + p)*Appell
F1[(1 + p)/2, p, 1, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2
*b*(6*a*b*(2 + p)*AppellF1[(1 + p)/2, p, 2, (3 + p)/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2] - 6*a*b*(2 + p)*AppellF1[(1 + p)/2, p, 3, (3 + p)/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 + p)*(3*a^2*AppellF1[1 + p/2,
p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*b^2*(AppellF1[1
 + p/2, p, 3, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - AppellF1[
1 + p/2, p, 4, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]))*Tan[(e +
 f*x)/2]))*(g*Tan[e + f*x])^p*(-1/8*(b^3*Sin[3*(e + f*x)]*Tan[e + f*x]^p) -
 a^3*Sin[e + f*x]^3*Sin[3*(e + f*x)]*Tan[e + f*x]^p + ((3*I)/8)*b^3*Sin[2*(
e + f*x)]*Sin[3*(e + f*x)]*Tan[e + f*x]^p + (3*b^3*Sin[2*(e + f*x)]^2*Sin[3
*(e + f*x)]*Tan[e + f*x]^p)/8 - (I/8)*b^3*Sin[2*(e + f*x)]^3*Sin[3*(e + f*x
)]*Tan[e + f*x]^p + Cos[e + f*x]^3*(a^3*Cos[3*(e + f*x)]*Tan[e + f*x]^p - I
*a^3*Sin[3*(e + f*x)]*Tan[e + f*x]^p) + Cos[2*(e + f*x)]^3*((I/8)*b^3*Cos[3
*(e + f*x)]*Tan[e + f*x]^p + (b^3*Sin[3*(e + f*x)]*Tan[e + f*x]^p)/8) + Sin
[e + f*x]^2*((-3*a^2*b*Sin[3*(e + f*x)]*Tan[e + f*x]^p)/2 + ((3*I)/2)*a^2*b
*Sin[2*(e + f*x)]*Sin[3*(e + f*x)]*Tan[e + f*x]^p) + Sin[e + f*x]*((-3*a*b^
2*Sin[3*(e + f*x)]*Tan[e + f*x]^p)/4 + ((3*I)/2)*a*b^2*Sin[2*(e + f*x)]*Sin
[3*(e + f*x)]*Tan[e + f*x]^p + (3*a*b^2*Sin[2*(e + f*x)]^2*Sin[3*(e + f*x)]
*Tan[e + f*x]^p)/4) + Cos[2*(e + f*x)]^2*((-3*b^3*Sin[3*(e + f*x)]*Tan[e +
f*x]^p)/8 - (3*a*b^2*Sin[e + f*x]*Sin[3*(e + f*x)]*Tan[e + f*x]^p)/4 + ((3*
I)/8)*b^3*Sin[2*(e + f*x)]*Sin[3*(e + f*x)]*Tan[e + f*x]^p + Cos[3*(e + f*x
)]*(((-3*I)/8)*b^3*Tan[e + f*x]^p - ((3*I)/4)*a*b^2*Sin[e + f*x]*Tan[e + f*
```

x]^p - (3*b^3*Sin[2*(e + f*x)]*Tan[e + f*x]^p)/8)) + Cos[3*(e + f*x)]*((-1/8*I)*b^3*Tan[e + f*x]^p - I*a^3*Sin[e + f*x]^3*Tan[e + f*x]^p - (3*b^3*Sin[2*(e + f*x)]*Tan[e + f*x]^p)/8 + ((3*I)/8)*b^3*Sin[2*(e + f*x)]^2*Tan[e + f*x]^p + (b^3*Sin[2*(e + f*x)]^3*Tan[e + f*x]^p)/8 + Sin[e + f*x]^2*(((-3*I)/2)*a^2*b*Tan[e + f*x]^p - (3*a^2*b*Sin[2*(e + f*x)]*Tan[e + f*x]^p)/2) + Sin[e + f*x]*(((-3*I)/4)*a*b^2*Tan[e + f*x]^p - (3*a*b^2*Sin[2*(e + f*x)]*Tan[e + f*x]^p)/2 + ((3*I)/4)*a*b^2*Sin[2*(e + f*x)]^2*Tan[e + f*x]^p)) + Cos[e + f*x]^2*((3*a^2*b*Sin[3*(e + f*x)]*Tan[e + f*x]^p)/2 + 3*a^3*Sin[e + f*x]*Sin[3*(e + f*x)]*Tan[e + f*x]^p - ((3*I)/2)*a^2*b*Sin[2*(e + f*x)]*Sin[3*(e + f*x)]*Tan[e + f*x]^p + Cos[3*(e + f*x)]*(((3*I)/2)*a^2*b*Tan[e + f*x]^p + (3*I)*a^3*Sin[e + f*x]*Tan[e + f*x]^p + (3*a^2*b*Sin[2*(e + f*x)]*Tan[e + f*x]^p)/2) + Cos[2*(e + f*x)]*(((-3*I)/2)*a^2*b*Cos[3*(e + f*x)]*Tan[e + f*x]^p - (3*a^2*b*Sin[3*(e + f*x)]*Tan[e + f*x]^p)/2)) + Cos[e + f*x]*(((3*I)/4)*a*b^2*Sin[3*(e + f*x)]*Tan[e + f*x]^p + (3*I)*a^3*Sin[e + f*x]^2*Sin[3*(e + f*x)]*Tan[e + f*x]^p + (3*a*b^2*Sin[2*(e + f*x)]*Sin[3*(e + f*x)]*Tan[e + f*x]^p)/2 - ((3*I)/4)*a*b^2*Sin[2*(e + f*x)]^2*Sin[3*(e + f*x)]*Tan[e + f*x]^p + Cos[2*(e + f*x)]^2*((-3*a*b^2*Cos[3*(e + f*x)]*Tan[e + f*x]^p)/4 + ((3*I)/4)*a*b^2*Sin[3*(e + f*x)]*Tan[e + f*x]^p) + Sin[e + f*x]*((3*I)*a^2*b*Sin[3*(e + f*x)]*Tan[e + f*x]^p + 3*a^2*b*Sin[2*(e + f*x)]*Sin[3*(e + f*x)]*Tan[e + f*x]^p) + Cos[3*(e + f*x)]*((-3*a*b^2*Tan[e + f*x]^p)/4 - 3*a^3*Sin[e + f*x]^2*Tan[e + f*x]^p + ((3*I)/2)*a*b^2*Sin[2*(e + f*x)]*Tan[e + f*x]^p + (3*a*b^2*Sin[2*(e + f*x)]^2*Tan[e + f*x]^p)/4 + Sin[e + f*x]*(-3*a^2*b*Tan[e + f*x]^p + (3*I)*a^2*b*Sin[2*(e + f*x)]*Tan[e + f*x]^p)) + Cos[2*(e + f*x)]*(((-3*I)/2)*a*b^2*Sin[3*(e + f*x)]*Tan[e + f*x]^p - (3*I)*a^2*b*Sin[e + f*x]*Sin[3*(e + f*x)]*Tan[e + f*x]^p - (3*a*b^2*Sin[2*(e + f*x)]*Sin[3*(e + f*x)]*Tan[e + f*x]^p)/2 + Cos[3*(e + f*x)]*((3*a*b^2*Tan[e + f*x]^p)/2 + 3*a^2*b*Sin[e + f*x]*Tan[e + f*x]^p - ((3*I)/2)*a*b^2*Sin[2*(e + f*x)]*Tan[e + f*x]^p))) + Cos[2*(e + f*x)]*((3*b^3*Sin[3*(e + f*x)]*Tan[e + f*x]^p)/8 + (3*a^2*b*Sin[e + f*x]^2*Sin[3*(e + f*x)]*Tan[e + f*x]^p)/2 - ((3*I)/4)*b^3*Sin[2*(e + f*x)]*Sin[3*(e + f*x)]*Tan[e + f*x]^p - (3*b^3*Sin[2*(e + f*x)]^2*Sin[3*(e + f*x)]*Tan[e + f*x]^p)/8 + Sin[e + f*x]*((3*a*b^2*Sin[3*(e + f*x)]*Tan[e + f*x]^p)/2 - ((3*I)/2)*a*b^2*Sin[2*(e + f*x)]*Sin[3*(e + f*x)]*Tan[e + f*x]^p) + Cos[3*(e + f*x)]*(((3*I)/8)*b^3*Tan[e + f*x]^p + ((3*I)/2)*a^2*b*Sin[e + f*x]^2*Tan[e + f*x]^p + (3*b^3*Sin[2*(e + f*x)]*Tan[e + f*x]^p)/4 - ((3*I)/8)*b^3*Sin[2*(e + f*x)]^2*Tan[e + f*x]^p + Sin[e + f*x]*(((3*I)/2)*a*b^2*Tan[e + f*x]^p + (3*a*b^2*Sin[2*(e + f*x)]*Tan[e + f*x]^p)/2)))))/(f*(1 + p)*(2 + p)*((2*p*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^p*Sec[e + f*x]^2*Tan[(e + f*x)/2]*(a^3*(2 + p)*AppellF1[(1 + p)/2, p, 1, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*b*(6*a*b*(2 + p)*AppellF1[(1 + p)/2, p, 2, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 6*a*b*(2 + p)*AppellF1[(1 + p)/2, p, 3, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 + p)*(3*a^2*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*b^2*(AppellF1[1 + p/2, p, 3, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - AppellF1[1 + p/2, p, 4, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]))*Tan[(e + f*x)/2]))*Tan[e +

```
f*x]^(-1 + p))/((1 + p)*(2 + p)) + (Sec[(e + f*x)/2]^2*(Cos[e + f*x]*Sec[(e
 + f*x)/2]^2)^p*(a^3*(2 + p)*AppellF1[(1 + p)/2, p, 1, (3 + p)/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*b*(6*a*b*(2 + p)*AppellF1[(1 + p)/2, p,
 2, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 6*a*b*(2 + p)*App
ellF1[(1 + p)/2, p, 3, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
+ (1 + p)*(3*a^2*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2] + 4*b^2*(AppellF1[1 + p/2, p, 3, 2 + p/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2] - AppellF1[1 + p/2, p, 4, 2 + p/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2]))*Tan[(e + f*x)/2]))*Tan[e + f*x]^p)/((1 + p)*(2 +
 p)) + (2*p*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^(-1 + p)*Tan[(e + f*x)/2]*(-(
Sec[(e + f*x)/2]^2*Sin[e + f*x]) + Cos[e + f*x]*Sec[(e + f*x)/2]^2*Tan[(e +
 f*x)/2])*(a^3*(2 + p)*AppellF1[(1 + p)/2, p, 1, (3 + p)/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2] + 2*b*(6*a*b*(2 + p)*AppellF1[(1 + p)/2, p, 2, (3
 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 6*a*b*(2 + p)*AppellF1[
(1 + p)/2, p, 3, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 +
 p)*(3*a^2*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2] + 4*b^2*(AppellF1[1 + p/2, p, 3, 2 + p/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2] - AppellF1[1 + p/2, p, 4, 2 + p/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2]))*Tan[(e + f*x)/2]))*Tan[e + f*x]^p)/((1 + p)*(2 + p)) +
 (2*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^p*Tan[(e + f*x)/2]*(a^3*(2 + p)*(-(((
1 + p)*AppellF1[1 + (1 + p)/2, p, 2, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p)) + (p*(1 + p
)*AppellF1[1 + (1 + p)/2, 1 + p, 1, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p)) + 2*b*(((1 +
 p)*(3*a^2*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2] + 4*b^2*(AppellF1[1 + p/2, p, 3, 2 + p/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2] - AppellF1[1 + p/2, p, 4, 2 + p/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2]))*Sec[(e + f*x)/2]^2)/2 + 6*a*b*(2 + p)*((-2*(1 + p)*App
ellF1[1 + (1 + p)/2, p, 3, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p) + (p*(1 + p)*AppellF1[
1 + (1 + p)/2, 1 + p, 2, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p)) - 6*a*b*(2 + p)*((-3*(1
 + p)*AppellF1[1 + (1 + p)/2, p, 4, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p) + (p*(1 + p)*
AppellF1[1 + (1 + p)/2, 1 + p, 3, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p)) + (1 + p)*Tan[
(e + f*x)/2]*(3*a^2*((-2*(1 + p/2)*AppellF1[2 + p/2, p, 3, 3 + p/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(2 +
 p/2) + ((1 + p/2)*p*AppellF1[2 + p/2, 1 + p, 2, 3 + p/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(2 + p/2)) + 4
*b^2*((-3*(1 + p/2)*AppellF1[2 + p/2, p, 4, 3 + p/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(2 + p/2) + (4*(1 +
 p/2)*AppellF1[2 + p/2, p, 5, 3 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(2 + p/2) + ((1 + p/2)*p*AppellF1
[2 + p/2, 1 + p, 3, 3 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(
```

```
e + f*x)/2]^2*Tan[(e + f*x)/2])/(2 + p/2) - ((1 + p/2)*p*AppellF1[2 + p/2,
1 + p, 4, 3 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2
]^2*Tan[(e + f*x)/2])/(2 + p/2)))))*Tan[e + f*x]^p)/((1 + p)*(2 + p))))
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(3\,ab^2\cos\left(fx+e\right)^2-a^3-3\,ab^2+\left(b^3\cos\left(fx+e\right)^2-3\,a^2b-b^3\right)\sin\left(fx+e\right)\right)\left(g\tan\left(fx+e\right)\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(g*tan(f*x+e))^p,x, algorithm="fricas")
```

```
[Out] integral(-(3*a*b^2*cos(f*x + e)^2 - a^3 - 3*a*b^2 + (b^3*cos(f*x + e)^2 - 3
*a^2*b - b^3)*sin(f*x + e))*(g*tan(f*x + e))^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)+a\right)^3\left(g\tan\left(fx+e\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(g*tan(f*x+e))^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3*(g*tan(f*x + e))^p, x)
```

maple [F] time = 2.63, size = 0, normalized size = 0.00

$$\int \left(a+b\sin\left(fx+e\right)\right)^3\left(g\tan\left(fx+e\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3*(g*tan(f*x+e))^p,x)
```

```
[Out] int((a+b*sin(f*x+e))^3*(g*tan(f*x+e))^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)+a\right)^3\left(g\tan\left(fx+e\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(g*tan(f*x+e))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3*(g*tan(f*x + e))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(g \tan \left(e + f x\right)\right)^p \left(a + b \sin \left(e + f x\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*tan(e + f*x))^p*(a + b*sin(e + f*x))^3,x)
[Out] int((g*tan(e + f*x))^p*(a + b*sin(e + f*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(g \tan \left(e + f x\right)\right)^p \left(a + b \sin \left(e + f x\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3*(g*tan(f*x+e))**p,x)
[Out] Integral((g*tan(e + f*x))**p*(a + b*sin(e + f*x))**3, x)
```

3.204 $\int (a+b\sin(e+fx))^2(g\tan(e+fx))^p\,dx$

Optimal. Leaf size=186

$$\frac{a^2(g\tan(e+fx))^{p+1}\,{}_2F_1\left(1,\frac{p+1}{2};\frac{p+3}{2};-\tan^2(e+fx)\right)}{fg(p+1)}+\frac{2ab\sin(e+fx)\cos^2(e+fx)^{\frac{p+1}{2}}(g\tan(e+fx))^{p+1}\,{}_2F_1\left(\frac{p+1}{2}\right.}{fg(p+2)}$$

```
[Out] a^2*hypergeom([1, 1/2+1/2*p],[3/2+1/2*p],-tan(f*x+e)^2)*(g*tan(f*x+e))^(1+p
)/f/g/(1+p)+2*a*b*(cos(f*x+e)^2)^(1/2+1/2*p)*hypergeom([1+1/2*p, 1/2+1/2*p]
,[2+1/2*p],sin(f*x+e)^2)*sin(f*x+e)*(g*tan(f*x+e))^(1+p)/f/g/(2+p)+b^2*hype
rgeom([2, 3/2+1/2*p],[5/2+1/2*p],-tan(f*x+e)^2)*(g*tan(f*x+e))^(3+p)/f/g^3/
(3+p)
```

Rubi [A] time = 0.24, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2722, 3476, 364, 2602, 2577, 2591}

$$\frac{a^2(g\tan(e+fx))^{p+1}\,{}_2F_1\left(1,\frac{p+1}{2};\frac{p+3}{2};-\tan^2(e+fx)\right)}{fg(p+1)}+\frac{2ab\sin(e+fx)\cos^2(e+fx)^{\frac{p+1}{2}}(g\tan(e+fx))^{p+1}\,{}_2F_1\left(\frac{p+1}{2}\right.}{fg(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2*(g*Tan[e + f*x])^p,x]
```

```
[Out] (a^2*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e +
 f*x])^(1 + p))/(f*g*(1 + p)) + (2*a*b*(Cos[e + f*x]^2)^((1 + p)/2)*Hyperge
ometric2F1[(1 + p)/2, (2 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(g
*Tan[e + f*x])^(1 + p))/(f*g*(2 + p)) + (b^2*Hypergeometric2F1[2, (3 + p)/2
, (5 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e + f*x])^(3 + p))/(f*g^3*(3 + p))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
```

```
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int (a+b\sin(e+fx))^2(g\tan(e+fx))^p\,dx = \int \Big(a^2(g\tan(e+fx))^p + 2ab\sin(e+fx)(g\tan(e+fx))^p + b^2\sin^2(e$$

$$= a^2\int (g\tan(e+fx))^p\,dx + (2ab)\int \sin(e+fx)(g\tan(e+fx))^p\,dx +$$

$$= \frac{\left(a^2g\right)\text{Subst}\left(\int \frac{x^p}{g^2+x^2}\,dx, x, g\tan(e+fx)\right)}{f} + \frac{\left(b^2g\right)\text{Subst}\Bigg(\int \frac{x^{2+p}}{\left(g^2+x^2\right)^2}}{f}$$

$$= \frac{a^2\,{}_2F_1\left(1,\frac{1+p}{2};\frac{3+p}{2};-\tan^2(e+fx)\right)(g\tan(e+fx))^{1+p}}{fg(1+p)} + \frac{2ab\cos^2(e}{}$$

Mathematica [C] time = 14.24, size = 2464, normalized size = 13.25

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])^2*(g*Tan[e + f*x])^p,x]
```

```
[Out] (2*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^p*Tan[(e + f*x)/2]*(a^2*(2 + p)*Appell
F1[(1 + p)/2, p, 1, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4
*b*(b*(2 + p)*AppellF1[(1 + p)/2, p, 2, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2] - b*(2 + p)*AppellF1[(1 + p)/2, p, 3, (3 + p)/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2] + a*(1 + p)*AppellF1[1 + p/2, p, 2, 2 + p/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]))*(g*Tan[e + f*
x])^p*(-1/4*(b^2*Cos[2*(e + f*x)]^3*Tan[e + f*x]^p) + (I/4)*b^2*Sin[2*(e +
f*x)]*Tan[e + f*x]^p + I*a^2*Sin[e + f*x]^2*Sin[2*(e + f*x)]*Tan[e + f*x]^p
 + (b^2*Sin[2*(e + f*x)]^2*Tan[e + f*x]^p)/2 - (I/4)*b^2*Sin[2*(e + f*x)]^3
*Tan[e + f*x]^p + Cos[e + f*x]^2*(a^2*Cos[2*(e + f*x)]*Tan[e + f*x]^p - I*a
^2*Sin[2*(e + f*x)]*Tan[e + f*x]^p) + Cos[2*(e + f*x)]^2*((b^2*Tan[e + f*x]
^p)/2 + a*b*Sin[e + f*x]*Tan[e + f*x]^p - (I/4)*b^2*Sin[2*(e + f*x)]*Tan[e
+ f*x]^p) + Sin[e + f*x]*(I*a*b*Sin[2*(e + f*x)]*Tan[e + f*x]^p + a*b*Sin[2
*(e + f*x)]^2*Tan[e + f*x]^p) + Cos[2*(e + f*x)]*(-1/4*(b^2*Tan[e + f*x]^p)
 - a*b*Sin[e + f*x]*Tan[e + f*x]^p - a^2*Sin[e + f*x]^2*Tan[e + f*x]^p - (b
^2*Sin[2*(e + f*x)]^2*Tan[e + f*x]^p)/4) + Cos[e + f*x]*((-I)*a*b*Cos[2*(e
+ f*x)]^2*Tan[e + f*x]^p + a*b*Sin[2*(e + f*x)]*Tan[e + f*x]^p + 2*a^2*Sin[
e + f*x]*Sin[2*(e + f*x)]*Tan[e + f*x]^p - I*a*b*Sin[2*(e + f*x)]^2*Tan[e +
 f*x]^p + Cos[2*(e + f*x)]*(I*a*b*Tan[e + f*x]^p + (2*I)*a^2*Sin[e + f*x]*T
an[e + f*x]^p))))/(f*(1 + p)*(2 + p)*((2*p*(Cos[e + f*x]*Sec[(e + f*x)/2]^2
)^p*Sec[e + f*x]^2*Tan[(e + f*x)/2]*(a^2*(2 + p)*AppellF1[(1 + p)/2, p, 1,
(3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*b*(b*(2 + p)*Appell
F1[(1 + p)/2, p, 2, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - b
*(2 + p)*AppellF1[(1 + p)/2, p, 3, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2] + a*(1 + p)*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]))*Tan[e + f*x]^(-1 + p))/((1 + p)*(
2 + p)) + (Sec[(e + f*x)/2]^2*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^p*(a^2*(2 +
 p)*AppellF1[(1 + p)/2, p, 1, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2] + 4*b*(b*(2 + p)*AppellF1[(1 + p)/2, p, 2, (3 + p)/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2] - b*(2 + p)*AppellF1[(1 + p)/2, p, 3, (3 + p)/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + a*(1 + p)*AppellF1[1 + p/2, p,
2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]))*Tan
[e + f*x]^p)/((1 + p)*(2 + p)) + (2*p*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^(-1
 + p)*Tan[(e + f*x)/2]*(-(Sec[(e + f*x)/2]^2*Sin[e + f*x]) + Cos[e + f*x]*S
ec[(e + f*x)/2]^2*Tan[(e + f*x)/2])*(a^2*(2 + p)*AppellF1[(1 + p)/2, p, 1,
(3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*b*(b*(2 + p)*Appell
F1[(1 + p)/2, p, 2, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - b
```

```
*(2 + p)*AppellF1[(1 + p)/2, p, 3, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2] + a*(1 + p)*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]))*Tan[e + f*x]^p)/((1 + p)*(2 + p))
 + (2*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^p*Tan[(e + f*x)/2]*(a^2*(2 + p)*(-(
((1 + p)*AppellF1[1 + (1 + p)/2, p, 2, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p)) + (p*(1 +
 p)*AppellF1[1 + (1 + p)/2, 1 + p, 1, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p)) + 4*b*((a*
(1 + p)*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2]*Sec[(e + f*x)/2]^2)/2 + a*(1 + p)*Tan[(e + f*x)/2]*((-2*(1 + p/2)*Ap
pellF1[2 + p/2, p, 3, 3 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec
[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(2 + p/2) + ((1 + p/2)*p*AppellF1[2 + p/2
, 1 + p, 2, 3 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)
/2]^2*Tan[(e + f*x)/2])/(2 + p/2)) + b*(2 + p)*((-2*(1 + p)*AppellF1[1 + (1
 + p)/2, p, 3, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[
(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p) + (p*(1 + p)*AppellF1[1 + (1 + p)/
2, 1 + p, 2, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e
 + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p)) - b*(2 + p)*((-3*(1 + p)*AppellF1[1
 + (1 + p)/2, p, 4, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p) + (p*(1 + p)*AppellF1[1 + (1
+ p)/2, 1 + p, 3, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*S
ec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p))))*Tan[e + f*x]^p)/((1 + p)*(2
+ p))))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(b^2\cos\left(fx+e\right)^2-2\,ab\sin\left(fx+e\right)-a^2-b^2\right)\left(g\tan\left(fx+e\right)\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(g*tan(f*x+e))^p,x, algorithm="fricas")
```

```
[Out] integral(-(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)*(g*tan(f*x
+ e))^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)+a\right)^2\left(g\tan\left(fx+e\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(g*tan(f*x+e))^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2*(g*tan(f*x + e))^p, x)
```

maple [F] time = 2.49, size = 0, normalized size = 0.00

$$\int \left(a+b\sin \left(fx+e\right)\right)^{2} \left(g\tan \left(fx+e\right)\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2*(g*tan(f*x+e))^p,x)
[Out] int((a+b*sin(f*x+e))^2*(g*tan(f*x+e))^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin \left(fx+e\right)+a\right)^{2} \left(g\tan \left(fx+e\right)\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(g*tan(f*x+e))^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e) + a)^2*(g*tan(f*x + e))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(g\,\tan\left(e+f\,x\right)\right)}^p\,{\left(a+b\,\sin\left(e+f\,x\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*tan(e + f*x))^p*(a + b*sin(e + f*x))^2,x)
[Out] int((g*tan(e + f*x))^p*(a + b*sin(e + f*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(g\tan{\left(e+fx \right)}\right)^{p} \left(a+b\sin{\left(e+fx \right)}\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2*(g*tan(f*x+e))**p,x)
[Out] Integral((g*tan(e + f*x))**p*(a + b*sin(e + f*x))**2, x)
```

3.205 $\int (a+b\sin(e+fx))(g\tan(e+fx))^p\,dx$

Optimal. Leaf size=129

$$\frac{a(g\tan(e+fx))^{p+1}\,{}_2F_1\left(1,\frac{p+1}{2};\frac{p+3}{2};-\tan^2(e+fx)\right)}{fg(p+1)}+\frac{b\sin(e+fx)\cos^2(e+fx)^{\frac{p+1}{2}}(g\tan(e+fx))^{p+1}\,{}_2F_1\left(\frac{p+1}{2},\right.}{fg(p+2)}$$

```
[Out] a*hypergeom([1, 1/2+1/2*p],[3/2+1/2*p],-tan(f*x+e)^2)*(g*tan(f*x+e))^(1+p)/
f/g/(1+p)+b*(cos(f*x+e)^2)^(1/2+1/2*p)*hypergeom([1+1/2*p, 1/2+1/2*p],[2+1/
2*p],sin(f*x+e)^2)*sin(f*x+e)*(g*tan(f*x+e))^(1+p)/f/g/(2+p)
```

Rubi [A] time = 0.15, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2722, 3476, 364, 2602, 2577}

$$\frac{a(g\tan(e+fx))^{p+1}\,{}_2F_1\left(1,\frac{p+1}{2};\frac{p+3}{2};-\tan^2(e+fx)\right)}{fg(p+1)}+\frac{b\sin(e+fx)\cos^2(e+fx)^{\frac{p+1}{2}}(g\tan(e+fx))^{p+1}\,{}_2F_1\left(\frac{p+1}{2},\right.}{fg(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])*(g*Tan[e + f*x])^p,x]
```

```
[Out] (a*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e + f
*x])^(1 + p))/(f*g*(1 + p)) + (b*(Cos[e + f*x]^2)^((1 + p)/2)*Hypergeometri
c2F1[(1 + p)/2, (2 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(g*Tan[e
 + f*x])^(1 + p))/(f*g*(2 + p))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int (a+b\sin(e+fx))(g\tan(e+fx))^p\,dx &= \int \left(a(g\tan(e+fx))^p+b\sin(e+fx)(g\tan(e+fx))^p\right)dx\\ &= a\int (g\tan(e+fx))^p\,dx+b\int \sin(e+fx)(g\tan(e+fx))^p\,dx\\ &= \frac{(ag)\,\text{Subst}\left(\int \frac{x^p}{g^2+x^2}\,dx,x,g\tan(e+fx)\right)}{f}+\frac{\left(b\cos^{1+p}(e+fx)\sin^{-1-p}(e\right.}{}\\ &= \frac{a\,{}_2F_1\left(1,\frac{1+p}{2};\frac{3+p}{2};-\tan^2(e+fx)\right)(g\tan(e+fx))^{1+p}}{fg(1+p)}+\frac{b\cos^2(e+fx)}{}\end{aligned}$$

Mathematica [C] time = 9.53, size = 849, normalized size = 6.58

$$\frac{}{f\left(-16p\cos\left(\frac{1}{2}(e+fx)\right)\csc^3(e+fx)\sec(e+fx)\left(a(p+2)F_1\left(\frac{p+1}{2};p,1;\frac{p+3}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])*(g*Tan[e + f*x])^p,x]
```

```
[Out] (2*(a + b*Sin[e + f*x])*Tan[(e + f*x)/2]*(a*(2 + p)*AppellF1[(1 + p)/2, p,
1, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*b*(1 + p)*Appell
F1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e
+ f*x)/2])*(g*Tan[e + f*x])^p)/(f*(Sec[(e + f*x)/2]^2*(a*(2 + p)*AppellF1[(
1 + p)/2, p, 1, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*b*(
1 + p)*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2]*Tan[(e + f*x)/2]) - 16*p*Cos[(e + f*x)/2]*Csc[e + f*x]^3*Sec[e + f*x]
*Sin[(e + f*x)/2]^5*(a*(2 + p)*AppellF1[(1 + p)/2, p, 1, (3 + p)/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*b*(1 + p)*AppellF1[1 + p/2, p, 2, 2 +
 p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]) + 2*p*Csc[
e + f*x]*Sec[e + f*x]*Tan[(e + f*x)/2]*(a*(2 + p)*AppellF1[(1 + p)/2, p, 1,
 (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*b*(1 + p)*AppellF1
[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e +
f*x)/2]) + 2*(1 + p)*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]*(b*AppellF1[1 + p/
2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (a*(2 + p)*(-A
ppellF1[(3 + p)/2, p, 2, (5 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
] + p*AppellF1[(3 + p)/2, 1 + p, 1, (5 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2])*Tan[(e + f*x)/2])/(3 + p) + (2*b*(2 + p)*(-2*AppellF1[2 + p/2
, p, 3, 3 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + p*AppellF1[2 +
p/2, 1 + p, 2, 3 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e +
f*x)/2]^2)/(4 + p))))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sin\left(fx+e\right)+a\right)\left(g\tan\left(fx+e\right)\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(g*tan(f*x+e))^p,x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e) + a)*(g*tan(f*x + e))^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)+a\right)\left(g\tan\left(fx+e\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(g*tan(f*x+e))^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)*(g*tan(f*x + e))^p, x)
```

maple [F] time = 1.53, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(fx+e\right)\right)\left(g\tan\left(fx+e\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))*(g*tan(f*x+e))^p,x)
```

```
[Out] int((a+b*sin(f*x+e))*(g*tan(f*x+e))^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)+a\right)\left(g\tan\left(fx+e\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(g*tan(f*x+e))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)*(g*tan(f*x + e))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(g\tan\left(e+f\,x\right)\right)^{p}\,\left(a+b\,\sin\left(e+f\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*tan(e + f*x))^p*(a + b*sin(e + f*x)),x)
```

```
[Out] int((g*tan(e + f*x))^p*(a + b*sin(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(g\tan\left(e+fx\right)\right)^{p}\left(a+b\sin\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(g*tan(f*x+e))**p,x)
```

```
[Out] Integral((g*tan(e + f*x))**p*(a + b*sin(e + f*x)), x)
```

3.206 $\int \frac{(g\tan(e+fx))^p}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=284

$$\frac{b\cos(e+fx)\sin^2(e+fx)^{-p/2}(g\tan(e+fx))^pF_1\left(\frac{1-p}{2};-\frac{p}{2},1;\frac{3-p}{2};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)}{f(p-1)\left(b^2-a^2\right)}+\frac{ag\sin^2(e+fx)^{\frac{1-p}{2}}(}{}$$

```
[Out] a*g*(1-b^2*cos(f*x+e)^2/(-a^2+b^2))^(-1/2+1/2*p)*Hypergeometric2F1(1/2-1/2*
p,1/2-1/2*p,3/2-1/2*p,(cos(f*x+e)^2-b^2*cos(f*x+e)^2/(-a^2+b^2))/(1-b^2*cos
(f*x+e)^2/(-a^2+b^2)))*(sin(f*x+e)^2)^(1/2-1/2*p)*(g*tan(f*x+e))^(-1+p)/(a^
2-b^2)/f/(-1+p)+b*AppellF1(1/2-1/2*p,-1/2*p,1,3/2-1/2*p,cos(f*x+e)^2,b^2*co
s(f*x+e)^2/(-a^2+b^2))*cos(f*x+e)*(g*tan(f*x+e))^p/(-a^2+b^2)/f/(-1+p)/((si
n(f*x+e)^2)^(1/2*p))
```

Rubi [F] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(g\tan(e+fx))^p}{a+b\sin(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(g*Tan[e + f*x])^p/(a + b*Sin[e + f*x]),x]
```

```
[Out] Defer[Int][(g*Tan[e + f*x])^p/(a + b*Sin[e + f*x]), x]
```

Rubi steps

$$\int \frac{(g\tan(e+fx))^p}{a+b\sin(e+fx)}\,dx=\int \frac{(g\tan(e+fx))^p}{a+b\sin(e+fx)}\,dx$$

Mathematica [B] time = 13.59, size = 858, normalized size = 3.02

$$\frac{}{a^2bf(p+1)(p+2)(a+b\sin(e+fx))\left(\frac{\sec^2(e+fx)\left(\left(a^2-b^2\right)(p+1)F_1\left(\frac{p+2}{2};-\frac{1}{2},1;\frac{p+4}{2};-\tan^2(e+fx),\left(\frac{b^2}{a^2}-1\right)\tan^2(e+fx)\right)\tan(e+fx)+a\left(b(p-\right.\right.}{a^2}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Tan[e + f*x])^p/(a + b*Sin[e + f*x]),x]
```

```
[Out] (Tan[e + f*x]^(1 + p)*(g*Tan[e + f*x])^p*((a^2 - b^2)*(1 + p)*AppellF1[(2 +
 p)/2, -1/2, 1, (4 + p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*
Tan[e + f*x] + a*(b*(2 + p)*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, ((-a
^2 + b^2)*Tan[e + f*x]^2)/a^2] - a*(1 + p)*Hypergeometric2F1[1/2, 1 + p/2,
2 + p/2, -Tan[e + f*x]^2]*Tan[e + f*x])))/(a^2*b*f*(1 + p)*(2 + p)*(a + b*S
in[e + f*x])*((Sec[e + f*x]^2*Tan[e + f*x]^p*((a^2 - b^2)*(1 + p)*AppellF1[
(2 + p)/2, -1/2, 1, (4 + p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]
^2]*Tan[e + f*x] + a*(b*(2 + p)*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2,
((-a^2 + b^2)*Tan[e + f*x]^2)/a^2] - a*(1 + p)*Hypergeometric2F1[1/2, 1 + p
/2, 2 + p/2, -Tan[e + f*x]^2]*Tan[e + f*x])))/(a^2*b*(2 + p)) + (Tan[e + f*
x]^(1 + p)*((a^2 - b^2)*(1 + p)*AppellF1[(2 + p)/2, -1/2, 1, (4 + p)/2, -Ta
n[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2 + (a^2 - b^2)*(
1 + p)*Tan[e + f*x]*((2*(-1 + b^2/a^2)*(2 + p)*AppellF1[1 + (2 + p)/2, -1/2
, 2, 1 + (4 + p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e +
 f*x]^2*Tan[e + f*x])/(4 + p) + ((2 + p)*AppellF1[1 + (2 + p)/2, 1/2, 1, 1
+ (4 + p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2
*Tan[e + f*x])/(4 + p)) + a*(-(a*(1 + p)*Hypergeometric2F1[1/2, 1 + p/2, 2
+ p/2, -Tan[e + f*x]^2]*Sec[e + f*x]^2) - 2*a*(1 + p/2)*(1 + p)*Sec[e + f*x
]^2*(-Hypergeometric2F1[1/2, 1 + p/2, 2 + p/2, -Tan[e + f*x]^2] + 1/Sqrt[1
+ Tan[e + f*x]^2]) + b*(1 + p)*(2 + p)*Csc[e + f*x]*Sec[e + f*x]*(-Hypergeo
metric2F1[1, (1 + p)/2, (3 + p)/2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2] + (1
- ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2)^(-1)))))/(a^2*b*(1 + p)*(2 + p))))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(g\tan\left(fx+e\right)\right)^{p}}{b\sin\left(fx+e\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((g*tan(f*x + e))^p/(b*sin(f*x + e) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\tan\left(fx+e\right)\right)^{p}}{b\sin\left(fx+e\right)+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((g*tan(f*x + e))^p/(b*sin(f*x + e) + a), x)
```

maple [F] time = 0.80, size = 0, normalized size = 0.00

$$\int \frac{\left(g \tan\left(f x + e\right)\right)^{p}}{a + b \sin\left(f x + e\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*tan(f*x+e))^p/(a+b*sin(f*x+e)),x)
```

```
[Out] int((g*tan(f*x+e))^p/(a+b*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g \tan\left(f x + e\right)\right)^{p}}{b \sin\left(f x + e\right) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((g*tan(f*x + e))^p/(b*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g \tan\left(e + f\, x\right)\right)^{p}}{a + b\, \sin\left(e + f\, x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*tan(e + f*x))^p/(a + b*sin(e + f*x)),x)
```

```
[Out] int((g*tan(e + f*x))^p/(a + b*sin(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g \tan\left(e + f x\right)\right)^{p}}{a + b \sin\left(e + f x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))**p/(a+b*sin(f*x+e)),x)
```

```
[Out] Integral((g*tan(e + f*x))**p/(a + b*sin(e + f*x)), x)
```

3.207 $\int \frac{(g\tan(e+fx))^p}{(a+b\sin(e+fx))^2}\,dx$

Optimal. Leaf size=737

$$-\frac{2ab\cos(e+fx)\sin^2(e+fx)^{-q/2}(g\tan(e+fx))^q F_1\left(\frac{1-q}{2};-\frac{q}{2},2;\frac{3-q}{2};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)}{f(q-1)\left(a^2-b^2\right)^2}+\frac{a^2\sin(e+fx)\mathrm{co}}{}$$

[Out] 1/2*a^2*cos(f*x+e)*(1-cos(f*x+e)^2)^(-1/2+1/2*q)/(1-b^2*cos(f*x+e)^2/(-a^2+b^2))*((2*a^2-2*b^2+b^2*(1+q)*cos(f*x+e)^2)*HurwitzLerchPhi(a^2*cos(f*x+e)^2/(a^2-b^2)/(-1+cos(f*x+e)^2),1,1/2-1/2*q)-b^2*(-1+q)*cos(f*x+e)^2*HurwitzLerchPhi(a^2*cos(f*x+e)^2/(a^2-b^2)/(-1+cos(f*x+e)^2),1,3/2-1/2*q))*sin(f*x+e)*(sin(f*x+e)^2)^(-1/2-1/2*q)*(g*tan(f*x+e))^q/(a^2-b^2)^2/(-a^2+b^2)/f-a^2*cos(f*x+e)*(1-b^2*cos(f*x+e)^2/(-a^2+b^2))^(-1/2+1/2*q)*Hypergeometric2F1(1/2-1/2*q,1/2-1/2*q,3/2-1/2*q,(cos(f*x+e)^2-b^2*cos(f*x+e)^2/(-a^2+b^2))/(1-b^2*cos(f*x+e)^2/(-a^2+b^2)))*sin(f*x+e)*(sin(f*x+e)^2)^(-1/2-1/2*q)*(g*tan(f*x+e))^q/(a^2-b^2)^2/f/(-1+q)+b^2*cos(f*x+e)*(1-b^2*cos(f*x+e)^2/(-a^2+b^2))^(-1/2+1/2*q)*Hypergeometric2F1(1/2-1/2*q,1/2-1/2*q,3/2-1/2*q,(cos(f*x+e)^2-b^2*cos(f*x+e)^2/(-a^2+b^2))/(1-b^2*cos(f*x+e)^2/(-a^2+b^2)))*sin(f*x+e)*(sin(f*x+e)^2)^(-1/2-1/2*q)*(g*tan(f*x+e))^q/(a^2-b^2)^2/f/(-1+q)-2*a*b*AppellF1(1/2-1/2*q,-1/2*q,2,3/2-1/2*q,cos(f*x+e)^2,b^2*cos(f*x+e)^2/(-a^2+b^2))*cos(f*x+e)*(g*tan(f*x+e))^q/(a^2-b^2)^2/f/(-1+q)/((sin(f*x+e)^2)^(1/2*q))

Rubi [F] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(g\tan(e+fx))^p}{(a+b\sin(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

[In] Int[(g*Tan[e + f*x])^p/(a + b*Sin[e + f*x])^2,x]

[Out] Defer[Int][(g*Tan[e + f*x])^p/(a + b*Sin[e + f*x])^2, x]

Rubi steps

$$\int \frac{(g\tan(e+fx))^p}{(a+b\sin(e+fx))^2}\,dx=\int \frac{(g\tan(e+fx))^p}{(a+b\sin(e+fx))^2}\,dx$$

Mathematica [A] time = 14.40, size = 866, normalized size = 1.18

$$a^3\left(a^2-b^2\right)f(p+1)(p+2)(a+b\sin(e+fx))^2\left(\frac{\sec^2(e+fx)\left(a(p+2)\left(\left(a^2+b^2\right){}_2F_1\left(1,\frac{p+1}{2};\frac{p+3}{2};\left(\frac{b^2}{a^2}-1\right)\tan^2(e+fx)\right)-2b^2\,{}_2F_1\left(2,\frac{p+1}{2};\frac{p+}{2}\right.\right.\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Tan[e + f*x])^p/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (Tan[e + f*x]^(1 + p)*(g*Tan[e + f*x])^p*(a*(2 + p)*((a^2 + b^2)*Hypergeome
tric2F1[1, (1 + p)/2, (3 + p)/2, (-1 + b^2/a^2)*Tan[e + f*x]^2] - 2*b^2*Hyp
ergeometric2F1[2, (1 + p)/2, (3 + p)/2, (-1 + b^2/a^2)*Tan[e + f*x]^2]) + 2
*b*(-a^2 + b^2)*(1 + p)*AppellF1[(2 + p)/2, -1/2, 2, (4 + p)/2, -Tan[e + f*
x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]))/(a^3*(a^2 - b^2)*f*(1 +
 p)*(2 + p)*(a + b*Sin[e + f*x])^2*((Sec[e + f*x]^2*Tan[e + f*x]^p*(a*(2 +
p)*((a^2 + b^2)*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, (-1 + b^2/a^2)*T
an[e + f*x]^2] - 2*b^2*Hypergeometric2F1[2, (1 + p)/2, (3 + p)/2, (-1 + b^2
/a^2)*Tan[e + f*x]^2]) + 2*b*(-a^2 + b^2)*(1 + p)*AppellF1[(2 + p)/2, -1/2,
 2, (4 + p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]
))/(a^3*(a^2 - b^2)*(2 + p)) + (Tan[e + f*x]^(1 + p)*(2*b*(-a^2 + b^2)*(1 +
 p)*AppellF1[(2 + p)/2, -1/2, 2, (4 + p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)
*Tan[e + f*x]^2]*Sec[e + f*x]^2 + 2*b*(-a^2 + b^2)*(1 + p)*Tan[e + f*x]*((4
*(-1 + b^2/a^2)*(2 + p)*AppellF1[1 + (2 + p)/2, -1/2, 3, 1 + (4 + p)/2, -Ta
n[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(
4 + p) + ((2 + p)*AppellF1[1 + (2 + p)/2, 1/2, 2, 1 + (4 + p)/2, -Tan[e + f
*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(4 + p))
 + a*(2 + p)*(-2*b^2*(1 + p)*Csc[e + f*x]*Sec[e + f*x]*(-Hypergeometric2F1[
2, (1 + p)/2, (3 + p)/2, (-1 + b^2/a^2)*Tan[e + f*x]^2] + (1 - (-1 + b^2/a^
2)*Tan[e + f*x]^2)^(-2)) + (a^2 + b^2)*(1 + p)*Csc[e + f*x]*Sec[e + f*x]*(-
Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, (-1 + b^2/a^2)*Tan[e + f*x]^2] +
 (1 - (-1 + b^2/a^2)*Tan[e + f*x]^2)^(-1)))))/(a^3*(a^2 - b^2)*(1 + p)*(2 +
 p))))
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(g\tan\left(fx+e\right)\right)^p}{b^2\cos\left(fx+e\right)^2-2\,ab\sin\left(fx+e\right)-a^2-b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-(g*tan(f*x + e))^p/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2
 - b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\tan\left(fx+e\right)\right)^{p}}{\left(b\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((g*tan(f*x + e))^p/(b*sin(f*x + e) + a)^2, x)
```

maple [F] time = 1.57, size = 0, normalized size = 0.00

$$\int \frac{\left(g\tan\left(fx+e\right)\right)^{p}}{\left(a+b\sin\left(fx+e\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*tan(f*x+e))^p/(a+b*sin(f*x+e))^2,x)
```

```
[Out] int((g*tan(f*x+e))^p/(a+b*sin(f*x+e))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\tan\left(fx+e\right)\right)^{p}}{\left(b\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((g*tan(f*x + e))^p/(b*sin(f*x + e) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g\tan\left(e+f\,x\right)\right)^{p}}{\left(a+b\,\sin\left(e+f\,x\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*tan(e + f*x))^p/(a + b*sin(e + f*x))^2,x)
```

```
[Out] int((g*tan(e + f*x))^p/(a + b*sin(e + f*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\tan\left(e+fx\right)\right)^{p}}{\left(a+b\sin\left(e+fx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))**p/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Integral((g*tan(e + f*x))**p/(a + b*sin(e + f*x))**2, x)
```

3.208 $\int (a+b\sin(e+fx))^m(g\tan(e+fx))^p\,dx$

Optimal. Leaf size=26

$$\text{Int}\left((g\tan(e+fx))^p(a+b\sin(e+fx))^m,x\right)$$

```
[Out] Unintegrable((a+b*sin(f*x+e))^m*(g*tan(f*x+e))^p,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (a+b\sin(e+fx))^m(g\tan(e+fx))^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[e + f*x])^m*(g*Tan[e + f*x])^p,x]
```

```
[Out] Defer[Int][(a + b*Sin[e + f*x])^m*(g*Tan[e + f*x])^p, x]
```

Rubi steps

$$\int (a+b\sin(e+fx))^m(g\tan(e+fx))^p\,dx=\int (a+b\sin(e+fx))^m(g\tan(e+fx))^p\,dx$$

Mathematica [A] time = 3.13, size = 0, normalized size = 0.00

$$\int (a+b\sin(e+fx))^m(g\tan(e+fx))^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[e + f*x])^m*(g*Tan[e + f*x])^p,x]
```

```
[Out] Integrate[(a + b*Sin[e + f*x])^m*(g*Tan[e + f*x])^p, x]
```

fricas [A] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sin\left(fx+e\right)+a\right)^m\left(g\tan\left(fx+e\right)\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(g*tan(f*x+e))^p,x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e) + a)^m*(g*tan(f*x + e))^p, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)+a\right)^{m}\left(g\tan\left(fx+e\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(g*tan(f*x+e))^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e) + a)^m*(g*tan(f*x + e))^p, x)
```

maple [A] time = 1.52, size = 0, normalized size = 0.00

$$\int \left(a+b\sin\left(fx+e\right)\right)^{m}\left(g\tan\left(fx+e\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^m*(g*tan(f*x+e))^p,x)
[Out] int((a+b*sin(f*x+e))^m*(g*tan(f*x+e))^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)+a\right)^{m}\left(g\tan\left(fx+e\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(g*tan(f*x+e))^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e) + a)^m*(g*tan(f*x + e))^p, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int {\left(g\,\tan\left(e+f\,x\right)\right)}^p\,{\left(a+b\,\sin\left(e+f\,x\right)\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*tan(e + f*x))^p*(a + b*sin(e + f*x))^m,x)
[Out] int((g*tan(e + f*x))^p*(a + b*sin(e + f*x))^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**m*(g*tan(f*x+e))**p,x)
```

```
[Out] Timed out
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
```

```
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
```

```
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
```

```
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)
   ))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
   (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
   Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
```

```
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
```

```
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.
   func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
   expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
   is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
   leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
   expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
```

```
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
        else: #result contains complex but optimal is not
            return "C"
    else: # result do not contain complex, this assumes optimal do not as
well
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
else:
    return "C"
```